Limitations of Gravitational Physics in the Early Universe

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Abstract

Evidence from recent astrophysical experiments - including the James Webb Telescope (JWST) and Pulsar Timing Arrays (PTA) - reveal potential inconsistencies with standard Big-Bang cosmology [1-4]. Several competing explanations of these (and similar) anomalies have been suggested, but a conclusive resolution is yet to be seen. The object of this brief note is to bring up a key challenge faced by classical gravity in *far-from-equilibrium conditions*. These conditions exist in the very early stages of cosmological evolution and point to a paradigm shift in our understanding of foundational physics.

Key words: Big-Bang cosmology, Lambda-CDM model, cosmological anomalies, far-from-equilibrium phenomena, Einstein-Hilbert action.

Consider the Einstein-Hilbert (EH) action in the vacuum and in the absence of the cosmological constant. Spacetime coordinates are indexed using Latin characters (a,b,c,d). A helpful convention introduced below is,

$$16\pi G=1 \tag{1}$$

The four-dimensional EH action reads,

$$S_{EH}=\int d^4x\sqrt{-g}R=\int d^4x(\sqrt{-g}\,g^{ab}R_{ab}) \tag{2}$$

where g^{ab} represents the metric with determinant $\sqrt{-g}$ and R is the Ricci scalar. The first-order variation of (2) contains three terms, namely,

$$\delta S_{EH}=\sum_{i=1}^{3}(\delta S_{EH})_i \tag{3}$$

in which [5 - 7]

$$\delta S_{EH(1)}=\int d^4x\sqrt{-g}\,g^{ab}\delta R_{ab} \tag{4a}$$

$$\delta S_{EH(2)}=\int d^4x\sqrt{-g}\,R_{ab}\delta g^{ab} \tag{4b}$$

$$\delta S_{EH(3)} = \int d^4x \, R \, \delta\sqrt{-g} \tag{4c}$$

Carrying out a tedious string of manipulations renders (4a) in the form,

$$\delta S_{EH(1)} = \int_{\partial M} d^3x \sqrt{|h|} \, n_c \, J^c \tag{5}$$

Here, ∂M is the boundary of the integration region M, J^c is a vector field defined as

$$J^c = g^{ab}\delta\Gamma^c_{ab} - g^{ac}\delta\Gamma^b_{ab} \tag{6}$$

n_c is the normal unit vector on ∂M and the tensor h_{ab} represents the induced metric given by,

$$h_{ab} = g_{ab} + n_a n_b \tag{7}$$

Invoking the Stokes theorem, (5) vanishes away on the assumption that the boundary contribution *at infinity can be set to zero.* However, this assumption *fails to hold* if,

1) ∂M is not placed at infinity and gravitational contribution of ∂M and M mixes up and cannot be decoupled,

2) the contribution of ∂M is *sensitive to initial conditions*,

3) ∂M undergoes *fluctuations* that do not cancel upon integration.

Likewise, either (3) or the terms (4b) - (4c) fail to vanish if,

1) metric variations δg^{ab} are *sensitive to initial conditions*,

2) there is *irreducible mixing* of (4a) - (4c) as in

$$\delta S_{EH} = \sum_{i=1}^{3} (\delta S_{EH})_i + \sum_{i,j=1}^{3} (\delta S_{EH})_i (\delta S_{EH})_j \tag{8}$$

3) (4a) - (4c) represent ill-defined integrals on account of large and random *curvature fluctuations.*

These observations equally apply to the case where (2) includes the contribution of *boundary terms*. A standard example is adding the Gibbons-Hawking (S_{GH}) boundary term to (2), which yields the overall action [5-7]

$$S = S_{EH} + S_{GH} = \int_M d^4x \sqrt{-g}\, R + 2\int_{\partial M} d^3x \sqrt{-h}\, K \tag{9}$$

where the extrinsic curvature K is given by

$$K = h^{ab}\nabla_a n_b \tag{10}$$

One is led to conclude that, if either (4a) – (4c) or the variation of the Gibbons-Hawking term cannot be consistently cancelled out, deriving the General Relativity (GR) equations from the first order variation of either (2) or (9) *is bound to fail*.

These considerations - consistent with the singularity problem of GR - are clearly applicable in the *far-from-equilibrium regime* describing the dynamics of the early Universe. They are likely to impact several areas of foundational research, such as quantum gravity unification, inflationary cosmology, quantum cosmology, Black Hole physics, alternative gravity theories and models of the cosmic web formation. It is also worth pointing out that the expected breakdown of gravitational field equations in the early Universe is on par with the onset of *non-integrability* and *complex dynamics* far above the Fermi scale, as inferred in [8 - 13].

References

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