Minimal Length,Primary and Generalized Measurability and Classical Mechanics

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Abstract

In terms of the notion **measurability** introduced in previous works of the author, this work gives statement and construction of Classical Mechanics.

1 Introduction

This article is the continuation of the previous works of the author [1]–[8], the first of which [1] was published in autumn, 2014.
The main idea of these works is as follows. At the present time physics is using (not without success) the mathematical apparatus based on the use of infinitesimal space-time variations (increments)

$$dt, dx_i, i = 1, ..., 3 \tag{1}$$

This mathematical apparatus comes from calculus [9], calculus of variations [10] and classical mechanics [11],[12]. Continuous space-time forms the base thereof.
The article [6] shows that while going over to the quantum theory at natural assumptions mentioned in [8] **Principle of Bounded Space-Time Variations (Increments)** the notion of continuous space-time becomes

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empty. And this is related to the fact that measurement procedure and Heisenberg's Uncertainty Principle (HUP) [13] play a fundamental role in the quantum theory.
If **Principle of Bounded Space-Time Variations (Increments)** is correct, minimal length l_{min} and time $t_{min} = l_{min}/c$ appear in the nature, (where c is light speed). Then, based on l_{min} and t_{min} definitions of **measurability** and **measurable** quantities may be correctly input in theory. Some examples show, although in this case it becomes discrete, but in low energies, E, far from Planck $E \ll E_P$, it is close to the initial theory in continuous space-time. Real discreteness of the theory is manifested only at high energies E close to Planck $E \approx E_P$ [1],[6],[8].
The main objective (hypothesis) of the author is as follows [6],[2],[8]:
It is possible to correctly construct the quantum theory and gravity as discrete theories in terms of **measurable** *quantities.*
The word **correctness** in this case means the following:

I1. At low energies these theories must, to a high accuracy, represent the results of the corresponding continuous theories.

I2. This theories should not have the problems of transition from low to high energies and vice versa and, specifically, the ultraviolet (UV) and infra-red (IR) divergences problem.

In this work a preliminary step is made on the way to the above-mentioned objective:
Based on **measurable** quantities the construction of *Classical Mechanics* is given.
As the mathematical apparatus based on the use of infinitesimal space-time variations (increments) (1) for *Classical Mechanics* is absolutely adequate, then the main objectives of this work are as follows:

I3. To show how in the natural passage to the limit **measurable** quantities transform into the infinitesimal space-time variations (1) and fundamental ingredients of *Classical Mechanics*.

I4. To improve methods, and to make more precise and generalize main definitions and formulae from [1]–[8] to solve the problems set up in **I1.** and **I2.**

2 Previous Information and Some Specializing and Generalization

This section gives the necessary preliminary information from [1]–[8]. Part of previous results is presented in detail [6],[8], as without this it's not possible to understand more precise definition and generalization of the main definitions (**Definition 1** and **Definition 2**) and formulae (for example formula (40)).

2.1 Minimal Length and Definition of Primary and Generalized Measurability

The present study is based on two initial, simple and quite natural, suppositions [6],[8]:
I. Any small variation increment $\widetilde{\Delta} x_{\mu}$ of any spatial coordinate x_{μ} of the arbitrary point $x_{\mu}, \mu = 1, ..., 3$ in some space-time system R may be realized in the form of the uncertainty (standard deviation) Δx_{μ} when this coordinate is measured within the scope of Heisenberg's Uncertainty Principle (HUP) [13]

$$\widetilde{\Delta} x_{\mu} = \Delta x_{\mu}, \Delta x_{\mu} \simeq \frac{\hbar}{\Delta p_{\mu}}, \mu = 1, 2, 3 \qquad (2)$$

for some $\Delta p_{\mu} \neq 0$.

Similarly, for $\mu = 0$ for pair "time-energy" (t, E), any small variation (increment) in the value of time $\widetilde{\Delta} x_0 = \widetilde{\Delta} t_0$ may be realized in the form of the uncertainty (standard deviation) $\Delta x_0 = \Delta t$ and then

$$\widetilde{\Delta} t = \Delta t, \Delta t \simeq \frac{\hbar}{\Delta E} \qquad (3)$$

for some $\Delta E \neq 0$. Here HUP is given for the nonrelativistic case. In the relativistic case HUP has the distinctive features [14] which, however, are of no significance for the general formulation of Any small variation (increment) $\widetilde{\Delta} x_\mu$ of any spatial coordinate x_μ of the arbitrary point $x_\mu, \mu = 1, ..., 3$ in some space-time system R may be realized in the form of the uncertainty (standard deviation) Δx_μ when this coordinate is measured within the scope of Heisenberg's Uncertainty Principle (HUP)

$$\widetilde{\Delta} x_\mu = \Delta x_\mu, \Delta x_\mu \simeq \frac{\hbar}{\Delta p_\mu}, \mu = 1, 2, 3 \tag{4}$$

for some $\Delta p_\mu \neq 0$. Similarly, for $\mu = 0$ for pair "time-energy" (t, E), any small variation (increment) in the value of time $\widetilde{\Delta} x_0 = \widetilde{\Delta} t_0$ may be realized in the form of the uncertainty (standard deviation) $\Delta x_0 = \Delta t$ and then

$$\widetilde{\Delta} t = \Delta t, \Delta t \simeq \frac{\hbar}{\Delta E} \tag{5}$$

for some $\Delta E \neq 0$. Here HUP is given for the nonrelativistic case. In the relativistic case HUP has the distinctive features [14] which, however, are of no significance for the general formulation of **I.**, being associated only with particular alterations in the right-hand side of the second relation Equation (2).

It is clear that at low energies $E \ll E_P$ (momenta $P \ll P_{pl}$) **I.** sets a lower bound for the variations (increments) $\widetilde{\Delta} x_\mu$ of any space-time coordinate x_μ.

At high energies E (momenta P) this is not the case if E (P) have no upper limit. But, according to the modern knowledge, E (P) are bounded by some maximal quantities E_{max}, (P_{max})

$$E \leq E_{max}, P \leq P_{max}, \tag{6}$$

where in general E_{max}, P_{max} may be on the order of Planck quantities $E_{max} \propto E_P, P_{max} \propto P_{pl}$ and also may be the trans-Planck's quantities.

In any case the quantities P_{max} and E_{max} lead to the introduction of the minimal length l_{min} and of the minimal time t_{min}.

II. There is the minimal length l_{min} as *a minimal measurement unit* for all quantities having the dimension of length, whereas the minimal time $t_{min} = l_{min}/c$ as a minimal measurement unit for all quantities having the dimension of time, where c is the speed of light.

l_{min} and t_{min} are naturally introduced as $\Delta x_\mu, \mu = 1,2,3$ and Δt in Equations (2) and (3) for $\Delta p_\mu = P_{max}$ and $\Delta E = E_{max}$.

For definiteness, we consider that E_{max} and P_{max} are the quantities on the order of the Planck quantities, then l_{min} and t_{min} are also on the order of Planck quantities $l_{min} \propto l_P$, $t_{min} \propto t_P$.

I.,II. are quite natural in the sense that there are no physical principles with which these suppositions are inconsistent.

The combination of suppositions I, II will be called the **Principle of Bounded Space-Time Variations (Increments)** .
Then, since in fact Suppositions **I.,II.** introduce the minimal length l_{min}, instead of HUP, we can consider its widely known high-energy generalization—the Generalized Uncertainty Principle (GUP) that naturally leads to the minimal length l_{min} [15]–[26]:

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' l_P^2 \frac{\Delta p}{\hbar}. \tag{7}$$

Here α' is the model-dependent dimensionless numerical factor and l_P is the Planckian length. As Equation (7) is a quadratic inequality, then it naturally leads to the minimal length $l_{min} = \xi l_P = 2\sqrt{\alpha'} l_P$.
As the minimal unit of measurement l_{min} is available for all the quantities L having the dimensions of length, the "Integrality Condition" (IC) is the case

$$L = N_L l_{min}, \tag{8}$$

where $N_L > 0$ is an integer number.
In a like manner the same "Integrality Condition" (IC) is the case for all the quantities t having the dimensions of time. And similar to Equation (8), we get the for any time t:

$$t \equiv t(N_t) = N_t t_{min}, \tag{9}$$

Due to (8), we have

$$\Delta x = N_{\Delta x} l_{min}. \tag{10}$$

Then the transition from high to low energies in GUP, i.e. $(GUP, \Delta p \rightarrow 0) = (HUP)$, is nothing else but

$$(N_{\Delta x} \approx 1) \rightarrow (N_{\Delta x} \gg 1). \tag{11}$$

Substituting (10) into (7) and making the necessary calculations, we can see that in the general case

$$\Delta p \equiv \Delta p_{N_{\Delta x}} = \frac{\hbar}{(N_{\Delta x} - \frac{1}{4N_{\Delta x}})l_{min}}. \tag{12}$$

Whereas at low energies $E \ll E_P$

$$\Delta p \equiv \Delta p_{N_{\Delta x}} = \frac{\hbar}{N_{\Delta x} l_{min}}. \tag{13}$$

At the same time, for the corresponding energy E we get

$$\Delta E \equiv \Delta E(N_t) = \frac{\hbar}{(N_t - \frac{1}{4N_t})t_{min}} \tag{14}$$

or for low energies

$$\Delta E \equiv \Delta E(N_t) = \frac{\hbar}{N_t t_{min}}. \tag{15}$$

In the relativistic case the formulae corresponding to (20),(14) have been derived in [2],[6].
Note that the above-mentioned formulae may be conveniently rewritten in terms of l_{min} with the use of the deformation parameter α_a [6]. This parameter has been introduced earlier in the papers [27]–[34] as a *deformation parameter* (in terms of paper [35]) on going from the canonical quantum mechanics to the quantum mechanics at Planck's scales (early Universe) that is considered to be the quantum mechanics with the minimal length (QMML):

$$\alpha_a = l_{min}^2/a^2, \tag{16}$$

where a is the measuring scale.
Actually, with the equality ($\Delta p \Delta x = \hbar$) Equation (7) is of the form

$$\Delta x = \frac{\hbar}{\Delta p} + \frac{\alpha_{\Delta x}}{4}\Delta x. \tag{17}$$

In this case due to Equations (8), (11) and (17) takes the following form:

$$N_{\Delta x} l_{min} = \frac{\hbar}{\Delta p} + \frac{1}{4N_\Delta x} l_{min} \tag{18}$$

or

$$(N_{\Delta x} - \frac{1}{4N_{\Delta}x})l_{min} = \frac{\hbar}{\Delta p}. \tag{19}$$

That is

$$\Delta p = \frac{\hbar}{(N_{\Delta x} - \frac{1}{4N_{\Delta}x})l_{min}}. \tag{20}$$

From Equations (18)–(20) it is clear that HUP Equation (2) appears to a high accuracy in the limit $N_{\Delta}x \gg 1$ in conformity with Equation (11).

It is easily seen that the parameter α_a from Equation (16) is discrete as it is nothing else but

$$\alpha_a = l_{min}^2/a^2 = \frac{l_{min}^2}{N_a^2 l_{min}^2} = \frac{1}{N_a^2}. \tag{21}$$

At the same time, from Equation (21) it is evident that α_a is irregularly discrete.

It is clear that from Equation (20) at low energies ($|N_{\Delta}x| \gg 1$), up to a constant

$$\frac{\hbar^2}{l_{min}^2} = \frac{\hbar c^3}{4\alpha' G} \tag{22}$$

we have

$$\alpha_{\Delta x} = (\Delta p)^2, (i.e. \alpha_{\Delta x} \propto (\Delta p)^2). \tag{23}$$

Definition 1 (**Elementary or Primary Measurability.**)
(1) *Let us define the quantity having the dimensions of length L or time t* **elementarily or primarily measurable**, *when it satisfies the relation Equation* (8) *(and respectively Equation* (9)*).*
(2)*Let us define any physical quantity* **elementarily or primarily measurable**, *when its value is consistent with points (1) of this Definition.*

However, physical quantities complying with **Definition 1** won't be enough for the research of physical systems.
Indeed, such a variable as

$$\alpha_{Nl_{min}}(Nl_{min}) = l_{min}/N, \tag{24}$$

(where $\alpha_{Nl_{min}}$ is taken from formula (21) at $a = Nl_{min}$), is fully expressed in terms *only* **Primarily Measurable Quantities** of **Definition 1** and that's why it may appear at any stage of calculations, but apparently doesn't comply with **Definition 1**. That's why it's necessary to introduce the following definition generalizing **Definition 1**:

Definition 2. Generalized Measurability
We shall call any physical quantity as **generalized-measurable** or for simplicity **measurable** if any of its values may be obtained in terms **Elementary or Primarily Measurable Quantities** of **Definition 2**.

In what follows for simplicity we will use the term **Measurability** instead of **Generalized Measurability**.
It's evident that any **primarily measurable quantity (PMQ)** is **measurable**. Generally speaking, the contrary is not correct, as indicated by formula (24).

Naturally, of course that, a minimal possible **primarily measurable** and change of length is l_{min}. It corresponds to some maximal value of the energy E_{max} or momentum P_{max}, If $l_{min} \propto l_P$, then $E_{max} \propto E_P$,$P_{max} \propto P_{Pl}$, where $P_{max} \propto P_{Pl}$, where P_{Pl} is where the Planck momentum. Then denoting in nonrelativistic case with $\triangle_p(w)$ a minimal measurable change every spatial coordinate w corresponding to the energy E we obtain

$$\triangle_{P_{max}}(w) = \triangle_{E_{max}}(w) = l_{min}. \qquad (25)$$

Evidently, for lower energies (momentums) the corresponding values of $\triangle_p(w)$ are higher and, as the quantities having the dimensions of length are quantized Equation (8), for $p \equiv p(N_p) < p_{max}$, $\triangle_p(w)$ is transformed to

$$|\triangle_{p(N_p)}(w)| = |N_p| l_{min}. \qquad (26)$$

where $|N_p| \ > \ 1$ is an integer number so that we have

$$|N_p - \frac{1}{4N_p}| l_{min} = \frac{\hbar}{|p(N_p)|}. \qquad (27)$$

In the relativistic case the Equation (25) holds, whereas Equations (26) and (27) for $E \equiv E(N_E) < E_{max}$ are replaced by

$$|\triangle_{E(N_E)}(w)| = |N_E| l_{min}, \tag{28}$$

where $|N_E| > 1$ is an integer.

Next we assume that at high energies $E \propto E_P$ there is a possibility only for the nonrelativistic case or ultrarelativistic case.

Then for the ultrarelativistic case, formula (27) takes the form [6]:

$$|N_E - \frac{1}{4N_E}| l_{min} = \frac{\hbar c}{E(N_E)} = \frac{\hbar}{|p(N_p)|}, \tag{29}$$

where $N_E = N_p$.

In the relativistic case at low energies we have

$$E \ll E_{max} \propto E_P. \tag{30}$$

and formula (26) is of the form

$$|\triangle_{E(N_E)}(w)| = |N_E| l_{min} = \frac{\hbar c}{E(N_E)}, |N_E| \gg 1 - integer. \tag{31}$$

In the nonrelativistic case at low energies Equation (30) due to Equation (27) we get

$$|\triangle_{p(N_p)}(w)| = |N_p| l_{min} = \frac{\hbar}{|p(N_p)|}, |N_p| \gg 1 - integer. \tag{32}$$

In a similar way for the time coordinate t, by virtue of Equations (9)–(15), at the same conditions we have similar Equations (25)–(27)

$$\triangle_{E_{max}}(t) = t_{min}. \tag{33}$$

For $E \equiv E(N_t) < E_{max}$

$$|\triangle_{E(N_t)}(t)| = |N_t| t_{min}, \tag{34}$$

where $|N_E| > 1$ is an integer, so that we obtain

$$|N_t - \frac{1}{4N_t}|t_{min} = \frac{\hbar c}{E(N_t)}. \qquad (35)$$

In the relativistic case at low energies

$$E \ll E_{max} \propto E_P, \qquad (36)$$

equation (26) takes the form [6]:

$$|\triangle_{E(N_t)}(w)| = |N_t| l_{min} = \frac{\hbar c}{E(N_t)}, |N_t| \gg 1 - integer. \qquad (37)$$

We shall make two important **Commentaries**:

Comment 2.1.
What's the main difference between **Definition 1** and **Definition 2**?

2.1.1.**Definition 1** defines variables which may be obtained as a result of an immediate experiment.

2.1.2. **Definition 2** defines the variables which may be *calculated* based on **primarily measurable quantities**, i.e. based on the data obtained in previous clause 2.1.1.

Comment 2.2.
It's evident that HUP-derived (2) $\Delta p_i \doteq \Delta p_{i,HUP}; i = 1, ..., 3$ are **primarily measurable** quantities:

$$\Delta p_i \simeq \frac{\hbar}{\Delta x_i} = \frac{\hbar}{N_{\Delta x_i} l_{min}} \qquad (38)$$

However, variables $\Delta p_i \doteq \Delta p_{i,GUP}$ obtained from GUP (7) and defined by formula (20) are already obviously not the same, but only **measurable** quantities.
From formulae (22) and (23) follows that in case of correctness of HUP (2) i.e. in low energies $E \ll E_{max} \propto E_P$, in notations of formulae (26)–(37)

$$\alpha_{N_p l_{min}}(HUP) \doteq \alpha_{\Delta x} = p(N_p)^2 \frac{l_{min}^2}{\hbar^2} = \frac{1}{N_p^2} \qquad (39)$$

where $\Delta x = N_p l_{min}$ and $p(N_p)$ is calculated from formula (32).
However, in high energies $E \approx E_P$, HUP is replaced with GUP, **primarily measurable quantity** $p(N_p)$ from formula (32) is replaced with **generalized measurable quantity** $\Delta p_i \doteq \Delta p_{i,GUP}$ from formula (27).
Then $\alpha_{N_p l_{min}}(HUP)$ may be replaced with $\alpha_{N_p l_{min}}(GUP)$:

$$\alpha_{N_p l_{min}}(GUP) = p(N_p, GUP)^2 \frac{l_{min}^2}{\hbar^2} =$$
$$= \frac{l_{min}^2}{(N_p - \frac{1}{4N_p})^2 l_{min}^2} = \frac{1}{(N_p - \frac{1}{4N_p})^2} \tag{40}$$

When going over from high energies $E \approx E_P$ to low energies $E \ll E_P$ we have:

$$\alpha_{N_p l_{min}}(GUP) \stackrel{(|N_p|\approx 1)\to(|N_p|\gg 1)}{\longrightarrow} \alpha_{N_p l_{min}}(HUP) \tag{41}$$

In what follows all the considerations are given in terms of **measurable quantities** in the sense of **Definition 2** given in this Section.

2.2 Space-Time Lattice of Primary Measurable Quantities and Dual Lattice

For convenience, we denote the minimal length $l_{min} \neq 0$ by ℓ and $t_{min} \neq 0$ by $\tau = \ell/c$.
So, provided the minimal length ℓ exists, two lattices are naturally arising.
I. Lattice of the space-time variation—Lat_{S-T} representing, to within the known multiplicative constants, for sets of nonzero integers $N_w \neq 0$ and $N_t \neq 0$ in corresponding formulae from the set Equations (26) and (37) for each of the three space variables $w \doteq x; y; z$ and the time variable t

$$Lat_{S-T} \doteq (N_w \ell, N_t \tau). \tag{42}$$

Which restrictions should be initially imposed on these sets of nonzero integers?

It is clear that in every such set all the elements $(N_w \ell, N_t \tau)$ should be sufficiently "close", because otherwise, for one and the same space-time point, variations in the values of its different coordinates are associated with principally different values of the energy E which are "far" from each other.

Note that the words "close" and "far" will be elucidated further in this text.

Thus, at the admittedly low energies (Low Energies) $E \ll E_{max} \propto E_P$ the low-energy part (sublattice) $Lat_{S-T}[LE]$ of Lat_{S-T} is as follows:

$$Lat_{S-T}[LE] = (N_w \ell, N_t \tau); |N_x| \gg 1, |N_y| \gg 1, |N_z| \gg 1, |N_t| \gg 1. \quad (43)$$

At high energies (High Energies) $E \rightarrow E_{max} \propto E_P$ we, on the contrary, have the sublattice $Lat_{S-T}[HE]$ of Lat_{S-T}

$$Lat_{S-T}[HE] = (N_w \ell, N_t \tau); |N_x| \approx 1, |N_y| \approx 1, |N_z| \approx 1, |N_t| \approx 1. \quad (44)$$

We will call lattice Lat_{S-T} (42) as **primary (or primitive)** *lattice of the space-time variation.*

II. Next let us define the lattice momenta-energies variation Lat_{P-E} as a set to obtain $(p_x(N_{x,p}), p_y(N_{y,p}), p_z(N_{z,p}), E(N_t))$ in the nonrelativistic and ultrarelativistic cases for all energies, and as a set to obtain $(E_x(N_{x,E}), E_y(N_{y,E}), E_z(N_{z,E}), E(N_t))$ in the relativistic (but not ultrarelativistic) case for low energies $E \ll E_P$, where all the components of the above sets conform to the space coordinates (x, y, z) and time coordinate t and are given by corresponding formulae (25)–(37) from the previous Section.

Note that, because of the suggestion made after formula Equation (30) in the previous Section, we can state that the foregoing sets exhaust all the collections of momentums and energies possible for the lattice Lat_{S-T}. From this it is inferred that, in analogy with point I of this Section, within the known multiplicative constants, we have

$$Lat_{P-E} \doteq (\frac{1}{N_w - \frac{1}{1/4N_w}}, \frac{1}{N_t - \frac{1}{1/4N_t}}), \quad (45)$$

where $N_w \neq 0, N_t \neq 0$ are integer numbers from Equation (42). Similar to Equation (43), we obtain the low-energy (Low Energy) part or the sublattice

$Lat_{P-E}[LE]$ of Lat_{P-E}

$$Lat_{P-E}[LE] \approx (\frac{1}{N_w}, \frac{1}{N_t}), |N_w| \gg 1, |N_t| \gg 1. \qquad (46)$$

In accordance with Equation (44), the high-energy (High Energy) part (sublattice) $Lat_{P-E}[HE]$ of Lat_{P-E} takes the form

$$Lat_{P-E}[HE] \approx (\frac{1}{N_w - \frac{1}{1/4N_w}}, \frac{1}{N_t - \frac{1}{1/4N_t}}), |N_w| \rightarrow 1, |N_t| \rightarrow 1. \qquad (47)$$

It is important to note the following.

In the low-energy sublattice $Lat_{P-E}[LE]$ all elements are varying very smoothly enabling the approximation of a continuous theory.

3 Classical Mechanics in "Measurable Format"

3.1 Preliminary Information

We will preserve the lattice Lat_{P-E}, but **primary** lattice Lat_{S-T} will be replaced with "$\alpha - lattice$", **measurable space-time quantities**, which will be denoted by Lat^{α}_{S-T}:

$$Lat^{\alpha}_{S-T} \doteq (\alpha_{N_w\ell} N_w \ell, \alpha_{N_t\tau} N_t \tau) = (\frac{\ell}{N_w}, \frac{\tau}{N_t}). \qquad (48)$$

In the last formula by the variable $\alpha_{N_t\tau}$ we mean the parameter α corresponding to the length $(N_t\tau)c$:

$$\alpha_{N_t\tau} \doteq \alpha_{(N_t\tau)c}. \qquad (49)$$

As in this case low energies $E \ll E_P$ are discussed, $\alpha_{N_w\ell}$ in this formula is consistent with the corresponding parameter from formula (39):

$$\alpha_{N_w\ell} = \alpha_{N_w\ell}(HUP) \qquad (50)$$

As it was mentioned in the previous section, in the low-energy $E \ll E_{max} \propto E_P$ all elements of sublattice $Lat_{P-E}[LE]$ are varying very smoothly enabling the approximation of a continuous theory.
It is similar to the low-energy part of the $Lat^{\alpha}_{S-T}[LE]$ of lattice Lat^{α}_{S-T} will vary very smoothly:

$$Lat^{\alpha}_{S-T}[LE] = (\frac{\ell}{N_w}, \frac{\tau}{N_t}); |N_x| \gg 1, |N_y| \gg 1, |N_z| \gg 1, |N_t| \gg 1. \quad (51)$$

In sectin 5 of [6] three following cases were selected:

(a) *"Quantum Consideration, Low Energies"*:

$$1 \ll |N_w| \leq \widetilde{\mathbf{N}};$$

(b) *"Quantum Consideration, High Energies"*:

$$|N_w| \approx 1;$$

(c) *"Classical Picture"*:

$$1 \ll \widetilde{\mathbf{N}} \ll |N_w|.$$

Here $\widetilde{\mathbf{N}}$ is a cutoff parameter , defined by the current task [6].
In *"Classical Picture"* (c) the passage to the limit

$$|N_w| \rightarrow \infty, |N_t| \rightarrow \infty is correct. \quad (52)$$

That's why, if for three space coordinates $x_i; i = 1, 2, 3$ we introduce the following notation:

$$\Delta(x_i) \doteq \widetilde{\Delta}[\alpha_{N_{\Delta x_i}}] = \alpha_{N_{\Delta x_i}\ell}(N_{\Delta x_i}\ell) = \ell/N_{\Delta x_i};$$
$$\frac{\Delta[F(x_i)]}{\Delta(x_i)} \equiv \frac{F(x_i + \Delta(x_i)) - F(x_i)}{\Delta(x_i)}, \quad (53)$$

then it's evident that

$$\lim_{|N_{\Delta x_i}|\to\infty}\frac{\Delta[F(x_i)]}{\Delta(x_i)}=\lim_{\Delta(x_i)\to 0}\frac{\Delta[F(x_i)]}{\Delta(x_i)}=\frac{\partial F}{\partial x_i}. \tag{54}$$

Respectively, for time $x_0=t$ we have:

$$\begin{aligned}\Delta(t)\doteq\widetilde{\Delta}[\alpha_{N_{\Delta t}}]=\alpha_{N_{\Delta t}\tau}(N_{\Delta t}\tau)=\tau/N_{\Delta t};\\ \frac{\Delta[F(t)]}{\Delta(t)}\equiv\frac{F(t+\Delta(t))-F(t)}{\Delta(t)},\end{aligned} \tag{55}$$

then

$$\lim_{|N_{\Delta t}|\to\infty}\frac{\Delta[F(t)]}{\Delta(t)}=\lim_{\Delta(t)\to 0}\frac{\Delta[F(t)]}{\Delta(t)}=\frac{dF}{dt}. \tag{56}$$

We shall designate for pulses $p_i; i=1,2,3$

$$\begin{aligned}\Delta p_i=\frac{\hbar}{N_{\Delta x_i}\ell};\\ \frac{\Delta_{p_i}F(p_i)}{\Delta p_i}\equiv\frac{F(p_i+\Delta p_i)-F(p_i)}{\Delta p_i}=\frac{F(p_i+\frac{\hbar}{N_{\Delta x_i}\ell})-F(p_i)}{\frac{\hbar}{N_{\Delta x_i}\ell}}.\end{aligned} \tag{57}$$

From where similarly (54) we get

$$\begin{aligned}\lim_{|N_{\Delta x_i}|\to\infty}\frac{F(p_i+\Delta p_i)-F(p_i)}{\Delta p_i}=\lim_{|N_{\Delta x_i}|\to\infty}\frac{F(p_i+\frac{\hbar}{N_{\Delta x_i}\ell})-F(p_i)}{\frac{\hbar}{N_{\Delta x_i}\ell}}=\\ =\lim_{\Delta p_i\to 0}\frac{F(p_i+\Delta p_i)-F(p_i)}{\Delta p_i}=\frac{\partial F}{\partial p_i}.\end{aligned} \tag{58}$$

Therefore, in low energies $E\ll E_P$, i.e. at $|N_{\Delta x_i}|\gg 1; i=0,...,3$ in passages to the limit (54),(56),(58) it's possible to obtain known partial derivatives like in case of continuous space-time.

Definition Cl1.
Let some quantity Ξ depend on integers $N_{\Delta x_i}, N_{\Delta t}$, at all values of $N_{\Delta x_i}, N_{\Delta t}$

is **measurable** *and formula (52) is correct, i.e. we have "Classical Picture" (c). Then, if there are passages to the limit*

$$\lim_{|N_{\Delta x_i}|\to\infty} \Xi(N_{\Delta x_i}) = \Xi_{x_i}; \lim_{|N_{\Delta t}|\to\infty} \Xi(N_{\Delta t}) = \Xi_t, \tag{59}$$

then the respective limits Ξ_{x_i}, Ξ_t *shall be also called* **measurable** *quantities. Particularly, if* F *in formulae (53)–(58) is a* **measurable** *quantity, then from* **Definition 2** *follows directly that the values* $\frac{\Delta[F(x_i)]}{\Delta(x_i)}, \frac{\Delta[F(t)]}{\Delta(t)}, \frac{\Delta_{p_i}F(p_i)}{\Delta p_i}$ *are also* **measurable** *quantities. Then, according to this definition, the same are the quantities* $\frac{\partial F}{\partial x_i}, \frac{\partial F}{\partial p_i}, \frac{dF}{dt}$*. in formulae (54),(56),(58).*

Commentary to Definition Cl1.
By virtue of (52) it's evident that Definition.Cl1 is applicable only to case (c) above (Classical Picture) and not applicable to cases (a) and (b),(Quantum Consideration, Low Energies) and (Quantum Consideration, High Energies) respectively

We shall make two notes
Remark 3.1
There is a significant difference between formulae (54),(56) on the one hand and formula (58) on the other hand.
Limits in (54) and in (56) may be obtained also when going over to continuous space-time

$$\begin{gathered} \ell \to 0; \tau \to 0; \\ \lim_{\ell\to 0} \frac{\Delta[F(x_i)]}{\Delta(x_i)} = \frac{\partial F}{\partial x_i}; \\ \lim_{\tau\to 0} \frac{\Delta[F(t)]}{\Delta(t)} = \frac{dF}{dt}. \end{gathered} \tag{60}$$

But in formula (58) passage to the limit at $\ell \to 0$ is not possible, as it leads to an infinitely great denominator.
Remark 3.2 The above-mentioned calculations show that in this offered discrete approach by virtue of **Definition Cl1.** in low energies in classical consideration it's possible to obtain all the main attributes of the continued

theory, particularly, for any respective function F the quantities $\frac{\partial F}{\partial x_i}, \frac{dF}{dt}, \frac{\partial F}{\partial p_i}$ are defined correctly.
Here it's not necessary to observe the condition $\hbar \to 0$, i.e. $\hbar \neq 0$ remains also in the classical situation and is suppressed due to the passage to the limit $|N_{\Delta x_i}| \to \infty$.

3.2 Lagrangian Formalism and Principle of Least Action in Terms of Measurable Quantities

By virtue of **Definition Cl1.** and formulae (53)–(58), as well as some of their generalizations, it's possible to show that all the main provisions of classical mechanics both in Lagrangian and Hamiltonian formalism remain correct in terms of **measurable** quantity, in the presence of quite natural additional assumptions.
Hereinafter we will use standard terminology of classical mechanics [11],[12]. Let there be a Lagrangian $L \doteq L(q, \dot{q}, t)$, where q are generalized coordinates; $\dot{q}$ are generalized speeds and t is time. However, in the duscussed case t changes discretely, according to the formulae above.

Definition Cl2. We shall call L as a measurable analogue and denote by $L_{meas}(q, \dot{q}, t)$, the quantity satisfying the following properties:

Cl1.1. Time t, and the generalized coordinate q included into $L_{meas}(q, \dot{q}, t)$ are **primarily measurable quantities** in terms of **Definition 1**

Cl1.2. The quantity $\dot{q}$ is obtained from formulae (55),(56), (where $F(t) = q(t)$) and that's why according to **Definition Cl1.**, it's a **measurable** quantity.

Cl1.3. In case of fulfillment of conditions *Cl1.1.* and *Cl1.2.*

$$L_{meas}(q, \dot{q}, t) = L(q, \dot{q}, t) \tag{61}$$

Hereinafter we will assume that the Lagrangian $L(q, \dot{q}, t)$ is **measurable**, i.e.

$$L(q, \dot{q}, t) = L_{meas}(q, \dot{q}, t) \tag{62}$$

It's necessary to make an important note:
Remark 3.3.
In formulae (53)–(58) Cartesian coordinates x_i were used and respective pulses p_i in terms of **measurable** quantities. However, it's not difficult to obtain analogue (53),(54),(57),(58) for **measurable** generalized coordinates q and speeds $\dot{q}$.
Indeed, let $\widetilde{\Delta}q$ and $\widetilde{\Delta}\dot{q}$ be **measurable** small increments q and $\dot{q}$ respectively. We shall introduce the following notations for the measurable value of time t_i:

$$\Delta q(t_i) = \alpha_{t_i}\widetilde{\Delta}q(t_i); \Delta\dot{q}(t_i) = \alpha_{t_i}\widetilde{\Delta}\dot{q}(t_i), \tag{63}$$

where $\alpha_{t_i} = \alpha_{N_{t_i}\tau}$ from formula (49).
It's clear that as $\widetilde{\Delta}q$ and $\widetilde{\Delta}\dot{q}$ are **measurable** small increments of q and $\dot{q}$ respectively, then Δq and $\Delta\dot{q}$ will be the same, and as we are discussing low energies and, consequently, for each t_i from formula (63) $t_i = N_{t_i}\tau, |N_{t_i}| \gg 1$, then $|\Delta q(t_i)| \ll |\widetilde{\Delta}q(t_i)|, |\Delta\dot{q}(t_i)| \ll |\widetilde{\Delta}\dot{q}(t_i)|$.
In formula (63) **measurable** small increments are set with the help of the corresponding parameter α by actual generalization for the case $\widetilde{\Delta}q, \widetilde{\Delta}\dot{q}$ of "$\alpha - lattice$" **measurable space-time quantities** Lat^{α}_{S-T} (48).
However, it's possible to act in a more simple way: as under the definition q and $\dot{q}$ are **measurable** quantities, then $\Delta q(t_i) = \frac{1}{N_{t_i}}q(t_i) = (\frac{\ell}{\hbar}p_{N_{t_i}})q(t_i)$ as well as $\Delta\dot{q}(t_i) = \frac{1}{N_{t_i}}q(t_i) = (\frac{\ell}{\hbar}p_{N_{t_i}})\dot{q}(t_i)$ are **measurable** small increments q and $\dot{q}$ at $|N_{t_i}| \gg 1$, which go to zero, at $|N_{t_i}| \to \infty$.
Next, we shall define

$$\frac{\Delta F(q(t_i))}{\Delta q(t_i)} \equiv \frac{F(q(t_i) + \Delta q(t_i)) - F(q(t_i))}{\Delta q(t_i)} \tag{64}$$

and, respectively,

$$\frac{\Delta F(\dot{q}(t_i))}{\Delta\dot{q}(t_i)} \equiv \frac{F(\dot{q}(t_i) + \Delta\dot{q}(t_i)) - F(\dot{q}(t_i))}{\Delta\dot{q}(t_i)} \tag{65}$$

Then it's evident that for the **measurable** function F right parts (64) and (65) will also be **measurable** and according to **Definition Cl1.** it's possible to obtain **measurable** limits:

$$\lim_{|N_{t_i}|\to\infty} \frac{\Delta F(q(t_i))}{\Delta q(t_i)} = \lim_{\Delta q(t_i)\to 0} \frac{\Delta F(q(t_i))}{\Delta q(t_i)} = \frac{\partial F}{\partial q};$$
$$\lim_{|N_{t_i}|\to\infty} \frac{\Delta F(\dot{q}(t_i))}{\Delta \dot{q}(t_i)} = \lim_{\Delta \dot{q}(t_i)\to 0} \frac{\Delta F(\dot{q}(t_i))}{\Delta \dot{q}(t_i)} = \frac{\partial F}{\partial \dot{q}} \qquad (66)$$

As according to **Definition Cl2.** the time t is a **primarily measurable quantity** we shall denote as follows

$$\widetilde{t} - \widehat{t} = \Delta t = N_{\Delta t}\tau \qquad (67)$$

In this case it's possible to define a *measurable action* as a sum:

$$S_{meas,N_{\Delta t}}(q,\dot{q},t) = \sum_{1\leq i\leq N_{\Delta t},\widehat{t}\leq t_i\leq \widetilde{t}} L_{meas}(q(t_i),\dot{q}(t_i),t_i)\alpha_{N_{\Delta t}}(N_{\Delta t}\tau) =$$
$$= \sum_{1\leq i\leq N_{\Delta t},\widehat{t}\leq t_i\leq \widetilde{t}} L_{meas}(q(t_i),\dot{q}(t_i),t_i)\frac{\tau}{N_{\Delta t}}, \qquad (68)$$

where $L(q,\dot{q},t)$ satisfies (62).
However, by virtue of **Definition Cl1.** in this particular case of classical mechanics the passage to the infinite limit is correct:

$$S_{meas,N_{\Delta t}}(q,\dot{q},t) \stackrel{|N_{\Delta t}|\to\infty}{\longrightarrow} S_{meas}(q,\dot{q}) \doteq \int_{\widehat{t}}^{\widetilde{t}} L_{meas}\left(q,\dot{q},t\right) dt \qquad (69)$$

Based on **Definition Cl1**, (69) may be rewritten as

$$S_{meas,N_{\Delta t}}(q,\dot{q},t) \stackrel{|N_{\Delta t}|\to\infty}{\longrightarrow} S_{meas}(q,\dot{q}) = \int_{\widehat{t}}^{\widetilde{t}} L_{meas}\left(q,\dot{q},t\right) dt \qquad (70)$$

Next, quite a natural supposition will be taken:

Supposition.Cl1.
For each **measurable** *quantity κ and quite large Δt (or the same for quite*

large $|N_{\Delta t}|$ *and, naturally, for* $|N_{\Delta t}| \rightarrow \infty$*) there is a* **measurable** *variation of* $\delta\kappa$*.*

(Indeed, this is a very natural supposition. As q is a **primary measurable**, then $q/N = \frac{P_N \ell}{\hbar} q$ is a **measurable** quantity and at quite large N it may be made arbitrary close to the **measurable** variation of $\delta\kappa$).
Taking as κ a **measurable** quantity q, according to **Supposition.Cl1.** we may obtain a **measurable** variation δq. Considering **Definition Cl1.**, we obtain

$$\frac{d}{dt}(\delta q) = \delta \dot{q} \tag{71}$$

is a **measurable** quantity as well.
Next, step by step we may obtain *Principle of Least Action* [11],[12] in terms of **measurable** quantities. For this we need to make sure that at each step of proof of this principle only **measurable** quantities appear.
Indeed, as $S_{meas,N_{\Delta t}}(q, \dot{q}, t)$ is a **measurable** quantity, then by virtue of **measurability** δq and $\delta \dot{q}$, the sum $S_{meas,N_{\Delta t}}(q + \delta q, \dot{q} + \delta \dot{q}, t)$ will be also **measurable**. By virtue of **Definition Cl1.**, using the passage to the limit (70), but already for $S_{meas,N_{\Delta t}}(q + \delta q, \dot{q} + \delta \dot{q}, t)$, we obtain **measurable** quantity $S_{meas}(q + \delta q, \dot{q} + \delta \dot{q})$:

$$\begin{aligned} S_{meas,N_{\Delta t}}(q + \delta q, \dot{q} + \delta \dot{q}, t) &\overset{|N_{\Delta t}| \rightarrow \infty}{\longrightarrow} S_{meas}(q + \delta q, \dot{q} + \delta \dot{q}) = \\ &= \int_{\widehat{t}}^{\widetilde{t}} L_{meas} \left(q + \delta q, \dot{q} + \delta \dot{q}, t \right) dt \end{aligned} \tag{72}$$

From where it follows directly that the variation $\delta S_{meas}(q, \dot{q})$ is also **measurable**:

$$\delta S_{meas}(q, \dot{q}) = S_{meas}(q + \delta q, \dot{q} + \delta \dot{q}) - S_{meas}(q, \dot{q}) = [\delta S(q, \dot{q})]_{meas} \tag{73}$$

Equating the right part (73) to zero we obtain the equation in which all the components are **measurable** quantities:

$$[\delta S(q, \dot{q})]_{meas} = \delta \int_{\widehat{t}}^{\widetilde{t}} L_{meas} \left(q, \dot{q}, t \right) dt = \int_{\widehat{t}}^{\widetilde{t}} \left(\frac{\partial L_{meas}}{\partial q} \delta q + \frac{\partial L_{meas}}{\partial \dot{q}} \delta \dot{q} \right) dt = 0 \tag{74}$$

Indeed, $\partial L_{meas}/\partial q, \partial L_{meas}/\partial \dot{q}$ are **measurable** according to **Remark 3.3.** and respective formulae. $\delta q, \dot{q}$ are measurable according to **Supposition.Cl1.**,**Definition Cl1.** and formula (71).
That's why using formula (71) and an integration by parts [11], which evidently does not destroy **measurability** we obtain the following from (74):

$$\delta S = \left[\frac{\partial L_{meas}}{\partial \dot{q}} \delta q \right]_{\widehat{t}}^{\widetilde{t}} + \int_{\widehat{t}}^{\widetilde{t}} \left(\frac{\partial L_{meas}}{\partial q} - \frac{d}{dt} \frac{\partial L_{meas}}{\partial \dot{q}} \right) \delta q dt = 0 \ , \tag{75}$$

where as usually $q(\widehat{t}) = q(\widetilde{t}) = 0, \frac{d}{dt} \frac{\partial L_{meas}}{\partial \dot{q}}$ are **measurable** by virtue of **Definition Cl1.** and formula (56) at $F(t) = \frac{\partial L_{meas}}{\partial \dot{q}}$ and as it was already used in formulae (68), (69) in case of classical mechanics dt is also a **measurable** quantity, as according to **Definition Cl1.** it appears within the limits for a **measurable** quantity

$$\tau / N_{\Delta t} \stackrel{N_{\Delta t} \to \infty}{\longrightarrow} dt. \tag{76}$$

From where the following representation follows *Euler-Lagrange equations* [11] in terms of only **measurable** quantities :

$$\frac{\partial L_{meas}}{\partial q} - \frac{d}{dt} \frac{\partial L_{meas}}{\partial \dot{q}} = 0,$$

$$or$$

$$\frac{d}{dt} \left(\frac{\partial L_{meas}}{\partial \dot{q}_i} \right) - \frac{\partial L_{meas}}{\partial q_i} = 0, \qquad (i = 1, 2, ..., s) \tag{77}$$

3.3 Hamiltonian Formalism and Measurability

Using the results of the previous Subsection it's not difficult to obtain also *Hamiltonian Formalism* in terms of **measurable** quantities. As well as in the previous Subsection it's necessary to make sure that at each step all members in respective formulae are **measurable**.
Indeed, using **"measurable"** *Euler-Lagrange equations* (77) it's possible to

introduce **measurable** generalized momenta and their time derivatives:

$$p_{meas} = \frac{\partial L_{meas}}{\partial \dot{q}}; \dot{p}_{meas} = \frac{\partial L_{meas}}{\partial q} \tag{78}$$

From where, using Legendre transformation [11], the following appears **"measurable"** Hamiltonian H:

$$H_{meas}(q, p, t) = \sum_i \dot{q}_i (p_i)_{meas} - L_{meas}(q, \dot{q}, t) \tag{79}$$

Here we don't put subscript *meas* for variables $q, \dot{q}$, as they are **measurable** by virtue of **Definition Cl2**.
Total differential of left part (75) will be equal to:

$$d[H_{meas}(q, p, t)] = \sum_i [\dot{q}_i d[(p_i)_{meas}] - (\dot{p_i})_{meas} dq_i] - \frac{\partial L_{meas}}{\partial t} dt \tag{80}$$

In right part (80) the member $\frac{\partial L_{meas}}{\partial t} dt$ will be **measurable** by virtue of **Definition Cl1.** and formulae (56) and (76). $q_i, \dot{q}_i$ are **measurable** by virtue of **Definition Cl1.**,**Definition Cl2.**, **measurability** $(p_i)_{meas}$ and $(\dot{p_i})_{meas}$ was obtained in (78). Finally, dq_i and $d[(p_i)_{meas}]$ will be **measurable** according to **Definition Cl1.** and formulae (63)–(66).
Therefore, right part (80) is a **measurable** quantity, that's why also left part (80) is a **measurable** quantity. From where

$$d[H_{meas}(q, p, t)] = d[H_{meas}(q, p, t)]_{meas} \tag{81}$$

From (81) and standard representation of total differential for $H_{meas}(q, p, t)$, which will also evidently be a **measurable** quantity *Canonical Hamilton's Equations* in terms of **measurable** quantities follow immediately:

$$\frac{\partial H_{meas}}{\partial q_i} = -(\dot{p_i})_{meas}, \frac{\partial H_{meas}}{\partial (p_i)_{meas}} = \dot{q}_i, \frac{\partial H_{meas}}{\partial t} = -\frac{\partial L_{meas}}{\partial t}. \tag{82}$$

Next, it's tacitly supposed that $p = p_{meas}$. Then any function of canonical variables $G(q, p, t)$ will be a **measurable** quantity, in that sense that any of its meanings may be obtained in terms of **measurable** set of variables

$G(q,p,t)$.
By virtue of the results obtained above, *Poisson bracket* $[,]_{PB}$ of two **measurable** functions $G(q,p,t)$ and $\Phi(q,p,t)$ [11] will also be a **measurable** function:

$$[\Phi, G]_{PB} = \sum_j (\frac{\partial \Phi}{\partial p_j}\frac{\partial G}{\partial q_j} - \frac{\partial \Phi}{\partial q_j}\frac{\partial G}{\partial p_j}) \quad (83)$$

Particularly, if $\Phi(q,p,t) = H(q,p,t) = H_{meas}(q,p,t)$, we come to the basic equation of the Hamiltonian mechanics [11], obtained in terms of **measurable** quantities:

$$\frac{dG}{dt} = \frac{\partial G}{\partial t} + [H, G]_{PB} \quad (84)$$

Note.
It's evident that in this formalism *Canonical Hamilton's Equations* in terms of **measurable** quantities (82) may be obtained from *Principle of Least Action*, if in **Definition Cl2.** we make a replacement $L \rightarrow H, L_{meas} \rightarrow H_{meas}$ and add **measurable** generalized momentum p.

4 Final Commentaries,Explanations and Conclusion

F1. Primary measurable the generalized coordinates q and **measurable** the generalized velocities $\dot{q}$ from **Definition Cl2.** are standard quantities of classical mechanics [11],[12], on which only one limitation is imposed:
Changes of all parameters,(naturally including time t), on which q and $\dot{q}$ depend satisfy **Definition 1** *and* **Definition 2** *respectively.*
The exception is the procedure to obtain $\dot{q}$ by q, as here q cannot be considered as a **primary measurable** quantity, but only a **measurable** quantity. This is discussed in details in clause **F4.**

F2. If the theory supposes the passage to the infinite limit (52), then this theory may be considered a *Classical Theory* and then **Definition Cl1.** is absolutely correct, as for limits Ξ_{x_i} and Ξ_t from (59) at quite large

$|N_{\Delta x_i}|, |N_{\Delta t}|$ it's always possible to find **measurable** quantities $\Xi(N^*_{\Delta x_i}), \Xi(N^*_{\Delta t})$ arbitrary close to Ξ_{x_i} and Ξ_t. I.e. with high precision Ξ_{x_i} and Ξ_t may be replaced with primary **measurable** quantities $\Xi(N^*_{\Delta x_i})$ and $\Xi(N^*_{\Delta t})$.

F3. It may seem that "$\alpha - lattice$" Lat^{α}_{S-T} (formula (48)) is introduced in this work artificially. But in reality this is not true. It appears, but with *"factor"* 1/4 from equation (17) written in the form

$$\Delta x - \frac{\hbar}{\Delta p} = \frac{1}{4}\alpha_{\Delta x}\Delta x. \tag{85}$$

It's evident that factor 1/4 in right part (85) is not significant in this case.

F4. Despite the fact that the generalized coordinate q from **Definition Cl2.** is initially a **primarily measurable quantity** in terms of **Definition 1**, *"the speed of its variation in time"*, i.e. $\dot{q}$ already cannot be the same and is just a **measurable quantity** in terms of **Definition 2**. Moreover, for its definition, according to formulae (55),(56), (at $F(t) = q(t)$) the generalized coordinate q and time t should be also considered as **measurable quantities**. There is no contradiction here. If during definition of $\dot{q}$ we considered q as a **primarily measurable quantity**, then in formula (55) at larger $|N_{\Delta t}|$ and $F(t) = q(t)$ we would obtain generally a discrete divergent row of values

$$\widehat{\Delta(t)} \doteq N_{\widehat{\Delta(t)}}\tau;$$
$$\frac{\widehat{\Delta(t)}[q(t)]}{\widehat{\Delta(t)}} \equiv \frac{q(t + N_{\widehat{\Delta(t)}}\tau) - q(t)}{N_{\widehat{\Delta(t)}}\tau}, \tag{86}$$

where $N_{\Delta t} = N_{\widehat{\Delta(t)}}$.
And then the limit (56) , i.e. $\dot{q}$ would not even exist!

F5. It's clear that the passage to the limit (76) from a **measurable** quantity $\tau/N_{\Delta t}$ to infinitesimal quantity dt, which in case of *Classical Mechanics* by virtue of **Definition Cl1.** will also be a **measurable** quantity, may be generalized as space variables and written as follows:

$$\begin{aligned}
&(\frac{\tau}{N_{\Delta t}} = p_{N_{\Delta t}c}\frac{\ell^2}{c\hbar}) \stackrel{N_{\Delta t}\to\infty}{\longrightarrow} dt,\\
&(\frac{\ell}{N_{\Delta x}} = p_{N_{\Delta x}}\frac{\ell^2}{\hbar}) \stackrel{N_{\Delta x}\to\infty}{\longrightarrow} dx,\\
&(\frac{\ell}{N_{\Delta y}} = p_{N_{\Delta y}}\frac{\ell^2}{\hbar}) \stackrel{N_{\Delta y}\to\infty}{\longrightarrow} dy,\\
&(\frac{\ell}{N_{\Delta z}} = p_{N_{\Delta z}}\frac{\ell^2}{\hbar}) \stackrel{N_{\Delta z}\to\infty}{\longrightarrow} dz.
\end{aligned} \tag{87}$$

Left parts of all four limits given in formula (87), are **measurable** quantities, which depend on available energies. They will be necessary in the construction of the *Quantum Theory* in terms of **measurable** quantities.

F5. Remarks 3.1,3.2 and formula (58) show that in this formalism, as distinct from the standard case of continuous space-time it will be possible to keep $\hbar \neq 0$ during the passage from *Quantum Picture* to *Classical Picture.*

Therefore, summing up it should be stated that at some natural suppositions *Classical Mechanics* may be correctly formulated in terms of **measurable** quantities of the [1]–[8] and present paper.

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Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this article.

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