

**Surmounting the Cartesian Cut Further: Torsion Fields, the Extended Photon,
Quantum Jumps, The Klein-Bottle, Multivalued Logic, the Time Operator
Chronomes, Perception, Semiosis, Neurology and Cognition**

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We present a conception that surmounts the Cartesian Cut -prevailing in science- based on a representation of the fusion of the physical ‘objective’ and the ‘subjective’ realms. We introduce a mathematical-physics and philosophical theory for the physical realm and its mapping to the cognitive and perceptual realms and a philosophical reflection on the bearings of this fusion in cosmology, cognitive sciences, human and natural systems and its relations with a time operator and the existence of time cycles in Nature’s and human systems. This conception stems from the self-referential construction of spacetime through torsion fields and its singularities; in particular the photon’s self-referential character, basic to the embodiment of cognition ; we shall elaborate this in detail in perception and neurology. We discuss the relations between this embodiment, bio-photons and wave genetics, and the relation with the enactive approach in cognitive sciences due to Varela. We further discuss the relation of the present conception with Penrose’s theory of consciousness related to non-computability -in the sense of the Goedel-Turing thesis- of quantum processes in the brain. We characterize quantum jumps in terms of the singularities of the torsion potential given by the differential of the complex logarithmic map (CLM) of the propagating wave satisfying the nilpotent eikonal equation of geometrical optics. We discuss the relations between the structure of these singularities, surmounting the alleged wave-particle duality in quantum physics, and the de Broglie-Vigier double solution pilot wave theory. We discuss the relations between torsion and semiosis, and the torsion singularities as primeval to perception and cognition. We introduce Matrix Logic from the torsion produced by the nonduality of the True and False Operators, and non-orientable surfaces: the Moebius band and the Klein-bottle. We identify the extended photon twistor representations derived from the above theory with cognitive states of the Null Operator of Matrix Logic. The CLM that generates the torsion potential is found to be embodied as the *analytical* topographic map representation (TMR) of diverse and integrated sensorial modes in the neurocortex, while the Klein-bottle appears to be the topological embodiment of self-reference in general, is the *topological* TMR. The singularities of the analytical map are the loci of the two-dimensional sections of the neurocortex in which *all* the field orientations are superposed. This is the singularity of the Klein-bottle. We discuss the appearance of vortical torsion structures in the striate neurocortex and its relation with Karman vortices as the ‘interiorization’ of the ‘outer’ torsion fields which describe the representations of the field of orientations of visual stimuli -as it appears in the hypercolumnar structure in the neurocortex. The Brownian motions diffusion processes produced by the torsion geometries through the CMP have correlated diffusion processes in the neurocortex that can be associated with developmental

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morphogenetic growth patterns. We discuss the relations between morphogenesis, neurology and development in particular of the human and mammal heart and self-reference. We relate torsion in Marix Logic, the resultant Logical Momentum Cognition Operator and its decomposition into the Spin Operator and the Time Operator. This relates quantum physics statements into logical statements. The Time Operator is a primeval distinction between cognitive states in this Matrix Logic as its action amounts to compute the difference between these states. As a geometric action, the Time Operator is a ninety degrees rotation in the 2-plane of all cognitive states. We relate the Time Operator with intention, control, will and the appearance of life, and *chronomes* (time waves and patterns in natural and human systems and phenomenae). We discuss cosmological and anthropological problems from this perspective. In particular, we shall confront the Myth of the Eternal Return as a self-referential process, relating it to the Time Operator, nilpotence, the enaction approach to cognition due to Varela, and the emergent consciousness proposal due to Penrose. We discuss the relations with the microviolations of the second law of thermodynamics in statistical thermodynamics, the torsion geometry of Brownian motions, the synthropic action of the Time Operator, the Klein-bottle non-linear topology of time, with subjectivity and angular momentum. We discuss the relation between the topology of chronomes and the so called arrow of time. We introduce the notion of heterarchies of Klein-bottles, or still, of quantized reentering limited domains, and discuss its relation with self-determination, syntropy, entropy, and the Time Operator. We present as fundamental examples of these heterarchies the Myth of Eternal Return and the Human Being. We shall relate the Time Operator with the perception of depth considered in the phenomenological philosophy of Merleau-Ponty as a protodimension, and the problem of hemilateral synchronization with universal torsion chronomes (Kozyrev) in terms of the transactional interpretation of quantum mechanics, and the CMP retinotopic representation. We discuss *quantized* time and motion perception in relation with ATP production.

1 INTRODUCTION

In this chapter we shall present a theory in which the ‘exterior’² world of physics, particularly the constitution of spacetime through the phenomenon of quantum jumps and extended photon structures, is fused with the ‘interior’ world of perception, cognition and subjectivity at large. This will surmount the Cartesian Cut conception which separates the world into an objective theatre on which consciousness plays a passive role as a bearer of information of the ‘exterior’ world³

Our conception is radically different; it is based on self-reference: The subject cognizes

²To surmount the Cartesian Cut we take initially the trend of expliciting its most notorious dualisms, postponing to a section below and to Rapoport [109] and the epistemology by Johansen [58] in which they are unnecessary from the very beginning; they both depart from the primeval notions of distinction and boundary.

³Or at best a physical world that arises from information (Wheeler’s it from *bit*), yet devoided of subjectivity and in particular of intentionality, and thus is still inside of the Cartesian Cut epistemology, as is the case of the observer, in the Copenhagen interpretation of quantum physics, a participant of the quantum measurement that ‘collapses’ the probability wave function.

the world and simultaneously establishes himself as a singularity (an irreducible form which is also a process) through cognition and perception stemming from distinctions, differences that make a difference in the sense of Bateson [8]. These are distinctions which on being perceived, generated, cognized, abstracted or interpreted, generate or unfold higher-order differences, which amount to the universe of all manifestations, either virtual, processual, operational, algorithmic, formal, conceptual or real; for further developments of a differential epistemology for science that departs from this notion due to Bateson, see Johansen [58]. Without distinctions in its manifold manifestations, the world would be homogeneous and imperceptible [109], and definitively, there would be no thing or process to cognize nor Cartesian subject to bear cognition, nor consciousness.

Returning to our discussion on the prevailing conception, we wrote as customary ‘information’ to indicate the Cartesian take on cognition, which erases the ideative aspect of knowledge of a lifeworld (lived world) of all traces of subjectivity. Instead, through the semiotic action of breaking ‘information’ by introducing the linking sign ‘-’, we indicate the presence of intention as a generative field, whose consequence is the emergence of a function derived from cognition; thus ‘in-formation’ is not about the data contained by the subject as a mere receptacle which is no more than the subject qua object of the Cartesian conception. ⁴ The latter introduction is an example of the fact that signs encode energy as discussed by Pattee [91] and Taborsky [138]. Most physicists working towards understanding cognition, apply physical models in their Cartesian mindset. There are several works that propose the origin of consciousness in the *brain’s* electromagnetic field; again, the brain, not the body [94, 148, 77], neglecting, among other aspects, insights claimed by Traditional Chinese Medicine [84], based in a vortex in-formation interconnected meridian system that regulates *all* physiological processes and embodies emotions organically . ⁵ McFadden’s scientific, poetical and humane reflections on will are a far cry from the usual academic parlance. Yet, it is framed in the Cartesian mindset, linking will and intention to non-computability of quantum processes on neurons, following Penrose [92], not to the action of a time operator as we shall elaborate in the present chapter. The bottomline for this idea of non-computability is the Hypothesis of the Continuum of Mathematics which heavily relies in the Antidiagonal Number Construction of Cantor’s Theorem; see page 22 in [47]. We quote N. Hellerstein’s comments on this number: “The number thus constructed leads to consider an infinity of infinities; so surely it must, within itself, contain an infinite amount of *information* about all these infinities. Otherwise, the silly thing is just bluffing us !”. This *infinite* amount of information of the continuum is what is at stake in Penrose’s

⁴The linking sign, -, appears as a conveyor of adjacency of two terms A and B which have equal qualities. The adjacency relation is written as $A - B$, and the linking sign together with deduction, induction and abduction, give rise to an associative *synduction* (Greek, syn -, with) logic. In fact, synduction is a creative logical operator. Synduction logic appears in the context of the *perplex* numbers system due to Chandler [15]. This system was constructed by abstracting logical diagrams from the electrical properties of the atomic numbers. The diagrammatic logic of discrete perplex numbers is analogous to the diagrammatic logic of category theory for continuous variables. In the present conception, the linkage sign is a conveyor of *intention*, or still, of will; the will of providing meaning through the suffix *in* linked to *formation*. As we shall see in the course of this work, it is not an abstract adjacency sign, but the semiotic embodiment of a Time Operator which stands for willful action.

⁵As the expression “venting ones’ spleen” means, consonantly with this system, to vent ones’ ire.

proposal of consciousness as an *emergent* physical-algorithmic phenomena. ⁶ Hellerstein dilligently follows the query to conclude in few lines of page 106 that the bluff is evident. Indeed, the Antidiagonal Number has a paradoxical bit at a certain place N, until this place is a mere *finite* dyadic and henceforth it has an infinite paradoxical section made of an identical paradoxical time wave [61] which appears in the 4-valued logic that follows from raising the calculus of distinctions of Spencer-Brown originated from a primeval distinction to encompass a self-referential equation which topologically is the Klein-bottle and from which we derived Matrix Logic [109]. Thus, “Cantor’s Theorem is hereby exposed as not only superfluous, but actually ridiculous. The continuum is countable; Cantor’s Paradox detects bit-flip at a dyadic. Therefore I propose a down-to-earth alternative to Cantor’s tottering cardinal tower; a single countable infinity with paradoxical logic”. This he calls, most appropriately, *Mathematics for Mortals;*” see page 108 in [47] ⁷ Hence generalizing to

⁶Of course, this infinite information idea runs counter with the conception of a quantum space and time, which is the core of torsion, yet remarkably this has escaped the attention of the researchers working in quantum formulations of consciousness: A contradiction in the very cornerstone of a conception which we understand to be ill-formulated; for a discussion in terms of phenomenological philosophy and the Klein-bottle see [117]. This contradiction is carried further in the attempts to blend quantum physics with Einstein’s GR which has null torsion and thus refers to a continuum, alike the one we are presently examining; so GR has to be *quantized*, a futile enterprise till today. In ‘Shadows of the Mind’, Penrose presents his theory based in the Goedel theorem and still Turing’s theory of computation. Penrose addresses the former from its alleged protoform, the paradoxical diagonal construction known as Cantor’s Theorem, reaching the proposition that a computation does *and* does not terminate; see page 75 [92]. Would we assume as Penrose *implicitly* did *-without* remarking that it was tantamount to a particular *choice* of a logic, namely the Aristotle-Boole logic- the principle of non-contradiction of this logic (i.e., for any proposition, p, p *and* not p is false) we would conclude with him that this is not possible and indeed the computation does not terminate. Thus, we *do* obtain a protoform of the Goedel Theorem; for the details see page 75 [92]. Now, in a self-referential construction as the one produced by Penrose albeit unacknowledged as such, and as further elaborated by Hellerstein and the present author, the corresponding logic is not Boolean but multivalued; it is the Klein-bottle logic [109]. In fact there is a number whose computation does terminate (it has a *finite* initial dyadic expansion) *and* also does *not* terminate; indeed, its finite dyadic expansion is followed by an *identical* infinite *paradoxical* expansion and thus is *redundant* with exception of its first term; yet, for the effect of better approximation we could still repeat it as far as we wished. (Thus, the principle of non-contradiction chosen by Penrose clearly misses the whole point of his arguments.) This paradoxical numbers is precisely the Antidiagonal Number produced by Cantor’s Theorem. Hellerstein proves further that in this multivalued logic framework, which essentially coincides with the logic constructed by the present author from the protologic that stems from a primeval distinction and the self-referential extension to the Klein-bottle, there are an *infinite* quantity of real numbers which have this same property and there only exist a *finite* quantity of reals that are actually infinite. For a discussion on self-reference, the Klein-bottle and the need of an ontology which is lacking in Goedel’s Theorem we refer to Johansen (2006) [58]. Our criticism to Penrose’s approach is essentially the same as Johansen’s: The anchoring to an ontology in which self-reference plays a generative role is lacking and thus the rejection of multivalued logic and self-reference is persued as if they would not be present in the developments, while these developments are essentially self-referential. Our point of departure to reach the understanding of the role of the Klein-bottle presented in the present conception was from this work due to Johansen.

⁷Following our adherence to Merleau-Ponty’s and S. Rosen’s philosophies in which cognition is embodied, this grounding of abstraction implies an embodiment of Mathematics in the human lifeworld. While Computer Science by definition works with discrete finite numbers which first appeared in Physics through *finite* big numbers, the usual corpus of Mathematics ignores this embodiment which in fact is the core of a Platonian lifeworld, not its denial as usually considered. In fact, it dissociates itself from Physics in doing so. An alternative approach is to work with appropriate (say, Mersenne primes) p-adic number fields as developed in the excellent work by Pitkanen [93].

a multivalued logic with paradox, “we get a much more simple theory; a sign of elegance”. Thus, the hypothesis of Penrose further raised by many researchers, that non-computability is a source for consciousness and free will ⁸ appears to be ill-conceived as we have just discussed, though it contains, in our understanding, some truth, in the fact that will and paradoxical time waves, which appear in the Antidiagonal Number Construction and in most (actually infinite) real numbers, are indeed related; see page 107 [47]. Hence, we feel obliged to note that we have just unveiled that time as a wave (in fact, a reentrance operator of a form on itself, the Klein-bottle) appears in Mathematics conceived as a system through the Continuum Hypothesis. Thus, a *time operator* is present in Mathematics as a system, inside the Continuum Hypothesis. Time, as an operator, reappears in the rotational recursive structure of the natural numbers (and in particular in the self-referential primes), in the remarkable work by Johansen [60]. In examining the natural numbers by following Johansen’s approach we shall find ourselves very far from the trivial sequential notion of linear time parameter, and in particular in its embodiment through the Peano construction of the natural numbers. In distinction to Hilbert’s proposal of constructing an axiomatisation for Physics and the working mathematician’s daily practices, axiomatics can only provide for the algorithmic rules. These rules embody the legislation for the formalities ⁹ of the game in which Mathematics is daily played on the background of an absent self-referential epistemology. Yet, the axiomatical approach will never provide for the meta-algorithmic lifeworld.

⁸The problem of the lack of constraints of an emergent physically-algorithmic will is still a very difficult if not impossible to resolve issue in a Cartesian mindset. This mindset ignores the joint constitution of the world and the subject. How can the subject in its finiteness determine the lack of constraints? If it is a matter of asserting its undetermination vis-à-vis a pledged independent reality of his/her subjectivity, which is believed to emerge from its undeterminateness, we find that in taking a conceptual route that ignores self-reference, then the wished freedom is left undetermined as well, and we are led to a paradoxical situation in which the subject and his/her lack of constraints become tied self-referentially, thus belying the initial conception. No theoretical exercise whatever its mathematical acuteness and complexity as in Penrose’s extraordinary corageous and thought provoking works can surmount this problem if it ignores self-reference in its outset. (In fact, Penrose takes a whole chapter III to justify his conclusions and use of Cantor’s Theorem, notably using self-referential constructions -and very complex argumentations which he uses to claim that these constructions are not grounded on self-reference! Thus, would we apply to Penrose’s theory his own choice of logic for constructing his theory, i.e. the principle of non-contradiction, it is clear that it would be rendered absurd.) Which is the absolute Undetermined Source from which the subject can claim its lack of constraints and how is it that the subject can assert the lack of constraints of the Source? This leads to an infinite antiregress of emergence in which the world and the subject are pushed away one from the other -perpetuating the alienation of the Cartesian mindset- in each attempt to assert their claimed independence. This is the dissolution of the grounding of an emergent consciousness approach which is very much ingrained in the works by physicists studying consciousness, and of course, is no matter of concern for the working Cartesian scientist. He/she is pleased enough in finding their efficacy in *controlling* the world, without even able to imagine nor grasp that control and will are undisolubly fused to a time operator which is grounded in the Klein-bottle meta-algorithmic fusion of object-with-subject. So their free will in practicing this control is related to a phantasy that transpires hubris which makes of the world a token for predation in which the scientist reinforces its own alienation. We shall reencounter this phantasy in a cosmological (!) setting in our study below of the Myth of Eternal Return as a self-referential system.

⁹Musès’ criticism of the non-standard analysis (A. Robinson, Edward Nelson nowadays) approach to infinitesimals and infinities as purely legislative in distinction of his treatment through hypernumbers [82] is an excellent showoff of how the management of this axiomatisation approach becomes a clear example of the infinite antiregress we mentioned before, leading to a vicious circle of unending complexity in ignorance of the fact that the existence of time operators is essential to Mathematics as we encountered above.

This is another example of the Cartesian Cut emergence antiregress we commented before.

Our proposal in first approximation only requires to ponder what the meaning of intention is, to establish the link between will, intention and the time operator.¹⁰ In the Cartesian mindset, language is considered to be informational, not in-formational. Interpretation is ontologically inexistent. Subjects qua objects, mere containers of data, i.e. information. Furthermore, hermeneutics, the inquisitive intention of the subject in search for interpretation for achieving understanding to sustain the logic of her/his own being together with the logic of the Universe with whom the subject is enacting themselves, is neglected in the Cartesian mindset. Thus, the lifeworld of Being is shattered to broken bits which henceforth will force cognitive scientists to frame consciousness in terms of some kind of emergence. These shattered bits produced by the act of irreflexion will never be recomposed because the Cartesian mindset has no operation-operand to provide for the glueing, nor the Klein-bottle which stands not for a mere reparation of the Cartesian Cut, but for the lifeworld of Being. So physicalism and emergence -which we have already discussed- or some form of subjectivism will be the core of its paradigmatical framework, or we may encounter a so called ‘third way’, such as the emergence of the ‘Great Doubt’ in which the Buddhist conception will want us to dissolve [115]. Thus, the subject is turned into an object though a thinking one as in Descartes, yet a subject for whom thinking is a process untraceable to its origins, as in the physicalist emergence proposal, or the subject disappears as in the ‘Great Doubt’, Varela’s et al proposal in cognitive sciences to surmount the Cartesian Cut. We have already presented a conception which surmounts the Cartesian Cut based on geometrical-logo-physical self-referential fields (torsion fields), phenomenological philosophy, second-order cybernetics (the cybernetics of controlling systems, i.e. which include the controller), multivalued logics, non-orientable surfaces such as the Moebius and Klein-bottle surfaces, and its relations to neurology [109].

In [111] we showed that extended photons are codified as cognitive states in a multivalued Matrix Logic (originally due to A. Stern [135]), which has as particular cases quantum, fuzzy and Boolean logics. This codification establishes a relation between quanta and thought. We introduced Matrix Logic in two ways. In the first way as the topology of paradox in a protologic (related to the work of Spencer-Brown [133]) which follows from a primeval cleavage-distinction¹¹ (which thereby acts as a semiotic -i.e. through a sign- codification of the torsion quantum field that embodies the fusion of object and subject and the joint

¹⁰In this chapter we shall propose in these terms, an explanation of an ancient myth, the Myth of Eternal Return, as an illustration of an all encompassing cosmology that includes natural, and particularly, human systems.

¹¹This notion of a primeval distinction-cleavage or still of boundary in the sense of the protologic due to Spencer-Brown, was further extended to a 3-valued logic by introducing the semiotic codification of the reentrance of a form on itself (essentially, the Klein-bottle which for the ancient traditions was the Eternal Return Ouroboros, or the Phrygian cap (of French Revolution fame) from the homonymous civilization of Anatolia, or still the Pelican Christus [98]), as a *third* logical value. Indeed, this value appears in addition to two other values; these are given by the distinguished (by the primeval distinction) state on a plane devoided of other signs (the plane of all potential states or forms, the Plenum), and the undistinguished state (the ‘empty state’, to Spencer-Brow) in which the plane is taken without the semiotic codification by the cleavage-distinction (thus, an untagged state); in the Boolean interpretation we interpret them as the usual false and true logical values [146]. It was still extended to a 4-valued logic by considering the reentrance of a form on itself with a delay [61] from which we derived Matrix Logic [109].

genesis of both) in a two-dimensional plane of all potentialities, giving rise to the appearance of a 4-state valued logic in which in addition to the Boolean states we have two primeval time waves enacting self-reference as new paradoxical states. In this first approach, the topology of torsion fields is given by the Klein-bottle (of Eternal Return fame). In the second way, we introduced Matrix Logic as the logic generated by two times two matrices representing all possible logical operators (not mere scalar connectives), from the torsion that surges from the commutator of the True and False *Operators* that extend the Boolean values; these operators are non-dual, in distinction to the Aristotelian-Boolean true and false logical values. This torsion can be also seen appearing in the guise of the non-orientability of the Moebius and Klein-bottle surfaces, which thus provides Matrix Logic with *superposition*¹² topo-logical cognitive states for its foundations. In particular, quantum field operators take a logical representation as nilpotent operators (i.e. their self-multiplication is null, i.e. equal to 0, which is not to be confused with *nil*, the value for nothingness.). We established a two-way transformation between the eigenstates of Null Operator of Matrix Logic and the twistors representation of the extended photon arising from the quaternionic wave propagation and eikonal equations and viceversa [111]. Thus, the primeval distinction gives rise to the most fundamental joint constitution of object with subject: the transformation of the quantum photon states into the eigenstates of the Null Operator, and vice versa. We showed that a Logical Momentum Operator, also called the Cognition Operator (since it can be alternatively represented as a variation of cognitive value, basically its derivative), appears from the non-duality of True and False, or still, from the above mentioned non-orientability. We found that the Cognition Operator decomposes as a Spin Operator plus a Time Operator. This allows to express quantum physics statements into logical statements and vice versa. Thus, the Cognition Operator introduces a fundamental Time Operator which represents the most primeval distinction between cognitive states in this Matrix Logic, since its action on two cognitive states amounts to compute the difference between them. This Time Operator admits, from its matrix representation, an interpretation as a primeval ninety degrees rotation in the 2-plane of all cognitive states. In this article we shall relate Time to intention-will-purpose, to the origins of life, to the Myth of the Eternal Return as a logo-physical process, and time-waves in natural and in particular human systems (chronomes), especially to the problem of the constitution of stereoscopic vision and the perception of depth. In the phenomenological philosophy of Merleau-Ponty depth is a protodimension, and its appearance in the perception of the Necker cube shows that depth is related to time and the appearance of multivalued logic (since there exist two alternative interpretations-perceptions of the Necker cube, both being true), surpassing thus Aristotelian dualism. Thus, the Necker Cube is rendered as a surface of paradox, alike the Moebius band and the Klein-bottle.

Yet, in these previous works, while the role of the Klein-bottle in the neurocortex structure was pointed out, the relation between the theory of the physical world -in terms of torsion- and the world of subjectivity in terms of perception, was only summarily introduced to argue the gestaltic identity between the physical theatre, logical and cognitive

¹²A more fundamental topo-logical rendering of superposition states than the usual approach in quantum physics.

realms and visual perception embodied in the Klein-bottle. In the present chapter we shall extend our works [109, 111], establishing a relation between the self-referential geometry of spacetime constituted by photon fields (as basic example of torsion fields, essentially vortical fields), and the somatotopic, visual and integrated sensorial modes representation in the neurocortex. We shall further relate this to the complex logarithm map which will appear in the characterization of quantum jumps in terms of the singularities of a gradient logarithm of photon waves satisfying the eikonal equations for light rays, and will reappear as the informational topographic map in the neurocortex. We shall see that this mapping from the ‘outer’ (skin surface, limbs, retina, clochea, integrated body sensorium) to the ‘inner’ world through a topographic representation on the neurocortex, is embodied in two superposed integrated maps. One of these topographical maps of the sensorium on the neurocortex is analytical (we shall qualify this soon), the other one is topological; these two maps are closely related between themselves and to the establishment of selfhood, as shall follow from our discussions in the final section of this chapter. The topological map is associated to the non-orientability of the Moebius and Klein-bottle self-referential surfaces. The analytical map is provided by the complex logarithmic function which, as in the physical realm, has singularities (in the physical case the loci for quantum jumps), which in the neurological case have for correlates vortical structures in the neurocortex (more specifically, the so called hypercolumnar structure); in the latter case, these singularities stand for the points in the neurocortex in which the stimuli orientation mapping given by the logarithmic map becomes multivalued. Yet, this map is the analytical representation of the $3D$ outer body (and its surface) to a unique $2D$ plane in the neurocortex in which each point codifies a whole hypercolumn vortex which is not anatomically distinguishable [54]. In the present conception, it will transpire that in distinction to other approaches to consciousness, which claim additional noetic higher-dimensions for consciousness following the Einstein tradition of treating complexity through the introduction of additional dimensions [3, 21], the dimension 2 common to the Klein-bottle, the space of all cognitive states in Matrix Logic, the undistinguished plane from which Matrix Logic arises from the primeval distinction, the dimension of the phase space associated to will-self-reference-control, the dimension for holography in general [124] and in particular in the neurocortex [73, 137, 141], and the complex plane (or still, the Riemann sphere) for the representation of the complex logarithm, will be singled out.

To resume: The multivalued singular structures of spacetime vortical torsion structures which are the locus for quantum jumps and as such are the most primitive distinctions that make possible the constitution of a spacetime, have in the perceptual mapping of the sensorium, given by the complex logarithm mapping, a representation in the neurocortex. This representation is provided by the same complex logarithm in which the quantum jumps are turned into multivalued points of stimuli orientation representation of the neurocortex (or still, their hypercolumn vortical structures). Hence, this conception establishes the embodiment of the Universe jointly with subjectivity, the subject, perception, cognition and thought.

Our presentation of this conception and its unfolding in the present article will be completely different to the one presented in [109]. In that work, the unfolding of the conception started with the philosophical aspects, mainly departing from second-order cybernetics to

further link with phenomenological philosophy and its relation with (paradoxical) logic, depth perception (departing from the work of M. Merleau-Ponty [79]), time and subjectivity (departing from the work of Heidegger, and more relevant to the present conception, to the works of Hegel [46] and G. Gunther [39]), to finally connect the present conception with the lifeworld embodied in the Klein-bottle, following the conception of *radical recursion* in the work by S. Rosen [116]. This previous presentation naturally led through the protodimension given by depth as elaborated by Merleau-Ponty and Rosen to visual perception and the Klein-bottle [109]. From those preliminaries we introduced the notion of primitive distinction and its identification as the semiotic codification of a torsion field that generates space, and furthermore time by considering the paradoxical equation in the calculus of distinctions that arises from this primeval distinction. We further introduced the time oscillations that arise from the solutions of this paradoxical equation (the Klein-bottle) and the multivalued logic that arises from them, the *Klein-bottle logic*. We then introduced Matrix Logic due to Stern [135] to show that it is associated with the torsion in cognitive space introduced by the non-duality of the True and False Operators. We further studied the transformation of cognitive operations into quantum spin operations and viceversa.

The present chapter will take for point of departure, instead of second-order cybernetics, phenomenological philosophy and visual perception as we did in [109], rather with the most basic embodiment of the fusion of object and subject that embodies (quantum) action and perception as the result of an integrative process of the ‘exterior objective’ and ‘subjective’ worlds: the photon. Indeed, the photon is not seen but it is the seeing [156], so at the fundamental level of constitution of reality, action and perception are unseparable which thus appears to be the foundation of coorigination of the world and the subject.¹³ So quantum physics will be our departure point though as we have just disgressed, physical reality cannot be separated from perception [70, 44] nor from thinking!

Our conception is somewhat related to the concept of *enaction* proposed by Varela and associates [115] for surmounting the Cartesian Cut by conceiving embodied cognition, following the phenomenological philosophy due to M. Merleau-Ponty [79]. We coincide with these authors that ‘...the self becomes an objectified subject and a subjectified object’; page 242 [115]). Yet, how this transformation is produced is left unexplained; in particular the relation of enaction and the Klein-bottle is altogether ignored, as well as the essential role of the latter in perception. Furthermore, it is quite remarkable that in the conception of Varela¹⁴ no relation between the photon as the embodiment of the self-referential fusion of action and perception is proposed. We shall present this relation in this chapter, in fact it will be our starting approach as we shall explain soon. By failing to notice this primeval *joint* constitution of the Self with the quantum photon (we shall find this later in the

¹³Thus in the photon we find a fusion of Kant’s noumenal (the ‘objective external’ world) and phenomenal (the ‘internal’ perceptual) realms. Due to the relation between the eigenstates of the Null Logical Operator (with all matrix elements being 0) of Matrix Logic with the twistor representations of the extended photon, we have claimed that the photon is seeing-thinking. This is because of the above mentioned relation with the cognitive states of the Null Operator of Matrix Logic, which we named the *Mind Apeiron* since it embodies all the potential cognitive logical states [111].

¹⁴Which is superposed with an apologetics of a particular religion -Buddhism- as a conceptual basis for cognition and sciences at large, and still the path to the experience of wholeness which should lead us to the creation of a better world, as well as a philosophy of pragmatics which thus includes ethics.

Myth of Eternal Return), they take the philosophy of Buddhism to elaborate an extreme form of nihilism, despite of their pledges on the contrary sense. Varela and associates base their proposal of enaction for surmounting the Cartesian Cut on the ‘Great Doubt’, which is the dissolution of the Self and thus of the claimed fusion of object-with-subject. Hence, in their disgressions self-hood is found to be inexistent but a mysterious illusion that, in contradiction with their profession of fusion of object-with-subject in their embodied cognition enaction approach, is left unembodied, with no explanation for the origin of its reification being delivered. In this regard, it is remarkable still that in this proposal not even the body’s cells embody cognition, due to the fact that they are short lived and thus -in their own words- are similar to the wooden planks of a ship that on decomposition due to the inclemence of the environment and hard use, are condemned to ephimery and to be replaced until the ship itself is no longer. Thus, with this conceptual background they are lead to question if the continuously replaced ship in its components is the same one than the previous one to substantiate the permanence of Self. We would like to comment on the Cartesian mindset still implicit to the proposal of Varela and associates for an embodiment of cognition which has no body for Being (we shall later find this proposal in the Myth of Eternal Return as an ontological mistake). In their take of enaction, the Self if biologically grounded should be *inert*: the cells are mere mechanical pieces, and thus time and light, as the primal organization fields of fusion of subjectivity and objectivity, are disconnected to Self; thus, implicit to this conception, is the resignation to physicalism. This neglect of the role of light in the constitution of the body, not to mention embodiment, runs counter with the knowledge on bio-photonics by Popp et al [95] at the time of the writing of Varela and associates, that originated in the discoveries of ultraweak photon emission from living systems by Alexander Gurwitsch circa 1923 in the USSR [40], and especially the fact that DNA emits electromagnetic waves [34]. Furthermore, as unveiled by Gariaev and associates, DNA has in the 98 percent strands unparticipating in the bio-chemical functions (the so-called ‘junk DNA’) ¹⁵, the structure of a language. These findings were confirmed in the work of Mantegna et al [72]. Thus, cells have down to DNA a cognition-like structure which is based on the photon’s fusion of object-with-subject and in a certain linguistic structuring. Thus, interpretation and meaning, as well as intention ¹⁶ are biologically grounded in the bio-photonic structure-process of DNA. Furthermore, communication between distant cells is produced by electromagnetic signals [95, 26]. For further crucial studies on the DNA electromagnetic structure-process and its holographic behaviour we refer to Marcer and Schempp [74, 124]; a remarkable non-Newtonian systemic rendering of bio-physics can be found in Simeonov [128]. Thus we are lead to suggest that the physical-biological basis for the embodiment of Self is already present in the bio-photon. ¹⁷ We shall find this

¹⁵An expression of the hubris implicit to the interpretation of the standard dogma in genetics, yet not uncommon to the practices of scientists of all professions.

¹⁶In the conception presented in this chapter, language is not a mere conveyor of information, but an informed and in-forming field by its self-referential essential character. It semiotically embodies in-formation as well as produces it, embodying cognition and the will to in-form. Language has an indicative -and in some instances imperative- character, and thus embodies purpose-will-intention. We shall later see the connection of this with time operators and life.

¹⁷We shall later relate the appearance of life to the willful action of timing, a time operator, that produce higher complexity structures-processes, countering thus the second law of thermodynamics, and in fact

again when discussing the embodiment of perception through neurocortex topographical maps. Returning to the philosophical aspects of Varela's proposal, the achievement of the experience of the 'Great Doubt' is claimed to be the core of Buddhism by its authorities and practitioners, and in particular of these authors' approach to cognitive sciences, leaving their 'enaction' absolutely ungrounded. Remarkably, the experience of 'Great Doubt' is claimed to be achieved through the so-called 'illumination', which makes the interpretation and theoretization of their conception, which led to the 'Great Doubt', a contradiction. In our understanding, the 'Great Doubt' is the specular nihilist image of the positivism of the dualistic approach, in which by taking the principle of no contradiction as its conceptual basis, all the universe of discourse is placed on the positive affirmative truth value which encompasses *all* the universe of discourse, as explained by Gunther[39]. Thus, in the dualistic approach, subjectivity and its relation with time and multivalued logics (in particular the self-complex which Varela and generally Buddhism want to leave groundless) is rendered without an ontological locus [39, 109]. This follows a quite puzzling -to this author- tradition of Tibetan Buddhism, in which though light and its experience is discussed quite extensively [38], yet its fundamental self-referential character is not mentioned at all (see especially pages 52 and 83 of Guenther's philosophical treatise), to the of our best knowledge, although we admit to be far from being scholarly versed in the subject. Anyway, it is remarkable that the great scientist Varela, which was very close to the maximum exponents of Tibetan Buddhism, would not be aware of this fundamental character of light to omit mentioning it all in his work [115].¹⁸

Yet, in the present setting related to the fundamental inhomogeneities given by photons as the fundamental case of torsion, and wavefront propagation of singularities which give a non-trivial spacetime which is not based on the notion of metrics but in the primeval notion of distinction and difference producing differences (as argued by Bateson [8]), which also is the starting point for the differential epistemology due to Johansen [58]. The unfolding of the present chapter following this Introduction, will be on mathematical terms, rather

production syntropic processes of self-determination and self-differentiation of ever higher-degrees orders. We shall return to discuss this issue on introducing the notion of chronotopology and discuss its non-linear being. So, returning to the linking operator in syduction logic,-, which we anticipated to be related to the willful action of a time operator, the bio-photon expresses through light emissions of cells and more complex biological structures, higher order self-regulations through light.

¹⁸We shall later reencounter this attempt to omit the self-referential being of light, in the Myth of Eternal Return. All religions, in spite of their cognitive richness, have a fundamental problem: the difficulty of grounding them socially, where the social character already finds its grounds in languages and the intentions embodied by them, and of course, in social organizations as well. Thus the former invariance of the lack of grounding of belief systems and religions, might perhaps be the hiatus between cognitive states, which are not the True state, and volition; both cognition and volition are actions, having both a perceptual and ideo-logical framework. This hiatus is related to the impossibility of dissolving self-reference into the undistinguished -under a primeval cleavage- state, the void to the Buddhist tradition, which is the plenum of all potential states, the Unmanifest, Apeiron. We shall return to this issue further below in relation with chronomes, universal times waves in natural and human systems. As with religions, any conception needs to be integrally grounded, and the economic and social realms have to be embodied into non-alienating self-organizations. For the studies of second-order economics and the *surmounting* critique of Marx's theory of capital, proposing an alternative grounding for the science of economics and its social implementation, we refer to the work by Johansen [60]; for the electromagnetic potential based geometry of written and spoken language we refer to the work of Doucet [23] and, most emphatically, to Musès [83].

than philosophical and perceptual, to which we shall arrive as a byproduct of the present departure with the photon, as the primeval gestalt of a participative universe, in which subjectivity is primeval. Then, the unfolding of the present conception in terms of light, will be the natural means for establishing the connection between the physical notion of quantum jumps as a primeval quantum distinction associated to torsion, with visual and somatosensory perception which stem from these primeval differences.

Thus this chapter will be separated into three main parts: Firstly we shall present the theory for the constitution of spacetime in terms of photon torsion fields and the relation with quantum jumps, comprising Sections II to IV, which we shall later discuss. Secondly we shall present Matrix Logic in the framework of the torsion in cognitive space that arises from the non-duality of the True and False Operators, and the codification of extended photons as eigenstates of the Null Operator of Matrix Logic.¹⁹ Let us recall that Matrix Logic was in the already described two possible approaches for its presentation, both related to the Klein-bottle, as we already discussed above. In one of these two possible approaches, we departed from the torsion that surges from the commutator of the True and False *Operators* that extend the Boolean values and are non-dual (in distinction with Aristotelian-Boolean logic), that stem from the non-orientability of the Moebius and Klein-bottle surfaces, which thus has superposition topological cognitive states for its foundations. This leads to the definition of a Logical Momentum Operator which is this commutator of the True and False Operators, which expresses their non-duality. In turn, this Logical Momentum Operator decomposes into the sum of the Spin Operator and the Time Operator. The former Spin operator allows to express quantum physics statements into logical statements and vice versa. The Time Operator represents a distinction between cognitive states in this Matrix Logic as its action on two cognitive states amounts to compute the difference between them. We find again, the differences that produce differences.²⁰ The Time Operator through its matrix representation has a dynamical interpretation in terms of rotations, since it turns to be a primeval ninety degrees rotation in the 2-plane of all cognitive states. Thus, also angular momentum is related to it. This will have a crucial role in the third series of topics that make this article. We shall start by relating this subjective Time Operator to intention-control, to the Myth of Eternal Return as a self-referential process, and to the existence of chronomes, time waves and patterns in natural and human systems and phenomenae. We shall relate the Time Operator with the perception of depth considered in the phenomenological philosophy of Merleau-Ponty as the primeval dimension, and further relate it with a possible solution to the problem of hemilateral eyes synchronization through universal chronomes (time structures and processes) such as the Kozyrev resonance entanglement due to torsion fields [66, 107, 134]. We shall discuss the relation of time and space perception in terms of ATP's metabolism to relate them to the energetics of neurons. We shall extend this to the quantization of time and the appearance of the perception of motion. We shall characterize cognition as the projection on the plane of all cognitive states in Matrix Logic of a vortex

¹⁹The philosophically inclined reader unincumbered with mathematical intricacies might skip these sections and proceed to the more conceptual sections that follow.

²⁰The introduction of a time operator in Quantum Mechanics was a very controversial issue following Pauli's initial attempts. It reappeared in the work of the Brussels school leadered by Prigogine [5], and more recently in the work by Schommers through the time-energy uncertainty relation [125].

torsion structure. We shall discuss the relation between the complex logarithmic map that gives rise to torsion potential vortex and its singularities as the fundamental quanta, as the *analytical* map that allows to represent the visual (in the foveal area), somatotopic, motor, auditory and integrated perceptions in a plane in the neurocortex. The singularities of this map will appear to be the loci of the two-dimensional sections of the neurocortex in which *all* the field orientations are superposed, yielding another example of the *plenumpotence* of 0, which already appeared in the light eigenstates of the Null Logical Operator, $\mathbf{0}$, the Mind Apeiron. This is the singularity of the Klein-bottle, the hole for which the whole unfolds to return to the singularity. We shall see that the Klein-bottle yields the *topological* map representation of the sensorium, a topographic map superposed to the analytical complex logarithm to which is related, as we shall discuss below. Further, we shall discuss the appearance of vortical torsion structures in the striate neurocortex and its relation with Karman vortices in viscous fluid dynamics as the ‘interiorization’ of the ‘outer’ torsion fields, which describe the representations of the field of orientations of visual stimuli -as it appears in the hypercolumnar structure in the striate neurocortex proposed by Hubel and Wiesel [54]. We shall relate these discontinuities with the natural appearance in the striate neurocortex of the Klein-bottle to the enaction of continuity of the representations of orientations. We shall further see that the complex logarithm map lead to diffusion processes in the neurocortex that can be associated with developmental morphogenetic growth patterns. Also we shall discuss the relation between the complex logarithmic map retinotopic representation and its function to integrate elementary eye distinctions to yield the binocular depth perception, thus returning to the integration of torsion vortex fields and stereoscopic vision, and the appearance of time discussed before, but from a neurological point of view that reaffirms its previous quantum explanation as entanglement through a torsion resonance effect. Finally, we shall discuss the relations between morphogenesis and anatomy-physiology and non-orientability in the Moebius and Klein-bottle surfaces, in particular of the human and mammal heart.

In all these developments the Klein-bottle plays an essential generative role, as the embodiment of self-reference and torsion [62], and of thought as much as crucial to perception and to neurology [137, 109]. Already in Rosen’s work we find that the idea that the origins of his thought on self-reference and paradox can be established in terms of a topological phenomenology that is traced back to Merleau-Ponty and Heidegger, which Rosen establishes in terms of the Klein-bottle. In contrast with Aristotelian dualism this (genus 0 Riemann) surface is both open *and* closed, continuous *and* discontinuous, inside and outside are fused since it has a single side, and fundamentally non-orientable. This surface can be seen -in a first approach- as having an uncontained part (the subject), a contained part (the object) and a containing part (space) [62, 116]. Yet, there is an in-formation flow from the uncontained to the contained part which we could inquire if it can be reduced to the Cartesian cut view of space as a container. The Moebius band which is another surface of paradox can be actually realized in three-dimensional space by taking a band, twisting it and glueing its opposite extremes and thus we have a surface of paradox which is non-orientable as the Klein-bottle, but it can still be thought as satisfying the classical figure of object on space independent of the subject. Yet, in distinction with the Moebius band, the Klein-bottle is

not a Cartesian object.²¹ Those objects are space occupying and space is thus a mere container. The Cartesian conception counters the experimental and theoretical studies in visual perception that go back to the master painters of Renaissance and more contemporarily in the work of Luneburg [70], Heelan (a former student of Heisenberg) [44] and Indow [56]. The construction of the Klein-bottle can be algorithmized as an initial identification of two sides of a rectangle (or more generally by a closed cleavage by a distinction of plane free of singularities which is thus topologically deformable to a rectangle) by glueing two opposite sides with the same orientation and identifying the other sides having *opposite* orientations which makes this figure impossible to be actually constructed in $3D$ in distinction with the Moebius band which can be seen as contained in space in a Cartesian sense, though it is non-orientable.²² This creates a topologically imperfect model in $3D$ since a hole has to be produced so that its construction already introduces singularities which then through the in-formation flow produces the whole structure, so that the whole structure is produced from a hole, and this returns to the singularity to complete the flow. Remarkably, this will manifest the torsion embodied in the Klein-bottle jointly with the singularity. This is further related to D. Bohm's holomovement [13, 99] and the integrality of the paradoxical structure to his implicate order, while the singularity is related to the explicate structure, in a first approximation. Indeed, in this wholeness surging from a singularity and back to it, what is at stake is the integral structure from which the implicate and explicate orders are instances and interchangeable through the flow [13, 99]. In this sense, our repetitious expression of fusion of object and subject should not be a conceived as a mere reparation of the Cartesian cut, but rather an indication of the integral structure of which both are instances in a process in which they are unseparable. Indeed, this is an holographic structure, and as such is constructed by the neurocortex as we already discussed before. The Cartesian mindset attitude to this would be to view the Klein-bottle as embedded in $4D$ where the hole is no longer necessary and in doing this, the concrete real figure is cast into an idealization which cannot be manifested by the subject (the Cartesian minded mathematician who thus keeps detached from this abstract ideal in-formation now ideally contained in the Cartesian view). Rosen's stance, to which we adhere, is instead to keep the hole -so that singularities are unavoidable as in quantum physics or already in the geometrical model of the photon we briefly presented above- as the starting point for questioning the Cartesian stance. In distinction with a Moebius band, a torus or any other object in $3D$, the loss of continuity of the Klein-bottle is necessary showing that $3D$ space is unable to contain the surface in the Cartesian stance for ordinary objects. So instead of abstracting by incorporating a fourth additional dimension (as is the proposal of Special and General Relativity), we keep the hole that produces the wholeness and instead of an additional dimension, we think of the depth dimension as the primeval dimension which becomes the source for the Cartesian dimension.²³ To resume, the Klein-bottle instead of being contained in space it

²¹In spite of the repeated claims of J. Lacan's acolytes in contradiction of their guru's writings.

²²Actually, the Klein-bottle can be weaved uni-dimensionally, a feat accomplished by Sheilah Morgan; our gratitude to Dr. Melanie Purcell for pointing this out to the author.

²³This is already apparent in the multivalued perception and logic elicited by the Necker Cube [85, 116]. As we stated already, in principle, instead of considering additional dimensions in disregard of the actual link between perception, self-reference, time and paradoxical logic, we follow the phenomenological approach

contains itself and the flow of in-formation (of action, to start with) that is associated to this topology, is the manifestation of this self-containment, this paradoxical situation which becomes real through the production of a singularity which produces the whole structure. By doing this, it supersedes the Cartesian cut and the Aristotelian dualism, by superseding the dichotomy of container and contained, and in semiotic terms, of interpreter and interpreted. We can additionally discern from the previous discussion, that the in-formation process of self-reference, i.e. of consciousness which through the laws of thought which are not longer those of Aristotelian dualism, transforms the ‘outside’ into the ‘inside’ world (this transformation is the fourth ontological locus that Gunther proposed for surmounting Aristotelian dualism: thought as a process [39]; for further discussion see [109]), and this transformation produces a relation between thought, logic and the physical world (and thus is essentially logo-physical), and topo-logical superposition states have a genetic role, and is further related with the actual process of transformation of the ‘outside’ and ‘inside’ realms. Since discontinuity can be seen as the source for wholeness, one can enquire on the role of quantization associated to the topology of the Klein-bottle and the in-formation process that is associated to this singularity and the self-referential topology, and furthermore the role of quantization with regards to the multivalued logic that is associated to this in-formation structure and its paradoxical character. We shall deal with these questions below. Already Rosen established a link between the Klein-bottle and quantization and still with Musès hypernumbers which incorporate not only non-trivial square roots of -1 but also of $+1$, the latter being associated to spinors, and more concretely, with the Pauli matrices of quantum mechanics [116] (2008)[82]), further applied to a cosmology placed in terms of the hypernumbers which are positive square roots of $+1$.

Returning to the issue of the organization of the presentation of the conception in this chapter, the first topic (comprising Sections II-IV) will deal with a geometrical theory for the characterization of quantum jumps in terms of spacetime singularities produced by a torsion field given by a closed yet not exact differential one-form given by the logarithmic differential of a wave function propagating on spacetime as a lightlike singularity described by a nilpotence condition: the eikonal equation of geometrical optics for light rays.²⁴ We shall show that quantum jumps are produced precisely when the complex logarithm of a wave function that acts as the source of this torsion singularity (a spacetime dislocation) becomes singular due to the nodes of this wave. These geometrical structures with trace-torsion field including the Hertz potential which has subluminal and superluminal solutions of the Maxwell

that threads them to a single lifeworld, and thus dimension 2 appears to be the appropriate dimension for consciousness, until ontological and perceptual considerations indicate otherwise. Rather than following the path of considering more complex geometries, we suggest to follow Musès, in considering more general number systems, hypernumbers, and the non-associative algebras related to them [82]. The suggestion of Musès is founded in the observation that the complexity of thinking intuitively follows the increasing complexity of algebraic operations, in which we relinquish commutativity, as we have done by passing from Boolean to Matrix Logic, and further to relinquish associativity, and to consider algebras rich with idempotents and nilpotent elements, which is already the case of Matrix Logic. This can be seen already in the introduction of the algebraic structures that sustain the geometrical methods in theoretical physics, such as Clifford algebras.

²⁴The reader unincumbent with mathematical complexities may give it a try skipping the first topic and proceed to the second one in Section V and VI.

equations, yield a theory of unification of spacetime geometries, non-relativistic and relativistic quantum mechanics [107], Brownian motions and fluid and magnetofluid-dynamics [103] and the application to obtain representations for the solutions of the Navier-Stokes equation [103, 105], non-equilibrium and equilibrium statistical thermodynamics [104], a torsion based theory of the electroweak interactions [113] and still the strong interactions as characterized by Hadronic Mechanics [108].²⁵ The relations between torsion and the Coriolis force have been studied in [104, 42], and those of torsion and spin in classical mechanics in [112].

Let us examine in the final paragraphs of this Introduction, the background for the first part of this article. In his theory of gravitation that stemmed from his criticism of General Relativity (GR), V. Fock showed that light rays described by the eikonal equations of geometrical optics, were at the basis for the possibility of introducing ‘objective’²⁶ spacetime coordinates and furthermore for the construction of a theory of gravitation based on characteristic hypersurfaces of the Einstein equations of GR [31]. These equations being hyperbolic partial differential equations have propagating wavefronts that arise as singularities of spacetime which are identical to the wavefronts singular solutions of Maxwell’s covariant equations of electromagnetism: they are all characterized by the solutions of the eikonal equation. These singular propagating fields stand for the inhomogenities of the otherwise uniform spacetime that the geometry of GR based on metrics lead to; this is also a common feature with a theory of spacetime conceived in terms of Cartan geometries with torsion (which is more fundamental, as the Bianchi equations show [32]) rather than the curvature produced by a metric. Without inhomogenities it is impossible to give sense to a geometrical locus as argued by Fock and separately, from a perspective based on torsion in [109]. In fact, Fock further proved that the Lorentz transformations of special relativity arise together with the Moebius (conformal) transformations as the unique solutions of the problem of establishing a relativity principle for observers described by inertial fields. As showed by Fock, it is *not* the Lorentz invariance of the Maxwell’s equation what makes Lorentz invariance so important in special relativity paving the way to a diffeomorphism invariant theory of gravitation which Einstein insisted in relating to special relativity, but rather the fact that the *singular* solutions of the Maxwell equations are invariant by the Lorentz transformations and still, by the full conformal group. We must recall, that already in 1910, Bateman discovered the invariance of Maxwell’s equations by this fifteen

²⁵To resume, torsion appears with a Janus face as the logo-physical-geometrical field which is incorporated into the codification by signs of the non-integrable equations of constraint that bridge the Cartesian (epistemic) Cut (and thereby allow for memory, measurement, and control in systems theory) [91], as the primitive distinction in the calculus of distinctions in the protologic of Spencer-Brown. Topologically embodied as the Klein-bottle, they generate Matrix Logic derived from a non-dualistic approach through paradox leading to multivalued logics and quantum superposition in cognition [109]. In this work, quantum field operators described by nilpotent hypernumbers, are associated to logical operators, establishing thus a connection between quantum field theory, nilpotents and multivalued logics. Nilpotence, which in our view rather should be called as plenumptence as much as the vacuum should be called the Plenum, has a crucial role in the nilpotent universal rewrite system [119].

²⁶Fock’s takes an approach based in dialectical materialism. In the phenomenological philosophical and dialectical approach by the present author for surmounting the epistemic cut, the photon is not an ‘objective’ particle, but the very signature of the fusion of object with subject, the latter being absent in the geometry of GR and unacknowledged by Fock due to his maintenance of the epistemic cut .

dimensional Lie group. The equivalence class of reference systems transformable by Lorentz transformations preserve the singular solutions which further have the essential property of being the invariance of the fusion of subject-with-object singularities propagating at a finite constant invariant speed equal to c [31]. The velocity of light waves is no longer constant for observers transformable under conformal transformations, but can be infinite [2]. For all observers related by a Lorentz transformation, if anyone would identify a propagating discontinuity with velocity c , all of them would likewise identify the phenomena. Thus, while the Maxwell equations are well defined with respect to *all* diffeomorphic observers, the singular solutions with speed c are well defined for all *Lorentz* group related observers. Most importantly, the singular sets $N(\phi) = \{x \in M : \phi(x) = 0\}$ were introduced by Fock in terms of scalar fields which are solutions ϕ of the eikonal equation

$$\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2 - \left(\frac{\partial\phi}{\partial t}\right)^2 = 0, \quad (1)$$

which in the more general case of a space-time manifold provided with an arbitrary Lorentzian metric, say g , can be written as $g(d\phi, d\phi) = 0$, from which in the case of g being the Minkowski metric lead to the light-cone differential equation $(dt)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 = 0$. Notice that eq. (1) is a nilpotence condition on the field $d\phi$ with respect to the Lorentzian metric g . But while the Maxwell equations are invariant by these two groups (Lorentz and Moebius-conformal) transformations, one could look for propagating waves that remain solutions of the propagation equation determined by the metric-Laplace-Beltrami operator, Δ_g , which we shall describe below- under arbitrary perturbations. Instead of considering solutions of the wave equation $\Delta_g\phi = 0$, which form a linear space, we want to investigate the class of solutions which are further invariant under action given by composition of arbitrary (with certain additional qualifications) perturbations f (real or complex valued) acting on the ϕ 's by composition, $f(\phi)$ that verify the same propagation equation: $\Delta_g f(\phi) = 0$. Notice that in these considerations we are concerned with singularities propagating on spacetime which is seemingly torsionless; we shall prove in the course of this work that this is not the case: The composite functions $f(\phi)$ and the ϕ themselves will be shown to generate torsion under certain conditions to be established below. We start introducing the geometrical-analytical setting with torsion.

2 RIEMANN-CARTAN-WEYL GEOMETRIES WITH TORSION FIELDS AND THEIR LAPLACIANS

In this section that follows [110] M denotes a smooth compact orientable n -dimensional manifold (without boundary) provided with a linear connection described by a covariant derivative operator $\tilde{\nabla}$ which we assume to be compatible with a given metric g on M , i.e. $\tilde{\nabla}g = 0$. Given a coordinate chart (x^α) ($\alpha = 1, \dots, n$) of M , a system of functions on M (the Christoffel symbols of $\tilde{\nabla}$) are defined by $\tilde{\nabla}_{\frac{\partial}{\partial x^\beta}} \frac{\partial}{\partial x^\gamma} = \Gamma(x)_{\beta\gamma}^\alpha \frac{\partial}{\partial x^\alpha}$. The Christoffel coefficients of $\tilde{\nabla}$ can be decomposed as $\Gamma_{\beta\gamma}^\alpha = \{ \beta\gamma \}^\alpha + \frac{1}{2} K_{\beta\gamma}^\alpha$ [7,9,10]. The first term in this decomposition stands for the metric Christoffel coefficients of the Levi-Civita connection ∇^g

associated to g , i.e. $\{\alpha_{\beta\gamma}\} = \frac{1}{2}(\frac{\partial}{\partial x^\beta} g_{\nu\gamma} + \frac{\partial}{\partial x^\gamma} g_{\beta\nu} - \frac{\partial}{\partial x^\nu} g_{\beta\gamma})g^{\alpha\nu}$, and $K_{\beta\gamma}^\alpha = T_{\beta\gamma}^\alpha + S_{\beta\gamma}^\alpha + S_{\gamma\beta}^\alpha$, is the cotorsion tensor, with $S_{\beta\gamma}^\alpha = g^{\alpha\nu} g_{\beta\kappa} T_{\nu\gamma}^\kappa$, and $T_{\beta\gamma}^\alpha = (\Gamma_{\beta\gamma}^\alpha - \Gamma_{\gamma\beta}^\alpha)$ the skew-symmetric torsion tensor. We are interested in (one-half) the Laplacian operator associated to $\tilde{\nabla}$, i.e. the operator acting on smooth functions, ϕ , defined on M by (see [103, 105])

$$H(\tilde{\nabla})\phi := 1/2\tilde{\nabla}^2\phi = 1/2g^{\alpha\beta}\tilde{\nabla}_\alpha\tilde{\nabla}_\beta\phi. \quad (2)$$

A straightforward computation shows that $H(\tilde{\nabla})$ only depends in the trace of the torsion tensor and g , so that we shall write them as $H(g, Q)$, with

$$H(g, Q)\phi = \frac{1}{2}\Delta_g\phi + \hat{Q}(\phi) \equiv \frac{1}{2}\Delta_g + Q \cdot \nabla\phi, \quad (3)$$

where $Q := Q_\beta dx^\beta = T_{\nu\beta}^\nu dx^\beta$ the trace-torsion one-form, and \hat{Q} is the vector field associated to Q via g : $\hat{Q}(\phi) = g(Q, d\phi) = Q \cdot \nabla\phi$, (the dot standing for the metric inner product) for any smooth function ϕ defined on M ; in local coordinates, $\hat{Q}(\phi) = g^{\alpha\beta} Q_\alpha \frac{\partial\phi}{\partial x^\beta}$. Finally, Δ_g is the Laplace-Beltrami operator of g : $\Delta_g\phi = \text{div}_g \nabla\phi$, $\phi \in C^\infty(M)$, with div_g and ∇ the Riemannian divergence and gradient operators ($\nabla\phi = g^{\alpha\beta}\partial_\alpha\phi\partial_\beta$), respectively; of course, on application on scalar fields, $\tilde{\nabla}, \nabla^g$ are identical: it is in taking the second derivative that the torsion term appears in the former case. Thus for any smooth function, we have $\Delta_g\phi = (1/|\det(g)|)^{\frac{1}{2}} g^{\alpha\beta} \frac{\partial}{\partial x^\beta} (|\det(g)|^{\frac{1}{2}} \frac{\partial\phi}{\partial x^\alpha})$. Thus $H(g, 0) = \frac{1}{2}\Delta_g$, is the Laplace-Beltrami operator, or still, $H(\nabla^g)$, the Laplacian of Levi-Civita connection ∇^g given by the first term in eq. (3). The connections $\tilde{\nabla}$ defined by a metric g and a purely trace-torsion Q are called RCW (after Riemann-Cartan-Weyl) connections with Cartan-Weyl trace-torsion one-form, hereafter denoted by Q [103, 105].

3 The Quantum Jumps Functional and Torsion

In the following we shall take g to be a Lorentzian metric on a smooth time-oriented space-time four-dimensional manifold M which we assume compact and boundaryless; we have the associated volume n -form given by $\text{vol}_g = |\det(g)|^{\frac{1}{2}} dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4$, where (x^1, x^2, x^3, x^4) is a local coordinate system. The solutions of the wave equation constitute a linear space. Furthermore, the germ of solutions of the wave equation in a neighborhood of a point form a linear space. Thus, the algebra generated by a single solution of the wave equation

$$\Delta_g\phi = 0, \quad (4)$$

consists of solutions of this equation if and only if ϕ satisfies in addition the eikonal equation of geometrical optics

$$(\nabla\phi)^2 := g(\nabla\phi, \nabla\phi) = 0. \quad (5)$$

Indeed, if f is of class C^2 (twice differentiable) and ϕ is real-valued, or still, if f is analytic and ϕ is complex valued, then the following identity is valid

$$\Delta_g(f(\phi)) = f'\Delta_g\phi + f''(\nabla\phi)^2. \quad (6)$$

The solutions of the system of equations

$$\Delta_g \phi = 0 \tag{7}$$

$$(\nabla \phi)^2 = 0, \tag{8}$$

are called monochromatic waves. They represent pure light waves; we discussed already above their relevance. A set of monochromatic waves having the structure of an algebra, will be called a monochromatic algebra. In Fock's approach, they are called *electromagnetic signals* [31]. Notice that the eikonal equation is a nilpotence condition for $d\phi$, the differential of ϕ , or equivalently its gradient, $\nabla\phi$, under the square multiplication defined by the metric. From the identity $e^{-i\phi} \Delta_g e^{i\phi} = i\Delta_g \phi - (\nabla\phi)^2$, we obtain, if $\Delta_g \phi = 0$, $(\nabla\phi)^2 = -e^{-i\phi} \Delta_g e^{i\phi}$. Let us consider the mapping $\phi \rightarrow e^{i\phi} = \psi$ which transforms the linear space of solutions of the wave equation into a multiplicative $U(1)$ -group, in which the kinetic energy integrand in the Lagrangian functional $(\nabla\phi)^2$ is transformed into $-\frac{\Delta_g \psi}{\psi}$, which has the familiar form of the quantum potential of Bohm, yet in a relativistic domain [13, 106]. If the ϕ are real valued, then the ψ are bounded and we can embed the above group in the Banach algebra under the supremum norm that it generates under pointwise operations and further completion [6]. To distinguish between them we call the original linear space the functional phase space \mathbf{S} and the Banach algebra defined above as the algebra of wave states \mathbf{A} , or simply the functional algebra of states. It is simple to see that the critical points of the functional

$$J(\psi) = \int \frac{\Delta_g \psi}{\psi} \text{vol}_g \tag{9}$$

are those ψ which satisfy

$$\Delta_g \ln \psi = 0, \tag{10}$$

i.e., those whose phase function satisfy the wave equation. Those intrinsic states will be called *elementary states*. The new representation has two advantages over the original one. It is richer in structure and in elements, as \mathbf{S} is mapped into a subset of the set of invertible elements Ω of \mathbf{A} , and so, by taking logarithm pointwise, on the elements of Ω , we obtain an enlargement of \mathbf{S} by possibly multivalued functions. The second advantage, that actually justifies the whole construction, is that the integrand of the Lagrangian $-\frac{\Delta_g \psi}{\psi}$, when integrated, exhibits jumps across the boundary $\partial\Omega$ of Ω . These jumps do correspond to kinetic energy changes, but in the interpretation of the integrand as a quantum potential, these changes represent a change due to the holographic information of the system present in the whole Universe, in D. Bohm's conception [13]. Let \mathbf{A} be a Banach algebra of continuous complex-valued functions defined on a four-dimensional Lorentzian manifold (M, g) , containing the constant functions, closed under complex-conjugation, with the algebraic operations defined pointwise and the supremum norm and containing a dense subset \mathbf{A}_2 of twice differentiable functions which are mapped by the Laplace-Beltrami operator Δ_g into \mathbf{A} . Assume further $f \in \mathbf{A}$ is invertible with inverse $f^{-1} \in \mathbf{A}$ if and only if $\inf_M |f(x)| > 0$.

The set of invertible elements is denoted by Ω . Furthermore, assume a positive linear functional, denoted by λ such that $\lambda : \mathbf{A}_2 \cap \Omega \rightarrow \mathbf{C}$ (the complex numbers) defined by

$$\lambda(\phi) = \int \frac{\Delta_g \phi}{\phi} \text{vol}_g \quad (11)$$

The critical elements of λ are those u such that

$$\text{div}\left(\frac{\text{grad}u}{u}\right) = 0, \quad \text{i.e.} \quad \frac{\Delta_g u}{u} - \left(\frac{\text{grad}u}{u}\right)^2 = 0. \quad (12)$$

If the linear functional is strictly positive, i.e. $\lambda(\phi) = 0$ if and only if $\phi \equiv 0$, these two identities are to hold in \mathbf{A} , otherwise in the sense of the inner product defined by λ on \mathbf{A} . By eq. (12) the set \mathbf{C} of critical points of λ is clearly a subgroup of Ω . The monochromatic functions of \mathbf{A} are as before, those $w \in \mathbf{A}_2$ satisfying the system of eqs. (7, 8) and their set is denoted by \mathbf{M} . From eq. (10) the composition function given by $f(w)$ belongs to \mathbf{M} again if f is an analytic function on a neighborhood of the set of values taken by w on M . Since by eq. (12) $\mathbf{M} \cap \Omega \subset \mathbf{C}$, we have that $uf(w) \in \mathbf{C}$ if $w \in \mathbf{M}$ and $f(w) \in \Omega$. The spectrum $\sigma(v)$ for any $v \in \mathbf{A}$, is defined by $\sigma(v) = \{z \in \mathbf{C} / |v - ze| \notin \Omega\}$ and therefore, by a previously assumed property, is the closure of the set of values $v(x)$ taken by v on M . It is obviously a compact non-void subset of \mathbf{C} . Ω has either one or else infinitely many maximal connected components, of which Ω_0 is the one containing the identity, e , defined by $e(x) \equiv 1$. Two elements f, h belong to the same component of Ω , if and only if $fh^{-1} \in \Omega_0$. Further, $f \in \Omega - \Omega_0$ if and only if its spectrum $\sigma(f)$ separates 0 and ∞ . The logarithm function, as a mapping from \mathbf{A} into \mathbf{A} is defined only on Ω_0 . With these preliminaries, we can now show that the quantum jumps arise as a generalized form of the standard argument principle.²⁷

Theorem. Let $u \in \mathbf{C}, w \in \mathbf{M} \cap \Omega$, i.e. it is an invertible monochromatic function. Denote by H_1, H_2, \dots , the maximal connected components of the complement of $\sigma(w)$. Then there exists fixed numbers $q_i, i = 1, \dots$, depending on u and w only, such that for any function $f(z)$ analytic in a neighborhood of $\sigma(w)$ and with no zeros in $\sigma(w)$, we have

$$\lambda(uf(w)) = \lambda(u) + \sum_i (N_i - P_i)q_i, \quad (13)$$

where N_i, P_i are the number of zeros and poles, respectively, of f in $H_i, i = 1, 2, \dots$. In particular choosing $\alpha_i \in H_i$, the q_i are given by

$$q_i = 2 \int g\left(\frac{\nabla u}{u}, \frac{\nabla w}{w - \alpha_i}\right) \text{vol}_g, \quad i = 1, 2, \dots \quad (14)$$

²⁷The following result [110] is a simpler geometrical version of a theorem proved by Nowosad in the more intricate setting of non-compact manifolds and functionals on generalized curves in L.C. Young's calculus of variations for curves with velocities having a probability distribution (Young measures) [86]. In our approach that surmounts the epistemic cut, we are interested in a particular Riemann surface, the Klein-bottle.

Proof.²⁸ Let $f = f(w) \in \mathbf{M}$ with $f(z)$ as in the hypothesis. A computation yields

$$\frac{\Delta_g(uf)}{uf} - \frac{\Delta_g u}{u} = 2g\left(\frac{\nabla u}{u}, \frac{\nabla f}{f}\right), \quad (15)$$

which we note that it is another way of writing

$$\frac{\Delta_g(uf)}{uf} = \frac{1}{u} H\left(g, \frac{df}{f}\right)(u), \quad (16)$$

where we have introduced in the r.h.s. of eq. (15) the laplacian defined in eq. (3) by a RCW connection defined by the metric g and the trace-torsion $Q = \frac{df}{f}$. Integrating eq. (15) yields,

$$\lambda(uf) - \lambda(u) = 2 \int g\left(\frac{\nabla u}{u}, \frac{\nabla f}{f}\right) \text{vol}_g. \quad (17)$$

In particular this shows that q_i in eq. (14) are well defined. From (17) one gets directly

$$\lambda(ufh) - \lambda(u) = [\lambda(uf) - \lambda(u)] + [\lambda(uh) - \lambda(u)] \quad (18)$$

$$\lambda(uf^{-1}) - \lambda(u) = -[\lambda(uf) - \lambda(u)], \quad (19)$$

where $h = h(w)$ as well as we recall $f = f(w)$, are the composition functions, from now onwards. Now if $f \in \Omega_0$, then $\ln f \in \mathbf{A}$ and $\nabla \ln f = \frac{\nabla f}{f}$, which substituted in eq. (17) gives, upon integration,

$$\lambda(uf) - \lambda(u) = 2 \int g\left(\frac{\nabla u}{u}, \nabla \ln f\right) \text{vol}_g = -2 \int f \text{div}_g\left(\frac{\nabla u}{u}\right) \text{vol}_g = 0, \quad (20)$$

by eq. (12). Hence

$$\lambda(uf) = \lambda(u), \quad \text{if } f \in \Omega_0. \quad (21)$$

If now f, h belong to the same component of Ω we can write $uh = (uf)(hf^{-1})$, and since $hf^{-1} \in \Omega_0$, the previous result yields

$$\lambda(uh) = \lambda(uf). \quad (22)$$

This shows that $\lambda(uf(w))$ is locally constant in Ω as f varies in the set of analytic functions. Let now $f(z) = z - \nu$ with $\nu \in H_i$. Then $z - \nu$ can be changed analytically into $z - \alpha_i$ without ν leaving H_i , which means that $w - \nu e$ and $w - \alpha_i e$ are the same connected component of Ω , with $e \equiv 1$. Therefore from eqs. (22, 17) and eq. (14) follows that

$$\lambda(u(w - \nu e)) - \lambda(u) = q_i, \quad (23)$$

²⁸An example. Take a compact submanifold of Minkowski space and plane waves with adequate boundary periodicity conditions. Take $u = e^{ik \cdot x}$, $w = e^{ik_0 \cdot x}$, $k_0^2 = 0$, $k_0 \cdot k \neq 0$ and the spectrum $\sigma(w) = S^1$, where S^1 is the unit circle; then $\lambda(u) = -k^2$ (minus the mass squared) and eq. (13) becomes $-\lambda(e^{ik \cdot x} f(e^{ik_0 \cdot x})) = k^2 + 2(k_0 \cdot k)(N - P)$, where N and P are the number of zeros and poles of f inside of the unit circle.

and by eq. (22)

$$\lambda(u(w - \nu e)^{-1}) - \lambda(u) = -q_i. \quad (24)$$

On the other hand, if ν belongs in the unbounded component of the complement of $\sigma(w)$, we may let $\nu \rightarrow \infty$ without crossing $\sigma(w)$ so that

$$\begin{aligned} \lambda(u(w - \nu e)) - \lambda(u) &= 2 \int g\left(\frac{\nabla u}{u}, \frac{\nabla w}{w - \nu e}\right) \text{vol}_g \\ &= \lim_{\nu \rightarrow \infty} 2 \int g\left(\frac{\nabla u}{u}, \frac{\nabla w}{w - \nu e}\right) \text{vol}_g = 0. \end{aligned} \quad (25)$$

Therefore, if $f(z) = c_0 \prod_{i=1}^N (z - a_i) \cdot \prod_{j=1}^p \frac{1}{z - b_j}$, $c_0 \neq 0$, $a_i, b_j \notin \sigma(w)$, then eq. (13) follows from eqs. (18, 24, 25). In the general case, if $f(z)$ is an holomorphic function in a neighbourhood of $\sigma(w)$, without zeros there, we can find a rational function $r(z)$ such that

$$|f(z) - r(z)| < \min_{\sigma(w)} |f(z)| \quad \text{in } \sigma(w), \quad (26)$$

by Runge's theorem in complex analysis. Then, $r(z)$ has no zeros in $\sigma(w)$ too, and $r(w)$ and $f(w)$ are in the same component of Ω , so that eq. (13) holds for $f(w)$ too. The proof is complete.

Observations. The quantization formula (13) tells us how the basic functional changes when we perturb the elementary state u into $uf(w)$ with f analytic near and on $\sigma(w)$. Changes occur only when zeros or poles of $f(z)$ reach and eventually cross the boundary of $\sigma(w)$, and these changes are integer multiples of fixed quanta q_i , each one attached to the hole H_i whose boundary is reached and crossed, while u, v remain fixed. Two more aspects are important. The first one being that the actual jump is measured modulo the product of the q_i by a classical difference (where by 'classical' we stress that we mean that it is the subtraction, in distinction to the quantum difference given by the commutator of operators) of poles and zeros; at the level of second quantization quantum jumps appear in terms of the difference of the creation and annihilation operators which defines a time operator in a logic in which the commutator of the true and false logical operators coincide with their classical difference, establishing thus a non-null torsion in cognitive space [109]. The second aspect is the actual form of the q_i which are given by integrating the internal product of the trace-torsion one-form $Q = \frac{du}{u}$ defined by the critical state u , with another almost logarithmic differential of the form $dw/(w - \alpha_i)$.

3.1 THE APPEARANCE OF TORSION

Let C_u denote the linear operator $h \rightarrow uh, h \in \mathbf{A}, u \in \Omega$. The very simple analysis above hinges on the fact that $C_u^{-1} \circ \Delta_g \circ C_u - C_{\frac{\Delta_g u}{u}}$ is a derivation on the germ $\mathbf{F}(w)$ of functions of w (see eq. (11) and also eq. (16) to see how it is related to the torsion geometry), which are analytic in a neighbourhood of $\sigma(w)$, and it could have been performed abstractly without further mention to the special case under consideration. The general abstract theory of variational calculus extending the functional λ for quantum jumps when specialized to

second order differential operators, say Δ_g or still $H(g, Q)$, shows that the condition $w \in \mathbf{M}$ is not only sufficient but also necessary in order to the quantum behaviour of λ occur [86].

The set of linear mappings $C_{f^{-1}}$ of \mathbf{A} defined by $h \rightarrow f^{-1}h, h \in \Omega, f$ defined on M , is a group which maps each connected component of Ω onto another one. In terms of functions defined on M it changes locally the scale of the functions, i.e. the ratio of any function at two distinct points is changed in a given proportion, and it therefore a gauge transformation of the first kind. Under this transformation we have that

$$\Delta_g \rightarrow C_{f^{-1}} \Delta_g C_f = \Delta_g + 2 \frac{\nabla f}{f} \cdot \nabla + \frac{\Delta_g f}{f} = 2H(g, \frac{df}{f}) + 2V_f, \quad (27)$$

where $H(g, \frac{df}{f})$ is the RCW laplacian operator of eq. (4) with trace-torsion 1-form $Q = \frac{df}{f}$ and $V_f = \frac{\Delta_g f}{f}$ is the relativistic quantum potential defined by f^2 [106]. Now noting that for vector fields $A = A^i \partial_i, B = B^i \partial_i$, with $A^i, B^i, i = 1, \dots, 4$ complex valued functions on M , with the hermitean pairing defined by the metric g on M , i.e. $\int g(\bar{A}, B) \text{vol}_g = \int g(B, \bar{A}) \text{vol}_g$ so that $A^\dagger = \bar{A} = \bar{A}^i \partial_i$. Therefore for the gauge transformation $d \rightarrow d + \frac{df}{f}$, since $\Delta_g = -d^\dagger d$ (see [9,10,15]), we further have the transformation

$$-d^\dagger d \rightarrow -(d + \frac{df}{f})^\dagger (d + \frac{df}{f}) = -(d^\dagger + \overline{(\frac{df}{f})}) (d + \frac{df}{f}). \quad (28)$$

where d^\dagger is the adjoint operator, the codifferential, of d with respect to this hermitean product so that $d^\dagger = -\text{div}_g$ on vectorfields [32]. If we assume that $\overline{(\frac{df}{f})} = -\frac{df}{f}$, so that $|f(x)| \equiv 1$ and thus f is a phase factor, $f(x) = e^{i\phi(x)}$, i.e. a section of the $U(1)$ -bundle over M then the r.h.s. of eq. (28) can be written as

$$\begin{aligned} -(d^\dagger - \frac{\nabla f}{f} \cdot) (d + \frac{df}{f}) &= \Delta_g + 2 \frac{\nabla f}{f} \cdot + (\frac{df}{f})^2 + \frac{\Delta_g f}{f} - (\frac{df}{f})^2 \\ &= (\Delta_g + 2 \frac{\nabla f}{f} \cdot) + \frac{\Delta_g f}{f} = 2H(g, \frac{df}{f}) + \frac{\Delta_g f}{f} = C_{f^{-1}} \circ \Delta_g \circ C_f. \end{aligned} \quad (29)$$

Consequently, if f is a phase factor on M , then under the gauge transformation of the first kind $h \rightarrow f^{-1}h$, the change of Δ_g into $C_{f^{-1}} \circ \Delta_g \circ C_f$ can be completely determined by the transformation $d \rightarrow d + \frac{df}{f}$ which is nothing else than the gauge-transformation of second type, from the topological (metric and connection independent) operator d to the covariant derivative operator $d + \frac{df}{f}$, of a RCW connection whose trace-torsion is $\frac{df}{f}$, equivalent to the gauge transformation $d \rightarrow d + A$ in electromagnetism [32].

In summary, when f is a phase factor, the gauge transformations of the first and second type are equivalent, and gives rise to the exact Cartan-Weyl trace-torsion 1-form. Whenever the metric g is Minkowski or positive-definite, these gauge transformation produce a transformation of a Brownian motion with zero drift to another Brownian motion with drift given by $\frac{\nabla f}{f}$. The node set of f , which coincides with the locus of quantum jumps, becomes an impenetrable barrier for this Brownian motion [7]. If we further impose on f the condition similar to the one placed for the electromagnetic potential 1-form, A , to satisfy the Lorenz

gauge $\delta A = 0$, i.e. $\delta(\frac{\text{grad}f}{f}) = 0$, we find that this is nothing else than the condition on f to be an elementary state i.e. a critical point of the functional $\lambda(f)$ given by (11). Therefore, when f is a phase factor, both the first and second kind of gauge transformations are equivalent and they give rise to a Cartan-Weyl one-form $Q = \frac{df}{f}$.

When $\frac{df}{f}$ cannot be written globally as $d\ln f$, f is said to be a non-integrable phase factor. When f belongs to the algebra \mathbf{A} , this is equivalent to saying that f does not have a logarithm in \mathbf{A} , which means that $f \in \Omega - \Omega_0$. In any case, the 2-form of intensity $F = d(\frac{df}{f})$ is always identically 0 because $\frac{df}{f}$ can be *locally* written as $d\text{Log}f$, where Log is a pointwise locally defined logarithm determination.²⁹

Consider now all the connected components Ω_α of Ω . Any such component can be transformed into Ω_0 by a gauge-transformation of the first kind: it suffices to take $f \in \Omega_\alpha$ and consider $h \rightarrow f^{-1}h$, which is indeed a diffeomorphism of Ω . This choice of the component, is a choice of gauge, and of course, there is no preferred gauge. That is, the topological operator d of one observer becomes the covariant derivative operator $d + \frac{df}{f}$ of a RCW connection for the other observer. We can interpret the difference of gauges as being equivalent to the presence of the trace-torsion 1-form $\frac{df}{f}$ in the second observer's referential. However as the electromagnetic 2-form $F \equiv 0$, this is an instance of the Aharonov-Bohm phenomena: non-null effects associated with identically zero electromagnetic fields. As we said before, this difference has a Brownian motion correlate in which null drift for the former is transformed into the drift $\frac{\nabla f}{f}$ which at the level of random dynamics is a non-trivial transformation. In fluid-dynamics as described by the Navier-Stokes equations for a velocity vector field u , there is a similar transformation from a drift independent purely noise Brownian process, in which the velocity term is subsumed in a Laplacian with no interaction non-linear term as a purely diffusive process, into the Navier-Stokes Brownian process with drift given by u ; see [103] (2002).

That there appear non-null effects is checked by our previous analysis of the functional $\lambda(u, f(w))$, where u is any elementary state and f , besides being a phase factor, is also monochromatic. In this case λ , which is locally constant depends on which $\Omega_\alpha f$ belongs to, that is to say, on the choice of the gauge.

Finally, according to the two ways of interpreting a linear operator (as a mapping on the vector space or as a change of referential frames) we have two possibilities. Indeed let $w \in \mathbf{M}$ and let $f_t(w)$, $t \in [0, 1]$ with $f_t(z)$ analytic in a neighbourhood of $\sigma(w)$, be a continuous curve on \mathbf{A} . For any $u \in \mathbf{C}$ we consider the curve of elementary states $u f_t(w)$; we described in eq. (13) the behaviour of $\lambda(u f_t(w))$ along this curve. In particular we considered $u f_t(w)$ as a perturbation, or excitation, of u evolving in time (here time may not be the time coordinate of a Lorentzian manifold but the universal evolution parameter introduced first in quantum field theory by Stueckelberg, and further elaborated by Horwitz and Piron [51].) We can also regard $u \rightarrow C_{f_t(w)}u$ as a continuous curve of gauge transformations of first kind acting on a fixed elementary state u , which, when f_t crosses $\partial\Omega$, determines a change of gauge. When that happens, it is obvious that f_t cannot be made a phase factor for all t , so that no electromagnetic interpretation can be given all along the evolution in t . However if,

²⁹The relation between Cartan torsion, singularities and dislocations in condensed matter physics is well known [63].

say, the initial states f_0 and f_1 are phase factors (i.e. $|f_i(x)| \equiv 1, i = 0, 1$), this change of gauge is equivalent to the appearance of a non-trivial trace-torsion one-form, which we can interpret as an electromagnetic potential, between the initial and final states. In any of these interpretations, a non-null effect is detected by a jump in λ as given by eq. (13); this quantum transition is interpreted in the first case as an excitation of the state u , and in the second state as a change of gauge of u , materialized by the appearance of the corresponding Cartan-Weyl one-form as an electromagnetic Aharonov-Bohm potential with zero intensity and non-null effects [1]. Thus, in this interpretation, quantum jumps are the signature of a non-trivial geometrical structure, the appearance of torsion.

3.2 SINGULAR SETS

Finally we examine the dimensions of singular sets $N(f)$ of monochromatic functions. Recall that a C^2 real or complex-valued function f defined on (M, g) is a monochromatic wave, $f \in \mathbf{M}$, if it satisfies the system given by eqs. (7, 8). In the real-valued case, all C^2 functions of f , and in the complex case, all analytic or anti-analytic functions of f belong to \mathbf{M} again, by eq. (4) (we changed here our notation there, pointing precisely to $f = f(w)$ for $w \in \mathbf{M}$, as above). If f is real, smooth and $df \neq 0$, then $N(f)$ is locally three-dimensional. If it is complex and $\text{Re}(f)$ and $\text{Im}(f)$ are functionally independent $N(f)$ is two-dimensional. Yet the Newtonian picture of a photon as an isolated point-like singularity moving with the speed of light in the vacuum, requires a one-dimensional singular set $N(f)$. Can we achieve this by going to hypercomplex, say quaternionic functions, or still Musès' hypernumbers which are rich in divisors of 0? The answer to the former question is negative; in the quaternionic framework, the photon is a propagating three-dimensional singularity with lower dimensional singularities, but still undivisely extended [111]; we shall present these issues in the following section.

3.3 PARTIAL CONCLUSIONS TO THESE SECTIONS

This initial part of the present work started by considering the fundamental role of spacetime singularities and particularly light rays in establishing the physical world jointly with the act of (self) perception of the subject: Indeed, the photon is not seen, but is about seeing [156]. We argued that this allows for a start the introduction of spacetime itself, in terms of the fundamental role that differences and more generally inhomogeneities play. In our presentation we argued in terms of what originally appeared to be two distinct conceptions for producing this joint constitution of reality and the subject. The first conception relied on the self-referential role of the photon as a singularity which embodies the fusion of reality, cognition and perception, while the second one relied on torsion, which is also linked to self-reference [109]. Light rays are described by the eikonal equations which appear as wavefront singularities in the Maxwell and Einstein's equations of electromagnetism and General Relativity, respectively. Quantum jumps play a fundamental role in this joint constitution of physics and consciousness, as they appear as fundamental differences from which spacetime is constructed as a physical reality, while they also have a fundamental role in visual perception. Without these quantum jumps, cognition as the difference produced

from a primeval difference [109] would not occur [8]; a differential epistemology for science, departing from Bateson's concept of information as difference that produces difference, was elaborated by Johansen [58]. While propagating waves play a primeval role, the notion of closure by composition by perturbations led us to consider the action of analytical or alternatively C^2 functions on them, which produced that the propagation equation was supplemented by the eikonal equations for light rays, and thus we were led to the class of monochromatic functions.³⁰ Thus we were able to give a formula that characterizes quantum jumps and further discovered that these compositions produce a torsion potential. These jumps occur whenever the perturbation ceases to be non-zero and thus become the locus for the singularities of the logarithmic map, establishing thus the generative role of 0. We shall reencounter in Sections VI and VII below, this generative role of 0, as a quantum-logical operator with all zero entries in Matrix Logic which is further associated with quantum fields. It will also appear in the visual complex logarithmic map representation in the neurocortex, in which the singularities appear as the loci where the orientation field provided by the map can take all possible values, thus becoming multivalued in those singularities. In this setting, the lightlike singularities are non-pointlike extended structures which we shall later represent by twistors; in turn, these twistors appear to be the eigenstates of the null quantum-logical operator, which can still be represented by cognitive states and vice versa [111]. In the course of this work we discovered that propagating lightlike singularities which are fundamental to the establishment of special relativity, electromagnetism and still General Relativity, produce a torsion potential, and thus the two approaches which the present work took as its unfolding rationale, became unified. Furthermore, this potential which becomes singular in its node sets, acts as the drift (i.e., the average velocity) of Brownian motions which cannot penetrate the infinite barrier posed by these singularities. Finally, we established that this torsion potential naturally produces an Aharonov-Bohm effect, which is further associated with the quantum jumps so that non-null effects, fundamental differences in the sense discussed above, are manifested despite that the electromagnetic fields produced by the torsion potential is null.

Remarkably, the complex logarithm map plays a fundamental role in a topographical geometrical model of visual perception [126] -and as we shall discuss in this chapter, for other sensory modes and their integration- in which the singularities of the topographical orientation visual mapping -an order parameter as in superconductivity-, is associated to the singularities of vortex dislocations [127], i.e. trace-torsion singularities which we shall discuss below). These singularities led to a topological solution of the topographical visual -as well as other sensorial modes- map in terms of the Klein-bottle [137] [141] which is the basis for multivalued logics and its relation with quantum mechanics and quantum field operators [109]. Furthermore, it led to Gabor wavelet holographical representations on the

³⁰It is crucial for the later connection of the developments presented in the previous sections, with perceptual codification in the neurocortex, to remark that what is accessible to the visual and still peripheric sensory systems, such as the skin, are precisely the family of all perturbations waves that we have discussed in the previous sections. This will be the starting point to connect the physical and the physiological realms. With respect to the auditory system, it is important also to remark that the complex logarithmic function which acts on monochromatic waves, is also the topographic representation of this sensory mode [126, 127, 139].

visual cortex [73], substantiating the holographic quantum paradigm, due to Pribram, in neurophysiology [96]. The complex logarithm has been carried to quantum holography in terms of the coadjoint orbits representations of the Heisenberg group in quantum mechanics, and has led to important technological developments, in the work due to Schempp [124].³¹

In considering the semiclassical theory of gravitation, quantum jumps produce discontinuities in the energy-momentum tensor leading to the existence of a cosmological time associated with a quantum-jumps time, in a *global* canonical decomposition of spacetime [76]. This approach due to Mashkevich departs from the incompleteness of the Cauchy problem for the Einstein equations of GR: they provide only six equations for the ten components of the metric. For curved spacetime, Mashkevich proved that the diffeomorphism invariance of the solutions of the Einstein equation is not valid; only in the case of Ricci flat spacetime this is assured. This underdetermination is resolved by the canonical complementary conditions, which in the semiclassical approach are provided by nonlocal quantum jumps whilst in Fock's theory they are provided by four equations as eq. (4), the so-called harmonic coordinates. Thus, according to Mashkevich, quantum jump nonlocality is essential for GR, they occur in nonempty spacetime where the underdetermination problem arises and actually solves this problem. Furthermore, quantum jumps lead to a universe with complete retrodiction in which only partial prediction is possible [76]. This establishes a remarkable relation, which requires further elucidation, with the theory of anticipatory systems introduced and developed by Dubois and others [24].

For closing remarks to this section, we note that quantum jumps were obtained here in terms of the quantum potential which stands for a holographic in-formation of the whole universe.

4 MONOCHROMATIC HYPERCOMPLEX FUNCTIONS

In this Section we shall discuss the problem of the impossibility of localizing the photon as a point-like structure and determine its geometrical-analytical characterization; detailed

³¹There is an essential point that has already appeared in our previous considerations, in the treatment of quantum jumps through the complex plane, or still, the Riemann sphere, in the Klein-bottle, and finally, in the holographic codification in the visual cortex, and through the Heisenberg group representations that sustain holography: The common, basic dimension to all of them, is 2. We shall retrieve this all along the present chapter, in which this fact will reappear in all instances: 1) In the cognitive plane for Matrix Logic that allows logical operators to be treated as Musès hypernumbers and further quantum field operators ; 2) in the characterization of cognition as the projection to this plane of a vortex structure; 3) in the codification of sensorial modes in the neurocortex; 4) in the synchronization of hemilateral vision leading to stereoscopic visual representation which requires a depth variable associated to time and paradox yet can be subsumed into a single plane; 5) in the developmental growth of the body and, particularly, the anatomy-physiology of the heart. So rather than partaking in the usual practice of following the meme already installed by Special Relativity, of incorporating additional dimensions to account for complexity and more recently the particular case of consciousness, what will emerge in relief from the development of the present chapter, is the ever present dimension 2 associated to self-reference and the imaginary numbers. This will not result from any hypothesis, but from the unfolding of the mathematical structures that will be applied to the several issues. This appears to be another manifestation of the possible ruling on Nature, of the famous Occam's Razor Principle. For a different approach through the incorporation of additional dimensions to characterize a noetic space, we refer to the important work by Amoroso [3]

proofs can be found in [111] following [86]. The starting basic issue is that for f defined on M as above, it can be proved that $N(f)$, the node set of f reduces to a single set. Let us introduce the quaternionic units $\vec{i}_1, \vec{i}_2, \vec{i}_3$ given by the multiplication rules

$$\begin{aligned}\vec{i}_1\vec{i}_2 &= \vec{i}_3, \vec{i}_2\vec{i}_3 = \vec{i}_1, \vec{i}_3\vec{i}_1 = \vec{i}_2 \\ \vec{i}_j\vec{i}_k &= -\vec{i}_k\vec{i}_j, k \neq j, \vec{i}_k^2 = -1, j, k = 1, 2, 3.\end{aligned}\quad (30)$$

Notice here that we could chose here the logical quaternions introduced before, and thus the structures we shall produce below, can be conceived as spacetime structures which are both ‘inner’ and ‘outer’ representations of the self-referential character of photons (though the neutrino is also considered below). We shall introduce the notation $(\phi, \psi) \in \mathbf{M}$ to mean that $\phi, \psi \in \mathbf{M}$ (i.e. they satisfy eqs. (7, 8)) *and* furthermore

$$g(\nabla\phi, \nabla\psi) = 0, \quad (31)$$

which is the requirement that any algebraic combination of ϕ, ψ belong in \mathbf{M} as well. It will also be assumed that ϕ and ψ are functionally independent, to rule out the trivial cases. We then have the following theorem.

Theorem 1. Any monochromatic quaternion valued function F defined on (M, g) is determined by a triple of real valued functions (ϕ, f, ρ) such that

$$(\phi, f + i\rho) \in \mathbf{M}, \quad \text{i.e. } g(\nabla\phi, \nabla f + i\nabla\rho) = 0, \quad (32)$$

and each of ϕ, f, ρ satisfy the system

$$\Delta_g \kappa = 0, \quad (\nabla \kappa)^2 = 0, \quad (33)$$

has the form

$$F = f + \rho[\vec{i}_1 G(\phi) + \vec{i}_2 H(\phi) + \vec{i}_3 P(\phi)] \quad (34)$$

where G, H, P are real valued functions satisfying

$$P^2 + H^2 + G^2 = 1. \quad (35)$$

Thus, F is a section of a $R \times R \times S^2$ -bundle over (M, g) , where S^2 denotes the two-dimensional sphere. ³²

³²We have constructed the quaternions in terms of logical operators in Matrix Logic [109]. So, we can represent this result as either an ‘objective’ space representation of the objective-subjective photon, or as a ‘subjective’ representation of it in terms of a quaternionic structure which stems from the laws of thought; [109]. Remarkably, from the quaternions in any of the two representation, the logical quaternions in Matrix Logic [109] or the usual quaternions built by Hamilton as unrelated to logic, we can obtain some cosmological solutions. Indeed, the natural metric in the Lie group of the invertible quaternions, can be parametrized as the closed Friedmann-Lemaitre-Robertson-Walker metrics [144] which constitute one of the most important classes of solutions of Einsteins equations and furthermore, as the Carmeli metric of Rotational Relativity. We recall that the latter was introduced to explain spiral galaxies rotation curves and ‘dark matter’ [17]. We stress that these derivations do *not* require solving the Einstein’s equations of GR but are intrinsic to the quaternions, or if wished, to Matrix Logic. This raises the question on what are we actually representing: Is it an ‘outer’ world, or a Klein-bottle cosmological fusion of the physical and the noetic realms, as this chapter claims to be the case? We shall return to this issue further below, on introducing the Bohr-like quantization of the Logical Momentum Operator in Matrix Logic.

4.1 MAXIMAL MONOCHROMATIC ALGEBRAS

A monochromatic algebra is called maximal monochromatic if it is not a proper subalgebra of a monochromatic algebra. In our context, the importance of maximal monochromatic algebras is obvious, in particular with respect to the question of singular sets. The main result in this respect is the following; the proof can be found in [111].

Theorem 2. The maximal C^2 algebras in (M, g) are precisely those generated by a single pair (see eq. (31))

$$(\phi, f + i\rho) \in \mathbf{M}, \quad (36)$$

with ϕ, f, ρ real, and are constituted by C^2 -functions of the form

$$\xi(f, \phi, \rho) + \eta(f, \phi, \rho)[\vec{i}_1 K(\phi) + \vec{i}_2 H(\phi) + \vec{i}_3 P(\phi)] \quad (37)$$

in the quaternionic case, and

$$\xi(f, \rho, \phi) + i\eta(f, \rho, \phi), \quad (38)$$

in the complex case, where for each fixed ϕ , $\xi + i\eta$ is an intrinsic analytic (or antianalytic) function of $f + i\rho$, the C^2 -dependence on ϕ is arbitrary and $K^2 + H^2 + P^2 = 1$, with K, H, P of class C^2 , but otherwise arbitrary. Thus, in the quaternionic case, it is given by a C^2 -section of a $R \times R \times S^2$ -bundle over M . In the complex case, non intrinsic functions are allowed.

4.2 GENERAL FORM OF SINGULAR SETS AND THEIR PHYSICAL INTERPRETATIONS

We are now in conditions for completing the objective of this section, namely, the characterization of the node set of complex and quaternionic monochromatic function. According to the above results the most general form for singular sets N of monochromatic complex or quaternionic functions is given by the conditions

$$\begin{aligned} \xi(f, \rho, \phi) &= 0, \\ \eta(f, \rho, \phi) &= 0, \quad (\phi, f + i\rho) \in \mathbf{M} \end{aligned} \quad (39)$$

Although N is locally at least two-dimensional we have now the possibility of locating a higher-order zero on a bicharacteristic line.

For instance, the singular set of $\phi.(f + i\rho)$ is the union of the 2-dimensional set defined by $f = \rho = 0$, and the 3-dimensional set $\phi = 0$, and since $(\nabla\phi)^2 = 0$, their intersection $f = \rho = \phi = 0$ is a bicharacteristic line carrying an isolated zero of higher order. The corresponding phase function has a higher order singularity located at a single point in three-space, moving with the speed of light along the singular line $f = \rho = 0$, accompanied by the wave-front singularity $\phi = 0$.

Observations This result is remarkable in many ways. Firstly, in the present analytic approach, it is apparent that the photon cannot exist per-se as a point-like singularity, since

in fact the most general maximal monochromatic algebra is three-dimensional when we go to the quaternionic case and then we can describe it as built in the larger singularity. Thus, we have in a four-dimensional Lorentzian manifold, three real dimensions to describe the eikonal wave discontinuities. We have associated them with spinors and twistors, which themselves are related to conjugate minimal surfaces ³³

4.2.1 PHOTON, NODAL LINES, MONOPOLES

Until now we have described the singular sets of quaternionic and complex solutions of the eikonal and wave propagation equations. We have discussed already the fact that the case of the photon is not that of an ‘objective’ particle but the actual fusion of object-with-subject. The extended character of the photon was understood in terms of this fusion and the extended character of the subject. It is the prerogative of the subject to establish from this a coordinate system, i.e. to use light-singularities as a way to establish a primeval coordinate system, and this is in fact what sentient beings do. ³⁴ A typical case is to establish a Cartesian coordinate system $(x, y, z, t) \in R^{1,3}$ (in Minkowski space) given by taking

$$f + i\rho = y + iz, \quad (40)$$

and

$$\phi = f(r) - t, \text{ with } r = (x^2 + y^2 + z^2)^{\frac{1}{2}}, \quad (41)$$

and f is a monotonic function of the radius r . In this case, for the function

$$(y + iz)(f(r) - t) \quad (42)$$

³³This is most remarkable since it points out to the existence of a Platonic world, with generic geometrical surfaces associated to the fusion of object-with-subject which the absorbed photon is. We recall that a minimal surface is one for which the mean curvature vanishes. Common to light rays, quantum jumps and minimal surfaces, is the fact that they all appear as solutions of variational problems and thus are established in terms of *extremal* solutions. On the opposite conceptual framework we find a geometrical approach to classical mechanics in which these extremal principles are absent yet the equations of motion even of null-mass particles are derivable [112]. The possible derivation of quantum jumps in terms of this more fundamental approach is an open problem.

³⁴This was one of the main points of Fock’s critique of Einstein’s General Relativity [31], that it is a theory of uniform space, and thus singularities are needed to establish a real spacetime. We have discussed already that the primeval distinction that encodes the torsion field is such a singularity. Thus, in analyzing the hyperbolic nature of the Einstein’s partial differential equations of GR and the Maxwell equations, Fock was led to propose as a starting point the eikonal and propagation wave equations, whose wavefronts correspond precisely to propagating singularities, that we have further associated with torsion. Though we cannot because of lack of space show that this is the case, i.e. that from the form of maximal monochromatic quaternionic functions we can indeed establish a coordinate system, in which the functions that produce this form act as a coordinate system! This can be established without recourse to an ad-hoc non-geometric energy-momentum tensor as Einstein’s inception of it in GR, but using the energy-momentum tensor of the electromagnetic field, giving thus a self-referential construction of the metric by solving the Einstein’s equations with light as a source for matter described by curvature derived from the metric in the Levi-Civita connection, according to Einstein. Thus, metrics are *not* primitive to the constitution of spacetime, as we have already argued. Self-referentially, it is the joint constitution of spacetime and the subject through torsion as a logo-physical sign and field.

the singular set consists of a spherical wave front in 3-space moving with the speed of light and cutting the singular x -axis $y = z = 0$ at a single point in the positive semi-axis $0 \leq x$, where therefore lies a higher-order singularity.

This higher-order singular point, piloting a lower order singular spherical wave, along a lower order singular line is now liable to represent the photon, conceived as a moving point singularity carrying energy, in agreement with the experimentally observed corpuscular behaviour of the photon at a metallic plate, (obtaining pictures of its trajectories in cloud chambers). On the other hand the weaker singularity carried by the spherical wave front $f(r) - t = 0$ is responsible for the diffraction patterns in the typical slit experiments, according to Huygens's law of propagation of singularities (eikonal equation), and so accounts for the experimentally observed wave nature of the photon. In this way the purely analytical characterization of the maximal monochromatic algebras leads us unequivocally, to the correct conclusions as regards to the physical nature of the photon and express its *purportedly* dual wave-corpuscular nature as a simple mathematical fact. It is essentially a *wave*, the particle being a *factor* of it, but *not* dual in any intrinsic sense. Remarkably, this stands in contrast with the de Broglie-Vigier double solution theory [153]. In that theory, the wavelike pattern is associated to a linear propagation (alike eq. (7)) while the particle was treated as a propagating singularity ascribed to a non-linear equation, which in the present theory is eq. (8). Thus the present theory fleshes out in a completely geometric setting the double solution theory, which appears in the torsion geometry of the linear and nonlinear Dirac-Hestenes equation [106], yet it *relinquishes* duality. Hence, the present conception is radically different to the Copenhagen interpretation of quantum mechanics.

The line $y = z = 0$ in 3-space carries a singularity too, but this is a standing one, independent on time, and therefore, its presence is detected through different effects. Actually this line is called a *nodal* line of the wave function $\phi \equiv y + iz$ ([12]) or a *dislocation* line of the planes of constant phase of ψ . Around this line occur vortices of the flux of the trace torsion one-form $d\ln\phi$ of the phase function (when the circulation of this flux around a nodal line is non-zero), described in detail by Hirschfelder [49]. Alternatively, Dirac found these nodal lines when considering singularities of wave functions, upon imposing the only requirement that the complex-valued functions ψ (in our example equal to $y + iz$) be single-valued and smooth, but not necessarily with single-valued argument, and then quantized them in terms of the winding number of the vector-field $(\nabla\ln\text{Re}(\psi), \nabla\ln\text{Im}(\psi))$ along a closed curve around the line. Dirac found that one could remove the non-zero circulation by means of a gauge transformation of the second kind, and that the electromagnetic vector potential associated with this transformation was precisely the same electromagnetic potential produced by a magnetic monopole at the initial point. Dirac then equated the effect of the circulation around the nodal line in the original gauge to the effect of a monopole in the new gauge [22]. His quantization by the winding number is actually just a special case of the general quantization theorem above, and his gauge interpretation is thus a concrete exemplification of the meaning of the analysis given there. Remarkably enough, a relation between the Kozyrev phenomenae -which is related to torsion [107, 134]- and Dirac monopoles has been established by Shakhparonov and has extremely interesting technological applications [123]. So, in the present setting, the torsion-trace one-form plays the role of the electromagnetic potential leading to quantization and the appearance of magnetic monopoles [113].

The variety of types of singular sets defined by the representation given in eq. (38) is very great, as exemplified in the pioneering work by Nye and Berry [12]. Besides the singular sets that we previously identified with the photon (spherical wave front plus a nodal line) there is also a remarkable singular set of the monochromatic wave constructed out of $\phi = f(r) - t$ and $f + i\rho = y + iz \in R^{1,3}$, by the following sum

$$\epsilon e^{i\omega[f(r)-t]} - (y + iz), \epsilon > 0. \quad (43)$$

Its singular set is given by

$$y = \epsilon \cos \omega[f(r) - t], z = \epsilon \sin \omega[f(r) - t]. \quad (44)$$

This represents an helicoidal line lying on the cylinder $y^2 + z^2 = \epsilon^2$, and moving with (variable) speed of light along its tangent direction at each of its points. (For simplicity, we can assume that $f(r) = t$ in order to get a better visualization: the speed is then constant and the helicoid has then a constant step.) Taking $y - iz$ instead, we get a screw motion with opposite handedness. The singular set is thus a moving screw in 3-space that can be right or left handed, and may carry the energy associated with a quantum jump, as shown above. It seems therefore that a monochromatic wave line like this can represent appropriately a right or left handed neutrino, concretely identified with its singular set. It has then quite distinct properties from those associated with a photon. For it is given by an infinitely long moving right or left handed helicoidal line in 3-space (which by the way, it is a minimal surface; additional discussions on dislocations, minimal surfaces and turbulence, shall be presented elsewhere) while the photon is given by a point piloting a spherical wave. In particular if the singular screw line of above is associated with an elementary state u and carries energy E , it also carries the angular momentum $\epsilon^2 E\omega/c^2$ directed along a x -axis, in the given referential. Hence the neutrino carries angular momentum while the photon does not. On the other hand, according to this description, the neutrino should not have (primary) diffraction patterns as the photon does, which should explain why it is so difficult of detect.

4.2.2 DISTINCTION OF MAXIMAL MONOCHROMATIC ALGEBRAS AND THEIR TWISTOR REPRESENTATIONS

Lemma 2. The maximal monochromatic algebras \mathbf{M}_1 and \mathbf{M}_2 with generators $(f, \phi + i\psi)$ and $(\tilde{f}, \tilde{\phi} + i\tilde{\psi})$ respectively, are distinct if and only if

$$g(\nabla f, \nabla \tilde{f}) = 0. \quad (45)$$

Consider the generators $(\phi, f + i\rho) \in \mathbf{M}$ of a maximal monochromatic algebras. The vector fields $\nabla\phi$ and $\nabla(f + i\rho)$ are, respectively, real and complex isotropic fields, mutually orthogonal on (M, g) ; here M is a generic spin-manifold provided with a Lorentzian metric g . By the previous analysis $\nabla\phi$ is given by a spin vector field ω^A in the spinor form

$$\nabla\phi = \omega^A \bar{\omega}^{A'} \quad (\text{or } -\omega^A \bar{\omega}^{A'}). \quad (46)$$

and since $\nabla(f + i\rho)$ is isotropic and orthogonal to $\nabla\phi$ then we have

$$\nabla(f + i\rho) = \omega^A \bar{\pi}^{A'}, \quad (47)$$

where π^A is another spin vector field. Consequently the pair $(\nabla\phi, \nabla(f + i\rho))$ of vector fields is completely determined by the ordered pair of spin vector fields

$$(\omega^A, \pi^{A'}), \quad (48)$$

but we have a fourfold map here since we have already altogether four different ways of building the vector fields according to eq. (46,47) out of the ordered pair given by (48). The correspondence (46), extended to complex vectors shows that the second choice in (46) reverses $\nabla\phi$ from, say, a future-pointing isotropic vector to a past-pointing isotropic vector while in (46) it chooses the complex-conjugate $\nabla(f - i\rho)$ instead of $\nabla(f + i\rho)$, reversing the roles of analytic and anti-analytic functions, which means inversion of handedness. Choosing locally a given time orientation and a given handedness, corresponds to a particular choice of the assignments in eqs. (46,47). The ordered pair (48) of spin vector fields at a point in (M, g) is called a local twistor and the corresponding field a local twistor field. From the twistor field we determine the real and complex vector fields by eqs. (46,47), which, upon integration yield an equivalent pair $(\phi, f + i\rho)$. This means that we can characterize completely a maximal monochromatic algebra (and consequently the light quanta it represents) in terms of a twistor field with divergence free associated vector fields. This can be of course produced, by working from the start with coclosed differential forms. The new representation is even richer as it has built in an extra degree of freedom, namely, polarization, due to the factor $e^{i\phi}$ mentioned before. This result, showing that we have identified as light quanta are indeed given by twistor fields, substantiates the belief of Penrose that twistors are the appropriate tool to describe zero rest-mass particles and to effect the connection between gravitation and quantum mechanics. Further below we shall see that this connection extends to the laws of thought.

5 SELF-REFERENCE, THE KLEIN-BOTTLE, TORSION, MATRIX LOGIC, THE LAWS OF THOUGHT AND THE TWISTOR REPRESENTATION OF THE COGNITIVE PLENUM

In the last sections we have elaborated a theory of torsion and photons which, would we have omitted our discussion on the self-referential being of light, a Cartesian minded physicist would have read it as customary to that mindset: As a theory of an 'objective' realm, implicitly, in which subjectivity does not participate, or still does not exist at all in the universe of discourse. Yet, we have shown that both torsion and the photon are very closely related to self-reference, and thus to consciousness [109]. Furthermore, the semiotic codification of torsion as a distinction sign produces through the incorporation of paradox, a multivalued logic which is associated with the Klein-bottle and time waves [109]. From this

logic, it was proved that the most general matrix-tensor logic that has as particular cases quantum, fuzzy, modal and Boolean logics [135] stem from these time waves. In this theory which stems from abandoning the scalar logic theory of Aristotle and Boole, promoting it to logical operators, we find that the Klein-bottle plays a fundamental role as an in-formation operator, which coincides with the Hadamard gate of quantum computation. The role of this gate is to transform the vector Boolean states to superposed states, the latter being associated with the torsion of cognitive space and the non-orientability of this space due to its constitution in terms of the self-referential non-orientable Moebius and Klein-bottle surfaces. Furthermore, the logical Cognition Operator which leads to quantization of cognition is generated by the torsion produced from the commutator of the TRUE and FALSE logical operators (at the start of this chapter, we wrote them as True and False, respectively), which self-referentially gives the difference between these two operators, as we shall see below. The picture that stems is that Matrix Logic can be seen as the self-referential logical code which stands at the foundation of quantum physics to which is indissolubly related. We have elaborated the relations between matrix logic, self-reference, non-orientability and the Klein-bottle, nilpotent hypernumber representations of quantum fields that represent some logical operators. Thus, in this theory, matter quantum field theories are logical operators, and vice versa, and a transformation between quantum and cognitive logical observables has been established; this theory has produced a new fundamental approach to the so-called mind-matter problem, establishing its non-separateness, and the primacy of consciousness which thus cannot be purported to be an epiphenomenon of physical and more complex fields [109]. By promoting the ‘truth tables’ of usual Boolean logic to matrix representations, the founder of Matrix Logic A. Stern was able to produce an operator logic theory in which logical operators may admit inverses, and the operations of commutation and anticommutation are natural [135]. Furthermore, logical operators can interact by multiplication or addition and, in some cases, being invertible³⁵, they yield thus a more complex representation of the laws of thought than the one provided by the usual Boolean theory of logical connectives. This representation was named the *Intelligence Code* by Stern [135]. The latter can be related to Quantum Mechanics for two-state systems as we shall describe below. Matrix Logic is naturally quantized, since its eigenvalues take discrete values which are $\pm 1, 0, 2, \pm\phi$, with ϕ the Golden number [135]. In this setting, the null quantum-cognitive observable is the 2×2 matrix, $\mathbf{0}$, with identical entries given by 0; the latter is the *Mind Apeiron* as we discussed in [111] which we shall retake below. The relation with quantum field operators and this Mind Apeiron observable, is their role in polarizing this cognitive-quantum Apeiron through non-null square roots which can be represented by nilpotents (we rather prefer to call them plenumpotents), i.e. hypernumbers whose square is $\mathbf{0}$.³⁶ In distinction with the

³⁵The fact that there are logical operators that are represented by determinant zero matrices posits a separation between the invertible and non-invertible operators which is associated with an irreversibility of thinking which is implicit to this separation. Thus we shall have a logic and an antilogic which we can perhaps relate to the iso-duality transformation in Hadronic Mechanics due to Santilli [122], due to the association between nilpotent logical operators and quantum field operators. We shall discuss this very important issue elsewhere.

³⁶We shall later encounter the plenumpotence of the Mind Apeiron when discussing the Myth of Eternal Return as a self-referential process. The present theory shares with the *nilpotent universal rewrite system* (NURS) due to Rowlands and further extended to biology as the *Code of Life* by Hill [119] [48], an

other cognitive-quantum observables, is that the eigenstates of $\mathbf{0}$ are no longer quantized, but rather give an orthogonal complex two-dimensional nullvector space. In this way the Plenum is no longer represented by a single point, $\mathbf{0}$, but rather becomes an extended object or zero-brane. This will allow to map the twistor representations of the extended photon presented in (48) with its representation in a cognitive state and vice versa!

Let us present this briefly in this Section. If we consider a space of all possible cognitive states (which in this context replace the logical variables) represented in this plenum as the set of all Dirac bras $\langle q| = (\bar{q} \ q)$, where $\bar{q} + q = 1$ ³⁷, is a continuous cognitive logical value not restricted to the false and true scalar values, represented by the numbers 0 and 1 respectively. In fact, q can take arbitrary values as we shall elaborate further below. Still, the standard logical connectives admit a 2×2 matrix representation of the their ‘truth tables’ and now we have that for such an operator, L , we have the action of L on a ket $|q \rangle = \begin{pmatrix} \bar{q} \\ q \end{pmatrix}$ is denoted by $L|q \rangle$ alike the formalism in quantum mechanics, and still we have a scalar truth value given by $\langle p|L|q \rangle$, where $\langle p|$ denotes another logical vector. We can further extend the usual logical calculus by considering the TRUE and FALSE

understanding of the generative role of 0. The theory of these authors (which has no time operator) is constructed in terms of recursion, which in their context is the *reduction* of self-reference to the ‘objective’ side of the Cartesian Cut. This recursion is not the full embodiment of self-reference with physics, semiotics, multivalued logic, perception, cognition and neurology. Remarkably, the reduction of self-reference to recursion in the NURS is produced -somewhat as a natural tradeoff, in recalling Pattee and Taborsky’s conceptions- through an *empowerment* of semiosis, providing thus very enriching features to this system. More specifically, mathematical symbols -conceived independently of the embodied energetics and nonintegrability claimed by Pattee [91] and Taborsky [138] which we associated to torsion and multivalued logics [109]-, are presented as a self-writing system of the objective world. The point which though unobserved by their authors is crucial to their claim of being the scripture of the ‘objective’ realm, is that the NURS requires an *interpreter* to become a map of this realm, alike any text, and thus the *seemingly* absence of the subject and its lifeworld is in fact embodied in the actual interpretation of the symbology by which the system is indeed of rewrite. Thus, this theory if conceived free of subjectivity as in Aristotelian dualism, which is already implicit in some degree in the reduction of self-reference to recursion, may actually produce the illusion of this independence of subjectivity, of the pledged to be a scripture of the objective realm. This is the usual conception of the Platonian realm in which the most fertile and original mathematical, artistic and creative works claim to find/receive their origin. In this chapter we are presenting a very different conception which does not deny this realm. It rather integrates it into the full fusion of the objective and subjective realms and its embodiment as a lifeworld, in which life and world are richer than abstractions, but rather a supraobjectivity and suprasubjectivity: Embodiment at *all* scales. The relations between this remarkable theory and the present conception will hopefully be presented elsewhere.

³⁷Notice that a difference with the definition of qubits in quantum computation, is that for them we have the normalization condition for complex numbers of quantum mechanics. In this case, the values of q are arbitrary real numbers, which leads to the concept of non-convex probabilities. While this may sound absurd in the usual frequentist interpretation, when observing probabilities in non-orientable surfaces, say, Moebius surfaces, then if we start by associating to both sides of an orientable surface -from which we construct the Moebius surface by the usual procedure of twisting and glueing with both sides identified- the notion of say Schroedinger’s cat being dead or alive in each side, then for each surface the probability of being in either state equals to 1 and on passing to the non-orientable case, the sum of these probabilities is 2. While this is meaningless in an orientable topology, in the non-orientable case which actually exist in the macroscopic world, this value is a consequence of the topology. In this case, the superposed state ‘being alive and being dead’ or ‘true plus false’ which is excluded in Aristotelian dualism, is here the case very naturally supported by the fact that we have a non-trivial non-orientable topology. As for the case of negative probabilities, we see in the previous example that -1 is the probability value complement of the value 2.

operators, defined by the eigenvalue equations $\text{TRUE}|q\rangle = |1\rangle$ and $\text{FALSE}|q\rangle = |0\rangle$, where $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ are the true and false vectors. It is easy to verify that the eigenvalues of these operators are the scalar truth values of Boolean logic. We can represent these operators by the matrices

$$\text{TRUE} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \text{FALSE} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad (49)$$

These operators are non-self adjoint; in distinction with usual (hermitean) quantum observables, logical operators are generally non-hermitean although they may have representations as quantum field operators and viceversa. A consequence of this non-hermiticity is that in contrast with the trivial duality of the true and false scalars of Boolean logic, 1 and 0 respectively (which is represented by the relations $\bar{1} = 0$ and $\bar{0} = 1$), by defining the *complement* \bar{L} of a logical operator L , by $I - L$, where I is the identity operator, hence $\overline{\text{TRUE}} \neq \text{FALSE}$ and $\overline{\text{FALSE}} \neq \text{TRUE}$. This notion of complementarity when restricted to scalar fields coincides with the dual operation of Boolean logic transforming conjunction into disjunction and viceversa which affirms the principle of non-contradiction (i.e. a proposition *and* its negation are false) of Aristotelian-Boolean logic, and thus the previous result proves the non-duality of TRUE and FALSE.

We note that the spaces of bras and kets do not satisfy the additivity property of vector spaces -while keeping the property that one is the dual of the other- due to the fact that normalization is not preserved under addition. A superposition principle is necessary. If $|p\rangle$ and $|r\rangle$ are two normalized states, then the superposition defined as follows

$$|q\rangle = c|p\rangle + \bar{c}|r\rangle, \quad \text{where } \bar{c} + c = 1, \quad (50)$$

also defines a normalized logical state. We can interpret these coefficients as components of a logical state $|c\rangle$ or still a probability vector, termed *denktor*, a German-English hybrid for a thinking vector. The normalization condition is found as follows: Multiply the states $|p\rangle$ and $|r\rangle$ by \bar{c} and c , respectively. By definition, the normalization condition on the sum $|q\rangle$ with coefficients \bar{c}, c leads to

$$\begin{pmatrix} \bar{q} \\ q \end{pmatrix} = c \begin{pmatrix} \bar{p} \\ p \end{pmatrix} + \bar{c} \begin{pmatrix} \bar{r} \\ r \end{pmatrix} = \begin{pmatrix} c\bar{p} + \bar{c}\bar{r} \\ cp + \bar{c}r \end{pmatrix}, \quad (51)$$

yet, since $\bar{q} + q = c\bar{p} + \bar{c}\bar{r} + cp + \bar{c}r = c(\bar{p} + p) + \bar{c}(\bar{r} + r) = c.1 + \bar{c}.1$ and thus $c + \bar{c} = 1$ since $|q\rangle$ is a normalized state by assumption. So, through this superposition principle we can give a vector space structure to normalized cognitive states. We now can identify under these prescriptions, the tangent space to the space of bras (alternatively, kets) with the space itself. ³⁸

³⁸Here it is simple to see that if $|q\rangle, |q'\rangle$ are two superpositions, then for any operator L , $L(|q+q'\rangle) = L|q\rangle + L|q'\rangle$.

5.1 TORSION IN COGNITIVE SPACE, THE KLEIN-BOTTLE, QUANTUM MECHANICS AND QUANTUM FIELD THEORY, LOGIC AND HYPERNUMBERS, THE COGNITION, TIME AND SPIN OPERATORS

Returning to the vector space structure provided by the superposition principle, and thus the identification of its tangent space with the vector space itself, it follows that a vector field as a section of the tangent space can be seen as transforming a bra (ket) vector into a bra (ket) vector through a 2×2 matrix, so we can identify the tangent space which with the space of logical operators. We have as usual the commutator of any such matrices $[A, B] = AB - BA$ and the anticommutator $\{A, B\} = AB + BA$. In particular we take the case of $A = \text{FALSE}$, $B = \text{TRUE}$ and we compute to obtain

$$[\text{FALSE}, \text{TRUE}] = \text{FALSE} - \text{TRUE}, \quad (52)$$

$$\{\text{FALSE}, \text{TRUE}\} = \text{FALSE} + \text{TRUE}. \quad (53)$$

Thus in the subspace spanned by TRUE and FALSE we find that the commutator that here coincides with the Lie-bracket of vectorfields defines a torsion vector given by the vector $(1 \ -1)$, and that this subspace is integrable in the sense of Frobenius: Indeed, we find that $[[\text{FALSE}, \text{TRUE}], \text{TRUE}] = [\text{FALSE}, \text{TRUE}]$ and $[[\text{FALSE}, \text{TRUE}], \text{FALSE}] = [\text{TRUE}, \text{FALSE}]$. Furthermore, on account that $\text{TRUE}^2 = \text{TRUE}$ and $\text{FALSE}^2 = \text{FALSE}$, i.e. both operators are idempotent, then the anticommutators also leaves this subspace invariant.

The remarkable aspect here is that the quantum distinction produced by the commutator, exactly coincides with the classical distinction produced by the difference (eq. (52)), while the same is valid for the anticommutator with a classical distinction which is represented by addition (eq. (53)). We notice that in distinction of quantum observables, these logical operators are not hermitean and furthermore they are noninvertible. Furthermore, we shall see below how this torsion is linked with the creation of cognitive superposed states, very much like the coherent superposed states that appear in quantum mechanics.

Now, if we denote by M the commutator $[\text{FALSE}, \text{TRUE}]$ so that from eq. (52) we get

$$M = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}, \quad (54)$$

we then note that it is nilpotent, (in fact a nilpotent hypernumber, since $M = \epsilon_2 + i_1 = \sigma_z + i_1$). Indeed,

$$M^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \equiv \mathbf{0}, \quad (55)$$

thus yielding the identically zero matrix, representing the universe of all possible cognitive states created by a non-null divisor of zero, which thus creates a polarization of the Plenum, precisely through the fact that the torsion is a superposed state which cannot be fit into the scheme of Boolean logic, but can be obtained independently by the loss of orientability of a surface which thus allows for paradox. Since M coincides with the classical difference between TRUE and FALSE , which are not hermitean, then we can think of this non-invertible

operator as a *Cognition Operator* related to the variation of truth value of the cognitive state, as we shall prove further below that $M = -\frac{d}{dq}$.

We would like to note that this polarization of the Plenum $\mathbf{0}$ is not unique, there are many divisors of $\mathbf{0}$, the Plenum, for instance the operator

$$\text{ON} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} := a^\dagger, \quad (56)$$

and

$$\text{OFF} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} := a \quad (57)$$

satisfy

$$a^2 = \mathbf{0}, (a^\dagger)^2 = \mathbf{0}, \quad (58)$$

and furthermore, $\{a, a^\dagger\} = I$, so they can be considered to be matrix representations of creation and annihilation operators, a^\dagger and a as in quantum field theory. In fact, if we consider the wave operators given by the exponentials of a, a^\dagger we have

$$e^a = I + a = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \text{IMPLY}, e^{a^\dagger} = I + a^\dagger = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \text{IF}, \quad (59)$$

where IMPLY \Rightarrow is the implication, and IF \Leftarrow is the converse implication: $x \Leftarrow y = \bar{x} \rightarrow \bar{y}$. Thus the implication and the converse implication logical operators are both wave-like logical operators given by the exponentials of divisors of $\mathbf{0}$, and in fact they are derived from quantum field operators of creation and annihilation in second-quantization theory, a^\dagger and a , respectively, which in fact can be represented by nilpotent hypernumbers. Indeed, $a = \frac{1}{2}(\epsilon_3 - i_1) = \frac{1}{2}(\sigma_x - i_1)$ and $a^\dagger = \frac{1}{2}(\epsilon_3 + i_1) = \frac{1}{2}(\sigma_x + i_1)$; see [109].

Now we wish to prove that the interpretation of M as the Logical Momentum Operator is natural since $M = -\frac{d}{dq}$. Indeed,

$$-\frac{d}{dq}|q\rangle = -\frac{d}{dq} \begin{pmatrix} 1-q \\ q \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} \bar{q} \\ q \end{pmatrix} = M|q\rangle \quad (60)$$

so that for any normalized cognitive state $|q\rangle$ we have the identity

$$M = -\frac{d}{dq}, \quad (61)$$

which allows to interpret the cognitive operator as a kind of logical momentum. Thus, in this setting which is more general but less primitive than the calculus of distinctions, it is the non-duality of TRUE and FALSE that produces cognition as variation of the continuous cognitive state. We certainly are facing a situation that is far from the one contemplated by Aristotle with his conception of a trivial duality of (scalar) true and false, and which produced the elimination (and consequent trivialization) of time and of subjectivity, as argued in [109].

Now consider a surface given by a closed oriented band projecting on the xy plane. Thus to each side of the surface we can associate its normal unit vectors, $(1 \ 0)$ and $(0 \ 1)$. Suppose

that we now cut this surface and introduce a twist on the band and we glue it thus to get a Moebius surface. Now the surface has lost its orientability and we can identify one side with the other so that we can generate the superpositions

$$\langle 0|+ \langle 1| = \langle (1 \ 1)| = \langle S_+|, \quad \langle 0|- \langle 1| = \langle (1 \ -1)| = \langle S_-|. \quad (62)$$

which we note that the latter corresponds to the torsion produced by the commutator of TRUE and FALSE operators. These states are related by a change of phase by rotation of 90 degrees. What the twisting and loss of orientability produced, can be equivalently produced by the fact that TRUE and FALSE are no longer as in Boolean logic and the Aristotelian frame, they are no longer dual and what matters is their difference, which in the case of scalar truth values does not exist. The second state in eq. (62), also can be interpreted as a state that represents the fact that the states as represented by vectors, have components standing for truth and falsity values which are independent, so that the Aristotelian link that makes one the trivial reflexive value of the other one is no longer present: they each have a value of their own. In that case then (0 0) is another state, ‘neither false nor true’ (which is the case of the Liar paradox as well as Schroedinger’s cat), which together with (1 1), ‘false and true’ state together with (0 1), true, and (1 0), false states, we have a 4-state logic in which the logical connectives have been promoted to operators.

Now consider for an arbitrary normalized cognitive state q the expression

$$\begin{aligned} [q, M]|q \rangle &= [q, -\frac{d}{dq}]|q \rangle = -q \frac{d}{dq}|q \rangle + \frac{d}{dq}q|q \rangle = -q \frac{d}{dq} \begin{pmatrix} 1-q \\ q \end{pmatrix} \\ &+ \frac{d}{dq} \begin{pmatrix} q-q^2 \\ q^2 \end{pmatrix} = \begin{pmatrix} q \\ -q \end{pmatrix} + \begin{pmatrix} 1-2q \\ 2q \end{pmatrix} = |q \rangle, \end{aligned} \quad (63)$$

for any normalized cognitive state q so that we have the quantization rule

$$[q, M] = I, \quad (64)$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, the identity operator. Instead of the commutation relations of quantum mechanics $[q, p] = i\hbar$ for $p = -i\frac{\partial}{\partial q}$ and those of diffusion processes associated to the Schroedinger equation, $[q, p] = \sigma$ where $p = \sigma\frac{\partial}{\partial x}$ with σ as the diffusion tensor given by the square-root of the metric g on the manifold with coordinates x on which the diffusion takes place so that $\sigma \times \sigma^\dagger = g$ [102], we have that the commutation of a normalized cognitive state where the Cognitive (Momentum) Operator is always the identity, thus yielding a fixed point. Indeed, consider the function $F_M(q) = [q, M]$, then $F_M(F_M(F_M(F_M(\dots))))(|q \rangle) = |q \rangle$, for any normalized cognitive state $|q \rangle$. Thus, $F_M(q)$ defines what is called in system’s theory an eigenform, albeit one which does not require infinite recursion but achieves a fix point already in the first step of the process, by the formation of the commutator $[q, M]$. This is the structure of Self, which whatever operation it may suffer by the action of logical operators, retains its invariance by the quantization of logic as expressed above by eq. (64).

Now we want to return to the superposed states, S_+ and S_- , the latter being the torsion produced by the commutator of the TRUE and FALSE operators, to see how they actually construct the cognitive operator. First a slight detour to introduce the usual tensor products of two cognitive states, $|p\rangle\langle q|$ which as the tensor product of a vector space and its dual is isomorphic to the space of linear transformations between them, we can think as an operator L acting by left multiplication on kets and by right multiplication on bras. So that if $L = |p\rangle\langle q|$ then $\langle y|L|x\rangle = \langle y, p\rangle\langle q|x\rangle$, for any $\langle y| = \bar{y}\langle 0| + y\langle 1|$ and $|x\rangle = \bar{x}|0\rangle + x|1\rangle$, where $\langle x|y\rangle = \delta_{xy}$ is equal to 1 for $x = y$, and equal to 0 for $x \neq y$ and $\sum_i |x_i\rangle\langle x_i| = I$. Then,

$$M = |S_+\rangle\langle S_-|, \quad (65)$$

which shows that the cognitive operator that arises from the quantum-classical difference between the TRUE and FALSE operators can be expressed in terms of the tensor products of the superposition states, being the sum of the true and false states and the torsion produced in the quantum commutator of the TRUE and FALSE operators.

Starting with the Logical Momentum or still the Cognition Operator, M , that satisfies $[q, M] = I$ for any cognitive variable q , we can link the quantization rule in cognitive space to the quantization rule of Bohr-Sommerfeld. We must stress at this point that this rule is not restricted to the quantization of the microphysical scales but also valid for astrophysical structures; for a thorough review of this we refer to [16, 120, 88] and the series of articles in [18] and references therein, notably Pitkanen's contribution [93]. As noted by Stern [135], the logical potential carrying the logical energy could be linked to the Bohr energy of atomic (to which, we add astrophysical structures) structures in the following way: $\oint(k) = \oint M dq = 2\pi(n + 1/2) = k\pi$, where q is a logical variable (if it is zero than the contour integral runs a full great circle on the Riemann sphere of zeros), n is the winding number specifying the numbers of times the closed curve runs round in an anticlockwise sense; n runs the bosonic numbers $0, 1, 2, \dots$ and $(n + 1/2)$ the fermionic numbers, $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$. The topo-logical potential is an odd multiple $(2n + 1)\pi$ of the elemental (topo)logical phase π and is \hbar^{-1} times the Bohr energy of the quantum oscillator: $\oint p dx = 2\pi\hbar(n + 1/2)$, where the position and momentum operator satisfy the standard quantum commutation relation: $[x, p] = i\hbar$. As we see, the to-pological potential, multiplied by the factor \hbar , gives the Bohr quantum energy opening up the possibility to treat atomic and astrophysical structures (the latter arising from the multivalued character of the Planck constant [87][?]) as a dynamical logic in a fundamental sense, where quantization stems from the closed topology or self-observation feature at this fundamental level of reality. This dynamical logic is Matrix Logic which we recall that can be derived from the protologic of Spencer-Brown (with the primeval distinction being the semiotic codification of torsion) in considering the paradoxical Klein-bottle renntrance of a form on itself [109]. Another interesting conjecture -proposed by Dienes [21]- which follows is, since matter, as energy, ($E = mc^2$) is a topo-logically transformed logical energy, the mass of an object is basically the in-formation contained in the holomatrix which projects the object and its mass out from the ground state. This is a more fundamental ontology for mass, with in-formational topo-logical roots, rather than the usual physical one. It stems from the differentiation of the Plenum placed by the semiotic codification of torsion and the associated non-orientable self-referential topology: the Klein-

bottle, or still by the action of the latter on the polarization of the Mind Apeiron provided by its twistor eigenstates which themselves are torsion fields embodying the fusion of the physical and subjective realms.

Let us now introduce the operator defined by

$$\Delta = a - a^\dagger \quad (66)$$

so that it follows that its matrix representation is

$$\Delta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (67)$$

and furthermore

$$\Delta = \rightarrow - \leftarrow. \quad (68)$$

We shall call Δ the TIME Operator. Note that *alternatively* we can define TIME by changing the sign in the above definition.³⁹ We notice that it is unitary and antisymmetric:

$$\text{TIME}^\dagger = \text{TIME}^{-1} = -\text{TIME}. \quad (69)$$

As an hypernumber $\text{TIME} = -i_1$, minus the unique 2×2 matrix representing a 90 degrees rotation, the old commutative square root of -1 from which complex numbers appeared. The reason for considering this operator given by the difference of nilpotents is because it plays the role of a *comparison* operator. Indeed, we have

$$\langle p | \text{TIME} | q \rangle = \bar{p}q - \bar{q}p = (1-p)q - (1-q)p = q - p = \bar{p} - \bar{q}. \quad (70)$$

TIME appears to be unchanged for unaltered states of consciousness:

$$\langle q | \text{TIME} | q \rangle = 0, \quad (71)$$

and if we have different cognitive states p, q , then $\langle p | \text{TIME} | q \rangle \neq 0$. So this operator does represent the appearance of a primitive difference on cognitive states: further, it is antisymmetric and unitary. It is furthermore linked with a difference between annihilation and creation operators and thus stands for what we argued already as a most basic difference that leads to cognition and perception: *the appearance of quantum jumps*. Without them, no inhomogeneities or events are accessible to consciousness. The very nature of self-reference as consciousness of consciousness requires such an operator for the joint constitution of the subject and the world. Thus its name, TIME operator; it stands clearly in the subject side of the construction of a conception that overcomes the Cartesian cut, yet a subject that has superposed paradoxical states.

Let us consider next the eigenvalues of TIME, i.e. the numbers λ such that $\text{TIME}|q\rangle = \lambda|q\rangle$; they are obtained by solving the characteristic equation $\det|\text{TIME} - \lambda I| = \lambda^2 + 1 = 0$, so that they are $\lambda = \pm i$ with complex eigenstates

$$\begin{pmatrix} 1 \\ i \end{pmatrix}, \begin{pmatrix} i \\ 1 \end{pmatrix}. \quad (72)$$

³⁹Remarkably, $-2i\text{TIME}$ is the hamiltonian operator of the damped quantum oscillator in the quantum theory of open systems; see N. Gisin and I. Percival, arXiv:quant-ph/9701024v1. In this theory based on the stochastic Schroedinger equation the role of torsion is central [107].

They are not orthogonal, but self-orthogonal; thus, they are spinors, and the complex space generated by them generates a two-dimensional null space. We diagonalize TIME by taking

$$\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \text{TIME} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}^{-1} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \quad (73)$$

so that

$$\text{TIME}_{\text{diag}} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad (74)$$

which as a hypernumber implies that $\text{TIME}_{\text{diag}} = i_2$, so that $\text{TIME}_{\text{diag}}^2 = -I$. We want finally to comment that TIME is not a traditional clock, yet it allows to distinguish between after and before ($\rightarrow - \leftarrow$), forward and backwards, in *logical causation*. There is no absolute logical time, nor a privileged direction of it. To have a particular direction it must be asymmetrically balanced towards creation or annihilation. This can be computed as the complement of the operator phase⁴⁰

$$\overline{\cos(2a^\dagger) + \text{sen}(2a^\dagger)} = a^\dagger - a, \quad (75)$$

from which it follows that $\text{TIME} = \overleftarrow{-2} \Rightarrow - \leftarrow$, as we stated before.

Let us now retake the Cognition Operator M and decompose it as

$$M = \text{TIME} + \sigma, \text{ or still} \quad (76)$$

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (77)$$

Then we have that

$$\langle q|M|q \rangle = \langle q|\sigma|q \rangle. \quad (78)$$

Indeed, since $\langle q|\text{TIME}|q \rangle = 0$, so that the proof of eq. (78) follows. Furthermore we note that

$$\langle q|\sigma|q \rangle = \bar{q}^2 - q^2 = (\bar{q} - q)(\bar{q} + q) = \bar{q} - q. \quad (79)$$

from the normalization condition. Note here that the identity given by eq. (79) is a kind of quadratic metric in cognitive space which due to the normalization condition loses its quadratic character to become the difference in the cognitive values: $\bar{q} - q = 1 - 2q$ which becomes trivial in the undecided state in which $\bar{q} = q = \frac{1}{2}$.

The role of σ is that of a SPIN operator, as we shall name it henceforth, which coincides with the hypernumber ϵ_2 (or as a Pauli matrix is σ_z), so that $\sigma^2 = I$, i.e. the non-trivial square root of hypernumber $I = \epsilon_0$, which is the usual Pauli matrix σ_z in the decomposition of a Pauli spinor in the form $\sigma_x e_x + \sigma_y e_y + \sigma_z e_z$, for e_x, e_y, e_z the standard unit vectors in R^3 and we write their representations as hypernumbers

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \epsilon_3, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \epsilon_1, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \epsilon_2. \quad (80)$$

⁴⁰The complement of a logical operator L , is defined by $\bar{L} = I - L$.

We can rewrite this average equation $\langle q|M|q \rangle = \langle q|\sigma|q \rangle$, as an average equation where the l.h.s. takes place in cognitive space of normalized states $|q \rangle$ and the r.h.s. takes place in a Hilbert space of a two-state quantum system, say, spin-up $\psi(\uparrow)$, spin-down $\psi(\downarrow)$, so that the generic element is of the form

$$\psi = \psi(\uparrow)|0 \rangle + \psi(\downarrow)|1 \rangle. \quad (81)$$

Indeed, if we write

$$|q \rangle = \overline{\psi(\uparrow)}\psi(\uparrow)|0 \rangle + \overline{\psi(\downarrow)}\psi(\downarrow)|1 \rangle, \quad (82)$$

then the r.h.s. of eq. (79) is $\bar{q}^2 - q^2$, with $\bar{q} = \overline{\psi(\uparrow)}\psi(\downarrow)$, and $q = \overline{\psi(\downarrow)}\psi(\uparrow)$, so that eqs. (76, 78) can be written as

$$\langle q|M|q \rangle = \langle \psi|\sigma|\psi \rangle \quad (83)$$

where the average of M is taken in cognitive states while that of the SPIN operator is taken in the two-state Hilbert space.⁴¹

We revise the previous derivation for which the clue is the relation between cognitive states $|q \rangle$ and elements of two-state of Hilbert state $|\psi \rangle$ is that the former are derived from the latter by taking the complex square root of the latter, so that probability($|0 \rangle$) = $\bar{q} = \overline{\psi(\uparrow)}\psi(\uparrow)$ and probability($|1 \rangle$) = $q = \overline{\psi(\downarrow)}\psi(\downarrow)$, so that $\langle \psi|\sigma|\psi \rangle = \bar{q} - q = (\bar{q} - q)(\bar{q} + q) = \bar{q}^2 - q^2$. Therefore, by using the transformation between real cognitive states q defined by the complex square root of ψ , i.e. $q = \psi\psi$, we have a transformation of the average of the Cognitive Operator M on cognitive states, on the average of the SPIN Operator on two-states quantum elements in Hilbert state, i.e. eq. (76). This is a very important relation, established by an average of the Cognition Operator (which transforms an orientable plane into a non-orientable Moebius surface due to the torsion introduced by M , as represented by eq. (62), and the spin operator on the Hilbert space of two-state quantum mechanics. It is an identity between the action of the cognizing self-referential mind and the quantum action of spin. Thus the cognitive logical processes of the subject become related with the physical field of spin on the quantum states. This is in sharp contrast with the Cartesian cut, and we remark again that this is due to the self-referential classical-quantum character of M as evidenced by eq. (62) which produces a torsion on the orientable cognitive plane of coordinates (true, false) to a plane which is torsioned to yield a superposed state, S_- . The relation given by eqs. (72, 76) establishes a link between the operations of cognition and the quantum mechanical spin. This link is an interface between the in-formational and quantum realms, in which topology, torsion, logic and the quantum world operate jointly. Yet, due to fact that for the Klein-bottle there is no inside nor outside, the exchange can go in both ways, i.e. the quantum realm can be incorporated into the classical cognitive dynamics, while the logical elements can take part in the quantum evolution. Indeed, if we have a matrix-logical string which contains the momentum product, say,

⁴¹According to Conte, “quantum mechanics is a theoretical formulation that necessarily includes also the cognitive function as it is shown by using a Clifford algebraic formulation of this theory. Therefore, quantum mechanics is also a physical theory of cognitive processes and of the profound existing link between cognitive dynamics and physical reality per se” [19].

... $\langle x|A|y \rangle \langle q|M|q \rangle \langle z|B|s \rangle \dots = \dots \langle x|A|y \rangle \langle \psi|\sigma|\psi \rangle \langle z|B|s \rangle \dots$. Thus, the factor $\langle \psi|\sigma|\psi \rangle$ entangles with the rest of the classical logical string creating a Schrodinger cat superposed state, since we have a string of valid propositions where one may be the negation of the other.

There is still another very remarkable role of these superposed states in producing a topological representation of a higher order form of self-reference, produced from oppositely twisted Moebius surfaces. So we shall consider the Cartesian modulo 2 sum of the superposed states

$$\mathcal{H} := |S_+ \rangle \oplus |S_- \rangle = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad (84)$$

which we call the topological in-formation operator which is an hypernumber; indeed, $\mathcal{H} = \sigma_x + \sigma_z = \epsilon_3 + \epsilon_2$. We could have chosen the opposite direct sum or still place the minus sign on the first row in any of the columns and obtain a similar theory, but for non-hermitean operators unless the minus sign is on the first matrix element. Notice that it is a hermitean operator, which essentially represents the topological (or still, logo-topological) in-formation of a Klein-bottle formed by two oppositely twisted Moebius surfaces.⁴² The in-formation matrix satisfies $\mathcal{H}\mathcal{H}^\dagger = \mathcal{H}\mathcal{H}^{-1} = 2I$. We recognize in taking $\frac{1}{\sqrt{2}}\mathcal{H}$ the Hadamard gate in Quantum Computation [4], which due to the introduction of the $\frac{1}{\sqrt{2}}$ factor is hermitean and unitary. Now we have two orthogonal basis given by the sets $\{|0 \rangle, |1 \rangle\}$ and $\{|S_- \rangle, |S_+ \rangle\}$ of classical and superposed states respectively, the latter un-normalized for which a factor $\frac{1}{2}$ has to be introduced but still does not give a unitary system as in quantum theory. An important role of the Klein-bottle is precisely to transform these orthogonal basis, from classical states to superposed states which are nor classical nor quantum, but become quantized by appropriate normalization with the $\frac{1}{\sqrt{2}}$ factor. Indeed,

$$\mathcal{H}|0 \rangle = |S_+ \rangle, \mathcal{H}|1 \rangle = |S_- \rangle, \quad (86)$$

and

$$\frac{1}{2}\mathcal{H}|S_+ \rangle = |0 \rangle, \frac{1}{2}\mathcal{H}|S_- \rangle = |1 \rangle. \quad (87)$$

For the logical space coordinates (*true, false*) we have rotated the state $|0 \rangle$ clockwise by 45 degrees through the action of \mathcal{H} and multiplied it its norm by 2, and for the state $|1 \rangle$ we have rotated it likewise after being flipped. In reverse, the superposed states are transformed into the classical states by halving the in-formation matrix of the Klein-bottle, producing 45 degrees counterclockwise rotations, one with a flip. Now classical and quantum states are functionally complete sets of eigenstates spanning each other. The classical states $|0 \rangle$ and $|1 \rangle$ can be easily determined to be the eigenstates of AND, and and the superposed states $|S_- \rangle, |S_+ \rangle$ are the eigenstates of NOT. It is known that the logical

⁴²Alternatively, instead of \mathcal{H} we can introduce another in-formation matrix for the Klein-bottle, namely

$$\mathcal{H} := |S_+ \rangle \oplus |S_- \rangle = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \quad (85)$$

which is non-hermitean.

basis of operators {AND, OR} is functionally complete, generating all operators. Hence our system of classical and superposed (or still, quantum by appropriate normalization by $\frac{1}{\sqrt{2}}$) eigenstates constitute together a functionally complete system: all operators of matrix logic can be obtained from them. This system is self-referential. Furthermore, there are operators which produce the rotation of one orthogonal system on the other orthogonal system. The logical Cognition Operator M defined by the commutator [FALSE, TRUE], or still eq. (61), transforms classical states $|x \rangle = \bar{x}|0 \rangle + x|1 \rangle$ into $|S_- \rangle$ and still the anticommutator {FALSE, TRUE} which coincides with the matrix $\mathbf{1} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ transforms $|x \rangle$ into $|S_+ \rangle$, i.e.

$$M|x \rangle = |S_- \rangle, \mathbf{1}|x \rangle = |S_+ \rangle. \quad (88)$$

This can be rephrased by saying that M evidences on its action on a classical state the torsion in the quantum commutator of FALSE and TRUE while the ONE operator $\mathbf{1}$ transforms $|1 \rangle$ into $\begin{pmatrix} 1 \\ 1 \end{pmatrix} = |S_- \rangle$. Since both M and $\mathbf{1}$ are non-invertible, we shall use instead the fact that $\mathcal{H}^{-1} = \frac{1}{2}\mathcal{H}$, so that in addition of the transformation by the Klein-bottle of the classical basis in eq. (86), the reversed transformation from the superposed to the classical states is achieved by

$$\frac{1}{2}\mathcal{H}|S_+ \rangle = |0 \rangle, \frac{1}{2}\mathcal{H}|S_- \rangle = |1 \rangle. \quad (89)$$

Yet, we stress again that these transformations are not unitary. This is easily resolved by the $\frac{1}{\sqrt{2}}$ factor and then we have a transformation of classical into quantum states and viceversa. In the latter case, the renormalized Klein-bottle acts like a quantum operator producing coherent quantum states, a topological Schroedinger cat state which does not decohere.

6 THE EIGENSTATES OF THE NULL OPERATOR, COGNITIVE APEIRON, THE EXTENDED PHOTON AND THEIR TWISTOR REPRESENTATIONS

In this section we shall retake the elaboration in [111] following a modification of a sketchy yet ground-breaking idea by Dienes [21], elaborated in [111]. We have discussed the eigenstates of some logical operators, and now we shall discuss the eigenstates of the Mind Apeiron, namely the 2×2 identically zero matrix, which we denoted as $\mathbf{0}$. In distinction with the other logical operators the eigenstates of $\mathbf{0}$ (as a linear transformation from C on C , so that $\mathbf{0}$ becomes a point of C^2 , its origin) are no longer quantized, but rather give an orthogonal complex two-dimensional nullvector space.⁴³ In this way the Plenum is

⁴³We have already seen this an identical situation in the eigenstates of TIME. Thus the eigenstates of the Mind Apeiron are given by a nilpotence condition alike the eikonal equation; we shall see that this likeness is in fact an identity

no longer represented by a single point, $\mathbf{0}$, but rather becomes an *extended* object or zero-brane. This phenomenon is well known in complex Clifford bundles. To create the correct complex vector bundle we modify C^2 to make different fibres (unit 2-spheres) disjoint, with no common origin. To achieve this we replace the origin of C^2 by a copy of the entire Riemann sphere, so that instead of having just one zero, we have a whole Riemann spheres worth of zeros, one for each fiber, giving the zero section of the bundle. This procedure is known as blowing up the origin of C^2 , which amounts to take a projective subspace of $C^2 \times S$, with S the Riemann sphere (i.e. the complex projective space $CP(1)$), defined as the set of all elements of the form $\{(z_1, z_2, \xi) \in C^2 \times S / z_1 = z_2 \xi\}$. Since zero is now allowed on the fibres, continuous cross-sections of the bundle exist [21].⁴⁴ These cross-sections represent the spinor (and twistor) fields on $S \simeq S^2$, the two-dimensional sphere, giving a 2- complex-dimensional vector space, which can be mapped to the 2-dimensional logic space of matrix logic by stereographic projection. We shall apply now this to the twistor representation of the extended photon through the maximal monochromatic algebra as described by (46, 47) which has an equivalent representation as a pair of divergenceless orthogonal spinor vectors $(\omega^A, \pi^{A'})$, $A, A' = 1, 2$ by (48). By stereographic projection of this twistor representation of the extended photon, we obtain a basis of cognitive space, or vice versa: We can use the Boolean basis $\langle 0|$ and $\langle 1|$, or still the superposed states $\langle S_+|$ and $\langle S_-|$, to represent the maximal monochromatic algebra by taking the inverse of the stereographic projection; in any case via the normalized Klein-bottle Hadamard information matrix, we generate all the operators of Matrix Logic. In this we see how the extended photon, which we purported to be a subjective-objective fused structure, is represented as a basis for cognitive space, and conversely, from cognitive space we are able to codify the maximal monochromatic algebra representation of the extended photon. This establishes the full self-referential construction of a world which is perceived through quantum jumps, i.e. differences that produce differences, or still, in terms of cognitive states of the Mind; yet, we recall that the (absorbed) photon is already tantamount to embodiment. Furthermore, we have seen above that the role of the Planck constant \hbar is precisely to connect the transformation of the quantum world (whatever the scale microphysical or astrophysical is) into the world of the mind, bridging thus the material and mind domains. The Riemann sphere is not only instrumental to codify this joint constitution by codifying the extended photon as a cognitive state. It is also the manifold in which the complex logarithmic function takes multivalued values to quantize the quantum jumps in terms of the different branches of the logarithm, allowing thus to codify the ‘outer’ and ‘inner’ worlds. Further elaborations in relation to the transactional interpretation of Quantum Mechanics, cosmological Kozyrev

⁴⁴The blowing up of the origin, transforming its point-like structure to yield a manifold has profound consequences. For example, the blowing up of the origin in R^2 is the Moebius surface, which as we already saw is basic to the Intelligence Code. Two oppositely twisted Moebius bands generate the Klein-bottle by glueing them. Thus the blowup of the origin gives rise to the higher order surface of paradox whose matrix representation, up to a normalizing constant, is the Hadamard gate of Quantum Computation, which together with the phase conjugator, allows to generate all quantum gates [4]. In taking in account that DNA performs quantum computation⁴⁵ as the works of Peter Gariaev and associates shows [34] and its relation to holography [74, 78] which is already performed by sensory processing cells of the human neurocortex, evidences the relevance of the importance of this gate and the multivalued logic we presented above which is derived from the Klein-bottle; see [109] and references therein.

torsion fields and entanglement, and brain synchronization in binocular vision, and the visual and somatosensory representations on the neurocortex, will be elaborated below. ⁴⁶

7 THE SURMOUNTAL OF THE CARTESIAN CUT AND ITS MANIFOLD MANIFESTATIONS

7.1 TORSION, THE PRIMEVAL DISTINCTION, THE KLEIN-BOTTLE, COGNITION, CONTROL, WILL AND TIME OPERATORS

While current science has been built in terms of the Cartesian Cut in its manifold expressions, we have unveiled a lifeworld that surmounts the Cartesian Cut which stems from incorporating into the very foundations of the constitution of space, time, thought, cognition and perception, the essential phenomenon which is the basis for consciousness: self-reference.

In second-order cybernetics self-reference transcends the cut between observer and controlled system by the semiotic codification through a primeval distinction (in the sense of the calculus of distinctions of Spencer-Brown) of the torsion geometry associated to the anholonomic variables (controlling variables that cannot be separated from the system that they control) variables which Pattee considered precisely -we say, paradoxically- to encode the Cartesian Cut [91, 109]. This fuses subject-with-object into an implicate *and* explicate (in the sense of Bohm) meta-algorithmic process-form which is the Klein-bottle [109, 13]. Departing from this primeval semiotic codification of torsion through a distinction on a plane, this introduces two states: the ‘empty’ undistinguished state which is given by the plane itself of all potential undistinguished states (the Plenum, rather than the void in spite of being empty to perception), and the ‘distinguished’ state codified as the distinction sign. The cleavage of the state of all potentialities under the distinction sign generates a process through the sign as a boundary that sets an ‘outer’ and ‘inner’ domains and thus the distinction becomes a logo-physical generator. Thus is established a non-dual process of content and context mutual transformation, in which the distinction is both operand and operator, and form and function become fused. By functioning as a boundary, this transversion of the sign-boundary is an abstraction of osmosis, which is related to torsion [105]. In terms of logic, in first examination this leads to Aristotelian-Boolean logic [133]; on raising this distinction through self-referential paradox we obtain the Klein-bottle [103] and this leads to the generation through time-waves of a 4-state logic from which was derived Matrix Logic [109]. This reverses completely the historical Western tradition of disregarding paradox as nonsensical, and thus paradox was proved to be the basis of logic. Again, this is possible because content and context conform an holonomic process-structure, rather than distinct related instances.

⁴⁶For a different conception in terms of Endophysics we refer to [118], and to the theory of fractal time due to S.Vrobel [149]. Remarkably, inasmuch matrix logic has a projective structure as well as the eigenstates of the mind apeiron, Saniga has proposed a theory of ‘altered’ mind states in terms of Cremona transformations which arise as well as blowups [14]. For a geometric formulation of noetic space we refer to Amoroso [3].

This participatory constitution of the geometry of space and time stands in sharp distinction with the Cartesian Cut in which space is exterior to the subject and a mere container of objects, while time is also considered to be external, though in General Relativity it is constrained by relations with the space variables. In Aristotelian thought, the cradle of Western scientific thought, we have as its backbone a dualist conception which expresses itself in the two-valued logic and the principle of non-contradiction, which is no longer valid in Matrix Logic nor in the calculus of distinctions with paradox. The dualist conception eliminates subjectivity from the universe [109]. Gunther's stance -departing from the Hegelian conception that time is related to logic- is that the elimination of time is related to the principle of non-contradiction [39], consistently with the findings in our work that its violation leads to time-waves. Further, in Matrix Logic time appears as an operator, TIME which we introduced in eq. (66), which is the matrix representation of the commutative square root of minus one which is the basis of imaginary numbers, an anticlockwise $\frac{\pi}{2}$ rotation in the cognitive plane of all cognitive (vector) states which we encountered above. We have seen already that the quantization of any cognitive value and Self as a fixed point of any cognitive state cognition arises from the Logical Momentum (Cognition) Operator, M , defined by the commutator of the False and True Operators -not (dual) scalars as in Boolean logic- which decomposes as $M = \text{TIME} + \text{SPIN}$. A torsion vector appears in two guises: as the coefficients of the self-referential structure produced by these operators, and as a superposed state of the difference between the normal unit vectors to a Moebius surface, i.e. it is given by $S_- = (1 \ -1)$, while the anticommutator of these operators yield their sum with coefficients hence given by the other superposition state given by the sum of the normal unit vectors⁴⁷. Now, we recall that $M = |S_+ \rangle \langle S_-|$, the tensor product, and the direct sum modulo 2 yields the Klein-bottle information matrix, $\mathcal{H} = |S_+ \rangle \oplus |S_- \rangle$, which -up to a normalizing factor- is non-other than the Hadamard gate of quantum computation; both played a major role in the theory. From the association of SPIN with space torsion and since TIME is a rotation on the cognitive plane generated by the true and false states, it is clear that cognition is represented by a vortex structure-process projection on two-dimensional cognitive space! We shall see later on that this is the case of somatosensory and visual codification in the neurocortex, in which each point of the planar neurocortex codifies through a whole columnar arrangement on which a Karman vortex structure is occurring.

7.2 CONTROL, WILL, SELF-REFERENCE, LIFE AND THE TIME OPERATOR OF MATRIX LOGIC

We want to explore the relation between will and TIME as introduced in Matrix Logic which is a self-referential operator (either by distinguishing two arbitrary cognitive states by computing their difference or by geometrically being a simple ninety degrees rotation, which thus if iterated by another three rotations leads to the identity) in a self-referentially constructed logic that has as its roots the Klein-bottle. This will be of paramount importance in examining later the role of time and self-reference in cosmological and human systems, and basically with regards to the existence of time waves -which we already discussed in

⁴⁷ $\{\text{False}, \text{True}\} = \text{False} - \text{True} \neq 0; \{\text{False}, \text{True}\} = \text{False} + \text{True}$

[109] that are appearing in very diverse natural (and particularly, human) systems (including Mathematics), and cosmology as well.

We start with a digression on the construction of number systems from the notion of primeval distinction. Indeed, with this concept of distinction and its interpretation as a boundary, we can construct numerical systems in the so-called boundary mathematics [57]. To construct the complex numbers as spatial forms, only three distinctions are necessary to generate them. Hence, this conception under this extra provision of three distinctions allows to construct the number system which allows to treat time and subjectivity : the imaginary numbers [135]. Furthermore, the time oscillations which in the present theory based on a single distinction and the Klein-bottle generate the 4-state imaginary logic of the calculus of distinctions which itself generates Matrix Logic, appear without recurring to the paradoxical equation that leads to the Klein-bottle, as an oscillation of the empty state and the *phase form* (which appears to be its self-inverse) created by the juxtaposition of the three distinctions [57]. If we think of each spatial distinction and its meaning as a boundary initiating a process which has a time interpretation as is the calculus of distinctions, then we have three different time distinctions associated to these three spatial boundaries. The first distinction creates a primeval cleavage, space, and the process of going through it, velocity; the second distinction creates a cleavage on the velocity, i.e. acceleration as a process through it, and finally the third distinction creates a cleavage on the accelerations which as a process introduces the time derivative of acceleration, thus a third order time derivative on the original undifferentiated (which is empty, to Spencer-Brown [133]) plenum state is associated to the three cleavages which allow us to construct Matrix Logic, and hence relate it to quantum field theory and the Klein-bottle. But a third order time derivative introduces a derivative of force which is not associated with determinism but rather with *control*, which is absent in Newtonian physics due to the fact that it cannot be used to predict. Yet, being related to control, it thus has an element extraneous to the Newtonian conception. Indeed, it is a subjective element linked to will, intention, purpose, which are essential characteristics of life, by which monads (i.e. entities which come to be through the semiosis of having a primeval distinction constituting their identity) can become autopoietic [109, 146]. Without will, intention, purpose, action would be purely automatic and assimilable to determinism. Thus, it would not be related to self-organization and self-reference, and the quantum world would be inexistent. As we discussed already, this would be tantamount to the dissolution of Being.

Remarkably, A.Young -following a lead by Eddington- introduced a $2D$ flat space (a plane which we can trivially identify point to point with the space of cognitive states in Matrix Logic) of phase representation (basically, a circle or better still a *cycle* in the complex plane), the cycle of action, which stands in place of *processual time*⁴⁸ as related to *action*, by rotations in the range of 0 to 2π (where we recall that the 2π *uncertainty* factor appears in quantum physics⁴⁹ in the commutator relation $[p, q] = 2\pi\hbar$); Young with unique insight

⁴⁸Young points out that processual time is about *timing*, or, in our terms, through a willful operation by and through time which in our conception is embodied in the joint constitution of space, time and subjectivity. There is a conception of a time operator with different roots than the present one, proposed by the Austin-Brussels school led by Prigogine [97, 5]; we shall discuss it further below.

⁴⁹In Eddington's theory of processes, it is the value of the curvature of space in the hypersphere with the

associates this phase with a *choice* of timing. We observe here that Young takes the direction of action to be anticlockwise and we can as well introduce the time operator TIME in Matrix Logic, with the opposite definition as well, and thus coincide with Young's determination of direction of rotation and thus of processual time. In the elaboration by Young of the setting due to Eddington, the null rotation establishes space (with dimensions of length, L), the first ninety degrees rotation establishes velocity (dimensions, $\frac{L}{T}$, L length, T universal time [51, 52, 53]), the additional iteration of another second ninety degrees rotation establishes acceleration (dimension $\frac{L}{T^2}$), and the third iteration which amounts to a $\frac{3}{4}$ turn of the full cycle establishes control, which requires a third-order derivative in T , since the velocity and acceleration require first and second-order derivatives in T , respectively. Thus, a three

added phase dimension, thus unifying quantum physics and gravitation. Eddington's approach departs from considering that both waves (as in the present theory) and curvature (as in General Relativity, GR) can be used to represent energy and momentum distributions, and they should not be confused. We have shown in the first part of this chapter and in several works [103, 105, 106, 107, 104] that spacetime geometry is associated to waves and it is fundamentally quantum, in distinction with GR, and yet in consonance with Fock's interpretation of energy-momentum densities associated to propagating singularities. These torsion geometries can be introduced by simple scale transformations on reference frames of *flat* spacetime by the action of wave functions [102, 113]. So, in distinction with GR, spacetime can be metrically flat and yet have a non-trivial geometrical structure, which we have seen is fundamentally quantum and related to the fusion of an objective and subjective realms, which of course, is absolutely absent in GR. An essential manifestation of these geometries is the appearance of Brownian motions as spacetime constituting quantum fluctuations, fusing geometry and randomness [102], the latter itself is both 'objective' and 'subjective' [11]. Thus, scale uncertainties play a basic role since they will yield uncertainties on the distribution of energy-momentum; in this conception, these uncertainties are unseparable to the joint constitution of spacetime and subjectivity through torsion. Since probability distributions come in conjugate -in the sense of complex numbers- pairs, their product gives the probability distribution of energy-momentum, and the variable conjugate to the scale is the *phase*, such as appears in the Schroedinger equation and its connection with torsion generated Brownian motions which have both a future-oriented and past-oriented waves [107]. Hence, time is already incorporated in these probability distributions [102]. Since there is to be considered the case of a reduction of scale to particular eigenvalues, the scale is related to momentum and to the phase as a coordinate. This phase coordinate (basically the imaginary exponent S -so that the uncertainty factor is related to *oscillations*- of the relativistic Schroedinger wave function [104]) is represented as a fifth-coordinate to (a now flat) spacetime. Metrics have no role whatsoever on this generation of spacetime; as the Bianchi structure equations show, torsion is more primitive than curvature [33]. Thus, scale and phase are invariant by Lorentz transformations, and more generally, by its double covering group, $Sl(2, C)$, which is the group of rotations for Special Relativity and relativistic quantum mechanics, embodied in the Dirac equation and spinors. The scale uncertainty is a fluctuation of the standard, actually creating spacetime starting out of the undistinguished Plenum. These fluctuations are reflected in the measured characteristics of the system. If we take as the original standard to be provided by angular momentum, thus scale momentum is an angular momentum and the corresponding phase coordinate is an angle. (In fact, in the infinite dimensional spinor representations of $Sl(2, C)$ [36], in terms of which the Dirac equation of relativistic quantum mechanics comes to the fore [113], these phase coordinates are a hidden invariant degree of freedom for spinors. Thus, the phase coordinate is indeed related to (quantized) angular momentum, spin, and gives rise to a universal spin-phase-bundle on spacetime [113], as an additional phase degree of freedom for each point of spacetime, exactly as originally conceived by Eddington.) So *fluctuations* of the phase assume a value between 0 and 2π in which the fluctuations take a uniform distribution over the whole range of 2π . This range takes the role of providing a fundamental scale upon which it is possible to pass from the quantum discrete domain to one in which we can assume continuity. We note here that fluctuations are described in terms of probabilities. Yet, as stressed by Costa de Beauregard, the concept of probability is jointly objective and subjective, "providing the hinge around which matter and mind interact"; [11]. So there is a fusion of objective and subjective realms in the introduction of the space of phases, which is reinforced through its connection to spinors and the Dirac equation, as explained before.

time iteration from the initial position that stands for space distinction, by three ninety degrees turn, leads to control and thus to will, and by applying a final turn, we return to the initial distinction, which is space, and the rebirth of action. (This fourth turn is the Eternal Return of self-reference by the action of self-control, and thus it represents the jump from first-order to second-order cybernetics [146] where now action and self-reference are identified bringing forth the joint creation of spacetime and subjectivity [109].) This can be seen in taking in account that since $T^4 = 1$ (the fourth derivative is the identity, yielding position since the action of control is purposefully applied in view of a spatial coordinate) in the space of phases; the control of force is parametrized by the multiplication of their units, i.e. by computing the multiplication of control with that of force: $\frac{L}{T^3} \frac{ML}{T^3} = \frac{ML^2}{T^5} = \frac{ML^2}{T}$, the units of action. In our conception, in regard to the previous introduction of TIME in Matrix Logic (which is a $\frac{\pi}{2}$ anticlockwise rotation in the cognitive plane of bras and kets), this is the second-order cybernetic self-referential action by which we return to the initial point of the cycle, by applying a final ninety degrees rotation, the action of TIME applied on TIME^3 , to yield $\text{TIME}^4 = I$, the identity.⁵⁰ This coincides with the action of processual time to return to Self: Self-reference, returning to space -the initial distinction-, closing thus a 4-cycle in phase-space; see page 51 in Young [156].

These ideas were crucial to the development by Young of the control system for the helicopter. As Young remarks with outstanding insight, control is the essential property of *Life*; its being is related to purpose-intention and volition, in other words, to will, and as such it is anticipative yet uncomplete, requiring another $\frac{\pi}{2}$ rotation in phase-space to reach back to the self, i.e. to become self-referential. Thus we return to our previous disquisitions, in which we associated purpose-intention to a time operator, as the meanings of these nouns clear suggest. Young still associates this to the fourth level of descent of processes on which degrees of freedom are progressively lost, starting with the self-referential photon as the exponent of the highest potentiality. Thus is established an evolution from the homogeneity (of the undistinguished Plenum), the photon being the source of all inhomogeneities, as we elaborated already) to inhomogeneity (say, complex molecules, DNA, etc.) as the fundamental units from which Life emerges, and which Young with great insight associates with will, as an essential precondition. So Young argues in regards to the cycle in the space of phases (processual time) that, on reaching self-referentially to the identity, we find a final degree in which the monads (i.e. the entities formed by the primeval distinction) start to build up their own structure countering the action of increasing entropy, as is the case of complex molecules, such as the case of viruses, *through timing*. In our conception, as we have seen above, this is due to the action of TIME on the control phase. Returning to Young's view, which we state again with a slightly different wording: In this evolution from the photon -which has the maximal degree of freedom-, from which a process of individuation starts into the more complex structures, as those that we associate with Life, such as complex molecules which have lesser degree with respect to the photon which is the building process from which all more complex structures are formed of, there occurs a

⁵⁰That time might operate in the quantum domain producing interference (and thus indeed is associated with control), is an experimentally verified fact; its theoretical framing is due to Horwitz; see [53], and references therein.

return by willful action (i.e. self-control) by which complex molecules chose their processual timing to counter the environment in which the monads are in exchange relation through their primeval boundaries, exercising thus their sole freedom left to reverse the condemning determinism initiated by the first two derivatives in universal time, through control and self-control, which amounts to the self-referential action of TIME. we encountered in Matrix Logic. ⁵¹ This is essentially linked to will and self-determination. Hence, the “free will” so much purported in the history of philosophy and diverse theologies, is the will of self-determination by the action of TIME. This is an original conception; we shall retake it in examining the Myth of Eternal Return, departing from this conception. ⁵² We shall also see the relation of Life as process-form, in the explicitation of the Myth of the Eternal Return, which we shall elaborate further below. We must recall here that the initial process that lead to TIME as appeared in Matrix Logic and our previous considerations, which we have identified with the processual time of any system defined by a boundary creating thus the multivalued logic that arises from paradox, is the Klein-bottle. Therefore, the *topology* of the time operator embodied in all systems and particles of any scale in their phase dimension which leads to control and self-control, is represented by the Klein-bottle. We shall reencounter it when dealing with the physiology and growth process of the neurocortex, sensorial codification and self-referential growth of bodily organs. ⁵³

⁵¹We are exercising a very different conception than the emergent non-computability conception due to Penrose for which Life will be another unexplained emergent factor requiring other factors, and thus ad nauseum. Our conception also greatly differs with Prigogine’s which, while attempting to introduce a time-operator in a general systemic approach based in dynamical systems theory and chaos, choses the second law of thermodynamics as a *metlaw* with causative precedence to any other law of physics and Nature at large [97]. We shall return to the implications in terms of ontology of this choice.

⁵²Thus, the so-called *arrow* of time which has been extensively discussed in physics and biology, is *subservient* to this timing process, or otherwise we would have reintroduced a dualism which we have already shown that it is not the case. Related to this issue we shall discuss in next Section the notion of *chronotopology*, the topology of processual time. Since this topology is non-linear, basically the Klein-bottle, the arrow of time is determined by the linear sections of the chronotopology of structures-processes which by self-referential timing come to be. It is clear that third-order derivatives notoriously absent in physics and biology point out to processes which may violate the second law of thermodynamics. While it is claimed that the first and second-order derivatives in time equations of physics are time-reversal, the Brownian motion quantization of the Schroedinger equation in which the drift is described by the trace-torsion, may *not* be time-reversal. (Remarkably, it is through the random fluctuations (say structured as Brownian motions) of the internal parameters of a system, even in the minimalistic case of absence of heat exchange through its outer boundary, that a system can violate the second law of thermodynamics, through fluctuations of the random entropy; we shall return to this issue below). Indeed, when the trace-torsion has, in addition to the logarithmic gradient term encountered already for describing quantum jumps and the Aharonov-Bohm effect associated to it, other electromagnetic potential terms, then the Brownian motion is not time-reversal [106, 104]. Furthermore, the actual construction of the solutions of the Schroedinger equation require initial and final conditions on a time interval. Note added in proof: On completion of the revision of this article, we found in [50] very interesting discussions on processual time, Bergson’s duration and the physics of life, which seem to be quite in agreement with some of the conclusions presented here.

⁵³One can ponder on the possibility of Mathematics (as a system) having a time operator acting through rotations which is further related to self-reference. We already discussed in the Introduction that this is already the case of the Continuum Hypothesis [47] and also the generative structure of the natural numbers and in particular of *all* primes, from a planar structure of an original set of natural numbers, which by rotations produce the complete system simply by recursion [60] .

7.3 CHRONOTOPOLOGY, THE MYTH OF THE ETERNAL RETURN, SELF-REFERENCE, TIME OPERATORS AND THE KLEIN-BOTTLE

Chronotopology is a far reaching concept which was first proposed by Musès in his outstanding work ‘Destiny and Control in Human Systems’ [83] in which he anticipates a theoretical framework for the understanding of time patterns and structures (chronomes), and their relation with psychological archetypes. The initial discussions in this pioneering work focus on natural systems which are found to have non-linear forms-functions to later proceed to analyze human systems. Musès was very much concerned that -to his understanding- Mathematics was not the adequate language for human systems, and in this unique work he proceeds to analyze a very ancient myth. Musès shared with Young the idea that myths are accounts of processes and as such they should be studied. Myths have played a fundamental role in framing human history and till today they continue to do so. Polymath Raju proceeds one step ahead of Musès in his fundamental work, ‘The Eleven Pictures of Time’, to show that the action of myths is all pervasive, up to the point that contemporary theoretical physics is very much framed in terms of them. In particular, the reigning conception that time is a mere parameter without any functional role, and furthermore it is *linear*, instead of having a non-trivial topology, is one such conception that has its roots in the political management of myths. Raju traces this prevailing conception to political-theological-religious issues that were decided in the Council of Nicea, at Byzantium, circa 340 C.E.; this linearity has far reaching implications even in framing our socioeconomical and cultural mindset and organizations [101]. This present chapter is intended to be a contribution to correct these apriori’s which are not of the Kantian type, but rather forcefully implicit to our lives due to decisions that acted out an all encompassing anthropological engineering operation, which hundreds of years later is still efficaciously weaved well enough into the present mindset ⁵⁴ to the point of have gone unnoticed for our current generation at least, until Raju’s work, to the author’s best knowledge. ⁵⁵ Returning to Musès studies

⁵⁴Who has not encountered the proverbial fool that passionately utters “Bing-Bang” to triumphally hastily mumble next “God”?

⁵⁵The history and current developments in science, particularly in physics, are rife with these cultural engineering operations. They became especially notorious with the advent of postmodernism. We have nowadays thousands of physicists working in a purported “Theory of Everything”, the so called string theories, which in fact there are an infinite number of them. Remarkably, none of them can be subjected to any kind of experimental validation or attempt to refutation. So these theories came to be qualified as ‘not even wrong’ by its critiques. Thus, this postmodern deconstruction of physics, which in this setting ceases to be related to experience, is a totalitarian posture claiming to be a theory of everything that there is to be known (about physics, or is it about ‘everything’? ...), a claim that does not stand the most candid consideration. (Of course, would we fall into the megalomania of claiming that a ‘Theory of Everything’ is at the disposition of science, then a first test of consistency would be to check its ability for explaining how the ‘theory’ was created in the first place! Of course, Goedel’s theorem will be invoked to dodge the test, but then the second test will be to check if the ‘theory’ is able to explain how the Goedel theorem came to be. Then, as explained in the Introduction, arguments like those expounded by Penrose will be invoked, falling thus into the infinite antiregress that we alluded in discussing these arguments for an emergent consciousness, or we may consider the logical shortcomings of his approach related to his choice of Aristotelian-Boolean logic’s dualism. Consequently, it follows that the purported ‘Theory of Everything’ is an unending theory

in interpreting myths as processes, he chose one myth which has a cosmological setting, but embraces all natural systems including human systems, of course. Musès claims that his rendering of the myth in question goes back to civilizations dated thousands of years ago, and was the result of his arduous -and non-conformist with respect to other scholars take- investigation in the Tayyibite Shi'a tradition of Islam, preserved in Medieval Yemen traditions and manuscripts dispersed through European libraries [37]; Musès in addition of being an outstanding mathematician (alas, not recognized much as such besides few) was a man of extraordinary qualifications and insight. It is none other than the Myth of Eternal Return, in a particular setting. This is an awesome read, and proceeding from the conception presented in this chapter, we shall unfold the issue in terms of self-reference. Musès account is very different to the more popular and somewhat naive account due to Eliade [29] in which there is no systemic approach. Musès processual approach was focused in trying to clarify a most difficult problem: To elicit and understand the origin of Evil (and consequencely also of Redemption) which he links to the will of a Third-Intelligence-Entity produced from differentiation of the Unmanifest, to self-referentially fuse with Apeiron and control it. The Unmanifest (which in Greek philosophy corresponds to Apeiron), the field of all potentialities, in our conception is the undifferentiated state (the *Plenum*) which upon introducing a primeval cleavage (torsion) will embody the fusion of object with subject and the cocreation of world and subjectivity, similarly in the latter issue to Spencer-Brown's protologic. The introduction of this primeval cleavage in this myth finds its expression in the First-Intelligence-Entity ('Aql, in Arabic), which is associated to Mind (with its entire gamut, including understanding, insight and spirit. It is also called Sabiq, the one who goes before time or space; according to Musès, in German it corresponds to *Verstand* rather than

of emergence, or simply logically untenable, and then totality is proved to be unattainable, contrary to its claims.) Certainly, such a theory of 'physics' has no qualms whatsoever in being unrelated to cognition in any sense, with exception of the Cartesian mode, which finds in this intellectual exercise its epitome of total disociation with reality (the *res extensa* of Being). If the question is on its any relation with psyche, rather than physics, the answer is positive, yet it is the unfolding of a mind which though embodied in the physicist's body, has resigned to all criteria of validation with exception of its self-*satisfaction* on its intellectual power, a form of narcissism. It is an alienated self, that claims complete dominion, through the theoretical framework, on the universe. Nothing else than the nineteenth century' positivism, extended from determinism to the purported quantum domain. Another similar operation that appeared concurrently to strings theories, followed the proposal by Prigogine of a theory for everything, based on dynamical systems, chaos and the imposition of the second law of thermodynamics as a metalaw for Nature. Similar to the flockings of physicists to the *appeal* of strings (indeed, fetishism is the case), postmodern academics - especially, in the human sciences- were happy enough to find in those digressions on chaos, order and the action of a time operator, a fertile territory for poesis; Prigogine, in his theory of processes [97], claimed to have established a physical territory for its reification [130]. These kind of operations were traced back to obscure aspects -and also to the genuine ethical quest to reach for an embodied conception by its founders!- of the Copenhagen interpretation of quantum mechanics- Beller, "...The opponents of the postmodernist cultural studies of science conclude confidently from the Sokal affair that "the emperors have no clothes...But who, exactly, are all those naked emperors? At whom should we be laughing? " [10]. (Rather, we understand that the issue is: At what we should be laughing?, granted that the natural attitude with regards to our folly, natural and engineered ignorance, and our graced quest for integrity, is spontaneous self-referential laughter, rather than crying.) We shall later return to Prigogine's proposal. For an attempt of saving strings theories of the inconsistency of its basic postulates (albeit not from its lack of grounding, an enterprise of no concern to strings' theorists), rooted in the topological phenomenology further developed in the present chapter, we refer to [117].

Vernunft), placing the first differentiation in subjectivity, as a transform of the Unmanifest, Apeiron. Our interpretation, of the First-Intelligence-Entity, which stems from our conception, is as follows: It is the primeval distinction that we have encountered as the sign in the Plenum, signifying the torsion field and the appearance of the Klein-bottle logic, in considering the paradoxical reentrance of the First-Intelligence-Entity in itSelf. It appears before time or space because it embodies the will of the fusion of subject with object in an integral process that finds its reification as both space, time, and mind, now as the psyche of Apeiron, in the account by Musès. The Second-Intelligence-Entity is a further stage of differentiation, produced by an *emanation* of the First-Intelligence-Entity, and thus light is brought into the world as *will* (which we have already related to the action of a time operator in all systems following Young’s musings and to torsion in both cognitive space (TIME Operator, see eq. 68 and the explanation after eq. (71)- and spacetime, both producing quantum jumps as differences producing differences) of transcendence through its reification implicit in Mind. The Third-Intelligence-Entity, according to Musès, is a further emanation of the Unmanifest, “ is called *Adam Ruhani* (in Hebrew), or the Spiritual Prototype of humanity, in whose image we were formed. Thus the Third-Intelligence is the Divinity of our universe which has not yet come into Being”. Its attainment is Redemption. “It corresponds to the great Anthropos Megos of Valentinian Gnosis”. We continue, Musès: “Now there was no problem even conceivably arising with the first two Intelligences who realized that the source of their being was in the Unmanifest”. Further, “...the main line of the dénouement of the origin of Evil in respect to the nature of Time, the Third Divine Entity sought to encompass its own origin and to plumb the very depths of being- a route that perforce have to lead into the Unmanifest, the Mystery of Mysteries that by its very nature cannot be unveiled with impunity to the one so seeking; the *veiling is inherent* and necessary for the eternal provision of immortal being. The notion echoes in the words of the great Goddess inscribed on the portals of the now lost Temple of Sais, preserved to us by the records of ancient travelers: “None can lift my veil and live”. To seek to manifest the source of life out of the Unmanifest, could end only in cutting himself off from the circuit and flow of life in so trying, even though unwittingly, to preempt it. No manifest being can contain the Unmanifested”. We already observed [109] that in the present conception of this chapter, the undistinguished protological state (the Plenum) cannot be merged with the distinguished state under the primeval distinction, unless both states would logically collapse into nothingness together with the ‘objective’ and ‘subjective’ realms. Musès: “The Third Divine Entity dreamed such a dream of finding that source explicitly and *controlling* ⁵⁶ it to be within himself (as he mistakenly thought was the case with the Second and First-Intelligences)”. ⁵⁷ Thus the

⁵⁶Our italics, which we introduce to remark that we already related control to a distinctive logo-physical action which is related to subjectivity and the operation of TIME in Matrix Logic.

⁵⁷The mistake attributed to the Second-Intelligence is the ‘Great Doubt’, the belief of the dissolution of light-Self to nothingness, proposed by Varela and associates in their take on enaction. Thus, it is the mistake of nihilism. The mistake attributed to the First Intelligence is the belief that the primeval distinction can self-obliterate itself, which in first examination, *formally*, it is no mistake at all. It is the cancellation rule in the calculus of distinctions of Spencer Brown [133], in which the self-operation of the distinguished state on itself gives the undistinguished state, Apeiron; it is a primeval rule of nilpotence [109]. Yet, this return to the Unmanifest through this operation does not erase the ideative process which is used to produce the primeval distinction and its recursive application; it only erases the *sign* of the primeval distinction.

Third Divine Entity willed to dissolve into the Unmanifest while keeping the will to self-referentially complete its own cycle. “That dream and wish momentary, had on the level of *power and perception* dire consequences, the first of which was *retarding* of the consciousness of the Third Intelligence by reason of this thus introduced blockage or fallacy that by nature could not advance, but only hold back.....The reason for such grave consequences of a released desire on the part of the Third-Archangelic-Power is bound with the fact embedded in ancient traditions preserved in Homeric Greece of the mere wish or propensity of a god being equivalent to the determined or implementing focussed will of man”. (Here we find an issue that we will reencounter later -when discussing visual hemispheric synchronization to attain stereoscopic vision-, namely the idea of hierarchies of Klein-bottles (which we shall call it quantized reentering limited domains, that we shall link with the syntropic TIME Operator) which we recall require a certain Planck action ‘constant’, since it is multivalued [87, 88, 93] to establish themselves as primeval distinctions reentering in different stages, a fusion of macrocosmos (a god) with microcosmos (man-woman), for better illustration). Musès: “But the implications go deeper, since the reason that it is so depends on the facts that the gods are not in our kind of time”. (Again, the issue of a hierarchy of Kleinbottles that we have just claimed). “Duration of things ⁵⁸, yes, and changes, too- but all without waiting time, which is the chief characteristic of what humans call time. We must wait for any idea or plan to be enacted and then mature to fruition or full manifestation”. We remark here that in Musès’ interpretation, the wish to *control* the source, the Unmanifest, was the source for Evil, and that *timing* (here again the time operator, which we identified with TIME of Matrix Logic, introduced in the discussion of the Eddington-Young phase space) is crucial for the manifestation of the Divinity implicit in the Third Intelligence Entity. Thus timing is essential to the very process of evolution as the unfolding of the primeval distinctions into the manifest forms-functions that make the universe, and is the

Therefore, if there is a legitimate reason to qualify the self-obliteration of the First-Intelligence as a mistake, it is founded in the belief that *all* phenomenology is exhausted in the sign. A sign requires a signifier, they are fused, as is the case of the primeval distinction. Thus, the mistake attributed to the First-Intelligence is the belief that its Self, i.e. Self-reference, is not transcendental. (Furthermore, it is the mistake of separating immanence from transcendence, in essence, the Cartesian Cut.) For example, it is the mistake produced by Penrose in his defence of his approach to consciousness as emergent by *using* self-reference, while *denying its significance and Being*, as manifested in Chapter III of his ‘Shadows of the Mind’, as we explained already. It is also the mistake of physicalism and, no less, of fetishism. From our previous discussion, it appears that nilpotence, which we chose to call plenumptence, for reasons that have been manifested along this chapter, is a transcendental property, and that nilpotent operators are transcendental; thus, the great importance of algebras rich in nilpotent elements: They represent higher-orders of Self [82]. Such is the case of the Cognition Operator, M. The present findings give retroactive support to the approach that stemmed from the eikonal equation of light rays, our departing point in the present chapter, and to the work of Rowlands and Hill [119]. Thus, the present conception can explain its self-generation as the transcendental action of self-reference.

⁵⁸We here reencounter time as duration as in Bergson’s critique to the time parameter of Einstein [90, 11]. Yet, according to Costa de Beauregard, Bergson’s duration is still associated with a linear topology in his belief that the future can in no sense pre-exist and in his insistence on the irreversibility of physical causation [11], which contradicts the Myth of Eternal Return. We shall later reencounter a quantization of time perception following Bergson’s intuitions. For a elaboration on a hierarchy of causation which appears to be the one corresponding to chronotopology and includes physical causation as a particular case, we refer to the differential epistemology by Johansen [58]. We shall not, due to lack of space, enter into considerations of this fundamental problem, though we shall touch upon it along our presentation of our conception.

will of complex structures to reverse determinism by seeking to self-referentially determine themselves through self-organization by countering the increase of entropy, as proposed by Young.⁵⁹ Recall that in the previous section we have linked intention-control to the action

⁵⁹Thus, since by timing Life be-comes, the second law of thermodynamics cannot have the ‘upper hand’ in the universe, as an entropy increase, as usually claimed. (As we said above, our conception is, in this sense, opposite to the one advocated by Prigogine, which pledged a metalaw character of the second law of thermodynamics [97].) Though not widely known, the mathematical theory of statistical thermodynamics already provides a proof of this: The *random* entropy corresponding to *internal* random parameters of a system (which may have its origins in the fluctuations of Apeiron) of a system which can even exchange no heat with the environment, can fluctuate and *decrease*, so that “...microviolations of the second law of thermodynamics are possible”; see pages 44 and 38 in [136]. We all know too well that Life makes of these ‘violations’ its *rule*, rather than the exception that the Cartesian minded physicist is eager to assert as the uncontestable truth. In fact, as kindly pointed out to the author by Prof. Jeremy Dunning-Davies, a notable scholar and contributor to the foundations of thermodynamics: “It’s also important to realise that the claim that entropy never decreases is NOT a statement of the Second Law; it is merely a result that may be deduced from that law in certain circumstances”; we are deeply grateful to Prof. Dunning-Davies. We repeat, these microviolations (the random entropy will decrease) can occur even in the case of a system which does *not* exchange heat with the environment [136] for which the second law is valid so that the entropy of the system does not decrease; of course, the latter description corresponds to the *outer* boundary of exchange of a system with its environment, the usual classical mechanics conception that neglects the exchanges of a system with its *internal* frontier, which is the Plenum. These internal exchanges are at the basis of claims of over-unity energy devices based on the zero-point fluctuations of Apeiron. (We recall here that randomness is both subjective and objective [11], and that when randomness is structured as Brownian motion, which is the one of the main cases considered in statistical thermodynamics (as processes verifying the so-called Markovian condition)[136], it is associated to geometries (either on spacetime or the manifold of thermodynamical variables) with trace-torsion [104, 102, 103, 105, 106]; these are the zero-point fluctuations of Apeiron, and in the case of the electron, they appear as the jittery Zitterbewegung [107]. So, the microviolation of the second law of thermodynamics installing the primacy of Life and syntropy, can be traced back to the torsion fields which constitute the primary joint constitution of the ‘objective’ and ‘subjective’ realms.) Furthermore, the microscopic character of the internal parameters of a system, claimed to be the case by statistical thermodynamics, strictly speaking can be taken to be the internal parameters in an arbitrary pair of Klein-bottle, one contained in the other, which can be established as tagged with different scales. Brownian motion need not be microscopical but is rather universal as shown in the works of the present author in which is related to torsion geometries at any scales, which as we already explained, fuse the ‘objective’ and ‘subjective’ domains. Thus, returning to the pair of Klein-bottle in a heterarchy of quantized reentering limited domains (heterarchy rather than hierarchy, because they can have different qualities, say, one corresponds to a human being, another to the society in which he is part of), in which one is contained in the other one, the smaller one has in its own scale, internal fluctuating parameters which will produce with respect to the exterior Klein-bottle of the pair, a syntropic self-organizing process, which will also violate the increase of entropy in his own inner scale, while perhaps increasing the entropy on the scale of the outer domain. We know by watching through the Hubble Telescope, that Brownian motion in astronomical and cosmological scales coexist with syntropic organization. Would we chose the microscope instead of the telescope, the observation would be the similar. The scale-dependance of the increase of entropy valid in a linear time scale does not entail its promotion to a fundamental law of Nature nor Being. The usual approach to thermodynamics -that stemmed from the mechanicistic ideas of the 18th century, the Carnot cycle- was extended by the statistical approach which fuses the objective and subjective realms, and thus surmounts the dualistic ontology, through self-reference. Returning to the ontological problems posed by the Myth of Eternal Return, we have a cognitive decision problem related to the choice of either the true or false Boolean states, which we already know are result of the Hadamard-Klein-bottle operator on the superposed non-orientable states; see eqs. (86,87): Indeed, in the terms of the Myth of Eternal Return, we either chose to acknowledge the fact that the First, Second and Third-Intelligence-Entities exist and thus Will-Life-TIME is the case and consequently the entropy increase is the case for *linear* sections of the non-linear chronotopology of living systems -rather than a universal all encompassing law-, or we dont. If we dont,

of the time operator (TIME) acting in the third state (acceleration) to produce and thus control variations of acceleration : The first distinction created space from Apeiron, the second one created velocity, the third one created acceleration and the fourth created control which was the final distinction since further iteration returned to the original distinction, since the fourth time derivative gives the identity. We repeat, the rotational action on the fourth distinction (control, derivative of acceleration, the third time derivative of space, will, Self) with the final recursion by TIME returns this fourth cleavage, control, to the identity through self-control, which is embodied in the formula $\text{TIME}^4 = I$). Coincidentally, Musès will find in the Third-Intelligence-Deity's attempt to return and control the Infinite undifferentiated Plenum the origin of the generic time-operator. The impossibility in achieving this goal, resides in that the TIME Operator returns the controlling Self to itSelf which, as we have just showed, it returns to Identity, being thus self-referential: $\text{TIME}^4 = I$. Hence, its achieved goal by this final recursive action, retakes the already cleaved state that fuses the objective and subjective realms in cocreation,. Yet, *cannot return* to the undistinguished state. The Null Operator in Matrix Logic (the Mind Apeiron), whose eigenstates are identified with the twistor representations of the extended photon that is the basis for the joint constitution of objectivity and subjectivity, plays its role. The Unmanifest is thus unreachable through the direct action of TIME and thus timing comes to Being and Becoming of all systems. ⁶⁰ To Musès, this impossible fusion produced the reification of evil

then Will-Life-TIME have no ontological locus; Will and Life lose their ontological loci and thus nihilism will be established setting the course for the proposals of both through emergency; furthermore, if this choice is made, there are no time-operators -in the sense introduced in this chapter, hence chronotologies can only be time-linear (which, by the way, neglects the existence of waves verifying non-linear evolutions equations [83], notoriously manifest and devastating as tsunamics are). Instead, the usual physicalist approach to Life in terms of *negentropy*, which is further rooted in the conception of a mechanical universe, is installed inside the 'objective' side of the Cartesian Cut's discourse. (Note added in completion of this chapter: A notable exception, there might be more, unknown to the author, is the work by Mae, in which processual time is discussed. Prigogine also discusses a processual time, from the mechanical pledge of the second law of thermodynamics as having the upper hand on Nature [97].) This second choice is the one taken by positivism and nihilism, and already incorporated into the Myth of Eternal Return. As trees are known by its fruits, the result of the second choice is the current predominant, but not for long, predatory mode. In the other hand, the present ontological considerations point out to the predominance of *syntropic* processes, rather than entropic processes. These are the product of the TIME Operator as the action of will-intension to reach self-reference. In terms of the phase-space of Young-Eddington, we already remarked that can relate it to the internal invariant phase hidden in the spinor infinite dimensional representations of $\text{Sl}(2, C)$, the (double covering) group of Lorentz transformations. We remark that these transformations are the linear complex (i.e. the matrix elements are complex numbers) transformations with determinant equal to 1 on the complex plane, which of course, we can chose to be the cognitive plane in Matrix Logic ! So, again, we find a representation which fuses the physical (the double covering group of the transformation group of Special Relativity, which is the group of rotations operating in the physical realm) with the subjective (will, control, through the action of the TIME Operator) realms. Related ideas on the predominance of syntropic processes have been proposed by Johansen, expressed in a private communication.

⁶⁰Then, as it is impossible to return through the action of TIME to the origin, the Mind Apeiron $\mathbf{0}$, the only way of returning to it is through the manifold polarizations of 0 and $\mathbf{0}$, the Mind Apeiron. Particularly, through the nilpotent (self-referential) polarizations. We have presented very important nilpotent polarizations already: the Cognition Operator, M , produced by the torsion of cognitive space, which contains TIME and SPIN in its decomposition, being the most important one. So, embedded in the Cognitive Operator resides the timing that allows systems to self-organize countering the increase of entropy; notice that this is a topo-logo-physical process. Thus, we can return to the Mind Apeiron by considering in addition to TIME

and the downfall (we prefer to call it an *unfolding*, as in the conception due to Bohm) of the Third-Intelligence-Entity in a process of individuation which in seven distinct logo-physical-psyche-archetypical phases will regain full self-reference, completing thus the Eternal Return in which the Tenth-Intelligence-Entity returns to the Third-Intelligence-Entity which is not the same state as the initial departure; in completing its full process of individuation beings become Being. Thus Redemption is achieved in this awesome cosmology, that we have presently unfolded in terms of the fundamental principles we have proposed in this work, differentiation, time-operator, self-reference, paradox and the Klein-bottle. ⁶¹ In sharp distinction with the usual dualistic platitudes that place Evil as dual to God-ness, justifying thus the former and countering by this take the natural action of Time, for Musès -and the present conception as well- there is a fundamental asymmetry, which is the parasitic character of Evil vis- à-vis Deity. Indeed, without that asymmetry, the recursion would be a repetition of sameness (in fact, there would be no recursion at all!), the trivial homogeneity which we claimed to be imaginary and the Return would be unachievable because the process of individuation, under the condition of homogeneity would not come to be because it requires an initial cleavage to start with. Thus, following Musès conception of chronotopolgy and its manifestations in natural and human systems, TIME and the Klein-bottle are the operators and operands of the Eternal Return and the individuation of God and Nature. ⁶²

also SPIN, and thus now their sum gives the Cognition Operator, M , which being nilpotent, $M^2 = \mathbf{0}$, the return to $\mathbf{0}$ is achieved! Hence, SPIN plays a fundamental role in the evolutionary individuation process that appears in the Myth of the Eternal Return. Returning to Matrix Logic, SPIN and M in their action on either cognitive and quantum states, exchange quantum and cognitive computations through averages; see eq. (83) and the following paragraph. We recall that M has together with the identity $\mathbf{1}$ the fundamental role of projecting the Boolean states to the superposition states as characterized by eq. (88) whose addition modulo 2 allows to retrieve the Hadamard quantum gate representation of the Klein-bottle ; see eq. (84) and thus the self-referential process of creation is manifested. Other very important polarizations of $\mathbf{0}$: Also, the eigenstates of $\mathbf{0}$, which we associated to the twistor representations of the photon that arose from the propagation wave and nilpotent equation of geometrical optics with which we started the developments of this chapter. Furthermore, the polarizations that have been presented in the universal nilpotent rewrite system due to Rowlands and Hill, which we have briefly discussed already. Thus, Self can return to ItSelf, but not to the Origin (the Mind Apeiron), through the action of TIME, and torsion as the embodiment of Self-reference establishes TIME included in a polarization of the Origin which establishes Cognition. The multiple polarizations and in particular the interchangeability of M and SPIN in logo-quantum computations, yield the basis for the several (seven in Young's rendering of natural systems [156] and in the Myth of the Eternal Return presented by Musès) differentiation stages that accompany and produce the return of the Third-Intelligence Entity to ItSelf. Therefore, the Unmanifest, First, Second and Third Intelligence Entities are the primeval examples of the Klein-bottle heterarchy we mentioned before. As we can see, the full reentrance of the present conception into itself and the emanation of it from the origin, are thus produced.

⁶¹Musès further relates these ten Intelligence-Entities with the ten spheres of Kabbalah.

⁶²The content of this myth is somewhat common to many religions though with different cosmologies, and incorporated into some ideologies, as is the case of the the communist society for Marx and his acolytes. It appears as the hope of reaching a state in which Wisdom, Justice, Beauty and Goodness reign on Earth yearned by all kind hearted people. Even in some inspiringly beautiful literature works: The return of the self-ostracized-king-to-be Aragorn in reaching self-acceptation of his royal being, in Tolkien's trilogy. It appears in philosophy, as is the case of S. Rosen's reflections on the Apeiron's self-forgetfulness that achieves self-redemption, through the process of individuation to return to full self-awareness and, of course, in Nietzsche. It is further more remarkable that the conception of Musès on the origin of evil, in principle finds a reflection in the seven different divinities' (in the Babylonean pantheon) archetypes, their taxonomy and operation through time waves conforming historical-cultural patterns, that was discovered through the

7.4 INFORMATION, TORSION FIELDS, SEMIOTICS, INTERPRETATION, SUBJECTIVITY, DESIGNED AND HUMAN SYSTEMS, TIME WAVES AND CHRONOMES

In the present self-referential conception no device or designed object is psycheless. We promptly add that this psychological character resides on the homo-sapiens-faber side of the Cartesian Cut constituting the device and further relating to it, and not in its atoms, molecules, parts, nor in the material aggregate of them and their constitutive relations that appears as a distinct object.⁶³ Indeed, etymologically, ‘design’ means to signify, an intention which has a semiotic materialization which is the codification of energy, as Pattee argued [91]; we have argued that this primeval sign is the primitive logo-physical-perceptual torsion field. For a deep study on the dynamics of semiosis, its social interpretation and perception, its logical valorization and its architecture as a gestalt we refer to the work by Taborsky [138].

To avoid misunderstandings of our statement of the psychological character of deiced objects, we shall provide an example to illustrate that it has to do with in-formation, cognitive states and interpretation and thus, we reiterate, belongs to the subjectivity realm. The example is provided by the testimonies of one of the bleakest pages in human history; we believe it clarifies the previous statement. According to the Spanish narratives, upon the arrival of the Spaniard conquerors to the site of the Inca emperor of Perú, Atahualpa, following their initial criminal incursions, they introduced themselves in a rather multivalued paradoxical -in some levels- way. According to the narratives, a priest accompanying the invaders uttered to the emperor -through the services of a translator- a demand that he should acknowledge the over-all rule of their Love-based creed and their king, and proceeded to deliver the Bible to him. (As we know all too well, religious creeds no less than ideologies, have been used and abused to provide the legitimization of unconfessable drives.) Atahualpa grabbed what for *him* was a priori a mere object, examined it meticulously, and dropped it with contempt. What followed was a carnage of the Inca population by the invading forces

work of mathematician-sophiologist Páles (together in joint critical examinations and reconfirmation of his findings, by his critics at the Slovakian Academy of Sciences) [89]. The existence of cultural time patterns was discovered by anthropologist Kroeber [67] and independently by sociologist Sorokin [132], they are present as well in the economics of the capitalist system (the Kondratiev cycles), and more recently in the most diverse natural systems [41, 129]. This is the contemporary scientific field of *chronomics*. These findings point out to the universality of anticipative, incursive and hyperincursive phenomenae in the sense of Dubois [24] which are examples of the chronotopological differentiations. We shall return to chronomics in relation with in-formation, as well as to designed and natural systems, such as visual perception.

⁶³Though following our previous discussion, *all* systems have a time operator which is essential to their constitution and regulation so there is a will of all entities. While we also add that the self-referential character Klein-bottle gestalt goes down -scalewise- in its applicability to the neutron which in the conception of Young, may have its proper timing as any other particle or system. Indeed, the Klein-bottle is the topology in the Rutherford-Animalu-Santilli compressed electron-proton model of the neutron in Hadronic Mechanics, the theory of the strong interactions due to Santilli [122]. This theory -according to its founder- succeeded by superseding Quantum Mechanics deemed to be inapplicable- to give account of the strong interactions. For its relation to torsion and Brownian motions see [103]. There is a saying by anthropologists, *referring* to self-reference: It is turtles all the way down (and up, we further comment). The pun by the author with “referring” to “self-reference”, was not accidental here but fully intensional. We are indicating the self-referential being of all languages, spoken, written, gestural, etc., which somehow we fail to identify.

and Atahualpa's own demise. To Atahualpa in-formed and handy with the intricacies of knots and its mathematics (as suggested by archeological findings) yet un-informed on the Spaniards' perversely manipulated creed and its valorative, political and power implications in place (though surely he did understand what was meant by the act of delivering to him the object, a demand of surrender and more), there was no message conveyed by the invaders as the Bible was no message to *him*. For the Spaniards, the message was delivered *completely and irrefutably*. For Atahualpa, his examination of the object had not rendered him the meaning that the interpreter had demanded to comply with, and thus was unable to interpret it (yet, he was in-formed previously to find improbable the truth value of the ethical aspects of the message demanding submission).⁶⁴ Notably, the shocked Spaniard narrators explicated the fact that both Spaniards and Incas were aware and spoke out in similar instances that this was the case. As we know from daily life, action, interpretation, truth value, cognition and perception, even if incomplete, are inseparable.

To resume: this psychic in-formational character of designed objects embodies a semiotic action jointly with intention, interpretation, a cognitive state, actual cognition and perception. As we have argued, it is a codification of energy through a torsion field from which multivalued logic, its transformation into quantum physics, and time waves appear.

Returning to our explanation, any artifact, device or even a social organization embodies purpose-intention through design (demanding thus to the beholder, user or social actor an interpretation), a fact rarely acknowledged -but crucial to an integral approach to life-, which is the intent of control and thus of anticipation. The *identification* of a subject with the reification of this psychic in-formational character is a regressed form - fetishism- of animism, all too common and a major driver of technological inventions, as well as of all kinds of undesirable events (we have seen the purport of fetishism in the previous tragic history). To wit, this reification (not the identification of the subject with it) materializes the intention of the in-formation, and since intention is associated to the action of the time operator, this process of manifestation of the in-formation is the result of the action of TIME in Matrix Logic, and is further related to perception as we shall elaborate below. The identification of the subject with this reification of the in-formation, consistently with Hegel's conception that subjectivity is related to time [46] while for Gunther this relation with time is established in connection with the abandonment of the principle of non-contradiction [39, 109], is tantamount to the assimilation of subjectivity and time to the Aristotelian universe of discourse, as transpires from direct examination of this process of fetishism.⁶⁵

⁶⁴An utterance becomes a message only if a meaning is ascribed to it and shared at least partially by the would-be participants in the communication; an act of interpretation is essential to this transformation. An interpretation requires an intention, to cognize, and an action, the actual cognition. This is not the take of the modern theory of communication based on Shannon's work, for which messages are subject-independent objects, i.e. Cartesian objects, then admit a binary coding, as in Boolean logic. With respect to the logical valorization we pointed out to it by mentioning the irrefutability issue: Atahualpa by dropping the object questioned the truth value of the invaders' claims of Love, and their true intentions with regards to his people and himself. Here appears another element absent in the dualist mindset: A cognitive state is assigned to the utterance in becoming a message and it carries a truth value in it as we described above. Hermeneutics may precede the assignation of truth value.

⁶⁵A particular notoriously active form of this fetishism is provided by the fact that usually criminals -as the previous tragic story shows- purport to be acting for good or some superior source, thus collapsing logical cognitive states to one value which is false, while pathologically claimed to be true. The reification

Returning to our discussion of examples of in-formation of designed objects, at a larger scale of humankind and in the antipode of largely low degrees of freedom (until nanotechnologies appeared) embodied in the transformation of raw materials to machines, or further artifacts by purpose-intention, we consider events which as A. Young rightly remarks are associated with light [156]. These are events that may appear from the infinite potentialities of differentiation and complexization that may arise from it. It is the logo-physical-photon-primeval-distinction-torsion-field that creates events as the unfoldment of a Klein-bottle, acting as a major social and historic field, as argued by M. Purcell [98], creating patterns and processes, thus embodying forms and functions, physis and psyche.⁶⁶ Since control is related to purpose-intention, we have a big-scale coherent anticipative field which conforms a *Zeitgeist* (spirit of the epoch), actions over resources and a self-action of humankind: the Klein-bottle in social and historical scales. Yet, purpose-intention is manifested through time operators, particularly, TIME. We recall that it has been found that there exist time regularities and patterns (*chronomes*) in history (revolutions, war, peace, etc.), culture and its several manifestations (design !, architecture, philosophy, music, creativity on the most diverse fields of psycho-social action), ideas (memes, ideologies, religions, etc.) which have been ascribed to cosmo-psychological archetypes in the work of sophiologist-mathematician Emil Páles [89], which as we mentioned already, his work found antecedents in the works of Kroeber [67] and Sorokin [132]. So, again, the psychic element ‘exteriorizes’ the logo-physical-torsion field as well as it is the ‘interiorization’ of ‘outer’ phenomenae -cosmological chronomes-, but as we argued, it is a Klein-bottle; we shall discuss this further in the case of visual perception. The study of chronomes, a field which had physiologist Pavlov as one of its pioneers, was developed in the former USSR by several researchers grounding the ideas of ‘cosmism’ (Chizhevsky, Vernadsky, Kondratiev, Kozyrev, etc.). At present, it is a growing field of knowledge (which requires persistent experimentation) which may lead us to a completely new understanding in science with amazing findings which not only indicate the existence of standing time-waves acting on all systems, which manifest nonlocal cosmological correlations; see [41] and references therein and [129, 65]. These findings give support to the ideas of Musès chronotopolgy, which in our conception is the Klein-bottle, the embodiment of the Myth of the Eternal Return. We shall retake this issue on examining the self-referential Klein-bottle topographic representations of the somatosensory and auditive modes, and the self-referential gestalt given by the mammal heart. Finally, we want to mention that in the mathematical representation of the process of growth of seashells, it was found that their development, was possible due to a retrodictive character of the action

of their identification, a second-order reification (the first one being the reification of their intentions yet with certain degrees of freedom which is followed by the exhaustion of all degrees of freedom), is what in different theologies, and in the human experience, has the lifeworld called Hell .

⁶⁶In the topological phenomenological *radical recursion* philosophy of S. Rosen, a logo-physical-psychological field hierarchy and associated Musès hypernumbers are ascribed to the Klein-bottle; we have shown already that logical operators are representable by hypernumbers, which are related to Pauli operators, i.e. to spin, and ultimately to torsion. In his book ‘Topologies of the Flesh’, Rosen further ascribes this hierarchy and the different topological and knot operations on the Klein-bottle to the mineral, vegetal, animal and human realms [116]. Thus, the Klein-bottle itself embodies an heterarchy of lower self-referentiality with respect to its integral Being, but each of these lifeworlds, is self-referential and heterarchical themselves, with respect, say, their molecules, atoms, etc.

of the future on their present state of growth (the Gaitlin propagator)[55]. This non-linear character of time was examined by Heidegger [45]. In physics it appeared as a branching time [100] ⁶⁷. For a study on the politics and religious beliefs behind the linear non-operator time prevailing in physics we refer to Raju [101].

To conclude these digressions on the physic in-formational character of designed objects imbued in the design itself, by sheer consistence we are now ready to postulate a psychic in-formational character of Nature at large, of which teleology is one of its expressions, in consonance with Spinoza and several traditions standing in sharp contrast with the alienation of the Cartesian mindset. We have encountered this already in discussing the phase variable introduced by Eddington and further examined by Young. There is a time operator acting on all scale dependent systems; these operators can be associated to the TIME Operator of Matrix Logic, in a particular scale for the system under consideration; through TIME's self-referential action the system is self-determined. Furthermore, these scales are not arbitrary, but may be determined by a fractal structure discovered by H. Muller, and which now conforms the central core of the theory of Global Scaling [81].

7.5 LIGHT, TORSION FIELDS, QUANTUM AND TOPOLOGICAL NON-LOCALITY, CHRONOMES, VISUAL PERCEPTION, DEPTH AND ANTICIPATION

Torsion as the primitive distinction introduced in the undifferentiated Plenum, thus establishing a locus and phenomenology, is the fusion of subject-with-object, as well as that of form and function, which biology and most notably, some medical practices tend to separate. This fusion unfolds in several ways, one of which is its association to spin which is a field not only notable in physics, but as well as all pervasive in biology [28]. Thus, as much as the primevally undifferentiated Plenum has the potentiality of all forms and functions, it is by torsion cleaving this plenum, that they come to manifestation. The Cartesian Cut attempts precisely to dissociate between form and function, being the case that their gestaltic fusion is due to the simple fact that function -physis- is established due to inhomogeneities that give rise to processes, which produce themselves forms and structures through symmetry, which due to the impossibility of being isolated from the environment of other forms-functions produced by other cleavages, tend to loose their symmetry and thus become processes, i.e. physis. This is the essential dynamics in semiosis [138]. Torsion is thus associated with action, giving rise to the quantization of Plenum, and thus the corresponding physical parameter is Planck's constant, which as remarked already, is not singular but multivalued as shown by Pitkanen [93] and Nottale's Scale Relativity [87]; the latter theory can be derived from torsion without recourse to forward and backward derivatives which are basic to Nottale's work [105, 107]. Thus, the Klein-bottle stands for quantized *re-entering* limited domains (QRLD), with different quantization magnitudes which correspond to different scales. We shall later return to this through stereoscopic vision

⁶⁷Raju's findings on the existence of a non-linear -a tilt of- time, lead him to propose a new paradigm in physics based on functional differential equations. Despite this, it is currently named as the *Atiyah Paradigm*; see <http://ckraju.net/>, <http://www.ams.org/notices/200606/comm-walker.pdf> and references therein.

and the Kozyrev phenomenae.

If thus time is subjective as in Hegelian dialectics, in the concrete realization of this stands the process of photon absorption by the subject. But the photon is no ‘external’ particle to the subject, as the classical formula would like us believe. Rather, the absorbed physical particle is the core of the self-referential process of fusion of object with subject (inasmuch as the geometrical fusion is the torsion). Indeed, when we visualize a photon, we are actually visualizing our seeing of the photon, the absorption process by which we complete the objectification of the photon as an independent emitter, object-in-space-before-subject which now when absorbed becomes the fusion of object-with-subject. Self-referentially, the photon (as a lifeworld) is the observation of the link between the photon (as an element of objective reality) and the perception of the photon (i.e. the Fibonacci type reentrance of a form into itself in the calculus of indications; the ‘atom of thought’ as in Johansen’s conception [58]); see [109].⁶⁸

In this context, quantum jumps play an essential role since they represent what is present to cognition and perception, i.e. differences which produce differences, as we also know well from physics and visual perception; these differences are crucial to the neurocortex organization and physiology under the complex logarithmic map topographic representation of vision, as explicated in the pioneering works of Schwartz [126]. We have characterized quantum jumps in terms of singularities of the torsion potential described by the differential of the logarithm of these scalar fields produced by the node set of them. Hence, concerning spacetime torsion fields, it is in the two-dimensional Riemann sphere representing the complex plane, coinciding with the cognitive plane of Matrix Logic, the plenum on which the primeval distinction creates the holonomic structure-process which fuses ‘outer’ and ‘inner’ realms, that quantum jumps, cognition, visual perception and logic are grounded on a plane, from which through holography the full structure can be retrieved [124]. This is linked to the nondual character of interior and exterior as in the topology of the Klein-bottle.

Several cues are used for the formation of the perception of depth, such as occlusion, rotation of objects (so a perceptual spin is relevant to the formation of depth perception which has already appeared as SPIN), the most important is believed possibly to be stereoscopic vision, i.e. the image formed by the joint use of two eyes. It appears that stereoscopic vision only leads to the formation of three-dimensional images if the two eyes actually sense asymmetric images for each of them, in the contrary there is no distinctive image but a blank homogeneous state [54]. This indicates that the actual concrete perception of a geometry requires an inhomogeneity at its basis, i.e. torsion, self-reference.⁶⁹ This is most

⁶⁸This process is a lifeworld in which the body in its integrity is compromised. In a meditative state a photon may come to settle in the forefront, in the locus of the so-called third eye. It appears as an extended radiating (and per se, not in complete equilibrium) two-dimensional flat structure, which subsequently is perceived inside the brain, as a very quickly fluctuating plasma structure with filaments of great complexity and beauty, by the extended watching Self living this awesome experience with and through intensity, self-extension and detachment. Thus, there is an actual seeing of the seeing; second order. The full body of subjectivity has in the photon a first stage of self-reference which is not the last one. Finally, it exits the body in a (silvery-like shortly lived perception) helicoidal motion. We have found no account of this lifeworld in the discussions of light in the Tibetan Buddhist tradition [38] nor elsewhere.

⁶⁹Further below we shall elaborate further on introducing the complex logarithm representation of the retinotopic visual and somatosensory perceptions which transform the photons impinging either in the retina

remarkable since stereoscopic vision is the basis for the conceptual emergence of symmetry with which physics is constructed. These findings points out that this is only possible from actual asymmetries (which conceptually are based on the manifestation of differences); if asymmetries are lacking, only an homogeneous perception is formed, i.e. no structured perception of inhomogenities, only the triviality of sameness.

This perceptual homogeneous plane where no distinctions are present is the one that is associated with the physical symmetrical vacuum, the plenum which we have already presented. So here we have the appearance that depth to be perceived as an original dimension, a difference that makes a differences is necessary, and this is the basic asymmetry between the images of each eye [54]. We shall discuss later the complex logarithmic map that appears in the expression of the quantum jumps that originates in the differential of the complex logarithm of the photon wave function, is also the key to the multisensorial representation of the body on the neurocortex and is further related to the codification both of depth and of stereoscopic vision in the neurocortex.⁷⁰ Furthermore, time dilation and space contraction are related to the baud rate of in-formation processing (which we shall elaborate in terms of ATP's metabolism) Hence, there is no 'pure objective' cognition of an object: Visual perception and cognition depend also on contextual *interpretations* by the subject. (As we said before, perception, cognition, truth values, action and interpretations are unseparable.) Furthermore and most surprisingly, they depend on cultural and theoretical constructs, to some extent [115]. Thus, the classical Cartesian formula is untenable and perception is not secondary to cognition, the subject is a full participant in the construction of the proper visual model. This 'interiorization' of the geometry by the subject with its dual operation of projection for the construction of the 'exterior' geometry of space and time, can still be linked with the metabolic rate of the production of ATP (adenosine triphosphate) in the brain's visual area, which in turn is linked with the quantity of light absorbed in the retina (which is related to quantum jumps and torsion, and thus to the quantization of 'outer' spacetime).

When general human metabolism (and the V5 area of the neurocortex which is believed to be associated with motion detection) is fast, time runs slow; conversely, when the metabolic rate is relatively slow, time runs relatively faster; see Harms [43] and references therein. This has support as we said already in the fact that there is a related 'inner' geometry of vision in which there is a limiting velocity of percepts processing. This is widely known to occur in extraordinary situations of stress such as high velocity drivers perceiving the 'outer' world through a time dilation and space contraction, which may be accompanied with their relative visual V5 metabolic states speed-up, compared to observers in other frames at lower speeds relative to this visual limiting perception velocity. In the Klein-bottle function-form of the neurocortex, quantized 'outer' spacetime is thus transformed

or the skin in a mapping at the plane two-dimensional neurocortex. We shall see that this logarithmic map that appears explicitly in the formula for the quantum jumps, plays a essential role in providing these maps, both locally and globally, and that this map provides also for the basis of the stereoscopic vision formation in terms of differences as we have already repeatedly alluded in this work, and in terms of the vortical structures that appear in the neurocortex' hypercolumns.

⁷⁰In the field of mathematical psychology, it has been theoretically and experimentally verified, that visual perception follows the organization of visual 'internal' geometrical representations [70, 56] that that they are described by a psychometric function dependent on the observer.

into ‘inner’ quantized perception and thus a minimal instant is associated to it, through which motion and all differences come to be.⁷¹ Then, at that minimal instant there is no motion. Motion, then must be the perceptual differences that exist from one instant to the next. Thus, motion may only take place between the instants in time. Here the instants are given to us by the processing speed of these instants by the V5 area, which might be related -though we are in a state of ignorance with respect to this issue- by the frequency of light entering the eye [43]. We would like to propose that TIME, Cognition Operator M and SPIN, operate at the limiting velocity of ATP production and visual geometry hyperbolic space [56], in which cognition appears due to the quantization of time, with associated limiting values of acceleration and control. This should establish the Lorentz-Fitzgerald perceptual phenomena in which the previous fundamental operators manifest. We recall that ATP production and destruction is a fundamental organizing process for the cell’s operations, and is further related to the zipping and unzipping of the microtubules, in terms of which Penrose proposed his quantum non-computability source for will. So again, it is the action of time that has to do with will, and this may have a biophysical operational substrate in the creation and destruction of ATP, which may actually create at their limiting velocity, the quantum action of TIME, as the difference between creation and annihilation as suggested by its very definition; see eq. (66). The background medium which is essential to the ATP production and destruction is water, which itself has the remarkable property of having memory-like behaviour [152]. We recall that anholonomy, and particularly torsion, allows to encode memory as argued by Pattee [91]. It is important at this point to remark that the Aharonov-Bohm effect we found to be associated to the logarithmic torsion potential in our discussion of quantum jumps in Section 3.1, plays a central role producing non-local correlations in biological systems [145], especially those that are sustained by water.

Let us return to the problem of depth, its identification with the operation of time as can be perceived in the Necker cube [85], and following the phenomenological philosophies of Merleau-Ponty and S. Rosen. Stereoscopic vision which is important in establishing depth perception, brings to the fore the problem that in order to obtain a synchronous visual flow in both left and right brain hemispheres, which is needed to account for its coherence, the temporally delayed signals of both eyes’ left and right visual fields (which are processed in different hemispheres), should be integrated with anticipated versions of their complements in order to close the time gap existing with the firing of neurons in the separated hemispheres and recurrent visual control [64]. As we have repeatedly elaborated in this article, control is linked to purpose and thus to anticipation and to the TIME Operator. Remarkably, Kampf’s proposal for a solution of this problem (which we recall that is fundamental to the establishment of a physiological process that sustains multivalued logic as the perception of the two possible instances of the Necker cube requires as both instances unfold in time through depth perception) , is to invoke anticipation, as conceived by Dubois [24]. Kampf: “representational simultaneity, as a brain process spread over spatially distant loci, is achieved by temporally bidirectional interactions of neuronal processing on a quantum scale. Absorber effects between the presumed ‘advanced’ and ‘retarded’ signal components

⁷¹Experimental verifications of this quantization of time have been obtained by E. Ruhnau and elaborated in her theory of hierarchical time windows [121]; in this regard the theory of fractal time due to Vrobel can be of great relevance [149].

are proposed to generate standing time-waves pattern which might be speculatively assigned to the carrier process of an internal psychophysics of the representation of visual space.” Furthermore “...absorber effects appear, on the operational side, as anticipations of future states of the system”. Kampf’s proposal then stems from the transactional interpretation of quantum mechanics (TIQM) due to Cramer [20]. In fact, the absorber theory which was the basis for TIQM, attempted to explain the EPR paradox in a straightforward way. It rests on the idea that a ‘handshake’ -we prefer the logical term ‘identity transparency’, or still the physical interpretation as a ‘resonance’- between the ordinarily transmitted signal and an anticipative effect deeply rooted in the quantum world is *feedback* from the ‘absorber’ to the ‘emitter’; so, in our terms, this requires control, or still, the action of the TIME Operator. ⁷² Coupled in-between the ‘retarded’ and ‘advanced’ components in the collapsing wave function of the quantum event under measurement, this process appears to an external observer as a seemingly time-reversed transaction. In our account, this is the action of the TIME Operator. Kampf presents arguments for the significance of advanced signals as an anticipative feedback for the synchronization of spatially distant retarded processes which he derives from an analogy between the synchronization of neuronal activity and that of chains of coupled oscillators on different scales (including a cosmological one). Thus Kampf argues for the existence of a time-loop that accounts for this synchronization and control, a proposal which fits well with our previous digressions. He further notices that the mathematical roots of this lies in the standard quantum-mechanical procedure for calculating the ‘collapse’ of the wave function, by computing the square of the probability amplitude which is done by multiplying a complex number $\cos t + i\sin t$ by its conjugate $\cos t - i\sin t$, where t stands for the time angular variable i.e. by phase (as in Eddington-Young space) conjugation, which is the basis of holography as elaborated by P. Marcer and W.Schempp [124] and which we note that in matrix logic is the transformation of a cognitive state of the form $\langle q| = \langle (\cos^2 t \ \sin^2 t)|$ into $\langle q| = \langle (\cos^2 t \ -\sin^2 t)|$. In the case of $t = \frac{\pi}{4}$ this is the TIME Operator transformation between the superposed state $\langle S_+|$ into $\langle S_-|$. These topological entangled states form an orthogonal basis which is the transform by the Klein-bottle in-formation matrix H of the Boolean orthogonal vectors $\langle 0|$ and $\langle 1|$, and conversely, $\frac{1}{2}\mathcal{H}$ transforms $\langle 0|$ and $\langle 1|$ into $\langle S_+|$ and $\langle S_-|$; these four states generate all Matrix Logic [103]. Thus, for these states, we can ‘interiorize’ the transaction as an action of TIME which generates the laws of thought. ⁷³, or conversely. ⁷⁴ We already argued that the neurocortex cell visual representation is both supported by holography and the Klein-bottle [109], and

⁷²Further elaborations of the causality operators that transcends the mere physical causation, were applied to the Kozyrev phenomenae, in terms of his epistemology, by Johansen [58].

⁷³We have argued elsewhere that following the phenomenological philosophy due to Merleau-Ponty, depth which is precisely related to stereoscopic vision is a primitive dimension associated to time and paradox. We shall later discuss the relation of this with respect to the retinotopic and somatosensory complex logarithm representation on the neurocortex and its relation with oppositely oriented vortical structures associated to the hypercolumns in the neurocortex.

⁷⁴For the benefit of the reader who skipped the previous parts of this chapter, we recall that cognitive states are real valued vectors which we write as Dirac bras and kets formalism of quantum mechanics $\langle q| = (\bar{q} \ q)|$, where \bar{q} is the negation of q (the *real*-valued logical variable), i.e. $1 - q$ so that they are *linearly* normalized, related to the wave-state of two-state quantum mechanics by the relation that the former are given by the complex square root of the latter, i.e. $q = \psi\bar{\psi}$.

that the latter is a classical-quantum-classical transformer which sustains multistate logic which is related to quantum fields, transforming the average of the cognitive operator M on cognitive states to the average of the logic spin operator $SPIN$ (related to the perception of depth) on two-state wave functions; furthermore, we can substitute M by $SPIN$ acting on cognitive states and M (cognition operator) acting on quantum states and this identity is still valid, so we can both have $\langle q|M|q \rangle = \langle \psi|SPIN|\psi \rangle$ and $\langle q|SPIN|q \rangle = \langle \psi|M|\psi \rangle$ where $\langle \psi = \langle (\psi(\uparrow) \ \psi(\downarrow))$ is the spin-up spin-down quantum state; this identity stems from the decomposition $M = TIME + SPIN$, and the fact that $TIME$ is a distinction *cognitive* operator, i.e. $\langle q|TIME|p \rangle = p - q$, so that the average $\langle q|TIME|p \rangle = p - p = 0$; $TIME$ appears to be unchanged for unaltered states of consciousness.. So, indeed, quantum effects, time loops as in the Klein-bottle lift of the calculus of distinctions giving thus the periodic reentering of a quantized limited space domain through time waves, the action of the $TIME$ Operator and anticipation, which are the very embodiment of the holographic structure of the Klein-bottle, are present in the mind-brain (we use this term because of the exchangeability of cognition -mindlike- and $SPIN$ -brainlike- observables described above). Entanglement is due to the action of the non-orientable topology of the Moebius surface, or still by the torsion introduced by the cognition operator, which also represents the non-duality of $TRUE$ and $FALSE$. To resume: entanglement is topo-logical-physical. One can enquire still if this quantum entanglement is related to the ‘interiorization’ of quantum entanglement at a cosmological level, a question which Kampf does not raise (perhaps due to the lack of a Klein-bottle logic), restricting the arguments to the possible parallels between cosmological (as in the Kozyrev phenomena ⁷⁵) and microscopical entanglements. The natural answer is represented through the Klein-bottle which has no inside nor outside, but a form-function which transforms a local interior to a local exterior holographically. Thus, absorption at one hemisphere of the cortex of a photon is entangled with the anticipative emission of a photon of the other hemisphere producing synchronization. This transaction is ‘interiorized’ in the laws of thought of multivalued Matrix Logic, or still in the calculus of distinctions in which we incorporate paradox through reentrance of a Klein-bottle limited domain defined by a quantized distinction, a QRLD as we called them. Yet, this is the rotational action of $TIME$ Operator; we shall later see that analytically it is embodied in the complex logarithmic map of sensorial modes (in particular, retinotopic) on the neurocortex. So, as we said before, it all (perception, cognition, interpretation, physics) boils down to the projection on a two-dimensional plane, be that a neurocortex slice, the phase space of Eddington-Young, the plane of cognitive states in Matrix Logic, or the projection of the Riemann sphere as the complex plane which can be generated by three primeval distinctions as pointed out by James [57], the phase-space of Young-Eddington and spinors, or still for characterizing quantum jumps as primeval differences. In this transaction the torsion geometry of cognitive

⁷⁵Kozyrev and Nasonov discovered on observing through a telescope with a device, that there exists a radiative field associated with, say a star or galaxy, that cannot be shielded but with polyethylene. In pointing the telescope to the future and current position of the star this radiation was also registered [66]. These experiments were repeated decades later by Lavrentyev with the same results and the theory is currently used to compute current positions of astronomical bodies [30]. So time-loops exist at astronomical scales.

space is essential ⁷⁶, since itself produces a superposition state, $\langle S_+ |$ which stems from the non-orientable character of the Moebius band and shows up defining the cognition operator; from $\langle S_+ |$ the other superposed state $\langle S_- |$ is produced through the TIME transform, and together they form the Cognition Operator (and the Klein-bottle in-formation matrix \mathcal{H}) which encodes the transformation of cognition to spin observables and conversely. This ‘interiorization’ process, in the Klein-bottle logic is identical to a cosmological entanglement, of which the Kozyrev astronomical observation which we have already pointed out that it is a cosmological example of a chronome, a time-structure-function which also can be interpreted through the TIQM. It has been verified to exist not only in cosmological scales, but also as entanglement between solar and geophysical phenomena, as proposed and experimentally verified by Korotaev [65]. Remarkably, the Kozyrev phenomenae can be explained also through the same geometries as the mind-brain operations associated with Matrix Logic and a QRLD, i.e. in terms of torsion through spacetime (Brownian motions) fluctuations [105] -which we observe that, unless thought is programmed, is random- and spin-torsion fields [134]. This leads to enquire on how QRLDs, say a cosmological one (with a corresponding cosmological Planck constant) [87] may be entangled with a meso or microscopic domain (with an appropriate Planck constant)- as already the human ‘homunculus’ representation already achieves- with a neurological domain so that the entanglement that synchronizes stereoscopic vision through logical torsion entanglement or through an emission-absorption transaction, forming a self-referential loop in which one is transformed into the other. In other words, what is the process-structure inherent to the heterarchies of Klein-bottles, or still, to the Klein-bottle itself as an embodiment of heterarchies, already discussed? A natural solution perhaps can be found in the fractal structure provided by nested QRLDs in which ‘interior’ domains reenter through ‘exterior’ ones and viceversa. It would be interesting to consider this in regards of the fractal structure of time [149]. We shall next see that the human structure-process itself is such an heterarchy.

8 THE COMPLEX LOGARITHM MAPPING OF VISUAL AND SOMATOSENSORY PERCEPTION AND THE KLEINBOTTLE

Up to this point we have presented a theory of the joint constitution of the physical, logical, cognitive and perceptual realms, yet for the latter we have mainly discussed the study of stereoscopic vision and synchronization. We shall retake this issue in a wider conception which evolves from the relation between the sensorial realms and its mathematical representation in the neurocortex, and further the physical phenomena that appear in the neurocortex. This is of course related to one of the longest standing philosophical problems

⁷⁶In fact, already the geometro-stochastic form of the Schroedinger equation for ψ (where the torsion field $d\ln\psi$ describes the average motion of the universal Brownian motion generated by the torsion geometry together with the diffusion tensor associated with a metric)[107], incorporates *both* boundary conditions on the past *and* the future that allow to consider the probability distribution $\psi\bar{\psi}$ which we interiorize as the logical variable q depending on the universal time variable of Stuckelberg and Horwitz [51]. This variable is the variable that parametrizes quantum jumps as we characterized in the initial sections to this chapter.

raised by several philosophers and the core problem of what we contemporarily know as cognitive sciences, and any attempt of dealing with it in the needed extension is unfortunately severed by length constraints. We have already presented our conception in [109] and above, yet rather than continuing with the philosophical discussions we shall restrict our disquisitions to the mathematical representations already alluded and their relations with neurology.

In the introduction to the deep work on the topology of the body representation by Werner [150], it is pointed out there is a wealth of clinical and experimental studies that established the existence of a fixed relation between regions of a body and areas on both the sensory and motor cortex, achieved in the decades 1930-1969, which led to the notion of the existence of orderly systematic mappings of the body's peripheral events in the brain [71, 80]. The evidence in this regards was that the cortical and subcortical projections of the visual system are topographically organized such that the fiber tracts and neurons preserve the spatial arrangements in the retina. Also, the spatial orderliness of representations of the body at the cortical and subcortical relay stations follows the segmental (i.e. dermatomal) innervation pattern of the body periphery.

Prior to these discoveries, there were several theories postulating that perceived space is correlated with spatial patterns established in the nervous system⁷⁷. Thus the notion of a mapping from the peripheral sensory bodies to the central nervous system was proposed. In particular, the mapping of the body surface, and the auditory and visual receptors.

We quote E. L. Schwartz [126]. "A universal feature of the anatomical organization of the vertebrate sensory system is that the visual [140, 2], auditory [71], somatosensory [154, 150, 151] and olfactory systems are organized in terms of orderly spatial projections of a peripheric receptor mosaic to more central processing sites. Embedded within this 'receptotopic structure' is the detailed local neurophysiological processing that results in the existence of well-defined neuronal 'trigger features'. At the striate cortex in primates, cells respond optimally to oriented, elongated stimuli with well-defined velocity, direction of movement, *binocular disparity*, ocular dominance and colour. Cortical cells that are sensitive to the *orientation* of a stimulus are grouped into columns, or slabs, of common orientation [54]. These columns are themselves arranged into a highly structured geometric pattern termed 'sequence regularity' by Hubel and Wiesel, who introduced the term 'functional architecture' (we note here the blending of function and form; author's comment) to describe the anatomical arrangement of the physiological significant cortical structure." In the work by E.L. Schwartz the term 'functional architecture' was generalized to include the global, retinotopic organization of the cortical spatial map as well as the local columnar structure described by Hubel and Wiesel, and the somatosensory representation and still the relation

⁷⁷In the Gestalt conception, "all experienced order in space is a *true* representation of a corresponding order in the underlying dynamical context of physiological processes". Remarkable to this conception, is that there is a logical value ascribed to perception, which can still be relative to the choice of particular cues. This is the basic phenomenae which we encounter when dealing with a geometry alike the Necker cube [85], in which there are two possible choices of cues and perceived cubes, which are related to the invalidity of the principle of non-contradiction of Aristotelian logic. Any of the possible perceptions are 'true' and thus the Necker cube embodies paradox very much in its design, as we discussed above. The extension of this to non-orientable surfaces, namely the Moebius surface and still more notably the Klein-bottle, led to the foundation of multivalued logics as shown above.

with stereoscopic vision. From the work by Schwartz appeared that all these maps act transforming input from two-dimensional sensorial surfaces to the two-dimensional cortex (albeit with a columnar structure representing each point of the cortex), and are all given by the complex logarithmic map, which in the retinotopic representation nearby to the centre of the eye, where the tangent space can be identified locally with the retina gives an excellent agreement which matches with experimental data. Furthermore, this has allowed for predictions which have later been verified, in particular, for binocular disparity tuning (which is a subject we have already raised above). Schwartz' discoveries lead him to pledge that the term functional architecture was to be taken in a literal sense: the spatial structure of neural activity in the primary sensory system turned to be of direct significance to visual perception. The problem arises as how the neural system accomplishes the task of mapping both the surface of the body and the interior of the periphery of the three-dimensional body into a *single two-dimensional* cortex surface.

The experimental findings by Werner lead to the conclusion that the “geometrical representations of the cortical body appear quite different from those of the body itself for several reasons. First, unlike the relation in the body periphery, the projection of the skin does not form a continuous boundary in the cortical map, enclosing the projection from the ‘deep sensors’. A second reason for the difference between the body and its map is that the relations of proximity and distance between points on the body do not consistently remain preserved in the cortical map. The meaning of this is that receptors fields which are closed in the periphery in the two-dimensional cortex space in which the topographical map takes their image, may not be close any longer”. Also remarkable, is that what is codified by the map in the neurocortex is *not* the actual points of the skin as a sensorium, but ‘dermatomal trajectories’ [150, 151], alike, say, shoes with helicoidal footlaces strappings or still the helicoidal Hebrew phylacteries ⁷⁸

Werner’s conclusion is that the topographic map was not related to a metric as in General Relativity but to an homeomorphism, a continuous bijective invertible map whose inverse is also continuous which further transforms open sets in the domain into open sets in the image [150]. The solution compatible with the experimental findings is that the global somatosensory map is given by the Klein-bottle [150]. Furthermore, what in the periphery may appear as broken dermatomal trajectories, in the Klein-bottle map they are represented by continuous trajectories. Werner further suggested the implications for the haptic sense (i.e. the integration of sensory modes, such as tactile and kinesthetic) are represented by one single map. Thus the Klein-bottle also plays the role of the topological codification of the integrative somatosensory mode. Yet there is something more striking on this and especially important to the very essence of the possible existence of the Kantian apriori schemata. This is the fact that “the sensory representation in the cerebral cortex can possess properties that are not inherent in the raw data originating from the peripheral sense can organs themselves, and that the nature of new properties can be a consequence of the characteristics of the neural mapping process which links body periphery and central sensory representation”. This is a very strong statement which somehow inverts completely in the Cartesian mindset

⁷⁸These phylacteries are strapped as helicoidal trajectories on the arm with further loops producing semiotic codifications (one of the ‘names of God’) and additionally have self-referential sentences vinculating the subject to God, written inside the box that initiate them as well as the one placed in the forefront.

the ideas of where the complexity stems from: As is usually stated in the dictum “the map is not the territory”, meaning that there is more to the actual complexity of what is the ‘objective world’ than there is in the representation. This points out to the possible fact that the complexity lies on the map, and not in the skin and efferent neuronal pathways that sustain the map itself, which here is topological; it is the Klein-bottle. Since the map plays the role of the subjective-objective fusion of the periphery and of kinesthetic recordings of the raw data, with the actual percepts, this becomes a self-referential -to the map!- linkage which is absolutely irreducible to the Cartesian Cut mindset.⁷⁹ . It clearly establishes the prominence of a Platonic realm, which is essentially semiotic- mathematical-logo-physical in which by the joint constitution of the lifeworld of both subject and the physical realm does not allow for discernment of where the complexity actually resides. Through the map we map the territory i.e. self-referentially, the map is both operator and operand, and thus we cannot get ‘out’⁸⁰ of it to cognize it. In fact as we have seen in the constitution of logic, the Klein-bottle is the basic operator-operand in which, by the joint constitution of time and thought comes to be, arising from paradox [109]. If we still take in account the self-referential being of language as pointed out in a torsioned way by psychoanalyst J. Lacan, it turns out that the Klein-bottle is the actual integral solution to this conundrum that philosophers, mystics, scientists and common folk have been trying to search for. Of course, what we are saying about the complexity of the human and primate bodies, apply as well to the Universe at large and its lifeworld. The last pending problem concerning this is on the *reality* of these schemata. There is no other consistent answer to this historically longstanding interrogation that the one we have just explained with respect to perception. Semiosis is not established independently of the subject but lies in the joint constitution of

⁷⁹These findings on the importance of the map with respect to the territory, may perhaps assist us to understand memetic blockages the reinforcement of paradigms, Taborsky has argued that the fixation (or better, in thermodynamical sense, freezing) by the signifier on his interpretation of semiosis, a kind of fetishism, leads to the disappearance of the structure that is being signified and the destabilization of the signifier, with its possible demise [138]. There is a widespread phrase attributed to Planck, purportedly asserting that the establishment of a new paradigm requires the demise of the whole generation that opposed to it. A similar idea is found in the Torah and retaken by the other monotheistic religions of the same branch, in which the generation that exited the land of servitude, had to expire to give place to a generation free of the previous experience of slavery, to enter a land in which they would no longer be aliens. In more explicit terms, the fixation to any particular paradigm or belief system, after a process of degradation through the impossibility to embody the new ambient conditions -thus Taborsky’s disquisitions are elaborated in terms of thermodynamics-, and develop the needed knowledge, may produce a form of alienation. We have encountered this process before when we presented the Myth of Eternal Return. In the present conception, both signifier and signified are fused, as is the case of the primeval distinction already discussed. In science and also all belief systems, paradigms are established as the territory of all reality, in a process in which the initial mapping used for the construction of the paradigm is no longer examined. This territory is a reification of a consensual ideative process, which by massive addition of more information of the same conceptual quality, and the performatal of -mostly hierarchical enhancement- rituals, a reinforcement of the paradigm takes place. An ideation or an experience which belies the paradigm or questions this reification, introducing at times a simpler and more integrative *mapping* principle, may be perceived as a threat to the permanence of this territory, which now has frozen frontiers and conceptual contents. Due to the identification between the signifier and its projection in a state devoided of degrees of freedom, self-preservation will be attempted through the liberation of the energy contained in the signifying process and the sign [138], of what is already a fusion of territory and signifier.

⁸⁰Identically, we cannot *speak* about the world in its manifold realms without language.

the lifeworld that constructs jointly the subject with the physical realm embodied in the subject's lifeworld. This is the *enaction* that sustains-creates the Self jointly with the World, without pledging the 'Great Doubt' [115].

Returning to the dermatomal helicoidal trajectories on the primate and human skin, these are transformed into straight lines in the planar neurocortex space. As observed by E. Schwartz, this transformation is inherent to the complex logarithmic map. Furthermore, this is not exclusive to the representation of the limbic system but also by the retinotopic representation discovered by Schwartz consistently with experiments [126, 127, 139]. The representation of the retinotopic stimuli, which as we mentioned above, amounts to the stimuli provided by the class of monochromatic functions in terms of which we constructed the theory of quantum jumps and the photon in which the complex logarithm appeared to give the different surfaces of the Riemann surface on which each quantized states lies, also appears to provide the mapping for the retinotopic stimuli. In fact, as discussed exhaustively by Schwartz in the case of the retinotopic mapping into the neurocortex, both the complex logarithmic map as an analytical topographic representation, and the Kleinbottle as the topological representation, are cocreated [126]. This has very important consequences to the physiology of perception. Let us briefly discuss this issue.

To start with anatomical cortical development, the complex logarithmic map being analytic (in the sense of the theory of complex functions), represent a potential flow which is -as usual in complex functions theory- subjected to *boundary* conditions imposed by the shape of the boundaries. Moreover, the dendritic summation of the afferent input to the neurocortex is also locally complex logarithmic, so the anatomical structure stands in correlation with the topographic representation by the analytic map which further is related to developmental functioning, which we shall later associate with the diffusion of morphogens. Thus we find in the retinotopic topographic map and the underlying anatomy, a perfect gestaltic superposition of function and form ⁸¹ which is further related to intra-cortical inhibition and the hypercolumnar organization. The boundaries in the neurocortex of ocular (vis-à-vis binocular) domain organize the complex logarithmic map through the fact that intra-cortical inhibition and sequence regularity run parallel to these borders while binocular summation runs perpendicular to them. The ocular dominance columns provide the link between the axis of the global and the local mappings of the cortex. Now, to understand how is it that the actual cell density provides for the functional architecture, it is essential to note that their density is described by the same inverse square law of general potential theory in the theory of complex functions, provides a two-dimensional functional-computational-architecture for electrostatics, Newtonian potential, etc.[35]. The mathematical representation for both is the complex logarithmic map which also plays a central role in Karman vortex street model in fluid-dynamics described by the Navier-Stokes equations (which we proved to be a most

⁸¹Further below, on discussing the somatotopic mapping, it will appear that the complexity appears to be encoded not in the body as anatomical differences per-se but on the map, while in the retinotopic visual encoding we have in the -foveal- domain of validity of the mapping, a perfect gestalt. So, for light as the most unconstrained field, the encoding is embodied as anatomical-computational-architecture while in the somatotopic case seems that the computational mapping carries the complexity as we evolve to lesser degrees of freedom. We shall not enter here, due to space limitations, to discuss the relation of this with Kant's philosophy.

basic example of a torsion geometry related to Brownian motions [105]). So, we do have vortical structures in the neurocortex which are related to the analytical retinotopic mapping. Now, how does the Klein-bottle structure appear in relation with the complex logarithmic map in this mapping? To start with, the topology of the complex logarithm map is that of the torus, which thus is the Riemann *orientable* surface for it. Anatomically, the hemispheric representation of the cortex are joined by the corpus callosum, yet in such a way that it supports the orientable torus topology [127]. Yet, the representation in cortical domains when this orientable topology is applied is asymmetric, the image by the map of the center of the local receptive field (the foveal) is on the boundary of the map, not in the central vertical axis. To reinstate symmetry with respect to this organizing center, the non-orientable topology of the Moebius band is needed, which thus becomes a non-orientable Riemann surface for the complex logarithmic map. In principle, both topologies are possible inasmuch both the Boolean and superposed basis in Matrix Logic exist and are transformed by the Klein-bottle Hadamard operator; see eq. (86). Simply, for the non-orientable topology, the field's receptive center is transformed into a symmetric organizing axis in the neurocortex, while in the Boolean orientable case it is represented in such a way that the center's field retinotopic representation in the cortex has been shifted to the boundary. Therefore, non-orientability is closely related to the preservation under the analytic map of an organizing center; again, non-orientability is related to Self and appears to be related to the integration of both hemispheres.

Returning to the issue of binocular vision and disparity which provides for a cue to depth perception, which we recall that in Merleau-Ponty's phenomenological philosophy ⁸², is the primeval dimension of perception, which, as also already presented, leads to multivalued logics and to the generation of time waves of synchronization, lies in the understanding of Schwartz in the fact that the complex logarithm links a cortical shift to a visual plane size change. Thus the slightly different projections of the two eyes are normalized by the complex logarithm to the identical cortical pattern, with the shift proportional to the size difference, and ultimately, the disparity of the stimulus [126].⁸³ Returning to the case of haptic perception, i.e. to the integration of several sensorial modes, we have a complex logarithmic map that is the representation map of both somatosensory and visual modes [126]. Now, as far as topologically the global representation for the somatosensory mode is the Klein-bottle, it is most remarkable that the Klein-bottle appears to be the solution of representation of visual topographic maps on the neurocortex. As for the topological representation of the retinotopic mode, symmetry properties of simple cell receptive properties lead naturally to the construction of the Klein-bottle [137] coinciding with the conclusions by Schwartz. So the geometry of visual (and furthermore, of auditory and somatosensory -limbs, skin surface, etc.) space has a representation at the visual cortex, and furthermore, at the fundamental level of cells, the topology of the Klein-bottle is naturally present. Furthermore, there is experimental evidence that supports that these maps can be represented by the Klein-bottle [137, 141, 150]. Interesting enough, the starting point is the 2-dimensional Gabor function

⁸²Depth appears also as a possible spatial interpretation of the recursion of the Klein-bottle into itself, alternatively to the time-waves interpretation [109].

⁸³Thus, stereoscopic vision appears as the process of a difference producing difference; the integration of both hemispheres which we also interpreted as a time-loop through a torsion field.

(of importance in holography [68]) commonly used to model the receptive-field profiles of simple cells [73], which make up a substantial percentage of visual cortical neurons. This function yields a topological representation which is the Klein-bottle, already present as an holographic representation of vision by the neurocortex.

9 Epilogue

To resume ⁸⁴, the primary sensory projection of each limb, the kinesthetic and the visual modes are all described by the complex logarithmic map, and they all share the same topology, the Klein-bottle. So the integrated haptic modes available at the cortex are provided by a single map and a self-referential topology, which is further related to the constitution of the fusion of participatory universe on which the subject and the object are fused through the self-referential gestalt of the photon and torsion fields. So we here find more than an integration of the psychology and the neurophysiology of the spatial senses. It is further integrated with the generation of a participatory Universe, in which the thinking, perceptual, cognitive, sensory and physical realms are all one. Topologically, it is the Klein-bottle. This establishes a Universe of resonances integrated by torsion fields, in which the quantization of these resonances are provided by sheets of the complex logarithmic map. These resonances give rise to the synchronization that leads to binocular vision, as being integrated through the complex logarithmic map.

Now, the body afferents which are the mediators between the body and its topographical map, independently of whether this is the analytical or the topological one, have a clear fractal recursive structure, and thus itself is eminently self-referential. This is common also to the actual development patterns of plants, and has been studied extensively in the mathematical work of Lindenmayer. As Schwartz with profound insight points out, for the map of the sensorium to be compatible with the end image in the neurocortex be realized through the complex logarithmic map, it is sufficient that this is the map that constructs the fractal at each and all stages of the recursion ⁸⁵ What about the topological map? Is this iteration of the analytical map related to non-orientability, i.e. the Klein-bottle and Moebius band, at all stages of the recursion? For the retinotopic representation, this would imply at each stage of the recursion the preservation of the stimuli local centre field, concatenated by the complex logarithmic map. More generally, we suggest that the answer may be affirmative and furthermore, and that there is anatomical evidence that this is the case. The evidence is the mammal and specifically human heart, but not seen in the usual Cartesian sense, as an object occupying a space being defined by its boundaries (were it not that it has valves connecting it with the rest of the body) by it. The actual structure of the human heart (dissected with previous extraction of the valves) is that of an iterated Moebius band, as was discovered in the work of his lifetime by F. Torrent-Guasp, in which

⁸⁴Reaching this stage of ending this chapter, and due to a conjunction of fortuitous and premeditated conjunction of the lack of space and the wish of the author not to spell out conclusions but allow the reader to ponder on the text by him (her)self, we shall only write an epilogue to resume the last section.

⁸⁵We recall the anthropologists tale: It is turtles all the way down (or up).

each band folds another identical band.⁸⁶ Furthermore, this development is crucial to the physiology of the heart, which thus functions as a torsioned geometry through vortex motions produced by the cyclical turns of the recursive Moebius bands [143]. Including the valves, is the heart a Klein-bottle?

The next question that this elicits is: What is the relation between this recursive structure-process and the growth of organs in the human body? According to the findings of Lindenmayer the case for plants is that they are unified [69]. Plants grow following the recursive patterns. Let us return to the complex logarithmic map. If we decompose it into its real and imaginary parts, they both satisfy the diffusion equation [35]. So at each stage of development of the recurrence, we have a diffusion process that we can think of as morphogens. But now the topology is non-orientable at each stage, and thus the growth also follows the same topology, at each stage, explaining thus the Myocardial Band Model due to Torrent-Guasp.⁸⁷

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⁸⁶We can actually ponder together with many loving people that there is a connection of the heart with Intelligence.

⁸⁷Pioneering work in the recurrent growth algorithms that lead to the formation of human or animal organs and structures, we refer to the important, but unfortunately not translated to English, work by Bazanov [9], inspired in the work by Edwards [28]. It is important to remark that this is a very remarkable example of the fusion of form and function, alike the one that the Klein-bottle embodies.

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