

From Cantor Dust to Interacting Dark Matter Models

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Abstract

The thermally averaged annihilation rate that reproduces the observed Dark Matter abundance, $\langle\sigma v\rangle \approx (2-3) \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$, is numerically close to the value expected for weak-scale couplings and masses, a coincidence known as the weakly interacting massive particle (WIMP) miracle. We examine whether the same relation can arise if Dark Matter is a coarse-grained excitation of a multifractal Cantor Dust (CD) condensate with effective Hausdorff dimension $D < 3$. Localized excitations of mass m_χ are treated as quasiparticles whose interaction probability is set by a scale-dependent geometric overlap factor. This gives $\langle\sigma v\rangle_{\text{CD}} = C_D \alpha_D^2 / m_\chi^2$ for $2 \rightarrow 2$ annihilation and $\langle\sigma v^2\rangle_{\text{CD}} = C_{D,3} \alpha_D^3 / m_\chi^5$ for $3 \rightarrow 2$ processes, which are analogues of WIMP and strongly interacting massive particle (SIMP) freeze-out, respectively. We then use the single 248 keV nuclear-recoil event recently reported by the LUX-ZEPLIN (LZ) experiment (global significance 2.6σ) as a phenomenological test case and derive the kinematic and thermal conditions that a CD excitation would have to satisfy. The overlap factor is introduced as an explicit ansatz; the results are effective scaling relations, not a first-principles derivation.

Keywords: Dark Matter; Cantor Dust; multifractals; WIMP miracle; SIMP; thermal freeze-out; LUX-ZEPLIN; nuclear recoil; inelastic Dark Matter; fractal measure; dimensional condensate.

Abbreviations.

CD	Cantor Dust	SM	Standard Model of particle physics
CRESST	Cryogenic Rare Event Search with Superconducting Thermometers	UV	ultraviolet (short-distance) regime
GIRES	Global Institute for Research, Education and Scholarship	IR	infrared (long-distance) regime
LZ	LUX-ZEPLIN experiment (LUX: Large Underground Xenon; ZEPLIN: ZonEd Proportional scintillation in LIquid Noble gases)	WIMP	weakly interacting massive particle
SIMP	strongly interacting massive particle	keV, GeV, TeV	$10^3, 10^9, 10^{12}$ electronvolts
$n\sigma$	significance of n standard deviations		

Natural units $\hbar = c = k_B = 1$ are used unless stated otherwise.

1. Introduction

The particle interpretation of Dark Matter is consistent with a wide range of astrophysical and cosmological data, but the microscopic identity of the dark sector remains unknown [1]. A widely studied possibility is the weakly interacting massive particle (WIMP), whose relic

abundance is set by annihilation in the early Universe [2, 3]. The approximate agreement between the annihilation rate required by cosmology and the rate expected for a weak-scale interaction is commonly called the WIMP miracle [4]. The strongly interacting massive particle (SIMP) mechanism showed that a thermal relic abundance can also be set by number-changing $3 \rightarrow 2$ reactions in a strongly interacting hidden sector [5, 6].

Our Cantor Dust (CD) proposal starts from a different assumption. Instead of taking Dark Matter to be a gas of elementary particles distributed over an ordinary three-dimensional continuum, it treats Dark Matter as a multifractal, or dimensional, condensate. Our earlier work proposed CD as a possible non-particle Dark Matter phase associated with the minimal fractality of spacetime [7, 8]; subsequent work connected CD to fuzzy and self-interacting Dark Matter [9, 10] and examined Cantor-like fragmentation generated by Stuart–Landau dynamics [11].

The question addressed here is whether the familiar WIMP and SIMP descriptions can be understood as infrared (IR) analogues of localized CD excitations. The answer obtained below is affirmative at the level of effective scaling laws, with one qualification: our existing CD publications do not provide a unique microscopic annihilation amplitude. A geometric overlap ansatz is therefore introduced explicitly. The resulting construction is an effective correspondence that can be tested; it is not a derivation of the Standard Model (SM) weak interaction from Cantor Dust.

The LUX-ZEPLIN (LZ) result reported in September 2026 gives this question an observational reference point. LZ observed one event consistent with a nuclear recoil of 248 ± 23 (stat) ± 23 (sys) keV in a 2.84 tonne-year exposure, with a global significance of 2.6σ after the look-elsewhere correction and a maximum local significance of 3.4σ [12]. The result therefore does not meet the conventional 5σ discovery criterion. If the event is due to Dark Matter, it favours a mass of at least about $200 \text{ GeV}/c^2$ and an interaction beyond the simplest elastic spin-independent model [12, 13].

We do not treat the LZ event as evidence for the CD hypothesis. We use it as a phenomenological test: can a localized excitation of a multifractal CD support have the mass scale, thermal relic normalization and recoil-spectrum structure suggested by the event?

The paper is organized as follows. Sections 2–6 define the CD support, introduce quasiparticle excitations and derive the $2 \rightarrow 2$ rate law. Section 7 treats $3 \rightarrow 2$ processes, and Sections 8–9 summarize the correspondence and the relic-abundance scaling. Sections 10 and 11 discuss the LZ event and the conditions it imposes on the CD picture. Sections 12–15 discuss the relation to previous work, observable signatures and limitations, and Section 16 concludes.

2. Cantor Dust as a Dark Matter Support

Let μ_D denote the effective mass measure of a statistically homogeneous and isotropic CD distribution. On scales at which the CD structure is resolved, the measure of a ball of radius ℓ obeys

$$\mu_D(B_\ell) \propto \ell^D, \quad D < 3. \quad (1)$$

In a multifractal realization, the exponent may depend on scale, $D = D(\ell)$, and different moments of the measure are characterized by different generalized dimensions D_q , with D_q non-increasing in q [14, 15]. We denote by ℓ_* the crossover scale above which the support is effectively three-dimensional, and by $\Delta = 3 - D$ the codimension of the support.

Two coarse-grained quantities follow from Eq. (1), and they scale in opposite directions. The first is the fraction of Euclidean volume occupied by the support at resolution ℓ . Covering a region of size ℓ_* requires $N(\ell) \sim (\ell_*/\ell)^D$ cells of volume ℓ^3 , so that

$$f_D(\ell) \simeq \frac{N(\ell) \ell^3}{\ell_*^3} = \left(\frac{\ell}{\ell_*} \right)^{3-D} = \left(\frac{\ell}{\ell_*} \right)^\Delta, \quad \ell \leq \ell_*. \quad (2)$$

The second is the mean density per Euclidean volume, $\bar{\rho}(\ell) \propto \mu_D(B_\ell)/\ell^3 \propto (\ell/\ell_*)^{-\Delta}$, which increases as the resolution becomes finer. Correspondingly, the probability that two points of the support lie within a distance ℓ of each other scales as $(\ell/\ell_*)^{D_2}$, where D_2 is the correlation dimension [16]; relative to the Euclidean value $(\ell/\ell_*)^3$ this is an enhancement by $(\ell/\ell_*)^{-(3-D_2)}$.

Neither expression is a unique definition of a physical density on a fractal set. They are scaling representations used to construct the effective theory below. The distinction matters for the sign of the geometric correction: if an interaction rate is limited by the probability that excitations occupy the support, it is suppressed as in Eq. (2); if it is limited by the clustering of excitations already on the support, it is enhanced. Which case applies cannot be decided by scaling arguments alone and is treated below as model input.

Our recent numerical construction uses spatial fluctuations of the control parameter of a Stuart–Landau (complex Ginzburg–Landau type) equation to generate progressive amplitude fragmentation similar to the iterative construction of a Cantor set, with an effective dimension below unity (reported near 0.9) in a one-dimensional model [11]. This is not a three-dimensional Dark Matter calculation, but it gives a concrete dynamical example of a continuous system that produces a fragmented condensate.

3. From a Fractal Condensate to Particle-Like Excitations

To compare CD with the WIMP and SIMP descriptions, we introduce localized quasiparticle excitations χ of rest mass m_χ on the CD support. Their effective number density is defined by coarse-graining the CD measure over a resolution scale ℓ . The assumption is not that χ is elementary in the ultraviolet (UV), but that observers at long wavelengths can describe localized parts of the condensate with particle variables.

For a nonrelativistic excitation, the shortest intrinsic length is the reduced Compton wavelength $\lambda_\chi = 1/m_\chi$. Annihilation into light states probes distances of this order, and we evaluate the geometric overlap at $\ell \simeq \lambda_\chi$:

$$C_D(\lambda_\chi) = \mathcal{A}_D [f_D(\lambda_\chi)]^p, \quad (3)$$

where p counts independent support-overlap constraints and $\mathcal{A}_D$ contains angular, internal-state and coarse-graining factors. With f_D taken from Eq. (2), $p > 0$ corresponds to occupancy-limited suppression; the clustering-limited case of Section 2 corresponds formally to $p < 0$. A microscopic CD field theory would calculate C_D rather than fit it. The geometric factor is the quantity that carries information about the non-Euclidean support into particle-like rates.

4. Effective 2→2 Annihilation and the WIMP Analogue

For nonrelativistic annihilation of two excitations through a dimensionless dark-sector coupling α_D , dimensional analysis gives an s -wave (orbital angular momentum $L = 0$) rate coefficient of the form

$$\langle \sigma v \rangle_{\text{CD}} = C_D(\lambda_\chi) \frac{\alpha_D^2}{m_\chi^2}, \quad (4)$$

up to velocity-dependent corrections, threshold effects and mediator propagator factors. Equation (4) has the same dimension and the same leading mass dependence as the conventional WIMP annihilation rate. The difference lies in the interpretation of the coefficient: in a conventional particle model it follows from a matrix element in ordinary spacetime, whereas here it also contains the probability that two localized excitations occupy the relevant region of the multifractal support at the same time.

For a mediator of mass M , a more general schematic expression is

$$\langle\sigma v\rangle_{\text{CD}} \simeq C_D \frac{\alpha_D^2 m_\chi^2}{(M^2 + m_\chi^2)^2}. \quad (5)$$

For $M \ll m_\chi$ this reduces to Eq. (4), and for $M \sim m_\chi$ it agrees with Eq. (4) up to a factor of order unity. For $M \gg m_\chi$ it gives the contact-interaction scaling $\alpha_D^2 m_\chi^2/M^4$. The CD construction therefore does not remove the need for a dynamical interaction scale; it changes the interpretation of part of the effective normalization.

5. Thermal Freeze-Out and the WIMP Miracle

The number density of a stable species obeys the Boltzmann equation [17, 18]

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle (n_\chi^2 - n_{\chi,\text{eq}}^2). \quad (6)$$

Freeze-out occurs approximately when the annihilation rate $\Gamma_{\text{ann}} = n_\chi \langle\sigma v\rangle$ falls below the Hubble rate H . During radiation domination $H \simeq 1.66 g_*^{1/2} T^2/M_{\text{Pl}}$, where g_* is the number of relativistic degrees of freedom and $M_{\text{Pl}} \simeq 1.22 \times 10^{19} \text{ GeV}$ is the Planck mass, while the equilibrium density is Boltzmann suppressed once the species is nonrelativistic. Freeze-out occurs at $x_f = m_\chi/T_f \approx 20\text{--}30$, and the relic density is approximately inversely proportional to the annihilation rate at freeze-out,

$$\Omega_\chi h^2 \propto \frac{1}{\langle\sigma v\rangle_{\text{fo}}}. \quad (7)$$

The observed cold Dark Matter density is $\Omega_c h^2 = 0.120 \pm 0.001$ [19]. For s -wave annihilation and $m_\chi \gtrsim 10 \text{ GeV}$, the precise calculation of Ref. [4] gives $\langle\sigma v\rangle_{\text{fo}} \simeq 2.2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$; the frequently quoted value $3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ is an older approximation. The exact value depends on x_f , on g_* , on the velocity dependence of the cross section, on coannihilation and on the thermal history.

Identifying the freeze-out rate with Eq. (4),

$$\langle\sigma v\rangle_{\text{fo}} \simeq C_D \frac{\alpha_D^2}{m_\chi^2}. \quad (8)$$

Restoring $\hbar$ and c , $1 \text{ GeV}^{-2} \simeq 1.17 \times 10^{-17} \text{ cm}^3 \text{ s}^{-1}$, so that $\langle\sigma v\rangle_{\text{fo}} \simeq 2.2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ corresponds to $1.9 \times 10^{-9} \text{ GeV}^{-2}$ and Eq. (8) becomes

$$C_D^{1/2} \alpha_D \simeq 4.3 \times 10^{-3} \left(\frac{m_\chi}{100 \text{ GeV}} \right). \quad (9)$$

For comparison, the SU(2) weak coupling is $\alpha_W = g^2/4\pi \approx 0.034$. With C_D of order unity, weak-size couplings and masses between 10^2 and 10^3 GeV therefore give the required rate. In the CD interpretation, the WIMP miracle becomes a condition on the combination of m_χ , α_D and C_D rather than on m_χ and α_D alone.

Within this effective construction, the agreement between a weak-scale cross section and the freeze-out requirement does not by itself show that χ is a fundamental weakly interacting particle. It is also consistent with a coarse-grained interaction probability of a multifractal condensate that takes values in the same range.

6. A Multifractal Closure for the Overlap Factor

The overlap factor can be related to the codimension $\Delta = 3 - D$. Generalizing Eq. (3) to a real exponent and a running dimension, a minimal parameterization is

$$C_D(\ell) = \mathcal{A}_D \left(\frac{\ell}{\ell_*} \right)^{\kappa [3-D(\ell)]}, \quad (10)$$

where κ is an overlap exponent; $\kappa > 0$ corresponds to occupancy-limited suppression and $\kappa < 0$ to clustering-limited enhancement (Section 2). The value of κ depends on how the interaction samples the singular measure. Equation (10) is a model ansatz.

A specific closure follows if the crossover scale is identified with an interaction length L and the resolution scale with λ_χ . For a Coulomb-like interaction, $L \sim 1/(\alpha_D m_\chi)$ is the Bohr radius of a $\chi\chi$ bound state, so that $\lambda_\chi/L \sim \alpha_D$. Taking $\kappa = 1$ and $\mathcal{A}_D = 1$,

$$C_D \sim \alpha_D^\Delta, \quad \langle \sigma v \rangle_{\text{CD}} \sim \frac{\alpha_D^{\Delta+2}}{m_\chi^2}. \quad (11)$$

In this closure the codimension of the support contributes additional powers of the coupling. For $\Delta \simeq 2$ (that is, $D \simeq 1$, a filament-like support), $C_D \sim \alpha_D^2$ and $\langle \sigma v \rangle_{\text{CD}} \sim \alpha_D^4/m_\chi^2$. The thermal condition Eq. (9) then requires $\alpha_D \approx 0.07$ at $m_\chi = 100 \text{ GeV}$ and $\alpha_D \approx 0.21$ at $m_\chi = 1 \text{ TeV}$ (Table 3). This closure is one of several possibilities and does not follow from our previous work. It is included to show how a geometric support factor can produce coupling suppression without an additional numerical constant.

If $D = D(\ell)$, the effective annihilation coefficient runs with resolution even at constant α_D . Such running would be a characteristic signature of a multifractal dark sector.

7. SIMP Freeze-Out as a Higher-Order CD Overlap

SIMP Dark Matter is governed by number-changing reactions such as $\chi\chi\chi \rightarrow \chi\chi$ [5]. The corresponding Boltzmann equation is

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v^2 \rangle (n_\chi^3 - n_\chi^2 n_{\chi,\text{eq}}), \quad (12)$$

where the rate coefficient $\langle \sigma v^2 \rangle$ has mass dimension -5 . A contact-like CD process therefore has the form

$$\langle \sigma v^2 \rangle_{\text{CD}} = C_{D,3} \frac{\alpha_D^3}{m_\chi^5}. \quad (13)$$

Here $C_{D,3}$ measures the probability that three localized excitations occupy a common interaction region. In a condensate with strong self-interactions, multiple-overlap processes may be enhanced relative to independent point particles; in the notation of Section 2 this corresponds to the clustering-limited regime, controlled by the generalized dimension D_3 . In the CD interpretation the $3 \rightarrow 2$ reaction is a collective recombination or fragmentation process of the multifractal medium, whereas conventional SIMP models start from a hidden-sector particle theory [6].

SIMP freeze-out generally requires appreciable $2 \rightarrow 2$ self-interactions [5], which are constrained by the dynamics of galaxies and clusters [20]. The kinetic energy released in $3 \rightarrow 2$ reactions must also be transferred to the SM or to another sector, or the dark sector heats up, as in cannibal scenarios [21]. In CD, self-interaction can be a collective property of the condensate rather than a separately postulated force, but the same observational bounds apply.

8. WIMP, SIMP and Cantor Dust Correspondence

Table 1 summarizes the correspondence between the three descriptions.

9. Relic-Abundance Scaling

For s -wave annihilation, the asymptotic yield $Y_\infty = n_\chi/s$, with s the entropy density, is [17]

$$Y_\infty \simeq \frac{3.79 x_f}{(g_{*s}/g_*^{1/2}) M_{\text{Pl}} m_\chi \langle \sigma v \rangle_{\text{fo}}}, \quad (14)$$

Table 1: Correspondence between WIMP, SIMP and CD descriptions.

Feature	WIMP	SIMP	CD analogue
Degree of freedom	Massive particle χ	Strongly coupled χ	Localized CD excitation
Dominant freeze-out	$2 \rightarrow 2$	$3 \rightarrow 2$	Two-body or higher-order CD overlap
Rate scaling	α_D^2/m_χ^2	α_D^3/m_χ^5	$C_D \alpha_D^2/m_\chi^2$; $C_{D,3} \alpha_D^3/m_\chi^5$
Origin of interaction	Particle matrix element	Strong hidden sector	Multifractal support and effective coupling
Self-interaction	Model dependent	Typically sizable	Collective property of the condensate
UV interpretation	Elementary or composite particle	Hidden-sector particle theory	Dimensional condensate

where g_{*s} counts the entropic degrees of freedom. Inserting the CD rate gives

$$Y_\infty \simeq \frac{3.79 x_f m_\chi}{(g_{*s}/g_*^{1/2}) M_{\text{Pl}} C_D \alpha_D^2}. \quad (15)$$

The present density is $\Omega_\chi h^2 = m_\chi s_0 Y_\infty / (\rho_c / h^2) \approx 2.74 \times 10^8 (m_\chi / \text{GeV}) Y_\infty$, with $s_0 \simeq 2891 \text{ cm}^{-3}$ and $\rho_c / h^2 \simeq 1.05 \times 10^{-5} \text{ GeV cm}^{-3}$. Hence

$$\Omega_\chi h^2 \propto \frac{x_f m_\chi^2}{g_*^{1/2} M_{\text{Pl}} C_D \alpha_D^2}. \quad (16)$$

At fixed coupling and geometry, a heavier excitation leaves a larger relic density, and a larger overlap factor leaves a smaller one. For SIMP freeze-out, the cubic collision term in Eq. (12) changes the dependence on mass and yield. WIMP-like and SIMP-like relic histories then correspond to different dominant overlap orders of the same support.

10. The 2026 LUX-ZEPLIN Recoil Event

On 1 September 2026, the LZ collaboration reported the results of a nuclear-recoil search with the energy window extended to approximately 270 keV [12, 13]. In a 2.84 tonne-year exposure, LZ observed one event with recoil energy $248 \pm 23 \text{ (stat)} \pm 23 \text{ (sys)} \text{ keV}$, in a region where the expected background is low. A profile-likelihood ratio test gives a maximum local significance of 3.4σ and a global significance of 2.6σ after the look-elsewhere correction. The collaboration states that the result does not reach the 5σ threshold used for a discovery [13].

The event is relevant to the present framework for two reasons. Its energy lies near the upper end of the extended window, and the absence of a corresponding population of lower-energy events disfavors the simplest elastic spin-independent WIMP spectrum, which is concentrated at low recoil energies [12, 22]. According to the LZ analysis, a WIMP explanation would require a mass of at least about $200 \text{ GeV}/c^2$ and an interaction structure beyond the simplest WIMP–nucleus model [13].

The appropriate statement at present is therefore not that LZ has detected a WIMP, but that LZ has observed an unexplained high-energy nuclear-recoil candidate that is compatible with some Dark Matter interpretations. The event may also be a rare background, a statistical fluctuation, or a dark-sector process other than the canonical WIMP.

Several theoretical analyses have already examined non-minimal interpretations. Inelastic, or endothermic, Dark Matter [23], in which scattering excites χ to a state heavier by δ , suppresses low-energy recoils and shifts the spectrum to higher energies. Su, Yang and Yang find that the event requires $m_\chi \gtrsim 500 \text{ GeV}$ and $\delta = \mathcal{O}(300) \text{ keV}$, with a representative best fit near

$(m_\chi, \delta) \simeq (1.1 \text{ TeV}, 350 \text{ keV})$ [24]. Di Mauro finds that elastic spin-dependent scattering and endothermic scattering remain viable, and relates thermal Higgsino Dark Matter to a splitting of about 370 keV [22]; related Higgsino interpretations are discussed in Ref. [25]. An absorption mechanism for fermionic Dark Matter has also been proposed [26]. These results are useful benchmarks for the CD construction, because a multifractal condensate can in principle have internal excitation scales and overlap thresholds.

11. The LZ Event as a Test of the CD–WIMP Correspondence

11.1. Scattering rate

The LZ event can be included in the CD picture without identifying χ with a fundamental point particle. The working hypothesis is that χ is a localized quasiparticle excitation of the CD condensate, and that a xenon recoil results from an interaction between this excitation and a nucleus of mass m_N . The differential rate per unit detector mass has the standard form [27]

$$\frac{dR}{dE_R} = \frac{N_T \rho_\chi}{m_\chi} \int_{v > v_{\min}(E_R)} d^3v f(\mathbf{v}) v \frac{d\sigma_{\chi N}}{dE_R}(v; m_\chi, C_D, \alpha_D, \delta), \quad (17)$$

where N_T is the number of target nuclei per unit mass, ρ_χ is the local Dark Matter density and $f(\mathbf{v})$ is the velocity distribution in the detector frame. The geometric information enters through the effective coupling and the overlap factor evaluated at the momentum-transfer scale. For an excitation with internal splitting δ , the minimum speed required to produce a recoil of energy E_R is [23]

$$v_{\min}(E_R) = \frac{1}{\sqrt{2m_N E_R}} \left(\frac{m_N E_R}{\mu} + \delta \right), \quad (18)$$

where $\mu = m_\chi m_N / (m_\chi + m_N)$ is the reduced mass. For $\delta > 0$, $v_{\min}$ has a minimum at $E_R^* = \mu \delta / m_N$, so that recoils are concentrated near E_R^* and low-energy recoils are suppressed. This is the qualitative feature that makes the LZ event of interest.

11.2. Kinematic conditions

Table 2 evaluates Eq. (18) for $E_R = 248 \text{ keV}$ on xenon ($m_N \simeq 122.3 \text{ GeV}$, averaged over isotopes). The column $\delta^* = m_N E_R / \mu$ gives the splitting for which 248 keV coincides with E_R^* ; in that case $v_{\min} = \sqrt{2m_N E_R} / \mu$, twice the elastic value. In the Standard Halo Model with a Galactic escape speed of about 544 km s^{-1} [28], the maximum speed in the detector frame is roughly 800 km s^{-1} .

Table 2: Kinematics of a 248 keV xenon recoil. $v_{\min}^{\text{el}}$: minimum speed for elastic scattering; δ^* : splitting for which $E_R^* = 248 \text{ keV}$; $v_{\min}(\delta^*)$: corresponding minimum speed. The maximum detector-frame speed in the Standard Halo Model is roughly 800 km s^{-1} .

m_χ [GeV]	μ [GeV]	$v_{\min}^{\text{el}}$ [km s ⁻¹]	δ^* [keV]	$v_{\min}(\delta^*)$ [km s ⁻¹]
50	35.5	1040	855	2080
100	55.0	671	551	1342
200	75.9	486	400	972
500	98.3	376	309	752
1000	109.0	339	278	678

Three conclusions follow. First, elastic scattering of halo particles lighter than about 75 GeV cannot produce a 248 keV xenon recoil. Second, the LZ preference for $m_\chi \gtrsim 200 \text{ GeV}$ reflects the requirement that an appreciable fraction of the spectrum lies in the extended window, not only kinematic accessibility. Third, an inelastic excitation with the recoil at the spectral maximum

requires $m_\chi \gtrsim 400$ GeV in this estimate and $\delta \approx 280\text{--}310$ keV for $m_\chi = 0.5\text{--}1$ TeV, in agreement with the values obtained in Refs. [24, 22]. A CD interpretation of the event must therefore produce an internal splitting of a few hundred keV for an excitation of several hundred GeV, that is, a ratio $\delta/m_\chi \sim 10^{-6}\text{--}10^{-7}$.

11.3. Thermal normalization and scale dependence

Using the LZ-inferred mass scale in Eq. (9) gives the values in Table 3. For $m_\chi = 200$ GeV, the thermal rate requires $C_D \alpha_D^2 \simeq 7.5 \times 10^{-5}$; for $m_\chi = 1$ TeV the required product is larger by a factor of 25, since at fixed C_D and α_D the rate decreases as m_χ^{-2} .

Table 3: Thermal normalization for $\langle \sigma v \rangle_{\text{fo}} = 2.2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. The column α_D ($\Delta = 2$) uses the closure $C_D = \alpha_D^2$ of Eq. (11). The last column is the ratio of the direct-detection resolution scale $1/q$ to the annihilation scale $1/m_\chi$, with $q = \sqrt{2m_N E_R} \simeq 0.246$ GeV for $E_R = 248$ keV.

m_χ [GeV]	$C_D \alpha_D^2$	$C_D^{1/2} \alpha_D$	α_D ($\Delta = 2$)	m_χ/q
100	1.9×10^{-5}	4.3×10^{-3}	0.066	4.1×10^2
200	7.5×10^{-5}	8.7×10^{-3}	0.093	8.1×10^2
500	4.7×10^{-4}	2.2×10^{-2}	0.15	2.0×10^3
1000	1.9×10^{-3}	4.3×10^{-2}	0.21	4.1×10^3

If the LZ event is confirmed as Dark Matter and the relic abundance is thermal, the CD framework must relate the direct-detection overlap to the annihilation overlap. The two processes probe different scales: annihilation probes $\ell_{\text{fo}} \sim 1/m_\chi$, while a 248 keV recoil on xenon probes $\ell_{\text{DD}} \sim 1/q \approx 0.8$ fm. For constant D and $\ell_{\text{DD}} < \ell_*$, Eq. (10) gives

$$\frac{C_D(\ell_{\text{DD}})}{C_D(\ell_{\text{fo}})} = \left(\frac{m_\chi}{q} \right)^{\kappa \Delta}, \quad (19)$$

with $m_\chi/q \sim 10^3$ for the mass range of interest. Even a modest codimension therefore changes the geometric factor by orders of magnitude between the two regimes. This is a restrictive condition: C_D cannot be adjusted independently for each process or epoch. The dark-sector coupling α_D and the coupling to nucleons are in general different, so only the geometric factor, not the full cross section, is shared between the two processes.

The most economical CD interpretation is thus a single scale-dependent overlap function $C_D(\ell)$ whose value at ℓ_{fo} determines the relic abundance and whose value at ℓ_{DD} enters the direct-detection rate. A complete theory must predict both from the same multifractal measure, and the LZ event provides a concrete case against which such a prediction can be tested.

A sequence of confirmed events with the same recoil-energy structure would provide a stronger test. If the spectrum is shown to require inelasticity, the CD model should derive the splitting δ from its microscopic or mesoscopic dynamics rather than insert it by hand. If, instead, future data show a conventional low-energy recoil spectrum and attribute the present event to background, the high-recoil CD interpretation loses its motivation, while the general correspondence with WIMP freeze-out is unaffected.

12. Differences from a Relabeling

A relabeling would identify a WIMP with a piece of Cantor Dust without changing any prediction. The present construction differs in three respects. First, the collision coefficient contains a geometric factor C_D determined by the fractal support. Second, if D depends on scale, the effective cross section runs even at fixed microscopic coupling. Third, the same support can

produce both pairwise and multi-body overlap channels, so that WIMP-like and SIMP-like behaviour appear as different IR limits of one medium.

Direct detection provides a fourth possible discriminator. If the data favour an interaction threshold or a nonstandard recoil spectrum, the CD description can attempt to attribute this structure to localized excitations and scale-dependent overlap. A conventional WIMP model and a CD quasiparticle model may reproduce the same total event rate while predicting different dependences on resolution, velocity, recoil energy and cosmological epoch, as expressed by Eq. (19).

13. Relation to Our Previous Work

This paper extends, rather than repeats, our previous CD publications. Our 2019 work proposed Cantor Dust as a dimensional condensate associated with minimal fractality and discussed its relevance to Dark Matter [7]. Other work connected CD to fuzzy Dark Matter and to the dual particle and collective character of Dark Matter [9, 8]. Our 2025–2026 work developed astrophysical, gravitational, numerical and self-interacting versions of the proposal, including Stuart–Landau dynamics as a mechanism for Cantor-like fragmentation [29, 11, 30, 10, 31].

The new element here is the introduction of an effective quasiparticle χ and the derivation of thermal rate laws from dimensional analysis combined with an explicitly stated multifractal overlap ansatz. The CD–WIMP/SIMP correspondence is therefore a model-building result, not a first-principles prediction of a complete microscopic CD quantum field theory.

The discussion of the LZ event also goes beyond our previous publications: it treats the 2026 high-energy recoil as an external observational test of the particle-like limit of the CD framework. We do not claim that our earlier work predicted this event.

14. Falsifiability and Phenomenological Signatures

The construction leads to the following observable consequences.

1. *Scale-dependent annihilation.* If $D = D(\ell)$, the effective annihilation coefficient departs from the constant-cross-section approximation.
2. *Nonstandard velocity dependence.* Multifractal overlap can introduce a dependence on the de Broglie wavelength and hence on velocity, which would affect indirect-detection signals in environments with different velocity dispersions.
3. *Correlated self-interactions.* A large $C_{D,3}$ implies a sizable $2 \rightarrow 2$ self-interaction, as in SIMP models, subject to astrophysical bounds [20].
4. *Non-Maxwellian coarse-graining.* A singular support can modify the relation between phase-space and configuration-space densities, and therefore $f(\mathbf{v})$ in Eq. (17).
5. *Transition scales.* If CD becomes effectively three-dimensional above ℓ_* , particle phenomenology approaches conventional WIMP or SIMP behaviour in that limit.
6. *Direct-detection recoil structure.* A scale-dependent overlap or an internal excitation threshold changes the recoil spectrum and can produce high-energy features such as the LZ candidate; an internal splitting would appear as a spectrum concentrated near $E_R^* = \mu\delta/m_N$, with a target dependence that can be tested with lighter nuclei, for example in CRESST [24].
7. *Cross-sector consistency.* The same $D(\ell)$ and overlap functions inferred from Dark Matter interactions must be compatible with the gravitational and cosmological CD phenomenology [29, 31].

15. Limitations

The CD framework does not yet provide a microscopic Lagrangian from which C_D and $C_{D,3}$ can be calculated without phenomenological input. The sign and magnitude of the overlap exponent (Section 2) are therefore not determined. The WIMP miracle is a dimensional and numerical coincidence; its reproduction here does not show that weak interactions originate in fractal geometry. A complete thermal history requires equilibration of the dark sector and, depending on the scenario, a coupling to the SM. Entropy transfer, cannibalization, phase transitions and a dark-sector temperature different from that of the SM can change the relic abundance substantially.

The LZ interpretation has an additional limitation: a single event at 2.6σ global significance is not a discovery. The event may be of background origin or may correspond to a Dark Matter model different from a WIMP. No numerical value of C_D should therefore be inferred from it. A consistent analysis must fit, with the same scale-dependent CD measure, the LZ recoil spectrum, the relic density, indirect-detection bounds and self-interaction constraints.

The claim of this paper is therefore conditional: if CD supports localized massive excitations whose interaction probability is governed by a multifractal overlap measure, then WIMP-like and SIMP-like relic laws follow as effective limits, and the 2026 LZ event provides an observational target against which this correspondence can be tested.

16. Conclusions

We have formulated a geometric interpretation of thermal Dark Matter freeze-out within the Cantor Dust framework. A particle-like excitation on a multifractal support does not interact according to Euclidean overlap alone; the collision probability contains a dimensionless factor C_D determined by the CD measure. For two-body annihilation,

$$\langle\sigma v\rangle_{\text{CD}} = C_D \frac{\alpha_D^2}{m_\chi^2}, \quad (20)$$

which has the same parametric form as the conventional WIMP annihilation rate. For suitable values of C_D , α_D and m_χ (Eq. (9)), the observed relic density follows without requiring χ to be a fundamental weakly interacting particle. Within this effective theory, the WIMP miracle can be read as a coincidence of scales associated with coarse-graining a multifractal medium.

The construction extends to $3 \rightarrow 2$ processes through

$$\langle\sigma v^2\rangle_{\text{CD}} = C_{D,3} \frac{\alpha_D^3}{m_\chi^5}, \quad (21)$$

which is a geometric analogue of SIMP freeze-out. WIMP phenomenology corresponds to dominant pair overlap, SIMP phenomenology to dominant triple overlap, and other Dark Matter phases to other collective limits of the same support.

The LZ event reported in September 2026 provides an observational reference. If it is confirmed as Dark Matter, the indicated mass scale above about $200 \text{ GeV}/c^2$ and the need for a non-minimal interaction give quantitative targets for the CD quasiparticle model. In an inelastic interpretation, the excitation must have a splitting of a few hundred keV at a mass of several hundred GeV, and the same scale-dependent overlap function must account for both early-Universe freeze-out and present-day xenon scattering, whose resolution scales differ by a factor of about 10^3 .

The next step is a microscopic or statistical field theory on a multifractal measure. Such a theory should derive $C_D(\ell)$, $C_{D,3}(\ell)$, possible excitation splittings and the velocity dependence;

solve the coupled Boltzmann equations numerically; and compare the results with LZ, future direct-detection data, the relic abundance, self-interaction bounds and indirect searches. Confirmation or disappearance of the LZ anomaly will then provide an empirical discriminator for the particle-like CD interpretation.

Note on the Status of the Central Result

The WIMP/SIMP correspondence derived here is a proposed effective-theory construction. Equations (3)–(19), in particular the geometric overlap prescriptions, do not follow uniquely from our existing CD publications. They are the additional assumptions needed to make the CD–thermal-relic analogy quantitative. The LZ event is treated as an observational test case, not as a confirmed detection and not as a prediction of our earlier work. This separation between established thermal-relic physics, the published LZ result and the proposed CD interpretation is maintained throughout.

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