Compton scattering in dielectric medium

Miroslav Pardy
Department of Physical Electronics
Masaryk University
Kotlářská 2, 611 37 Brno, Czech Republic
e-mail:pamir@physics.muni.cz

September 26, 2023

Abstract

We determine the Compton effect from the Volkov solution of the Dirac equation for a process in medium with the index of refraction n. Volkov solution involves the mass shift, or, the mass renormalization of an electron. We determine the modified Compton formula for the considered physical situation. The index of refraction causes that the wave lengths of the scattered photons are shorter for some angles than the wave lengths of the original photons. This is anomalous Compton effect.

PACS: 12.20Ds, 13.60Fz

Keywords: Dirac equation, Volkov solution, Compton effect, dielectric medium.

1 Introduction

The process of the photon scattering on electron is called the Compton process after Arthur Compton who made the first measurement of photon-electron scattering in 1922. He used the monochromatic X-rays for the determination of their scattering on the free electrons in a block of graphite. He observed that the scattered beam consisted of X-rays is of the longer wavelengths in addition to the original beam. This was explained successfully by applying the laws of conservation of energy and momentum in the collision between X-ray photon and free electron inside graphite block. Compton was awarded in 1927 Nobel Prize in Physics for his discovery.

He derived the equation

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta), \tag{1}$$

where λ' is wavelength of the scattered X-ray and θ is the angle between the incident and scattered X-ray. The scattering was considered in the laboratory frame where electron was at rest. The considered process was so called one photon process with the symbolic equation

$$\gamma + e \to \gamma + e. \tag{2}$$

At present time with the high power lasers there is possible to realize so called the multiphoton scattering according to equation:

$$N\gamma + e \to \gamma + e, \tag{3}$$

or,

$$N\gamma + e \to M\gamma + e, \tag{4}$$

where N and M are numbers of photons participating in the scattering.

The equation (3) is the symbolic expression of the two different physical processes. One process is the nonlinear Compton effect in which several photons are absorbed at a single point, but only single high-energy photon is emitted. The second process is the interaction where electron scatters twice or more as it traverses the laser focus. Similar interpretation is valid for equation (4).

In our article, the attention is concentrated to the situation where the Compton process is considered in dielectric medium with index of refraction n. The index of refraction is generated microscopically by the system of bound electrons and this system forms a medium called dielectric which enables transmitting of electromagnetic waves in this medium. There are also free electrons in this medium. The scattering of photons of the medium on free electrons in this medium then can occur. At present time, the beam of photons can be realized by lasers which play prestige role in all physical laboratories.

Our aim is to determine the Compton process in the dielectric medium as a result of the Volkov solution of the Dirac equation. The Volkov solution involves not only the one-photon scattering but also the multiphoton scattering of photons on electron. In time of Compton experiment in 1922, the Volkov solution was not known, because the Dirac equation was published in 1928 and the Volkov solution in 1935 (Volkov, 1935).

To be pedagogically clear, we remember in the next section some known ideas concerning the Volkov solution of the Dirac equation.

2 Volkov solution of the Dirac equation in vacuum

Let us remember the derivation of the Volkov solution of the Dirac equation in vacuum. We use here the method of derivation and metric convention by Berestetzkii et al. (1989):

$$(\gamma(p - eA) - m)\psi = 0, \tag{5}$$

where

$$A^\mu = A^\mu(\varphi); \quad \varphi = kx. \tag{6}$$

We suppose that the four-potential satisfies the Lorentz gauge condition

$$\partial_\mu A^\mu = k_\mu \left(A^\mu\right)' = \left(k_\mu A^\mu\right)' = 0, \tag{7}$$

where the prime denotes derivation with regard to φ. From the last equation follows

$$kA = const = 0, \tag{8}$$

because we can put the constant to zero. The tensor of electromagnetic field is

$$F_{\mu\nu} = k_\mu A'_\nu - k_\nu A'_\mu. \tag{9}$$

Instead of the linear Dirac equation (5), we consider the quadratical equation, which we get by multiplication of the linear equation by operator $(\gamma(p-eA)+m)$, (Berestetzkii et al., 1989). We get:

$$\left[(p-eA)^2 - m^2 - \frac{i}{2}eF_{\mu\nu}\sigma^{\mu\nu}\right]\psi = 0. \tag{10}$$

Using $\partial_\mu(A^\mu\psi) = A^\mu\partial_\mu\psi$, which follows from eq. (7), and $\partial_\mu\partial^\mu = \partial^2 = -p^2$, with $p_\mu = i(\partial/\partial x^\mu) = i\partial_\mu$, we get the quadratical Dirac equation for the four potential of the plane wave:

$$[-\partial^2 - 2i(A\partial) + e^2A^2 - m^2 - ie(\gamma k)(\gamma A')]\psi = 0. \tag{11}$$

We are looking for the solution of the last equation in the form:

$$\psi = e^{-ipx}F(\varphi). \tag{12}$$

After insertion of this relation into (11), we get with $(k^2 = 0)$

$$\partial^\mu F = k^\mu F', \quad \partial_\mu\partial^\mu F = k^2 F'' = 0, \tag{13}$$

the following equation for $F(\varphi)$

$$2i(kp)F' + [-2e(pA) + e^2A^2 - ie(\gamma k)(\gamma A')]F = 0. \tag{14}$$

The integral of the last equation is of the form:

$$F = \exp\left\{-i\int_0^{kx}\left[\frac{e(pA)}{(kp)} - \frac{e^2}{2(kp)}A^2\right]d\varphi + \frac{e(\gamma k)(\gamma A)}{2(kp)}\right\}\frac{u}{\sqrt{2p_0}}, \tag{15}$$

where $u/\sqrt{2p_0}$ is the arbitrary constant bispinor.

Al powers of $(\gamma k)(\gamma A)$ above the first are equal to zero, since

$$(\gamma k)(\gamma A)(\gamma k)(\gamma A) = -(\gamma k)(\gamma k)(\gamma A)(\gamma A) + 2(kA)(\gamma k)(\gamma A) = -k^2A^2 = 0. \tag{16}$$

Then we can write:

$$\exp\left\{e\frac{(\gamma k)(\gamma A)}{2(kp)}\right\} = 1 + \frac{e(\gamma k)(\gamma A)}{2(kp)}. \tag{17}$$

So, the solution is of the form:

$$\psi_p = R\frac{u}{\sqrt{2p_0}}e^{iS} = \left[1 + \frac{e}{2kp}(\gamma k)(\gamma A)\right]\frac{u}{\sqrt{2p_0}}e^{iS}, \tag{18}$$

where u is an electron bispinor of the corresponding Dirac equation

$$(\gamma p - m)u = 0, \tag{19}$$

with the normalization condition $\bar{u}u = 2m$

The mathematical object S is the classical Hamilton-Jacobi function, which was determined in the form:

$$S = -px - \int_0^{kx}\frac{e}{(kp)}\left[(pA) - \frac{e}{2}(A)^2\right]d\varphi. \tag{20}$$

The current density is

$$j^\mu = \bar{\psi}_p\gamma^\mu\psi_p, \tag{21}$$

where $\bar{\psi}_p$ is defined as the transposition of (18), or,

$$\bar{\psi}_p = \frac{\bar{u}}{\sqrt{2p_0}}\left[1 + \frac{e}{2kp}(\gamma A)(\gamma k)\right]e^{-iS}. \tag{22}$$

After insertion of ψ_p and $\bar{\psi}_p$ into the current density, we have:

$$j^\mu = \frac{1}{p_0}\left\{p^\mu - eA^\mu + k^\mu\left(\frac{e(pA)}{(kp)} - \frac{e^2A^2}{2(kp)}\right)\right\}. \tag{23}$$

which is in agreement with formula in the Meyer article (Meyer, 1971).

If $A^\mu(\varphi)$ are periodic functions, and their time-average value is zero, then the mean value of the current density is

$$\overline{j^\mu} = \frac{1}{p_0}\left(p^\mu - \frac{e^2}{2(kp)}\overline{A^2}k^\mu\right) = \frac{q^\mu}{p_0}. \tag{24}$$

3 Volkov solution of the Dirac equation for plane wave in a dielectric medium

The mathematical approach to the situation where we consider plane wave solution in a medium is the same, only with the difference that the Lorentz condition must be replaced by the following one (Schwinger et al., 1976):

$$\partial_\mu A^\mu = kA' = (\mu\varepsilon - 1)(\eta\partial)(\eta A) = (\mu\varepsilon - 1)(\eta k)(\eta A') \tag{25}$$

with the specification $\eta^\mu = (1, \mathbf{0})$ as the unit time-like vector in the rest frame of the medium (Schwinger et al., 1976).

For periodic potential A^μ we then get from equation (25) instead of $kA = 0$ the following equation:

$$kA = (\mu\varepsilon - 1)(\eta k)(\eta A). \tag{26}$$

Then, we get instead of equation (14) the following equation for function $F(\varphi)$:

$$2i(kp)F' + [-2e(pA) + e^2A^2 - ie(\gamma k)(\gamma A') - ie(\mu\varepsilon - 1)(\eta k)(\eta A')]F = 0. \qquad (27)$$

The solution of the last equation is the solution of the linear equation of the form $y' + Py = 0$ and it means it is of the form $y = C\exp(-\int Pdx)$, where C is some constant. So, we can write the solution as follows:

$$F = \exp\left\{-i\int_0^{kx}\left[\frac{e}{(kp)}(pA) - \frac{e^2}{2(kp)}A^2\right]d\varphi + \frac{e(\gamma k)(\gamma A)}{2(kp)} + \frac{e}{2(kp)}\alpha\right\}\frac{u}{\sqrt{2p_0}}, \qquad (28)$$

where

$$\alpha = (\mu\varepsilon - 1)(\eta k)(\eta A) \qquad (29)$$

The wave function ψ is then the modified wave function (18), which we can write in te form

$$\psi_p = \left[1 + \sum_{n=1}^{\infty}\left(\frac{e}{2(kp)}\right)^n (2\alpha)^n(\gamma k)(\gamma A)\right]\frac{u}{\sqrt{2p_0}}e^{iS}e^T, \qquad (30)$$

where

$$T = \frac{e}{2(kp)}(\mu\varepsilon - 1)(\eta k)(\eta A), \qquad (31)$$

and where we used in the last formula the following relation

$$[(\gamma k)(\gamma A)]^n = (2\alpha)^n(\gamma k)(\gamma A). \qquad (32)$$

So, we see that the influence of the medium on the Volkov solution is involved in $\exp(T)$, where T is given by equation (31) and in the new term which involves sum of the infinite number of coefficients.

4 Compton emission of photons by electron

Now, let us consider electromagnetic monochromatic plane wave which is polarized in circle. We write the four-potential in the form:

$$A = a_1\cos\varphi + a_2\sin\varphi, \qquad (33)$$

where the amplitudes a_i are the same and mutually perpendicular, or,

$$a_1^2 = a_2^2 = a^2, \quad a_1a_2 = 0. \qquad (34)$$

If we use the derived solution (30) of the Dirac equation in a medium, we get very complicated formalism. To avoid such complications we use the original Volkov approach and then we introduce index of refraction into definition of a photon in a medium. Such approach was used in the similar form by Schwinger et al. (Schwinger et al., 1976) in case of the description of the Čerenkov radiation with massless photons and by author (Pardy,

1994) in case of the Čerenkov effect with radiative corrections and of the Čerenkov effect with massive photons (Pardy, 2002). We hope, that this simple method can be used also in case of the application of the Volkov solution for photons in medium.

Volkov solution for the standard vacuum situation is of the form:

$$\psi_p = \left\{1 + \left(\frac{e}{2(kp)}\right)[(\gamma k)(\gamma a_1)\cos\varphi + (\gamma k)(\gamma a_2)\sin\varphi]\right\}\frac{u(p)}{\sqrt{2q_0}} \quad \times$$

$$\exp\left\{-ie\frac{(a_1p)}{(kp)}\sin\varphi + ie\frac{(a_2p)}{(kp)}\cos\varphi - iqx\right\}, \tag{35}$$

where

$$q^\mu = p^\mu - e^2\frac{a^2}{2(kp)}k^\mu. \tag{36}$$

because it follows from eq. (24).

We know that the matrix element M corresponding to the emission of photon by electron in the electromagnetic field is as follows (Berestetzkii et al., 1989):

$$S_{fi} = -ie^2\int d^4x\bar{\psi}_{p'}(\gamma e'^*)\psi_p\frac{e^{ik'x}}{\sqrt{2\omega'}}, \tag{37}$$

where ψ_p is the wave function of an electron before interaction with the laser pulse and $\psi_{p'}$ is the wave function of electron after emission of photon with components $k'^\mu = (\omega', \mathbf{k}')$. The quantity e'^* is the four polarization vector of emitted photon.

Then, we get the following linear combination in the matrix element:

$$e^{-i\alpha_1\sin\varphi + i\alpha_2\cos\varphi} \tag{38}$$

$$e^{-i\alpha_1\sin\varphi + i\alpha_2\cos\varphi}\cos\varphi \tag{39}$$

$$e^{-i\alpha_1\sin\varphi + i\alpha_2\cos\varphi}\sin\varphi. \tag{40}$$

where

$$\alpha_1 = e\left(\frac{a_1p}{kp} - \frac{a_2p'}{kp'}\right), \tag{41}$$

and

$$\alpha_2 = e\left(\frac{a_2p}{kp} - \frac{a_2p'}{kp'}\right). \tag{42}$$

Now, we can expand exponential function into the Fourier transformation where the coefficients of the expansion will be B_s, B_{1s}, B_{2s}. So we write:

$$e^{-i\alpha_1\sin\varphi + i\alpha_2\cos\varphi} = e^{-i\sqrt{\alpha_1^2+\alpha_2^2}\sin(\varphi-\varphi_0)} = \sum_{s=-\infty}^{\infty} B_s e^{-is\varphi} \tag{43}$$

$$e^{-i\alpha_1 \sin\varphi + i\alpha_2 \cos\varphi} \cos\varphi = e^{-i\sqrt{\alpha_1^2+\alpha_2^2}\sin(\varphi-\varphi_0)} \cos\varphi = \sum_{s=-\infty}^{\infty} B_{1s} e^{-is\varphi} \tag{44}$$

$$e^{-i\alpha_1 \sin\varphi + i\alpha_2 \cos\varphi} \sin\varphi = e^{-i\sqrt{\alpha_1^2+\alpha_2^2}\sin(\varphi-\varphi_0)} \sin\varphi = \sum_{s=-\infty}^{\infty} B_{2s} e^{-is\varphi}. \tag{45}$$

The Coefficients B_s, B_{1s}, B_{2s} can be expressed by means of the Bessel function as follows (Berestetzkii et al., 1989):

$$B_s = J_s(z) e^{is\varphi_0} \tag{46}$$

$$B_{1s} = \frac{1}{2}\left[J_{s+1}(z)e^{i(s+1)\varphi_0} + J_{s-1}(z)e^{i(s-1)\varphi_0}\right] \tag{47}$$

$$B_{2s} = \frac{1}{2i}\left[J_{s+1}(z)e^{i(s+1)\varphi_0} - J_{s-1}(z)e^{i(s-1)\varphi_0}\right], \tag{48}$$

where the quantity z is defined in (Berestetzkii et al., 1989) through α components as follows:

$$z = \sqrt{\alpha_1^2 + \alpha_2^2} \tag{49}$$

and

$$\cos\varphi_0 = \frac{\alpha_1}{z}; \quad \sin\varphi_0 = \frac{\alpha_2}{z} \tag{50}$$

Functions B_s, B_{1s}, B_{2s} are related one to another as follows:

$$\alpha_1 B_{1s} + \alpha_2 B_{2s} = s B_s, \tag{51}$$

which follows from the well known relation for Bessel functions:

$$J_{s-1}(z) + J_{s+1}(z) = \frac{2s}{z} J_s(z) \tag{52}$$

The matrix element (37) can be written in the form (Berestetzkii et al., 1989):

$$S_{fi} = \frac{1}{\sqrt{2\omega' 2q_0 2q'_0}} \sum_s M_{fi}^{(s)} (2\pi)^4 i\delta^{(4)}(sk + q - q' - k'), \tag{53}$$

where the δ-function involves the law of conservation in the form

$$sk + q = q' + k', \tag{54}$$

where, respecting eq. (24)

$$q^\mu = p^\mu - \frac{e^2}{2(kp)}\overline{A^2} k^\mu. \tag{55}$$

Using the last equation we can introduce the so called "effective mass" of electron immersed in the periodic wave potential as follows:

$$q^2 = m_*^2; \quad m_* = m\sqrt{\left(1 - \frac{e^2}{m^2}\overline{A^2}\right)} \tag{56}$$

Formula (56) represents the mass renormalization of an electron mass in the field A. In other words the mass renormalization is defined by the equation

$$m_{\text{phys}} = m_{\text{bare}} + \delta m \tag{57}$$

where δm follows from eq. (56). The quantity m_{phys} is the physical mass that an experimenter would measure if the particle were subject to Newton's law $\mathbf{F} = m_{\text{phys}}\mathbf{a}$. In case of the periodic field of laser, the quantity δm has the finite value. The renormalization is not introduced here " by hands " but it follows immediately from the formulation of the problem of electron in the wave field. We know the general opinion that renormalization is unavoidable in the quantum field theory ('t Hooft, 200), (Veltman, 2000). On the other hand Schwinger source theory works without renormalization at all (Dittrich, 1978).

We can write

$$q^2 = q'^2 = m^2(1 + \xi^2) \equiv m_*^2, \tag{58}$$

where for plane wave (35) with relations (36)

$$\xi = \frac{e}{m}\sqrt{-a^2}. \tag{59}$$

According to (Berestetzkii et al., 1989) the matrix element in (53) is of the form:

$$M_{fi}^{(s)} = -e\sqrt{4\pi}\bar{u}(p')\left\{\left((\gamma e') - e^2 a^2 \frac{(ke')}{2(kp)}\frac{(\gamma k)}{(kp')}\right) B_s + \right.$$
$$e\left(\frac{(\gamma a_1)(\gamma k)(\gamma e')}{2(kp')} + \frac{(\gamma e')(\gamma k)(\gamma a_1)}{2(kp)}\right) B_{1s} +$$
$$\left. e\left(\frac{(\gamma a_2)(\gamma k)(\gamma e')}{2(kp')} + \frac{(\gamma e')(\gamma k)(\gamma a_2)}{2(kp)}\right) B_{2s}\right\} u(p) \tag{60}$$

It is possible to show, that the total probability of the emission of photons from unit volume in unit time is (Berestetzkii et al., 1989):

$$W = \frac{e^2 m^2}{4q_0}\sum_{s=1}^{\infty}\int \frac{du}{(1+u)^2} \quad \times$$
$$\left\{-4J_s^2(z) + \xi^2\left(2 + \frac{u^2}{1+u}\right)\left(J_{s+1}^2(z) + J_{s-1}^2(z) - 2J_s^2(z)\right)\right\}, \tag{61}$$

where

$$u = \frac{(kk')}{(kp')}, \quad u_s = \frac{2s(kp)}{m_*^2}, \quad z = 2sm^2\frac{\xi}{\sqrt{1+\xi^2}}\sqrt{\frac{u}{u_s}\left(1 - \frac{u}{u_s}\right)}. \tag{62}$$

Variables $\alpha_{1,2}$ are to be expressed in terms of variables u and u_s from the equation (62).

When $\xi \ll 1$ (the condition for perturbation theory to be valid), the integrand (61) can be expanded in powers of ξ. For the first term of the sum W_1, we get

$$W_1 \approx \frac{e^2m^2}{4p_0}\xi^2 \int_0^{u_1} \left[2 + \frac{u^2}{1+u} - 4\frac{u}{u_1}\left(1 - \frac{u}{u_1}\right)\right] du =$$

$$\frac{e^2m^2}{4p_0}\xi^2 \left[\left(1 - \frac{4}{u_1} - \frac{8}{u_1^2}\right)\ln\left(1+u_1\right) + \frac{1}{2} + \frac{8}{u_1} - \frac{1}{2(1+u_1)^2}\right] \tag{63}$$

with

$$u_1 \approx \frac{2(kp)}{m^2}. \tag{64}$$

It is possible to determine the second and the next harmonics as an analogy with the Berestetzkii approach, however the aim of this article was only to illustrate the influence of the dielectric medium on the Compton effect.

Let us consider the equation (54) in the form:

$$sk + q - k' = q'. \tag{65}$$

The equation (65) has physical meaning for $s = 1, 2, ...n$, n being positive integer. For $s = 1$ it has meaning of the conservation of energy momentum of the one-photon Compton process, $s = 2$ has meaning of the two-photon Compton process and $s = n$ has meaning of the multiphoton interaction with n photons of laser beam with an electron. The multiphoton interaction is nonlinear and differs from the situation where electron scatters twice or more as it traverses the laser focus. It also means that the original Einstein photoelectric equation can be replaced by the more general multiphoton photoelectric equation in the form:

$$n\hbar\omega = \frac{1}{2}mv^2 + E_i, \tag{66}$$

where E_i is the binding energy of the outermost electron in the atomic system . It means (Delone et al., 2000) that the ionization effect occurs also in the case that $\hbar\omega < E_i$ in case that number of participating photons is $n > E_i/\hbar\omega$. We will not solve furthermore this specific problem.

We introduce the scattering angle θ between $\mathbf{k}$ and $\mathbf{k}'$. In other words, The scattering angle θ is measured with respect to the incident photon direction. Then, with $|\mathbf{k}| = n\omega$ and $|\mathbf{k}'| = n\omega'$, where n is index of refraction of the dielectric, we get from the squared equation (65) in the rest system of electron, where $q = (m_*, 0)$, the following equation:

$$s\frac{1}{\omega'} - \frac{1}{\omega} = \frac{s}{m_*}(1 - n^2\cos\theta), \tag{67}$$

which is modification of the original equation for the Compton process

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m}(1 - \cos\theta). \tag{68}$$

So, we see that Compton effect described by the Volkov solution of the Dirac equation differs from the original Compton formula only by the existence of the renormalized mass and the presentation of index of refraction.

We know that the last formula of the original Compton effect can be written in the form suitable for the experimental verification, namely:

$$\Delta\lambda = 4\pi\frac{\hbar}{mc}\sin^2\frac{\theta}{2}, \tag{69}$$

which was used by Compton for the verification of the quantum nature of light (Rohlf, 1994).

If we consider the Compton process in dielectric, then the last formula goes to the following form:

$$\Delta\lambda = 2\pi\frac{\hbar}{mc}(1 - n^2\cos\theta). \tag{70}$$

It is evident that relation $\lambda' - \lambda \geq 0$ follows from eq. (1). However, if we put

$$1 - n^2\cos\theta \leq 0 \qquad (71),$$

or, equivalently

$$\frac{1}{n^2} \leq \cos\theta \leq 1, \tag{72}$$

then, we see that for some angles determined by eq. (72) the relation $\lambda' - \lambda \leq 0$ follows. This surprising result is the anomalous Compton effect which is caused by the index of refraction of the medium. To our knowledge, it was not published in the optical or particle journals.

The equation $sk + q = q' + k'$ is the symbolic expression of the nonlinear Compton effect in which several photons are absorbed at a single point, but only single high-energy photon is emitted. The second process where electron scatters twice or more as it traverses the laser focus is not considered here. The nonlinear Compton process was experimentally confirmed for instance by Bulla et al. (Bulla et al., 1996).

The formula (67) can be also expressed in terms of λ as follows:

$$s\lambda' - \lambda = \frac{2\pi s}{m_*}(1 - n^2\cos\theta) \tag{73}$$

where we have put $\hbar = c = 1$.

Formula (73) can be use for the verification of the Compton effect in a dielectric medium and on the other hand the index refraction follows from it in the following form:

$$n^2 = \frac{1}{\cos\theta}\left[1 - \frac{m_*}{2\pi s}(s\lambda' - \lambda)\right]. \tag{74}$$

It means, if we know the $\theta, \lambda, \lambda', s, m_*$, we are able to determine index of refraction of some dielectric medium from the Compton effect. To our knowledge, this method was not published in the optical journals.

5 Discussion

We have considered the Compton effect in the framework of the Volkov solution of the Dirac equation assuming that the process occurred in medium with the index of refraction

n. We have determined the Compton formula for such physical situation, and we have got the formula from which we can determine the index of refraction of a dielectric medium. While the Compton was awarded in 1927 Nobel Prize in Physics, it is not excluded, the Compton effect in dielectric medium is in the state of preparation.

The index of refraction which is substantial in the considered process, is effective quantity which was introduced into physics from experiment. Following Lorentz, the contributions of large numbers of atoms can be averaged to represent the behavior of an isotropic dielectric medium with the index of refraction n. We know that the introducing the index of refraction as a constant physical quantity is inadequate, because it fails to account for the decomposition of white light in prism. In other words index of refraction depends on frequency. We can understand it only by learning more about the optical properties of matter. We know that the electric composition of matter is, that every atom of matter consists of positive nucleus and negative electrons. After application of the electric field, the polarization of the system occurs (Sommerfeld, 1954).

The electron charge is separated from the ion charge after application of electric field. The phenomenon is called polarization. In other words polarization means that the electric field displaces the electrons from their rest position. In the elementary theory of polarization it is supposed that electrons are bound elastically to their rest position. So, their motion can be described by the equation for harmonic oscillator with frequency ω_0 in the most simple case. Using some elementary physics and mathematics of the dispersion theory, we can derive the known formula for the index of refraction of matter.

$$n^2 = 1 + \frac{Ne^2}{m\varepsilon_0}\,\frac{1}{\omega_0^2 - \omega^2}, \tag{75}$$

where N is the number of dispersion electrons per unit volume, ε_0 is the dielectric constant of matter.

Let us remark that the above definition of the index of refraction is the classical one. On the other hand, there is the quantum theory of the dispersion and of the index of refraction. It is described in many textbooks of quantum mechanics and quantum optics. The modern aspects can be found in the article of Crenshaw (1989).

It is substantial of our approach that electromagnetic wave is transmitted by the bound electrons and not by the free electrons. In such a way the process can exist in matter where the electromagnetic wave is diffracted by existing free electrons. And this is the Compton effect.

In particle physics, we know the processes at the finite temperature where the medium is usually formed by the electron-positron pairs and photon gas at finite temperature (Ternov et al., 1986). Formulation and solution of the Compton process at finite temperature is, of course, possible, however we solve the problem Compton process in dielectric medium. We have proved that this problem is physically meaningful, because of the existence electromagnetic waves in medium and free electrons in medium. Free electrons in medium contribute also at the index of refraction, however this contribution is considered here as very small.

The interesting result of our article is the derivation that for some scattering angles given by eq. (73) there exist the so called anomalous Compton effect, where the wave lengths of scattered photons are shorter than the wave lengths of the original photons.

To our knowledge this effect was not published in the physical journals.

The present article is continuation of the author discussion on laser problems (Pardy, 1998; Pardy, 2001; Pardy, 2003; Pardy, 2004), where the discussion of the Compton process in the model of the laser acceleration was considered.

It is obvious that the Compton scattering is, at the present time, the elementary laboratory problem because for the monochromatic X-rays for $\lambda = 1\mathring{A}$ the shift of wave length is several percent. This is quantity which can be easily measured. On the other hand the Compton wave length shift for the visible light is only 0,01 percent. It means that the measurement of the Compton effect for the visible light in the dielectric medium involves some obstacles and it means that this problem can be solved only by the well educated experimenters.

References

Berestetzkii, V. B., Lifshitz, E. M. and Pitaevskii, L. P. *Quantum Electrodynamics*, Moscow, Nauka, 1989., (in Russian).

Bulla, C. (1996). *et al.*, Observation of nonlinear effects in Compton scattering, *Physical Review Letter*, 76, 3116.

Crenshaw, M. E. (2004). Quantum optics of dielectrics in momentum-space, *Optics Communications*, 235, 153.

Delone, N. B. and Krainov, V. P. *Multiphoton Processes in Atoms*, 2nd ed., Springer-Verlag, Berlin, Heidelberg, New York, 2000.

Dittrich, W. (1978) . Source methods in quantum field theory, *Fortschritte der Physik*, 26, 289.

't Hooft, G. (2000). Nobel lecture: Confrontation with infinity, *Reviews of Modern Physics*, 72 No.2, April 333.

Meyer, J. W. (1971). Covariant classical motion of electron in a laser beam, *Physical Review D: Particles and Fields*, 3, (2), 621.

Pardy, M. (1994). The Čerenkov effect with radiative corrections, *Physics Letters B*, 325, 517.

Pardy, M. (2002). Čerenkov effect with massive photons, *International Journal of Theoretical Physics*, 41,(5) 887.

Pardy, M. (1998) . The quantum field theory of laser acceleration, *Physics Letters A*, 243, 223.

Pardy, M. (2001). The quantum electrodynamics of laser acceleration, *Radiation Physics and Chemistry*, 61, 321.

Pardy, M. (2003) .Electron in the ultrashort laser pulse, *International Journal of Theoretical Physics*, 42(1), 99.

Pardy, M. (2004). Massive photons and the Volkov solution, *International Journal of*

Theoretical Physics, 43(1), 127.

Rohlf, J. W. *Modern Physics from α to Z^0*, John Wiley & Sons, Inc. New York 1994.

Schwinger, J., Tsai, W. Y. and Erber, T. (1976). Classical and quantum theory of synergic synchrotron-Čerenkov radiation, *Annals of Physics (New York)*, 96(2) 303.

Sommerfeld, A. *Optics*, Academic Press, New york 10, N. Y., USA, 1954.

Ternov, M. E., Zhukovskii, V. Ch., Midodashivili, V.P. G. and Eminov, P. A. (1986). The anomalous magnetic moment of electron at finite temperature, *Journal of Nuclear Physics*, 43 No. 3 764

Veltman, M. J. G. (2000). Nobel Lecture: From weak interactions to gravitation, *Reviews of Modern Physics*, 72 No.2, April 341.

Volkov, D. M. (1935).Über eine Klasse von Lösungen der Diracschen Gleichung, *Zeitschrift für Physik*, 94, 250.