2000 MSC: 35Q60, 83C5

Newton's laws for a biquaternionic model of the electro-gravimagnetic field,charges,currents and their interactions

L.A.Alexeyeva

Wave Dynamics Laboratory, Institute of Mathematics, Ministry of Education and Science of the Republic of Kazakhstan, 125 Pushkin Street, Almaty 050100, Kazakhstan, E-mail:alexeeva@math.kz

Abstract

With use the Hamiltonian form of the Maxwell's equations one biquaternionic model for electro-gravimagnetic (EGM) field is offered. The equations of the interaction of EGM-fields, which are generated by different charge and current, are built. The field analogs of three Newton's laws are offered for free and interacting charge-currents, as well as total field of interaction. The invariance of these equations at Lorentz transformation is investigated, and, in particular, of the charge-current conservation law. It is shown that, by fields interaction, this law differs from the well-known one. The new modification of the Maxwell's equations is offered with entering the scalar resistance field in biquaternion of EGM-field tension. Relativistic formulae of the transformation of density of the masses and charge, current, forces and their powers are built. The solution of the Cauchy problem is given for equation of charge-current transformations.

1 Introduction

In the present paper, a biquaternionic model of the electro-gravimagnetic (EGM) field is considered, which is called *energetic*. For this a complex Hamiltonian form of Maxwell equations (MEs) is used which allows to get the biquaternionic form of these equations [1,2]. Note that Maxwell gave his equations in quaternionic form, but the modern form belongs to Heaviside [5]. Quaternionic forms of MEs were used by some authors [5-7] for their solving. Kassandrov applied similar forms for building a united field model [8].

Here we use scalar-vector form of biquaternion, which is very impressive and can be adapted for writing the physical values and equations. Based on Newton's laws, the biquaternionic transformation equations of charges and currents at the presence of the EGM fields are constructed. The relation of these equations to hydrodynamics equations is considered. The energy conservations laws at the presence of the fields interaction is found.

The Lorentz invariance of equations of energetic field interaction is studied, and also the invariance of charge-current conservation law. It is shown that by the charges-current interaction this law differs from the well-known one for the closed electromagnetic (EM) field. The new modification of the MEs is proposed for open field. One has to introduce the scalar *field of resistance*. The relativistic transformation of mass and electric charge and current densities, acting forces and their powers are constructed.

2 Hamiltonian form of Maxwell equations

Let $\mathbf{M} = R^{1+3} = \{(\tau, x) = (ct, x) : t \in R^1, x = (x_1, x_2, x_3) \in R^3\}$ denote the Minkowski space. In $\mathbf{M}$ the symmetric form of Maxwell equations for electromagnetic field can be written as [1]

$$\partial_\tau A + i\, rot A + J = 0, \tag{2.1}$$

$$\rho = div\, A, \tag{2.2}$$

where A is complex vector of EM field tensions:

$$A = A^E + i\, A^H = \sqrt{\varepsilon}\, E + i\, \sqrt{\mu}\, H. \tag{2.3}$$

Vectors E and H are the tensions of electric and magnetic fields , ε and μ are constants of electric conductivity and magnetic permeability of the medium. Complex charge density ρ and current density J are expressed through the densities of electric and magnetic charges and currents by

$$\rho = \rho^E/\sqrt{\varepsilon} - i\, \rho^H/\sqrt{\mu}, \quad J = \sqrt{\mu}\, j^E - i\sqrt{\varepsilon}\, j^H, \tag{2.4}$$

$$\rho^E = \varepsilon\, div\, E, \quad \rho^H = -\mu\, div\, H. \tag{2.5}$$

Energy density W and the Pointing vector P of A-field read:

$$W = 0,5\left(\varepsilon \|E\|^2 + \mu \|H\|^2\right) = 0,5(A, \bar{A}), \quad P = c^{-1} E \times H = 0,5i\, [A, \bar{A}], \tag{2.6}$$

where $\bar{A} = \sqrt{\varepsilon}E - i\sqrt{\mu}H$, $c = 1/\sqrt{\varepsilon\mu}$ is the speed of light. Here and hereinafter

$$(a, b) = \sum_{i=1}^{3} a_i b_i, \quad [a, b] = a \times b = \sum_{i,j,k=1}^{3} e_{ijk} e_i a_j b_k$$

are scalar and vector products of the a and b respectively, e_{ijk} is Levi-Chivita pseudotensor and e_i $(i = 1, 2, 3)$ are the unit vectors of the cartesian coordinate system in R^3 .

One can see, that energy density W is simply the module of the complex vector A (half of it). Note that, differently from MEs , all relations for A-field don't contain the constants of EM medium. In particular, the velocity of electromagnetic waves in this coordinate system is non-dimensional and equal to 1.

Here some known statements are given which are due to Maxwell equations [1].

Theorem 2.1. *For given current and charge the solution of Eq.* (2.1) *satisfies to the wave equation*:

$$\Box\, A = (\partial_\tau^2 - \Delta)A = i\, rot\, J - grad\rho - \partial_\tau J, \tag{2.7}$$

and the conservation laws of charge and energy is hold:

$$\partial_\tau \rho + div\, J = 0, \tag{2.8}$$

$$\partial_\tau W + div\, P = -\mathrm{Re}(J, \bar{A}) = c^{-1}(j^H H - j^E E). \tag{2.9}$$

In MEs the density of the magnetic charges $\rho^H = 0$ .This means that the magnetic field is solenoidal one: $div\, H = 0$.

But it is known that the classical gravitation is scalar field. It can be described by a scalar gravitational potential, which depends on the masses. Here these two fields are united in a unique *gravimagnetic field*. It is possible to do so if to introduce a density of gravitational mass in Maxwell equations. In particular the following hypothesis can be proposed.

H y p o t h e s i s. The *density* ρ^H *is equal to density of gravitational mass.*

Hereinafter we will show that this hypothesis has a theoretical acknowledgements which bring us very plausible effect.

Thence follows that potential part of H describes the gravitational field and solenoidal part describes magnetic field. So H-field is *gravimagnetic field*. Consequently, A-field is *electro-gravimagnetic*. Since its dimensionality is defined by density of energy, it possible be called as *energetic field*.

We name j^H *gravimagnetic* current. If $\rho^H = 0$ it is *magnetic* current. If H-field is potential ($rotH = 0$), j^H is *mass* current.

Note, the system of MEs is unclosed. It allows for given charge and current to define the A-field, or for given A-field to find corresponding charges and currents. If they are unknown then for its closing usually the equations of mechanics of mediums are used. However we will enter here the other image, using biquaternionic form of these equations and Newton laws.

For going to this form and new equations we will give the thumbnail sketch on functional space of biquaternions and operation on it.

3 Differential algebra of biquaternions: Bigradients

The functional space of biquaternions is the space of complex quaternions:

$$\mathbf{K}(R^{1+3}) = \{\mathbf{F} = f(\tau, x) + F(\tau, x), \}$$

where f is complex function and F is three-dimensional vector-function with complex components. We assume further that f and F are local integrable and differentiable on $\mathbf{M}$.

The space $\mathbf{K}$ is *associative* but non commutative algebra with addition

$$\mathbf{F} + \mathbf{G} = (f + g) + (F + G),$$

and product($\circ$)

$$\mathbf{F} \circ \mathbf{G} = (f + F) \circ (g + G) = (fg - (F, G)) + (fG + gF + [F, G]). \tag{3.1}$$

The biquaternion $\bar{\mathbf{F}} = \bar{f} + \bar{F}$ is called *complex conjugated*, $\mathbf{F}^* = \bar{f} - \bar{F}$ is *conjugated*. If $\mathbf{F}^* = \mathbf{F}$, it is called *selfconjugated*. The example of selfconjugated biquaternion is $\mathbf{F} = f + iF$, when f and F are real functions.

Definition 3.1. *Scalar product* of $\mathbf{F}_1, \mathbf{F}_2$ is defined by

$$(\mathbf{F}_1, \mathbf{F}_2) = f_1 f_2 + (F_1, F_2)\,.$$

Definition 3.2. *Norma* of $\mathbf{F}$ is defined by

$$\|\mathbf{F}\| = \sqrt{(\mathbf{F}, \bar{\mathbf{F}})} = \sqrt{f \cdot \bar{f} + (F, \bar{F})} = \sqrt{|f|^2 + \|F\|^2}.$$

Definition 3.3. *Pseudonorma* of $\mathbf{F}$ is defined by

$$\langle \mathbf{F} \rangle = \sqrt{f \cdot \bar{f} - (F, \bar{F})} = \sqrt{|f|^2 - \|F\|^2}.$$

Hereinafter the *mutual complex gradients* are used:

$$\nabla^+ = \partial_\tau + i\nabla, \quad \nabla^- = \partial_\tau - i\nabla,$$

where $\nabla = grad = (\partial_1, \partial_2, \partial_3)$. The action of these differential operators on $\mathbf{K}$ is determined as in the biquaternions algebra : (accordingly to a sign)

$$\nabla^\pm \mathbf{F} = (\partial_\tau \pm i\nabla) \circ (f + F) = (\partial_\tau f \mp i\,(\nabla, F)) \pm \partial_\tau F \pm i\nabla f \pm i[\nabla, F] =$$

$$= (\partial_\tau f \mp i\, div\, F) \pm \partial_\tau F \pm i grad\, f \pm i\, rot\, F.$$

Further we call them *bigradients.*

4 Biwave equations: Cauchy problem

It is easy to check that wave operator can be presented in the form

$$\Box = \frac{\partial^2}{\partial \tau^2} - \triangle = \nabla^- \circ \nabla^+ = \nabla^+ \circ \nabla^-.$$

Using this property, it is possible to build the solution of the differential equations of the type:

$$\nabla^\pm \mathbf{K} = \mathbf{G}. \tag{4.1}$$

We call such equations the *biwave equations*. From (4.1) it is follow that

$$\Box \mathbf{K} = \nabla^\mp \mathbf{G}.$$

Its solution is the convolution

$$\mathbf{K} = \nabla^\mp \mathbf{G} * \psi, \tag{4.2}$$

where $\psi(\tau, x)$ is the fundamental solution of the wave equation

$$\Box \psi = \delta(\tau)\delta(x).$$

This solution is also the solution of (4.1). Really, using property of differentiation of convolution, we have

$$\nabla^\pm \mathbf{K} = \nabla^\pm \nabla^\mp (\mathbf{G} *\ \psi) = \Box\, (\mathbf{G} *\ \psi) = (\mathbf{G} *\ \Box\psi) = \mathbf{G} * \delta(\tau)\delta(x) = \mathbf{G}.$$

The fundamental solutions are defined for constructing solutions of wave equation. For value problems it is convenient to use as fundamental solution the simple layer on light cone $\tau = \|x\|$:

$$\psi = (4\pi \, \|x\|)^{-1} \delta(\tau - \|x\|).$$

In this case, as it is easy to show, by writing the convolution in integral type, the decision (4.2) will be equal to zero for $\tau = 0$. We use this for building of the solution of (4.1) with Cauchy data.

Cauchy problem. The initial data are given: $\mathbf{K}(0, x) = \mathbf{K_0}(x)$. It is require to find the solution of (4.1), which satisfies these data.

To solve the problem we use here methods of distribution theory [9]. We will consider regular generalized functions of the type $\widehat{\mathbf{G}} = H(\tau)\mathbf{G}(\tau, x)$, where $H(\tau)$ is the Heaviside function. By using differentiation of generalized function we obtain $\nabla^{\pm}\widehat{\mathbf{K}} = \widehat{\mathbf{G}} + \delta(\tau)\mathbf{K_0}(x)$. Hence

$$\mathbf{H}(\tau)\mathbf{K}(\tau, x) = \nabla^{\mp}\{H(\tau)\widehat{\mathbf{G}} * \psi\} + \mathbf{G}(0, x) \underset{x}{*} \psi + \nabla^{\mp}\{\mathbf{K_0}(x) \underset{x}{*} \psi\} \tag{4.3}$$

where the sign " $\underset{x}{*}$ "means that convolution is given only over x. Its integral form is

$$4\pi\mathbf{K}(\tau, x) = -\nabla^{\mp}\left\{\int\limits_{r\le\tau} \frac{\mathbf{G}(\tau - r, y)}{r} dV(y) + \tau^{-1} \int\limits_{r=\tau} \mathbf{K_0}(y) dS(y)\right\} - \tau^{-1} \int\limits_{r=\tau} \mathbf{G}(0, y) dS(y), \tag{4.4}$$

where $r = \|y - x\|$, $dV(y) = dy_1 dy_2 dy_3$, $dS(y)$ is a differential of sphere's area.

This formula is a generalization of the famous Kirchhoff formula for solution of Cauchy problem for wave equation [9].

5 Biquaternions of A-field

We introduce the following biquaternions:

$$\textit{potential} \quad \mathbf{\Phi} = i\phi - \Psi,$$

$$\textit{tension} \quad \mathbf{A} = 0 + A,$$

$$\textit{charge-current density} \quad \mathbf{\Theta} = -i\rho - J,$$

$$\textit{energy-pulse density} \quad \mathbf{\Xi} = 0{,}5\,\mathbf{A}^* \circ \mathbf{A} = W + iP.$$

In the biquaternions space the Maxwell equations (2.1)-(2.2) have the simple form [2]

$$\nabla^+\mathbf{A} = \mathbf{\Theta}. \tag{5.1}$$

If the potential satisfies to Lorentz calibration $\partial_\tau \phi - div\,\Psi = 0$, then

$$\mathbf{A} = \nabla^-\mathbf{\Phi}.$$

If we take corresponding complex gradient, we get the wave equations

$$\Box\mathbf{\Phi} = \mathbf{\Theta}, \tag{5.2}$$

$$\Box\mathbf{A} = \nabla^-\mathbf{\Theta}. \tag{5.3}$$

Hence it follows that the bigradient from A-field potential defines biquaternions, corresponding to the field tension, charge and current. The scalar part of bigradient of energy-pulse gives the law of the energy conservation [2].

One can see that the *charges and currents is simply physical appearance of the bigradient of EGM field* .

Cauchy problem for Maxwell equations. From equation (4.3) it follows that for given charge-current and initial data $\mathbf{A}(0, x) = \mathbf{A_0}(x)$, the solution of (5.1) is given by

$$4\pi\mathbf{A} = -\nabla^-\left\{\int\limits_{r\le\tau} \frac{\mathbf{\Theta}(\tau - r, y)}{r} dV(y) + \tau^{-1} \int\limits_{r=\tau} \mathbf{A_0}(y) dS(y)\right\} - \tau^{-1} \int\limits_{r=\tau} \mathbf{\Theta}(0, y) dS(y).$$

Hence it is easy to write the integral representations for vector of the EGM-field tension *E,H*.

6 Lorentz transformation on M

Denote

$$\mathbf{Z} = \tau + ix, \quad \bar{\mathbf{Z}} = \tau - ix.$$

It is easy to see that

$$\mathbf{Z} = \mathbf{Z}^*,\ \bar{\mathbf{Z}} = \bar{\mathbf{Z}}^*,\ \|\mathbf{Z}\|^2 = \|\bar{\mathbf{Z}}\|^2 = (\mathbf{Z}, \bar{\mathbf{Z}}),\ \langle \mathbf{Z} \rangle^2 = \langle \bar{\mathbf{Z}} \rangle^2 = \mathbf{Z} \circ \bar{\mathbf{Z}}.$$

Consider the selfconjugated biquaternions by using the hyperbolic sine and cosine:

$$\mathbf{U} = \cosh\theta + ie\sinh\theta,\ \bar{\mathbf{U}} = \cosh\theta - ie\sinh\theta, \quad \|e\| = 1.$$

Here θ is real number. It is easy to check that

$$\mathbf{U} \circ \bar{\mathbf{U}} = 1. \tag{6.1}$$

Lemma 6.1. *Classical Lorentz transformation* $L : \mathrm{Z} \to \mathrm{Z}'$ *has the form*

$$\mathbf{Z}' = \mathbf{U} \circ \mathbf{Z} \circ \mathbf{U}, \quad \mathbf{Z} = \bar{\mathbf{U}} \circ \mathbf{Z}' \circ \bar{\mathbf{U}},$$

Proof. The direct calculation proves this lemma. The pseudonorma is saved:

$$\langle \mathbf{Z}' \rangle^{\mathbf{2}} = \mathbf{U} \circ \mathbf{Z} \circ \mathbf{U} \circ \bar{\mathbf{U}} \circ \bar{\mathbf{Z}} \circ \bar{\mathbf{U}} = \langle \mathbf{Z} \rangle^{\mathbf{2}}.$$

Here the property of associativity and (6.1) we are used.

If we use the notations

$$ch2\theta = \frac{1}{\sqrt{1-v^2}}, \quad sh2\theta = \frac{v}{\sqrt{1-v^2}}, \quad |v| < 1,$$

then the scalar and vector part of biquaternions can be written in the form of known relativistic formulas:

$$\tau' = \frac{\tau + v(e,x)}{\sqrt{1-v^2}}, \quad x' = (x - e(e,x)) + e\frac{(e,x) + v\tau}{\sqrt{1-v^2}},$$
$$\tau = \frac{\tau' - v(e,x)}{\sqrt{1-v^2}}, \quad x = (x' - e(e,x')) + e\frac{(e,x') - v\tau'}{\sqrt{1-v^2}},$$

It corresponds to the motion of coordinate system X in the direction of vector e with velocity v.

Lemma 6.2. *The conjugated quaternions*

$$\mathbf{W} = \cos\varphi + e\sin\varphi,\ \mathbf{W}^* = \cos\varphi - e\sin\varphi,\ (\|e\| = 1)$$

define the group of transformation on **M** *which are orthogonal on vector part* Z :

$$\mathbf{Z}' = \mathbf{W} \circ \mathbf{Z} \circ \mathbf{W}^*,\ \mathbf{Z} = \mathbf{W}^* \circ \mathbf{Z}' \circ \mathbf{W}.$$

It is rotation around the vector e on the angle 2φ . As the result of these two lemmas we have the following.

Lemma 6.3. *The Lorentz transformation on* $\boldsymbol{M}$ *can be defined by* :

$$\mathbf{Z}^{'} = \mathbf{L} \circ \mathbf{Z} \circ \mathbf{L}^{*}, \;\; \mathbf{Z} = \bar{\mathbf{L}^{*}} \circ \mathbf{Z}^{'} \circ \bar{\mathbf{L}}, \tag{6.2}$$

$$\mathbf{L} = \mathbf{W} \circ \mathbf{U} = ch(\theta + i\varphi) + iesh(\theta + i\varphi), \;\; \mathbf{L}^{*} = \mathbf{U}^{*} \circ \mathbf{W}^{*} = ch(\theta - i\varphi) + iesh(\theta - i\varphi),$$

The pseudonorm is saved for Lorentz transformation : $\langle \mathbf{Z} \rangle = \langle \mathbf{Z}' \rangle$.

It is easy to see that $\bar{\mathbf{L}} \circ \mathbf{L}^{*} = \mathbf{L}^{*} \circ \bar{\mathbf{L}} = 1$, because the pseudonorm $\mathbf{Z}$ is saved.

7 Lorentz transformation of biwave equations

Let us consider how bigradients are transformed under Lorentz transformation.

Lemma 7.1. *If* $\mathbf{Z}^{'} = \mathbf{L} \circ \mathbf{Z} \circ \mathbf{L}^{*}$, *then*

$$\mathbf{D}' = \bar{\mathbf{L}}^{*} \circ \nabla \circ \mathbf{L}, \quad \mathbf{D} = \mathbf{L} \circ \nabla' \circ \bar{\mathbf{L}}^{*},$$

where $\mathbf{D} = \nabla^{+}$ *or* $\mathbf{D} = \nabla^{-}$.

Based on this lemma, consider how the biwave equation (4.1) is changed by Lorentz transformation. Using associativity of the product and characteristic of $\mathbf{L}$, we get

$$\nabla'\mathbf{K}' = \left(\bar{\mathbf{L}}^{*} \circ \nabla \circ \mathbf{L}\right)\left(\bar{\mathbf{L}}^{*} \circ \mathbf{K} \circ \mathbf{L}\right) = \bar{\mathbf{L}}^{*} \circ \nabla \circ \mathbf{K} \circ \mathbf{L} = \bar{\mathbf{L}}^{*} \circ \mathbf{G} \circ \mathbf{L} = \mathbf{G}'.$$

Hence, the form of equation is saved:

$$\left(\frac{\partial}{\partial \tau'} \pm i\nabla'\right)\mathbf{K}' = \mathbf{G}',$$

where $\mathbf{K}^{'} = \bar{\mathbf{L}}^{*} \circ \mathbf{K} \circ \mathbf{L}$, $\mathbf{G}^{'} = \bar{\mathbf{L}}^{*} \circ \mathbf{G} \circ \mathbf{L}$. From here we have the following result.

Theorem 7.1. *The Lorentz transformation of the Maxwell equations can be written as follows:*

$$\mathbf{D}^{+}\mathbf{A}' = \boldsymbol{\Theta}', \;\; \textit{where } \mathbf{A}^{'} = \bar{\mathbf{L}}^{*} \circ \mathbf{A} \circ \mathbf{L}, \;\; \boldsymbol{\Theta}^{'} = \bar{\mathbf{L}}^{*} \circ \boldsymbol{\Theta} \circ \mathbf{L}.$$

Relativistic formulas for tension, charge and current (when $\varphi = 0$ *):*

$$A' = (A - e(e,A)) + e\frac{(e,A)}{\sqrt{1 - v^{2}}} \tag{7.1}$$

$$\rho' = \frac{\rho - v(e,J)}{\sqrt{1 - v^{2}}}, \quad J' = (J - e(e,J)) + e\frac{(e,J) - v\rho}{\sqrt{1 - v^{2}}} \tag{7.2}$$

One can see that the tension of A-field always increases in direction of vector e. In the absence of current, the charge-mass will increase. At the presence current, depending on directions of their motion, the charge-mass can increase or decrease.

8. The third Newton law: The power and density of acting forces

Let us consider two EGM fields $\mathbf{A}$ and $\mathbf{A}'$. Their generating charges and currents are $\boldsymbol{\Theta}$ и $\boldsymbol{\Theta}'$. We call a *power-force density* biquaternion

$$\mathbf{F} = M - iF = \boldsymbol{\Theta} \circ \mathbf{A}' = -(i\rho + J) \circ A' = (A', J) - i\rho A' + [A', J] \tag{8.1}$$

which is acting from side of A' -field on the charge and current of A-field. Really, using (2.3) and (2.4), the scalar part is determined as power density of acting forces:

$$M = (A', J) = c^{-1}((E', j^E) + (H', j^H)) + i((B', j^E) - (D', j^H)) \tag{8.2}$$

Selecting the real and imaginary parts of vector form of biquaternion, we get expressions for density of acting forces $\left(F = F^H + i\,F^E\right)$:

$$F^H = \rho^E E' + \rho^H H' + j^E \times B' - j^H \times D' \tag{8.3}$$

$$F^E = c\left(\rho^E B' - \rho^H D'\right) + c^{-1}\left(E' \times j^E + H' \times j^H\right) \tag{8.4}$$

Here $B = \mu H$ is an analog of a vector to magnetic induction (in torsional part complies with it), $D = \varepsilon E$ is a vector of the electric offset.

The potentional part of H describes the tension of gravitational field. Torsional part of this vector describes magnetic field. The scalar part of $\mathbf{\Theta}$, $\mathbf{\Theta}'$ contains the densities of electric charge and mass, its vector part contains the densities of electric and mass currents.

Coming from these suggestions, in formula (8.3) the known forces are standing, consecutively: Coulomb force $\rho^E E'$, gravitational force $\rho^H H'$ (more exactly, it complies with it in potential part H'), Lorentz force $j^E \times B'$ (more exactly,it complies with it in torsional part B')) and new force $-D' \times j^H$, which we call *gravielectric*. In real part of the power (8.2) we see the powers of Coulomb force, gravitational and magnetic force.

The power of gravielectric force does not enter in real part of (8.2) as it does not work on the mass displacement, because it is perpendicular to its velocity. It is interesting that Lorentz force also does not enter in real part of (8.2). It proves that this force is perpendicular to mass velocity, though directly from Maxwell equations this does not follow.

Naturally, in analogy, to expect that equations (8.4) describe forces, causing change of electric current, but in imaginary part M the power stands which corresponds to it. On the virtue of the third Newton law about acting and counteracting forces, we suppose that must be executed: $\mathbf{F}' = -\mathbf{F}$. From here we get

The law of fields action and reaction :

$$\mathbf{\Theta} \circ \mathbf{A}' = -\mathbf{\Theta}' \circ \mathbf{A}. \tag{8.5}$$

It is interesting to note that in scalar part it requires the equality of the powers corresponding to forces, acting on charges and currents of the other field. That is befitted with what is known in mechanics as the identity reciprocity of Betti, which is usually written for the work of forces.

9. The second Newton law: Transformations equation

The charge-current field is changed under influence of the field of other charge and current. As it is well known, direction of the most intensive change of the scalar field describes its gradient. In analogy we can expect that the most intensive change of the charge-current field occurs toward its bigradient. Naturally expect, that this change must occur toward power-force, acting on the side of the second field on the first one. So the law of the change of the charge-current field under the action of the others (like second Newton law) is offered in the manner of the following equations.

The equations of the charge-current interaction :

$$\kappa\nabla^{-}\boldsymbol{\Theta} = \mathbf{F} \equiv \boldsymbol{\Theta} \circ \mathbf{A}', \quad \kappa\nabla^{-}\boldsymbol{\Theta}' = \boldsymbol{\Theta}' \circ \mathbf{A}, \tag{9.1}$$

$$\boldsymbol{\Theta} \circ \mathbf{A}' = -\boldsymbol{\Theta}' \circ \mathbf{A}, \tag{9.2}$$

$$\nabla^{+}\mathbf{A} = \boldsymbol{\Theta}, \quad \nabla^{+}\mathbf{A}' = \boldsymbol{\Theta}'. \tag{9.3}$$

Here (9.1) correspond to the second Newton law which is written for each charge-current of interacting field. Equation (9.2) is the third Newton law. Together with Maxwell equations for these fields (9.3) they give closed system of the nonlinear differential equations for determination $\mathbf{A}, \mathbf{A}', \boldsymbol{\Theta}, \boldsymbol{\Theta}'$. Entering the constant of interaction κ is connected with dimensionality. Revealing scalar and vector part in (9.1), we have

The equations of charge-current transformations:

$$i\,\kappa\,(\partial_\tau \rho + div\, J) = M, \tag{9.4}$$

$$i\,\kappa\,(\partial_\tau J - i\,rot\,J + \nabla\rho) = F. \tag{9.5}$$

At first let consider the second equation. By virtue (2.2), (2.3), (2.4), we obtain analog of

The second Newton law for charge-current field:

$$\kappa\left(\sqrt{\varepsilon}\,\partial_\tau j^H + \sqrt{\mu}\,rot\,j^E + \mu^{-0,5} grad\,\rho^H\right) = \rho^E E' + \rho^H H' + j^E \times B' - j^H \times D', \tag{9.6}$$

$$\kappa\left(\sqrt{\mu}\,\partial_\tau j^E - \sqrt{\varepsilon}\,rot\,j^H + \varepsilon^{-0,5} grad\,\rho^E\right) = c\left(\rho^E B' - \rho^H D'\right) + c^{-1}\left(E' \times j^E + H' \times j^H\right). \tag{9.7}$$

In (9.6) the value $\kappa\sqrt{\varepsilon} j^H$ is an analog of linear momentum. Equation (9.7) describes the influence of the external field on electric charge.

If one field is much stronger the second one, then it is possible to neglect the second field change under influence of charge and current from first field. In this case we get closed system of equations for determination the charge and current motion of the first field under action of second field:

$$\kappa\nabla^{-}\boldsymbol{\Theta} = \boldsymbol{\Theta} \circ \mathbf{A}',$$

where $\mathbf{A}'$ is given. The corresponding A-field is defined by Maxwell equations (9.3).

10. First Newton law: Free field

Let us consider A-field, which is generated by $\boldsymbol{\Theta}$, in the absence of other charges-currents. We call it a *free field*. In this case $\mathbf{F} = 0$. From (9.1) we get *inertia law*, which is analog of

The first Newton law for charge-current field:

$$\nabla^{-}\boldsymbol{\Theta} = \mathbf{0}, \tag{10.1}$$

which is equivalent to equations:

$$\partial_\tau \rho + div\, J = 0, \quad \partial_\tau J - i\; rot\, J + \nabla\rho = 0.$$

For initial designations we have following formulas:

$$\partial_t \rho^E + div\, j^E = 0, \quad \partial_\tau j^E = \sqrt{\varepsilon/\mu}\, rot\, j^H - c\, grad\, \rho^E, \tag{10.2}$$

$$\partial_t\rho^H + div\, j^H = 0, \quad \partial_\tau j^H = -\sqrt{\mu/\varepsilon}\, rot\, j^E - c\, grad\, \rho^H. \tag{10.3}$$

Consequently, charge-current conservation law (2.8) holds in the absence of the external field.

Cauchy problem. For free field the solution of this problem is given by the formula:

$$\kappa\boldsymbol{\Theta}(\tau, x) = \kappa\nabla^-\{\boldsymbol{\Theta}_0(x) \underset{x}{*} \psi\} = -\frac{\kappa H(\tau)}{4\pi}\nabla^-\left\{\tau^{-1}\int\limits_{r=\tau}\boldsymbol{\Theta}_0(y)dS(y)\right\}, \tag{10.4}$$

and tensions of A-field are defined in Section 5.

11 Modified Maxwell equations: Scalar resistance field

Let us consider the first equation (9.4). Evidently, it is the charge-current conservation law, which contains the power of external acting forces M in the write-hand site. When $M=0$, this law has well-known form (2.8), which we have had for Maxwell equation (see Theorem 2.1).

This means that by EGM fields interaction we must enter the scalar part in tensions biquaternion:

$$\mathbf{A} = i\, a(\tau, x) + A(\tau, x).$$

We call $a(\tau, x)$ the *A-field resistance*. From (9.1) -(9.3) follows that

$$\Box\mathbf{A} = \nabla^-\boldsymbol{\Theta} = \kappa^{-1}\mathbf{F}.$$

The scalar part of this is

$$\kappa\Box\, a = iM.$$

Remark 11.1. In system of Maxwell equations (2.1),(2.2) the first equation defines the currents, the second one determinate the charge, but the charges conservation law is due to these two equations. It can be get, if we take divergence in (2.1) with provision for (2.2). However, the biquaternionic approach, as it is shown here , brings to modification of the Maxwell equations, which, in what follows from (9.3), has a following type:

The modified Maxwell equations

$$J = grad\, a - \partial_\tau A - i\, rot A, \quad \rho = div\, A - \partial_\tau a. \tag{11.1}$$

If ρ and J are known, this system for determining a and A is closed. Only in closed system (in the absence of external field) $a = 0$ and it has the classical type (2.1)-(2.2).

Obviously, by introducing the *resistance scalar field* a, the form of scalar and vector parts of power-force biquaternion (8.1) is changed, as follows

$$\mathbf{F} = \boldsymbol{\Theta} \circ \mathbf{A}' = ((A', J) + a'\rho) - i(a'J + \rho A') + [A', J]. \tag{11.2}$$

We can see the additional summands which appears in the presence of powers ($a'\rho$) and force ($-ia'J$). The vector $a'J$ is called a *resistances force* of A'-field.

Selecting real and imaginary part of this vector we get additional summands in expressions for density of electric F^E and gravimagnetic F^H forces, forming F $(F = F^H + i\, F^E)$ in (9.5) with provision for the resistance force of fields, which we must add in right parts of the Eqs. (9.6), (9.7).

Cauchy problem for equation of transformation. Using formula (4.2), we get

$$\kappa\mathbf{\Theta}(\tau,x)=\nabla^{+}\{H(\tau)\mathbf{F}(\tau,x)*\psi\}+\mathbf{F}(0,x)\underset{x}{*}\psi+\kappa\nabla^{+}\{\mathbf{\Theta}(0,x)\underset{x}{*}\psi\} \tag{11.3}$$

This equation gives the system of integral equations for determining $\mathbf{\Theta}$, as the right-hand side contains $\mathbf{\Theta}$ in $\mathbf{F}$. It can be used for solving the problem if we neglect the second field change. In general case, we write the similar equation for second field $\mathbf{\Theta}'$. These two equations give us the full system of integral equations for determination of charges and currents by their interaction, if initial state of fields are known.

The Lorentz transformations of Transformation Equations. According to Theorem 7.2, Lorentz transformations for $\mathbf{A}$, $\mathbf{\Theta}$, $\mathbf{F}$ have such form:

$$\mathbf{A}^{'}=\bar{\mathbf{L}}^{*}\circ\mathbf{A}\circ\mathbf{L},\ \ \mathbf{\Theta}^{'}=\bar{\mathbf{L}}^{*}\circ\mathbf{\Theta}\circ\mathbf{L},\ \ \mathbf{F}^{'}=\bar{\mathbf{L}}^{*}\circ\mathrm{F}\circ\mathbf{L}. \tag{11.4}$$

(here the sign ' means the coordinates in moving coordinate system). Note, Lorentz transformation of power-force density at the presence of interaction of two fields of form (8.1) have the same form:

$$\mathbf{F}'=\mathbf{\Theta}_{\mathbf{1}}'\circ\mathbf{A}_{\mathbf{2}}'=\bar{\mathbf{L}}^{*}\circ\mathbf{\Theta}_{\mathbf{1}}\circ\mathbf{L}\circ\bar{\mathbf{L}}^{*}\circ\mathbf{A}_{\mathbf{2}}\circ\mathbf{L}=\bar{\mathbf{L}}^{*}\circ\mathbf{\Theta}_{\mathbf{1}}\circ\mathbf{A}_{\mathbf{2}}\circ\mathbf{L}=\bar{\mathbf{L}}^{*}\circ\mathbf{F}\circ\mathbf{L}.$$

For $\varphi=0$ relations (11.4) are equivalence to equalities (7.1)-(7.2) and

$$\mathbf{F}'=(M\cosh 2\theta-(e,F)\sinh 2\theta)+i\{F+2e(e,F)\sinh^{2}\theta-Me\sinh 2\theta\}\Rightarrow$$

Relativistic formula for power and force:

$$M'=\frac{M+v(e,F)}{\sqrt{1-v^{2}}},\quad F'=(F-e(e,F))+e\frac{(e,F)-vM}{\sqrt{1-v^{2}}}. \tag{11.5}$$

So, the power also depends on velocity of coordinate system. If in initial system it is equal to zero, but in other system it is equal zero if only external forces are absent ($F=0$). By this reason the charge conservation law is not postulated in traditional form (2.8) for open systems, which is subjected to external influence.

12. Stress pseudotensor: Equations of EGM-medium

The *stress pseudotensor* may be introduced from formula (9.6):

$$\sigma_{ik}^{H}=-\kappa\left(\frac{\rho^{H}}{\sqrt{\mu}}\delta_{ik}+\sqrt{\mu}\,j_{l}^{E}e_{ikl}\right),\ \ i,k,l=1,2,3. \tag{12.1}$$

It is analog of stress tensor of liquid (σ_{ik}).

Using this pseudotensor, (9.6) takes form, which looks like hydrodynamics equations:

$$\frac{\partial\sigma_{ik}^{H}}{\partial x_{k}}+F_{i}^{H}=\kappa\varepsilon\sqrt{\mu}\,\frac{\partial j_{i}^{H}}{\partial t}.$$

Here the second summand on the left-hand side is the density of mass forces:

$$F_{i}^{H}=\rho^{E}E_{i}^{'}+\rho^{H}H_{i}^{'}+j^{E}\times B_{i}^{'}-j^{H}\times D_{i}^{'}+Re(a^{'}J).$$

However there are not traditional index symmetries of the stress tensor: $\sigma_{ik} \neq \sigma_{ki}$.

Using (9.7) , we may similarly introduce the *electric stress pseudotensor*:

$$\sigma_{ik}^E = -\kappa \left(\frac{\rho^E}{\sqrt{\varepsilon}} \delta_{ik} - \sqrt{\varepsilon}\, j_l^H e_{ikl} \right).$$

By using this, the equation (9.7) can be written as

$$\frac{\partial \sigma_{ik}^E}{\partial x_k} + F_i^E = \kappa\mu\sqrt{\varepsilon}\,\frac{\partial j_i^E}{\partial t}. \qquad (12.2)$$

Here the second summand on the right-hand site is the density of electric forces:

$$F_i^E = \rho^E B_i' - \rho^H D_i' + c^{-1}\left(E_i' \times j^E + H_i' \times j^H \right) + Im(a' J).$$

The analog of this formula is unknown for author.

13 The first thermodynamics law

We introduce the energy-impulse density for charge-current field:

$$0,5\boldsymbol{\Theta} \circ \boldsymbol{\Theta}^* = \left(\frac{\|\rho_E\|^2}{\varepsilon} + \frac{\|\rho_H\|^2}{\mu} + Q \right) + i\left(P_J - \sqrt{\frac{\mu}{\varepsilon}}\rho^E j^E - \sqrt{\frac{\varepsilon}{\mu}}\rho^H j^H \right). \qquad (13.1)$$

It contains the current energy density:

$$Q = 0,5\,\|J\|^2 = 0,5\left(\mu \left\|j^E\right\|^2 + \varepsilon \left\|j^H\right\|^2 \right),$$

where the first summand includes the Joule heat $\left\|j^E\right\|^2$; the second one includes kinetic energy density of mass current $\left\|j^H\right\|^2$, also it contains the energy of torsional part of currents (magnetic current). The vector P_J is analog of Pointing vector, but for the current:

$$P_J = 0,5i\,J \times \bar{J} = c^{-1}\left[j^H, j^E \right]$$

Only if gravimagnetic and electrical currents are parallel or one from them is equal zero, then $P_J = 0$. If we take scalar product in (9.5) with $i\bar{J}$, we get

The charge-current conservation law:

$$\kappa\left(\partial_\tau Q - \,div\, P_J + \mathrm{Re}\left(\nabla\rho, \bar{J} \right) \right) = \mathrm{Im}\left(F, \bar{J} \right) = c^{-1}\left((F^H, j^H) + (F^E, j^E) \right). \qquad (13.2)$$

It is easy to see that this law is like the first thermodynamics law. Here the sum of second and third summands in left part is denote by $-U$. The function

$$U = \,div\, P_J - \sqrt{\mu/\varepsilon}\left(\nabla\rho^E, j^E \right) - \sqrt{\varepsilon/\mu}\left(\nabla\rho^H, j^H \right)$$

characterizes the self-velocity of the change of energy current density of $\boldsymbol{\Theta}$-field. The right-hand site (13.2), which depends on power of acting external forces, can to increase or decrease this velocity.

For the free field the first thermodynamics law:

$$\partial_\tau Q = U.$$

If we integrate (13.2) on $\{(S^- + S) \times (0,t)\}$ and use Gauss formula, then the integral representation of this law may be written as

$$\int\limits_{S^-} (Q(x,t) - Q(x,0))dV(x) = \int\limits_0^t dt \int\limits_S (P_J, n)\, dS(x) -$$

$$-\int\limits_0^t dt \int\limits_{S^-} \{\varepsilon^{-1}\left(\nabla\rho^E, j^E\right) + \mu^{-1}(\nabla\rho^H, j^H)\,\}dV(x) +$$

$$+c^{-1}\int\limits_0^t dt \int\limits_{S^-} \left\{(F^H, j^H) + (F^E, j^E)\right\} dV(x).$$

Here $n(x)$ is unit normal vector to boundary S of the region S^- in space R^3.

14 The total field equations and interaction energy

If there are some (N) interacting fields, generated by different charges and currents, then Eq. (9.1) can be written as

$$\kappa\nabla^+\mathbf{\Theta}^k + \mathbf{\Theta}^k \circ \sum_{m\neq k} \mathbf{A}^m = \mathbf{0}, \quad \nabla^+\mathbf{A}^k + \mathbf{\Theta}^k = \mathbf{0}, \quad k = 1, ..., \mathrm{N} \tag{14.1}$$

$$\nabla^+\mathbf{A}^m \circ \mathbf{A}^k + \nabla^+\mathbf{A}^k \circ \mathbf{A}^m = 0, \quad k \neq m. \tag{14.2}$$

The total field, as it is easy to see after summing (14.1) over k, is free, because all forces are internal, also as in mechanics of interacting solids.

Interacting fields satisfy to the analog of the second Newton law (14.1), (14.2) and for total charge-current there is the equality:

$$\nabla^+\mathbf{\Theta} = \nabla^+\sum_{m=1}^{M} \mathbf{\Theta}^m = \mathbf{0}. \tag{1.}$$

Let us consider the laws of energy transformation in the case of interaction of different charges-currents. Energy-pulse for total charge-current field reads

$$\mathbf{\Xi}_\Theta = 0,5\mathbf{\Theta} \circ \mathbf{\Theta}^* = 0,5\sum_{k=1}^{N} \mathbf{\Theta}^k \circ \sum_{l=1}^{N} \mathbf{\Theta}^{*l} = 0,5\left(\sum_{k=1}^{N} \mathbf{\Theta}^k \circ \mathbf{\Theta}^{*k} + \sum_{k\neq l} \mathbf{\Theta}^k \circ \mathbf{\Theta}^{*l}\right) =$$

$$= \sum_{k=1}^{N} W_\Theta^{(k)} + i\sum_{k=1}^{N} P_\Theta^{(k)} + \delta\mathbf{\Xi}_\Theta$$

Here the first summand is an amount of energy-pulse of interacting charge-current.

We can introduce biquaternion of *energy-pulse interaction* . Its real part describes energy-pulse interaction for the same name charge and current, but in the imagine part for different name ones :

$$\delta\Xi_\Theta = \delta W_\Theta + i\delta P_\Theta = \sum_{k\neq l} \Xi_\Theta^{kl}, \quad \Xi_\Theta^{kl} = 0,5\left(\boldsymbol{\Theta}^k \circ \boldsymbol{\Theta}^{*l} + \boldsymbol{\Theta}^l \circ \boldsymbol{\Theta}^{*k}\right)$$

$$\Xi_\Theta^{kl} = \mathrm{Re}\left(\rho^k\rho^{*l} + \left(J^k, J^{*l}\right)\right) - i\left\{\mathrm{Re}\left(\rho^k J^{*l} + \rho^{*l}J^k\right) + \mathrm{Im}\left[J^k, J^{*l}\right]\right\},$$

or in initial notation:

$$\Xi_\Theta^{kl} = \frac{\rho^{E(k)}\rho^{E(l)}}{\sqrt{\varepsilon_k\varepsilon_l}} + \frac{\rho^{(k)H}\rho^{H(l)}}{\sqrt{\mu_k\mu_l}} + \sqrt{\mu_k\mu_l}\left(j^{(k)E}, j^{(l)E}\right) + \sqrt{\varepsilon_k\varepsilon_l}\left(j^{(k)H}, j^{(l)H}\right) -$$

$$-i\left\{\sqrt{\frac{\mu_l}{\varepsilon_k}}\rho^{(k)E}j^{(l)E} + \sqrt{\frac{\varepsilon_l}{\mu_k}}\rho^{(k)H}j^{(l)H} + \sqrt{\frac{\mu_k}{\varepsilon_l}}\rho^{(l)E}j^{(k)E} + \sqrt{\frac{\varepsilon_k}{\mu_l}}\rho^{(l)H}j^{(k)H} - \right.$$
$$\left. -\sqrt{\varepsilon_k\mu_l}\left[j^{(l)E}, j^{(k)H}\right] + \sqrt{\varepsilon_l\mu_k}\left[j^{(k)E}, j^{(l)H}\right]\right\}$$

As result we get the conditions of energy transformation in the case charges-currents interaction: *energy separation* if $\delta W_\Theta > 0$; *energy absorption* if $\delta W_\Theta < 0$; *energy conservation* if $\delta\Xi_\Theta = 0$.

15 Conclusion

We considered a model of EGM-field (called *A-field*),which is founded on hypothesis on *magnetic charge=mass*, that has allowed to name such field *electro-gravimagnetic*.

We used Maxwell equations in biquaternionic form and constructed the new biquaternionic equations for description of charges-currents changing by their interaction. We name these equations as *analog of Newton laws*.

Investigation of invariance of these equations with respect to Lorentz transformation showed that it is necessary to enter the *scalar field of resistance* $a(\tau, x)$ in scalar part of biquaternion of EGM-field tension. One has to modify the Maxwell equations in the case, when charge and current are subjected to influence by external field. We call these equations *the modified Maxwell equation for open system*.

When constructing the equation to charge-current transformations aside from the known gravitational and electromagnetic forces we found the presence of new forces, which is needed in experimental motivation. Some suggestions on this cause were presented in [3,4], where this model was offered at first, but with charge-current conservation law in traditional form. But this is true only for closed system. As it is shown here, for open system we must take into account the power of external forces, which changes the form of this law.

Note also that essential at building and studying this models of EGM-field the algebra of biquaternions is using, without which construction of the differential equations, describing interaction of charge and current in such form would be practically impossible.

References

1. L.A. Alexeyeva. Hamilton form of Maxwell equations and its generalized solutions. Differential equations.**39**(2003). N 6. 769-776.

2. L.A. Alexeyeva. Quaternions of Hamilton form of Maxwell equations. Mathematical journal. **3**(2003).N 4.20-24.
3.L.A. Alexeyeva. About one models of electro-gravimagnetic field. The interaction equations and conservation laws. Mathematical journal. **4** (2004) N 2. 23-34.
4. L.A. Alexeyeva. The interaction equations of A-fields and Newton laws. Transactions of National Academy of Sci. of Rep. Kazakhstan. Physical and mathematical issue. 2004. N 3. 45-53.
5. R. Rastall. Quaternions in relativity. Rev. Modern Physics., **34**(1964). 820-832.
6. A.P.Efremov. Quaternions: algebra, geometry, and physical theories. Hypercomplex numbers in geometry and physics. **1**(2004). N 1. 111-127.
7. G.Casanova. *Vector algebra.* Mir. Moscow. 1979. (in Russian)
8. V.V.Kassandrov. Biquaternion electrodynamics and Weyl-Cartan geometry of space-time. Gravitation and cosmology. **1** (1995). N 3. 216-222.
9. V.S.Vladimirov. *Generalized functions in mathematical physics.* Nauka. Moscow. 1976.