

Complex Dynamics and Foundational Physics

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An Introduction

A Research Monograph on the Role of
Complex Dynamics in High-Energy Physics and
Cosmology

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Complex Dynamics and Foundational Physics: An Introduction

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Foreword

This book is a genuine attempt to engage with a period of stagnation that many in the field of foundational physics have come to acknowledge. For more than a couple of decades, two paradigms of modern physics—the Standard Model of elementary particles (SM) and the Λ -Cold Dark Matter (Λ CDM) model of cosmology—have continued to pass every precision test while, at the same time, facing a growing list of puzzles, anomalies, and tensions that neither model explains from first principles. This stagnation is not experimental; the data pipeline has been anything but quiet. It is theoretical: a wide lack of consensus on what, if anything, lies beyond these two frameworks, and a growing suspicion within the community that the standard toolkit—field theory built on the classical action principle, conventional consistency requirements, perturbation theory, and equilibrium statistical mechanics—may be approaching the limit of its validity.

This monograph does not claim to end that stagnation. It offers a single, well-motivated framework of ideas for why it persists, and a mathematical language in which to explore it. The hypothesis, stated plainly, is this: many of the open puzzles of the SM and of Λ CDM are not isolated failures of otherwise robust theories, but symptoms of a couple of common and largely unexamined idealizations,

- 1) Spacetime has a fixed, scale-independent integer number of dimensions at every conceivable energy scale,
- 2) Physical systems evolve in or near thermodynamic equilibrium, obey exact unitarity and locality, and follow the classical action principle at every conceivable energy scale.

We argue that these idealizations, however well they serve the infrared (IR), low-energy regime for which they were formulated and tested, are not guaranteed to survive extrapolation to the deep ultraviolet (UV) sector of particle phenomenology or to the far-from-equilibrium conditions of the primordial Universe. Where these assumptions break down, we propose that spacetime dimensionality turns into a dynamical, scale-dependent quantity, whose evolution is governed by the mathematics of *complex dynamics*—bifurcation theory, chaos and multifractal geometry, self-organized criticality, fractional dynamics and generalized entropy. As the book argues, aside from Dimensional Regularization of Quantum Field Theory (QFT), there are essentially *three independent routes* linking field theory to the continuously evolving dimensionality of spacetime far above the electroweak scale, namely:

- 1) Emergence of nontrivial attractors in the Renormalization Group (RG) theory of phase transitions,
- 2) Onset of Hamiltonian Chaos in the phase-space of nonintegrable systems, either strongly coupled or having many degrees of freedom,
- 3) Triviality of the Higgs sector and SM in exactly four-spacetime dimensions.

Dimensional Regularization goes beyond the conventional principles of QFT by analytically continuing spacetime dimension to non-integer values. Although this procedure has traditionally been viewed as a purely formal tool, recent mathematical research proves that such continuation is consistent *only if* loop integrals are interpreted as integrals over *fractal measures*, rather than smooth manifolds.

Independently, we argued over the years that the onset of a spacetime with evolving fractality is formally equivalent to *critical behavior in continuous dimensions*, where scale separation mandated by the Decoupling Theorem fails, nonlinear dynamics becomes inevitable and complexity takes over. The last two decades have consistently shown that both the Cantor Dust concept of Dark

Matter and evolving spacetime dimensionality can lead to key developments in high-energy theory, Statistical Physics, self-organized systems, fractional dynamics, as well as Astrophysics and Cosmology.

Readers already familiar with the technical literature that underlies this book will recognize the argument as a *synthesis*. Every substantive claim developed here has already been presented in previously cited research articles that this volume organizes and consolidates under a single line of reasoning. The contribution this book makes is not a new derivation buried in each chapter; instead, each chapter represents a specialized component of the overall picture. The book covers a variety of topics such as Hopf bifurcation in the Higgs sector, the multifractal foundation of Dimensional Regularization, a numerical study of Cantor Dust as a Dark Matter candidate. Taken together and organized around the thesis of evolving spacetime dimensionality above the electroweak scale, the book develops a rather ambitious framework of unifying ideas. Whether this conceptual framework ultimately survives scrutiny is, of course, for the reader—and for the wider research community—to judge. It is offered here not as settled doctrine but as “work in progress”, deliberately assembled to be evaluated as a whole.

It is worth being explicit, from the outset, about what kind of book this is and is not. It is not an undergraduate or introductory graduate textbook; it does not re-derive Quantum Field Theory (QFT), General Relativity (GR), or the standard cosmological model from scratch, and it assumes the reader is fluent in these subjects. It is not a popular-science account, and it does not shy away from technical jargon—Path Integrals, Renormalization Group (RG) flow, bifurcation diagrams, multifractal spectra, Rényi and Tsallis entropies—as required by content. It is also not a claim of empirical discovery: no new particles, no new observational signature, and no experimental confirmation are reported here. Instead, the book is an extensive argument that the well-developed methodology of *complex dynamics* deserves serious consideration as foundational tool in high-energy physics and cosmology.

ogy, complementing canonical principles such as gauge symmetry and general covariance. On account of its inherent *memory* and *long-range spatial properties*, - while being directly inaccessible in the deep UV sector of particle physics or primordial Cosmology - complex dynamics leaves a host of detectable signatures in low-energy phenomena, including (but not limited to) the many puzzles and anomalies seen today.

The book is organized into eight parts. The first introduces the conceptual framework of complex dynamics above the electroweak scale and reframes Dimensional Regularization—traditionally treated as a purely formal device for controlling divergences in QFT—as hinting to a genuine physical process, the so-called *dimensional flow*. The second and third parts examine the consequences of abandoning classical integrability and near-thermodynamic equilibrium conditions above the electroweak scale. It is argued there that, in extreme dynamical regimes, the stationary-action principle breaks down to be replaced by a description based on generalized entropy, chaotic attractors, and self-organized criticality. The fourth part turns to the origin of mass, fermion mixing, and the flavor structure of SM. There, we argue that these main attributes of SM follow from applying complex dynamics to the physics of dimensional flow. The fifth part shows how classical gravity of GR can be recovered from primordial dimensional fluctuations. The sixth and seventh parts apply the same framework to cosmology, arguing for a multifractal Cantor Dust reinterpretation of Dark Matter and tracing the σ_8 tension, recent DESI results, and other structure-formation anomalies to an evolving spacetime fractality in the primordial Universe. The eighth part steps back to assessing the framework as a whole and to lay out the open problems it leaves for future work. A set of appendices collects the mathematical background—multifractal and Rényi-entropy formalism, Hopf bifurcation theory, Cantor-set constructions, and the technique of Dimensional Regularization—needed to follow the development of these ideas without constant appeal to outside sources.

A word on how to read this book. It is not necessary to proceed strictly in order. A high-energy theorist primarily interested

in the origin of mass and the flavor puzzle may move directly to parts I, III, and IV; a cosmologist concerned with the Dark Matter problem and the σ_8 and DESI anomalies may prefer instead I, VI, and VII; a reader whose primary background is in nonlinear dynamics and who wants to see how bifurcation theory and multifractal geometry make contact with gauge theory and GR may want to begin with part I and read straight through. In every case, I is the recommended point of entry, since it establishes the shared vocabulary—dimensional flow, generalized entropy, multifractal measure and so on—on which every later part depends.

Finally, an acknowledgment of the obvious. A framework this broad, covering topics from the mass spectrum of the Standard Model (SM), the structure of the Higgs sector, the nature of Dark Matter, and the statistics of cosmic structure formation, invites—as it should—a correspondingly high standard of scrutiny. It is offered here in that spirit: not as a complete theory ready to be accepted, but as a coherent and, we hope, sufficiently well-argued hypothesis to be tested, criticized, extended, or discarded by the community of researchers equipped to do so. If this book succeeds, it will not be because it has settled any of the questions it raises, but because it clearly states what the challenges are, offering unforeseen answers and inviting independent evaluation by the research community at large.

— Why Complex Dynamics?

Traditional textbooks on Statistical Physics typically cover the topic of large many-body systems in *thermodynamic equilibrium*. An implicit assumption of this classical paradigm is that small fluctuations away from equilibrium relax to zero in a finite lapse of time. The assumption extends to transient perturbations that do not impact the global stability of the system or produce unpredictable outcomes in the long run.

By contrast, the last decades have consistently shown that most many-body systems driven by nonlinear interactions evolve in *out-of-equilibrium* conditions. These conditions apply to a vast array of collective phenomena, from early Universe cosmology to climate science, evolution of living organisms, social interactions, molecular dynamics, information networks, collective processes in fluids and plasma, and relativistic heavy ion collisions in high-energy physics. As of today, the consensus is that the science of complex many-body systems in non-equilibrium conditions has gained significant ground, with notable contributions across various disciplines.

Complex dynamics is customarily characterized by the following properties:

- a) nonlinear, dissipative and nonlocal interactions between components,
- b) enabling emergence of instability and chaos,
- c) enabling self-organization and adaptive behavior,
- d) generating unforeseen outcomes and patterns,
- e) unfolding on multiple scales, displaying memory effects and an endless game of cooperation and competition.

As alluded to in the Foreword, applying complex dynamics to theoretical physics requires a slew of unconventional tools such as the analysis of chaos and multifractal geometry, non-extensive statistics, self-organized criticality, and fractional dynamics.

I.1 A Confirmed Theory with an Unexplained Structure

The Standard Model of Particle Physics (SM) is, by any reasonable metric, the most successful theory in the history of science. Every elementary particle it predicted has been found; every coupling it fixes has, once measured, been confirmed to remarkable precision by subsequent experiments; the 2012 discovery of the Higgs boson closed its last major question related to the internal consistency of SM. And yet, to this day, SM remains an incomplete account of Nature. It requires about twenty-six free parameters—quark and lepton masses, mixing angles and CP-violating phases in both the quark and neutrino sectors, gauge couplings, the Higgs mass and self-coupling—whose numerical values are simply supplied by hand, fitted to experiment, with no principle internal to the theory that fixes them or explains why they take the values they do. The fermion mass spectrum alone spans roughly thirteen orders of magnitude, from the lightest neutrino to the top quark, with no accepted dynamical mechanism generating this hierarchy from a simpler underlying structure. The Higgs mass sits uneasily low relative to the quantum corrections a generic ultraviolet completion would induce—the hierarchy or naturalness problem—and the measured top-quark mass places the Higgs potential uncomfortably close to the boundary of vacuum stability, a coincidence with no accepted explanation. Also with no compelling explanation is the proven failure of the Higgs model in exactly four space-time dimensions (the so-called triviality problem). Meanwhile, direct searches for the new particles that were widely expected to resolve these puzzles—supersymmetric partners, Technicolor resonances, extra-dimensional Kaluza–Klein excitations—have re-

turned null result after null result at the Large Hadron Collider, pushing the scale of any new physics ever higher and straining the naturalness arguments that motivated the search for it in the first place.

None of these results represent a failure of the SM as an *effective field theory*, which continues to work spectacularly, within its range of validity. What it constitutes is the absence of a robust explanation for the origin of its free parameters, of a universally accepted resolution to its internal fine-tuning problems, and, so far, of any experimental hint pointing toward a specific completion. It is this absence, sustained now across several decades of increasingly precise null results, that motivates the central question of this book: not whether SM is correct, but whether its underlying assumptions—locality, unitarity, integrability, renormalizability, decoupling of the IR and UV observation scales, absence of anomalies and, most importantly, the constant integer dimensionality of spacetime background—remain valid all the way up to the energy scales at which its free parameters might, in principle, be recovered.

I.2 Observational Cosmology: A Standard Model with Growing Conceptual Issues

A structurally similar situation confronts observational cosmology. The Λ CDM model—cold dark matter plus a cosmological constant, evolving within General Relativity (GR)—fits an extraordinary range of observations, from the cosmic microwave background (CMB) power spectrum to large-scale structure surveys, with a mere handful of parameters. Yet each of its main ingredients remains either undetected or, increasingly, in tension with the data used to constrain it. Dark Matter, which makes up roughly five-sixths of the matter content of the Universe on the standard picture, has never been detected in any direct-detection, indirect-detection, or collider experiment, despite several decades of steadily improving searches; the space of viable particle candidates has been nar-

rowed but not closed off, and no laboratory signature has yet distinguished among the remaining possibilities. At the same time, a cluster of tensions internal to Λ CDM has deepened rather than being resolved with better data: the anomalous evidence for early massive galaxies and quasars in the deep field observations of the James Webb Space Telescope (JWST), the cusp-core and missing-satellites problems, the “too-big-to” fail problem, and the persistent S8 tension between weak-lensing and CMB-inferred clustering amplitudes – suggests that structure formation may depart from the scale-independent-gravity assumption of the standard Λ CDM paradigm. In particular, the amplitude of matter clustering inferred from weak gravitational lensing and cluster counts (parametrized by σ_8 , or its close relative S_8) sits persistently below the value predicted by extrapolating the CMB to late times; the locally measured expansion rate of the late Universe remains in tension with the value inferred from the early Universe; and small-scale structure observations—the cores of dwarf-galaxy halos, the census of satellite galaxies around the Milky Way, the apparent absence of the very largest sub-halos predicted by simulations—continue to sit awkwardly against the predictions of collisionless cold Dark Matter. Most recently, results from the Dark Energy Spectroscopic Instrument (DESI) have added a further, and arguably more fundamental, complication: baryon acoustic oscillation (BAO) measurements across multiple data sets show a preference, at increasing statistical significance, for a Dark Energy equation of state that evolves with cosmic time, directly challenging the Λ CDM cosmology.

Individually, each of these tensions allows for conventional solutions: better systematics, careful control of bias in modelling assumptions, an as-yet-undetected particle with slightly unusual properties, a still-undiscovered astrophysical feedback mechanism, and a statistical fluctuation that will regress toward the standard prediction with more data. Any of these solutions may yet prove to be correct for any given anomaly, taken in isolation. What is increasingly harder to dismiss is the pattern that emerges when the tensions are considered together: essentially every departure

from the Λ CDM baseline appears where the model is extrapolated furthest from the regime in which it was originally calibrated—to the smallest scales, to the earliest epochs, or to the most nonlinear stages of structure formation. This is precisely the pattern one would expect if the difficulty lay not in any single ingredient of the model but in an assumption common to all of them.

I.3 Two Paradigms, One Diagnosis

The particle-physics puzzles of Section I.1 and the cosmological tensions of Section I.2 are, in the conventional division of labor within Theoretical Physics, treated as belonging to entirely separate domains, addressed by largely non-overlapping communities, using largely non-overlapping tools of analysis. This book proceeds from the conviction that this separation, however natural administratively, is artificial and has long obscured a common root. Current puzzles arise at the extremes of the environments in which our two great frameworks were built and tested: the deep ultraviolet (UV) limit placed well above the electroweak scale and both primordial and present-day sectors of the Universe, where QFT, GR and standard Cosmology are routinely extrapolated to. A natural assumption is that these extreme scales, cosmic epochs, and nonlinear regimes are far removed from the near-equilibrium, weak-field conditions under which our paradigms were originally formulated for and evaluated. In both these dynamic regimes, the same four idealizations are tacitly at work: that physical systems evolve in or near thermodynamic equilibrium; that the number of spacetime dimensions is fixed at an integer value, conventionally four, at every conceivable observation scale; that quantum evolution and interactions stay exactly unitary and local at arbitrarily high energy; and that the classical dynamics underlying field theory is integrable, allowing a stationary-action (Hamiltonian) description with no genuine sensitivity to initial conditions. Each of these four assumptions is entirely reasonable—indeed, essentially forced—in the regime of directly tested physics: at collider-

accessible energies, in the late, linear Universe, and in weakly coupled, near-equilibrium settings of laboratory experiments. None of the four, however, is a hard proven assertion. Each act as an idealization whose validity has never been firmly established outside the boundaries of our current knowledge.

The key leitmotif of the book is that these four idealizations are the common background for the open puzzles described above, and that they necessarily lead to a specific, fully tractable framework provided by *complex dynamics*.

I.4 How Complex Dynamics Enters the Picture

Complex dynamics, in the sense used throughout this book, refers to a well-established body of mathematical tools developed originally to describe collective, far-from-equilibrium, nonlinear, and self-organizing physical systems; *bifurcation theory*, which classifies the qualitative transitions a dynamical system undergoes as a control parameter is varied; *deterministic chaos*, in which sensitive dependence on initial conditions produces effectively unpredictable long-term behavior from entirely deterministic equations of motion; *multifractal geometry*, which generalizes the notion of dimension from a fixed integer to a continuous spectrum of scaling exponents defining an inhomogeneous measure; *fractional dynamics*, which studies systems using differential operators of non-integer order suitable for modeling nonlocal phenomena endowed with memory, *self-organized criticality*, in which extended dissipative systems spontaneously evolve toward a critical, scale-invariant state without fine-tuning of any external parameter; and *generalized entropy measures* which extend the additive, equilibrium entropy of Boltzmann and Gibbs to systems with long-range correlations, multifractal structure, or genuine non-equilibrium dynamics. None of this mathematics is either new or speculative: it underlies well-developed, empirically confirmed descriptions of turbulence, phase transitions, population dynamics, condensed-matter applications, and a wide range of other collective phenom-

ena across science. The novelty of the viewpoint advanced here is that complex dynamics supplies the correct language for describing matter and spacetime in far-from-equilibrium settings, introduced in Sections I.1 and I.2, where the four idealizations of Section I.3 are most likely to fail.

The book is organized in the following way. Part I (*“On the Onset of Complex Dynamics in Foundational Physics”*) lays the conceptual groundwork, introducing complex dynamics in a theoretical physics context and developing dimensional flow as its central driving mechanism. Parts II (*“Breakdown of Hamilton’s Principle and Integrability above the Electroweak Scale”*) and III (*“Entropy Flow, Criticality, and Emergent Spacetime Dimensions”*) examine the breakdown of classical integrability and thermodynamic equilibrium above the electroweak scale and develop the generalized-entropy and self-organized-criticality as defining features of this regime. Part IV (*“Mass Generation and the Flavor Composition of the Standard Model”*) applies complex dynamics to mass generation and the flavor structure of the SM. Part V (*“Classical Gravity from Complex Dynamics”*) extends the formalism to the phenomenology of classical gravity. Parts VI (*“Dark Matter as Cantor Dust”*) and VII (*“Observational Cosmology and Primordial Structure Formation”*) turn to Cosmology, developing the Cantor Dust model of Dark Matter and its consequences for structure formation, the σ_8 tension, and the DESI results. Part VIII (*“Synthesis and Outlook”*) synthesizes the framework as a whole and outlines the open problems it leaves for future analysis, while a limited set of appendices collects the necessary mathematical background for readers who wish to review the formal content at hand without leaving the book.

In a nutshell, our book may be best characterized as a research monograph advancing a novel theoretical framework. Its main purpose is engaging the interested readers, stimulating further evaluation and development by experts in the field, rather than establishing a definitively settled theory.

PART I

**On the Onset of Complex
Dynamics in Foundational
Physics**

Introduction to Complex Dynamics of Non-Equilibrium Systems

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Abstract

Complex dynamics (CDYN) is present across many fields of human activity, from physics, social sciences, political and financial affairs, to biology, geophysics, internet traffic, social media and weather forecasting. It exhibits robust large-scale regularities despite operating in a diversity of contexts. This paper unveils a unified framework for describing CDYN, based on non-equilibrium statistical physics. Emphasis is placed on the role of continuous and scale-dependent dimensions, anomalous transport, generalized entropies, and Renormalization Group (RG) flows in organizing macroscopic behavior. We argue that complexity is not an emergent accident but the natural outcome of systems operating near critical manifolds in phase space, often maintained through self-organized criticality (SOC). The paper introduces a lexicon of foundational concepts to standardize language across disciplines and prepare the ground for a systematic theory of CDYN applicable primarily to field theory and cosmology.

Key words: complex dynamics, non-equilibrium systems, nonlinear science and complex phenomena, field theory, cosmology.

1 Introduction

Complex Dynamics (CDYN) studies the behavior of systems comprising many interacting degrees of freedom, whose collective behavior cannot be inferred from either linear superposition or equilibrium conditions. Hallmarks of such systems include scale invariance, long-range correlations, memory effects, intermittent fluctuations, anomalous transport, and sensitivity to initial conditions. Traditional theoretical frameworks—based on integer-dimensional geometry, local interactions, and Gaussian statistics—are often inadequate to describe these complex phenomena. In our view, it is for this reason that both contemporary cosmology and the Standard Model of particle physics (SM) continue to be challenged by many outstanding puzzles.

Over the past several decades, multiple lines of inquiry have converged toward a new paradigm. For example, fractional calculus has revealed that nonlocality in time and space provides a natural language for memory effects and anomalous diffusion. Multifractal analysis has shown that fluctuations are often organized on hierarchies of interwoven singularities rather than homogeneous sets. Self-organized criticality (SOC) has proven that systems far from equilibrium can dynamically tune themselves to critical states without external fine-tuning. Renormalization Group (RG) methods have clarified how the onset of macroscopic universality emerges from microscopic complexity.

More recently, the notion of dimensional flow, the scale or time dependence of effective dimensionality, has emerged as a unifying concept. In this picture, space, phase space, or configuration space need not possess a fixed integer dimension. Instead, the operational dimension associated

with physical processes evolves with scale, energy, or cosmic time. This insight has profound implications for field theory, particle physics, condensed matter phenomena, pattern formation, and cosmology.

The goal of this paper is twofold. First, articulate the operational principles underlying CDYN, emphasizing mechanisms rather than specific models. Second, establish a common lexicon that bridges disciplines and clarifies how concepts such as fractional dynamics, SOC, anomalous diffusion, and generalized entropies interlock within a single theoretical framework. This lexicon is not intended as a glossary of “buzzwords”, but as a transparent set of definitions based on dynamic and statistical principles.

2 Lexicon of Core Concepts

2.1 Critical Behavior in Continuous Dimension

Critical behavior in continuous dimension refers to phase transitions or scale-invariant regimes occurring in systems whose effective spatial, spectral, or phase-space dimension is non-integer and scale-dependent. Unlike classical criticality defined at fixed integer dimensions, critical exponents and universality classes here vary continuously, reflecting the underlying fractal or multifractal geometry of the system.

2.2 Fractional Dynamics and Fractional Field Theory

Fractional dynamics generalizes classical equations of motion by replacing integer-order derivatives with fractional (non-integer) derivatives in time and/or space. Fractional field theory extends this idea to fields, incorporating nonlocal operators that encode memory, long-range interactions, and anomalous scaling. These frameworks naturally arise in disordered media, turbulence, and spacetime with fractal structure.

2.3 Self-Organized Criticality (SOC)

Self-organized criticality describes the spontaneous evolution of driven, dissipative systems toward a critical state characterized by scale invariance and power-law statistics. Unlike equilibrium critical phenomena, SOC does not require fine-tuning of control parameters; instead, feedback between slow driving and fast relaxation maintains the system near a critical manifold.

2.4 Dimensional Flow

Dimensional flow is the scale-, energy-, or time-dependence of the effective dimension governing physical processes. It may involve the Hausdorff dimension, spectral dimension, or information dimension. Dimensional flow provides a unifying description of scale-dependent transport, field propagation, and entropy production in complex and quantum-gravitational systems.

2.5 Multifractal Analysis

Multifractal analysis characterizes systems whose measures are distributed over a hierarchy of intertwined fractal subsets, each with its own scaling exponent. Instead of a single dimension, a spectrum of generalized dimensions governs the statistics of fluctuations, intermittency, and singular structures, particularly in turbulence, deterministic chaos and critical phenomena.

2.6 Rényi and Tsallis Entropies

Rényi and Tsallis entropies generalize the Boltzmann–Gibbs entropy to systems with long-range correlations, multifractality, or non-ergodic dynamics. Rényi entropy provides a moment-based characterization of probability distributions, while Tsallis entropy introduces a nonadditive parameter that quantifies deviations from extensivity and underlies q -statistics.

2.7 Bifurcations and Pattern Formation

Bifurcations are qualitative changes in system behavior induced by smooth variations of control parameters. Pattern formation arises when instabilities amplify specific modes, leading to spatial or temporal structures. In complex systems, bifurcations often occur on fractal sets in parameter space and are strongly influenced by nonlocal interactions.

2.8 Complex Ginzburg–Landau Equation

The Complex Ginzburg–Landau Equation (CGLE) is a universal amplitude equation describing the evolution of slowly varying modes near instability thresholds. It unifies pattern formation, nonlinear waves, spatiotemporal chaos, and synchronization, and serves as a bridge between microscopic dynamics and macroscopic order.

2.9 Chaos and Hamiltonian Chaos

Chaos refers to deterministic dynamics exhibiting exponential sensitivity to initial conditions and long-term unpredictability. Hamiltonian chaos arises in conservative systems and is characterized by mixed phase space, resonances, and transport via stochastic layers. In many-body systems, chaos underpins thermalization and entropy production.

2.10 Renormalization Group (RG)

The RG formalism studies how system descriptions change under scale transformations. RG identifies fixed points, critical manifolds, and flow trajectories that govern universality and scaling laws. In complex dynamics, RG extends beyond equilibrium to non-equilibrium, fractional, and systems endowed with evolving dimensions.

2.11 Non-Equilibrium Dynamical Systems

Non-equilibrium dynamical systems operate far from thermodynamic equilibrium and lack detailed balance. They exhibit sustained fluxes, entropy production, and time-dependent structures. Many complex systems, including biological, ecological, and cosmological systems, belong intrinsically to this class.

2.12 Anomalous Diffusion

Anomalous diffusion describes transport processes in which the mean squared displacement scales nonlinearly with time. It reflects the presence of long-range correlations, trapping, or fractal geometry and is naturally modeled using fractional diffusion equations and continuous-time random walks (CTRW).

2.13 Lévy Flights/Transport

Lévy flights and Lévy transport are stochastic processes characterized by step-length distributions with heavy power-law tails, leading to rare but dominant large jumps. They underlie super-diffusive transport and are common in turbulent flows, financial systems, and foraging dynamics.

2.14 Fractional Statistics (q-Statistics)

Fractional or q-statistics generalize classical statistical mechanics to systems with long-range interactions, memory, or multifractal phase space. Governed by a deformation parameter q , these statistics interpolate between Gaussian and heavy-tailed distributions and provide a thermodynamic framework for complex systems.

3 Operational Axioms of Complex Dynamics

In what follows, we formulate CDYN in terms of a minimal set of operational axioms. These axioms do not specify the underlying microscopic behavior of complex systems but constrain the class of admissible dynamics capable of producing complexity.

Axiom I: Scale Dependence of Observables

Physical observables depend explicitly on the resolution scale at which they are probed. Effective laws are therefore not invariant under scale transformations but evolve continuously with scale. This implies that parameters, dimensions, and couplings are running quantities rather than constants.

Axiom II: Nonlocality in Time and/or Space

Complex systems exhibit memory and long-range interactions. Operationally, this requires the use of nonlocal operators—most naturally expressed through fractional derivatives or integral kernels. Local differential equations emerge only as limiting cases.

Axiom III: Breakdown of Extensivity

The additivity of entropy and energy fails in the presence of long-range correlations or multifractal phase space. Statistical descriptions must therefore be generalized beyond Boltzmann–Gibbs thermodynamics.

Axiom IV: Dynamical Access to Critical Manifolds

Complex systems evolve toward extended critical regions in phase space rather than isolated critical points. These regions act as attractors organizing large-scale behavior.

Axiom V: Emergent Dimensionality

The effective dimension governing dynamics is not fixed a priori. Instead, it emerges dynamically and may vary with scale, time, or energy. Integer-dimensional behavior is recovered only in restricted regimes.

Axiom VI: Universality Through Coarse-Graining

Despite microscopic diversity, macroscopic behavior falls into universality classes determined by symmetry, conservation laws, and effective dimensionality. Renormalization Group (RG) flows formalize this principle.

4 Dimensional Flow, RG Evolution, and Self-Organized Criticality (SOC)

This section establishes the structural equivalence between dimensional flow, RG evolution, and SOC.

4.1 Dimensional Flow as Order Parameter

Let $D(\ell)$ denote a generic effective dimension measured at the scale ℓ . In complex systems, $D(\ell)$ is not constant but flows between asymptotic values:

$$D_{\text{UV}} \longrightarrow D(\ell) \longrightarrow D_{\text{IR}}.$$

This flow governs transport, spectral properties, and entropy production.

4.2 RG Interpretation

RG transformations act by coarse-graining degrees of freedom. In systems with evolving geometry or multifractal structure, RG flow induces *dimensional renormalization*:

$$\frac{dD}{d \ln \ell} = \beta_D(D),$$

where β_D encodes the coupling between geometry and dynamics. Fixed points of this flow correspond to scale-invariant regimes, also referred to equilibrium states.

4.3 SOC as Fixed Manifold of the RG Flow

SOC corresponds not to a fine-tuned RG fixed point but to a self-sustained fixed manifold. Slow driving pushes the system away from criticality, while fast relaxation restores it. This feedback mechanism dynamically locks the system onto a critical RG trajectory.

By definition, SOC implies:

- no external tuning of control parameters,
- persistent dimensional flow,
- scale-free avalanches and transport.

4.4 Unified Picture

Dimensional flow provides the geometric foundation, RG provides the coarse-graining dynamics, and SOC supplies the feedback mechanism. Together, they define a closed operational framework for complexity.

5 Applications

We briefly illustrate how operational principles of CD manifest across various sectors of physics phenomena.

5.1 Turbulence and Intermittency

Turbulent flows exhibit multifractal energy dissipation, anomalous scaling of structure functions, and Lévy-like transport. Turbulent flows are similar to fractional dynamics, which capture non-local energy transfer, and to dimensional flows, which encode the transition from smooth flow to intermittent cascades.

5.2 Pattern Formation and Spatiotemporal Chaos

Near instability thresholds, the Complex Ginzburg–Landau Equation (CGLE) governs amplitude dynamics. Fractional generalizations of CGLE account for nonlocal coupling and fractal media, producing complex spatiotemporal patterns and chaotic attractors.

5.3 Transport in Disordered and Biological Media

Anomalous diffusion and Lévy flights dominate transport in porous media, cellular environments, and ecological systems.

5.4 Many-Body Chaos and Thermalization

Hamiltonian chaos underlies entropy growth and equilibration in isolated many-body systems. When phase space is fractal or weakly ergodic, generalized entropies and q-statistics replace conventional equilibrium thermodynamics.

5.5 Cosmology and Multifractal Spacetime

At cosmic scales, evolving dimensionality offers a natural explanation for scale-dependent clustering, anomalous growth of matter structures, and deviations from standard Gaussian statistics. From this standpoint, Cantor Dust thus becomes a fundamental ingredient of spacetime itself.

6 Outlook

The framework outlined so far suggests that complexity is not a special feature of select systems but a generic outcome of dynamics on evolving, non-integer dimensional structures emerging in far-from-equilibrium conditions. Hence, fractional dynamics, dimensional flow, and self-organized criticality are not auxiliary tools but foundational elements of a unified theory of complex systems.

7 Fractional RG and Evolving Dimensions

7.1 Fractional Coarse-Graining

Let a system be described by a field $\phi(x, t)$ living in an effective space with scale-dependent dimension $D(\ell)$. Assuming that variational principles are still applicable in mild non-equilibrium conditions, the dynamics is controlled by a fractional action

$$S = \int dt d^{D(\ell)}x \left[\phi^* \left(\partial_t + \kappa(-\Delta)^{\alpha/2} \right) \phi + \lambda |\phi|^{2q} \right],$$

where $\alpha \in (0, 2]$ determines spatial nonlocality and q encodes nonlinear coupling in multifractal phase space. Under the scale transformation

$$x \rightarrow bx, \quad t \rightarrow b^z t,$$

the fractional Laplacian scales as

$$(-\Delta)^{\alpha/2} \rightarrow b^{-\alpha} (-\Delta)^{\alpha/2},$$

yielding the **fractional dynamic exponent**

$$z = \alpha.$$

7.2 Dimensional RG Flow

We define the effective spectral dimension $d_s(\ell) = D(\ell)$ as,

$$D(\ell) = -\frac{d \ln P(\ell)}{d \ln \ell},$$

where $P(\ell)$ is the return probability of a diffusion process occurring on the underlying geometry. The dimensional RG flow equation takes the generic form

$$\frac{dD}{d \ln \ell} = \beta_D(D, \alpha, q),$$

with fixed points satisfying

$$\beta_D(D_*) = 0.$$

These fixed points correspond to scale-invariant cosmological regimes, while the slow drift between these regimes is a manifestation of dimensional flow.

7.3 Fixed Manifolds of RG Flows

Unlike equilibrium systems, complex dynamics generically exhibits fixed manifolds rather than isolated points:

$$\mathcal{M}_{\text{crit}} = \{(D, \alpha, q) \mid \beta_i = 0\}.$$

Self-organized (SOC) systems dynamically evolve toward this manifold through continuous feedback between driving (cosmic expansion) and dissipation (formation of galactic structures), which ensures stability against perturbations.

8 Generalized Entropy Production and Scaling Laws

8.1 Entropy Growth in Multifractal Phase Space

For a probability measure $\{p_i\}$ distributed on multifractal support, the generalized entropy of order q reads

$$S_q = \frac{1}{q-1} \left(1 - \sum_i p_i^q \right).$$

The rate of entropy production can be presented as

$$\frac{dS_q}{dt} \sim \sigma(\ell) \ell^{\eta(\ell)}$$

with exponent

$$\eta(\ell) = D(\ell)(1-q)$$

where $\sigma(\ell)$ is the scale-dependent entropy flux.

8.2 Scaling of Correlations

Two-point correlation functions scale as

$$C(r) = \langle \phi(0)\phi(r) \rangle \sim r^{-(D-2+\eta)}$$

with anomalous dimension

$$\eta = \eta(D, \alpha, q).$$

In SOC systems, η varies continuously with the observation scale, reflecting criticality in continuous dimension.

9 SOC and Cosmology

This section tailors the CDYN framework specifically to cosmological dynamics, viewed as a self-organizing, far-from-equilibrium system.

9.1 Cosmology as Non-Equilibrium Critical System

As previously stated, the Universe is continuously driven by expansion and dissipates through cosmic structure formation. These ingredients satisfy the defining conditions of SOC, namely,

- slow driving: cosmic expansion,
- fast relaxation: gravitational clustering,
- scale-free response reflected on the onset of power-law correlations.

In this interpretation, the large-scale structure of the Universe naturally evolves toward a cosmological critical manifold.

9.2 Dimensional Flow in Cosmic Structure Formation

Let the effective spatial dimension governing clustering be $D(z)$, evolving with redshift z . The growth of matter perturbations δ obeys

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho\delta \Rightarrow \delta(k, z) \sim k^{D(z)-3}.$$

As the Universe evolves toward late times,

$$D(z) = 3 - \epsilon(z), \quad \epsilon(z) > 0,$$

leading to:

- suppressed small-scale power,
- reduced clustering amplitude,
- scale-dependent growth rates.

This behavior is the cosmological analogue of SOC saturation.

9.3 SOC Interpretation of Large-Scale Anomalies

Within this framework:

- Power-law clustering arises from criticality,
- Deviations from standard growth are a consequence of dimensional flow,

The Universe does not sit at a fixed point of dimensional flow but self-organizes along a critical trajectory in parameter space.

9.4 Fractional Transport and Cosmic Diffusion

Effective transport of matter and information follows from an anomalous diffusion process,

$$\langle r^2(t) \rangle \sim t^{2/\alpha},$$

with $\alpha < 2$ at late cosmic times (low redshift z) due to the onset of multifractal geometry. This setting changes clustering statistics and entropy growth without violating the principles of local dynamics.

10 Unified Picture: Complexity as an Organizing Principle

Complex dynamics emerges when:

1. geometry becomes scale-dependent,
2. dynamics becomes nonlocal,
3. Entropy becomes nonadditive,
4. systems self-tune to criticality.

In this picture of CDYN, SOC, fractional dynamics, and dimensional flow are therefore not distinct dynamic mechanisms but complementary aspects of a single operational framework.

11 CDYN and Particle Physics

We put forward several contributions bridging the gap between CDYN and particle physics which has provided new insights into the foundational structure of the Standard Model (SM). Below is a concise overview of these contributions.

11.1 Bifurcations and the SM Composition

The bifurcation model of SM posits that the entire gauge and flavor makeup of particle physics emerge from sequential bifurcations induced by the self-interaction term of the Higgs potential. In this framework, classical fields develop through self-similar cascade of period doubling bifurcations.

11.2 Bifurcations and Feynman Diagrams

We draw a connection between the universal bifurcation scenario and Feynman diagrams. In this picture, self-similarity and scaling properties of bifurcation phenomena relate to the amplitudes of particle interactions described by Feynman diagrams.

11.3 Three Generations of Fermions

Triplification of fermion generations in the Standard Model follows from the onset of bifurcations in the neutrino sector of the bifurcation branching process.

11.4 The Origin of C and CP Violations

Both C (charge) and CP (charge-parity) violation in the electroweak sector appear to be direct consequences of the bifurcation dynamics.

11.5 The Square of Masses Relationship

We deploy the tools of multifractal analysis to derive a fundamental relationship between the square of particle masses and the square of the electroweak scale set by the vacuum expectation value of the Higgs boson, namely $v = 246$ GeV.

11.6 Ratios of Particle Masses and the Feigenbaum Constant

Using the RG flow driven by dimensional deviation from four spacetime dimensions, we link the ratios of particle masses to the Feigenbaum constant, which characterizes the scaling behavior of chaotic maps.

11.7 Higgs Mechanism and Fermion Mixing Angles

Our research also reveals that the Higgs mechanism and the generation of fermion mixing angles can be linked to dynamic onset of the Hopf bifurcation and the formation of limit cycles.

11.8 Ultraviolet Turbulence in Quantum Chromodynamics (QCD)

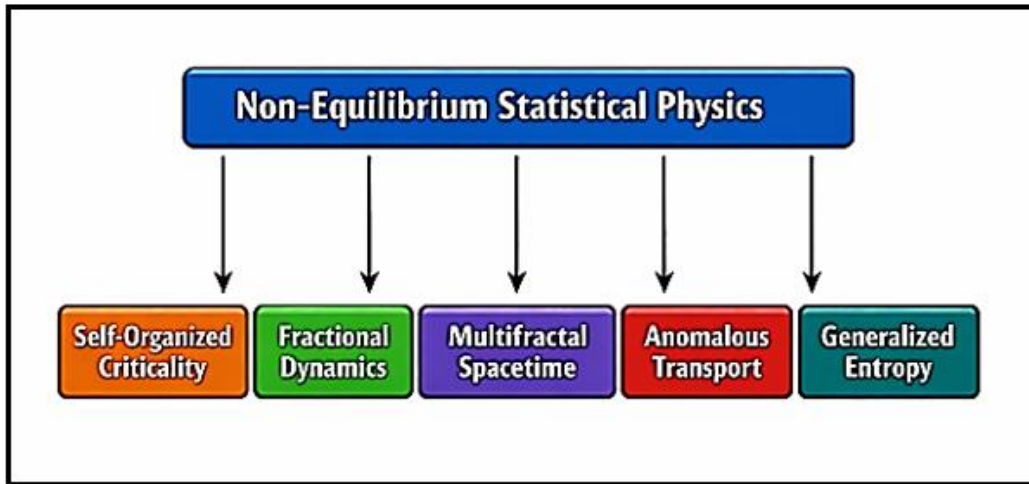
We point out that the asymptotically free model of QCD fails in the deep ultraviolet sector (UV); instead, the coupling exhibits intermittent behavior, stemming from the multiplicative noise induced by dimensional fluctuations. We find that this turbulent regime leads naturally to a multifractal attractor, consistent with a Cantor–Dust description of ultraviolet physics.

12 Conclusions

We have outlined a unified theory of CDYN based on operational principles rather than model-specific assumptions. By combining fractional field theory, RG methods, generalized entropies, and dimensional flow, we have pointed out how complexity arises naturally in systems evolving in out-of-equilibrium conditions.

When applied to cosmology, this framework suggests that large-scale structure formation is governed by SOC on evolving, multifractal geometries. The Universe operates near—but not exactly at—criticality, with dimensional flow encoding its long-term behavior. By the same token, when applied to particle phenomena, CDYN generates unexpected outcomes consistent with experimental observations and fit to explain some of the outstanding open questions facing SM.

This perspective opens a path toward a genuinely non-equilibrium, scale-adaptive theory of cosmological evolution and particle phenomenology. The diagram below is a condensed account of our paper.



References

(Note that the list is currently incomplete, to be updated in the near future).

Entropy production equation (multifractal / fractional kinetics)

1. G. Nicolis, I. Prigogine, (1977), *Self-Organization in Nonequilibrium Systems*, Wiley.
2. R. Zwanzig, (2001), *Nonequilibrium Statistical Mechanics*, Oxford.
3. A. Ottinger, (2005), *Beyond Equilibrium Thermodynamics*, Wiley.
4. E. Goldfain, (2026), “Emergence of Evolving Dimensions in Field Theory and Cosmology”, preprint <https://doi.org/10.13140/RG.2.2.14622.09282/1>
5. E. Goldfain, (2026), “Dimensional Regularization as Gateway to Complex Dynamics”, preprint available at the following site <https://doi.org/10.13140/RG.2.2.27631.50084/1>
6. E. Goldfain, (2025), “On the Gravitational Entropy of Cantor Dust”, preprint <https://doi.org/10.13140/RG.2.2.13547.04642>
7. M. Bologna, P. Grigolini, “Anomalous diffusion and entropy production”, Phys. Rev. E 66, 036130 (2002).

Dimensional flow in cosmology

8. G. Calcagni, (2010), “Fractal universe and quantum gravity”, Phys. Rev. Lett., 104, 251301.
9. E. Goldfain, (2025), “General Relativity from Primordial Dimensional Fluctuations”, preprint <https://doi.org/10.13140/RG.2.2.33155.16165/1>
10. J. Ambjørn et al., (2005), “Spectral dimension of the universe”, Phys. Rev. Lett. 95, 171301 (2005).

Matter power spectrum with anomalous scaling

11. S. Weinberg, (2008), *Cosmology*, Oxford, Chapters 6–8.
12. P. Coles, B. Lucchin, (2002), *Cosmology*, Wiley.
13. L. Amendola et al., (2018), “Observables and signatures of dimensional flow”, *Phys. Rev. D* 98, 103521.

Growth rate, SOC saturation equations and cosmological applications

14. P. Bak, (1996), *How Nature Works*, Springer.
15. H. J. Jensen, (1998), *Self-Organized Criticality*, Cambridge.
16. M. Aschwanden, (2011), *Self-Organized Criticality in Astrophysics*, Springer.
17. J. Buchert, (2008), “Non-equilibrium cosmology”, *Gen. Rel. Grav.* 40, 467.
18. A. Coniglio, (1981), “Cluster fluctuations and criticality”, *Phys. Rev. Lett.* 46, 250.

From Complex Dynamics to Foundational Physics

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Abstract

As of today, Quantum Field Theory (QFT) and General Relativity (GR) are broadly recognized paradigms of foundational physics. There are, however, growing suspicions that both paradigms fail to hold somewhere above the Standard Model (SM) scale and in the realm of primordial cosmology. Evidence collected on multiple fronts indicates that emergence and complexity are universal features of far-from-equilibrium systems with many degrees of freedom. In line with these findings, Part 1 of the report explores the complex dynamics of evolving dimensional fluctuations beyond the SM scale. Part 2 outlines the role of complex dynamics in the nonintegrable sector of particle physics, Dark Matter condensation and the gravitational regime of the early Universe.

Key words: Non-equilibrium critical phenomena, Self-Organized Criticality, Standard Model, fractal spacetime, multifractals, emergence, complex dynamics.

Cautionary remarks

We caution from the outset that the sole intent of this report is to lay the groundwork for further exploration of the topic. Exclusively presented in a preliminary form, our analysis is far from completion and far from meeting the quality standards of a formal research project. The style and presentation do not comply with traditional academic standards. Independent work is needed to develop, validate, or reject the hypotheses presented here. Readers unfamiliar with the topic are encouraged to carefully review the enclosed references prior to drawing premature conclusions.

1 Introduction and Motivation

The last decade has shown that large systems of nonlinearly interacting components are prone to slide outside thermodynamic equilibrium and become nonintegrable in the long run. Reliable modeling of such systems requires new concepts and methods inspired by chaos theory, multifractal geometry, non-equilibrium critical phenomena, self-organized criticality (SOC), and fractional calculus. These non-conventional tools form the mathematical basis of complex dynamics [1, 6-7, 9-14, 16-17, 23, 29].

A fundamental assumption of QFT is that its vacuum state is a large many-body system of quantum fluctuations whose dynamics follows the framework of equilibrium thermodynamics. There is no guarantee that this conjecture will continue to hold above the low-energy scale of the SM. On the contrary, many condensed matter studies consistently suggest the opposite: on high-energy scales, unbalanced vacuum fluctuations are likely to slide outside equilibrium and the perturbative treatment of conventional QFT is prone to break down. When driven far away from thermodynamic equilibrium, complex systems are known to exhibit emergent dynamics stemming from the interplay

between nonlinear interaction of components and steady dissipation. As paradigm of this type of emergent behavior, SOC has a vast range of applications extending from astrophysics, natural hazards and magnetospheric physics, to complex networks, internet dynamics, biophysics, and social sciences [15, 24].

According to [1], there is a tentative path leading from Dimensional Regularization procedure of QFT to fractal spacetime, on the one hand, and to fractional dynamics, on the other. Both concepts require passage to a spacetime endowed with continuous dimensions, which are conjectured to flow with the observation scale. This viewpoint offers unforeseen insights on the deep ultraviolet extension of Effective Field Theory (EFT). At the same time, they are manifestly at odds with most SM extensions, Quantum Gravity theories or field unification models.

In a nutshell, fractals and multifractals define the underlying geometry of complex behavior for systems evolving in far-from-equilibrium conditions. To appreciate the alleged impact of fractal spacetime beyond the boundaries of EFT, we bring up two textbook examples:

- 1) Perturbative QFT is an archetype of EFT which lies at the foundation of the SM for particle physics. Although astonishingly successful in matching results from collider data, QFT is confronted by many drawbacks. It is known, for instance, that amplitude computations are at best asymptotic expansions, meaning that Feynman diagrams fail to uncover the true dynamics of the theory. Non-perturbative QFT methods have been partially successful, yet it is fair to say that, for the most part, the underlying physics of QFT remains unknown. In addition, there are several caveats and patches needed to secure self-consistency demanded by the principles of QFT.
- 2) Another often cited paradigm of an EFT is General Relativity (GR), whose validity rests on the principle of general (or diffeomorphism) covariance. This principle asserts that all physical laws must take the same mathematical form regardless of the coordinate system used by observers in arbitrary relative motion. Stated differently, general covariance means invariance of physical laws under all possible coordinate transformations. An implicit assumption of general covariance is that any coordinate transformation and its inverse are smooth/analytic functions that can be differentiated arbitrarily many times. However, it is known that there is a plethora of non-differentiable curves and surfaces in Nature, as abundantly discovered since the introduction of fractal geometry in 1983. The unavoidable conclusion is GR assigns a preferential status to differentiable transformations, which is at odds with the very spirit of general covariance. In addition, the singularity problem, the relentless instability of the transient regime of GR, and the N-body problem are likely tied to the transition from deterministic to a self-sustained complex evolution in the primordial stages of Universe formation [28 - 30, 33-34].

Experimental and numerical evidence of recent years indicates that emergence and complexity are universal features of far-from-equilibrium systems having many degrees of freedom. In line with these findings, several independent research works point to indirect hints for a spacetime endowed with minimal fractality, likely to develop somewhere above the SM scale [5, 12-13, 17-22, 25-27].

The report is divided into two parts. Part 1 explores the complex dynamics of evolving dimensional fluctuations above the SM scale. In support of this scenario, the Appendix section bridges the gap between universal route to chaos and the flavor composition of the SM. Part 2 outlines the role of complex dynamics in the nonintegrable sector of particle physics, the formation of Dark Matter as large-scale dimensional condensate and the gravitational regime of the primordial Universe.

2 From Dimensional Regularization to fractal spacetime

There are several ways of introducing the concept of spacetime continuum whose non-integer dimensions evolve with the observation scale [1, 20-21]. To make the paper self-contained and for the sake of clarity, we begin by iterating here the arguments for fractal spacetime inspired by Dimensional Regularization program of QFT [1].

It is known that Euclidean formulation of the Path Integral in QFT enables a useful analogy between QFT and critical phenomena. To this end, consider the two-point function of massive scalar field theory. The Euclidean propagator in momentum space is given by

$$D_E(p) = \frac{1}{p^2 + m^2} \quad (1)$$

and the correlation function of the corresponding statistical system is

$$\langle \varphi(x)\varphi(0) \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{\exp(ipx)}{p^2 + m^2} \quad (2)$$

in which $|p^2| = p_\mu p_\mu$ and $px = p_\mu x_\mu$. In the limit $m|x| \gg 1$, (2) is well approximated by,

$$\langle \varphi(x)\varphi(0) \rangle \approx \frac{1}{|x|^2} \exp(-m|x|) \quad (3)$$

Let us assume that the field is placed on a four-dimensional lattice of points separated by a fixed spacing $a = \Lambda_{UV}^{-1}$, in which Λ_{UV} is the cutoff scale. The spatial coordinate is then,

$$|x| = Na = N\Lambda_{UV}^{-1}, \quad N \gg 1 \quad (4)$$

and (3) can be written as,

$$\langle \varphi_N \varphi_0 \rangle \propto \exp[-N(m/\Lambda_{UV})] \quad (5)$$

By analogy with statistical mechanics, (5) defines the dimensionless correlation length according to,

$$\langle \varphi_N \varphi_0 \rangle \propto \exp(-N/\xi) \quad (6)$$

where,

$$\xi = \frac{\Lambda_{UV}}{m} \quad (7)$$

Dimensional Regularization in momentum space sets up a relationship between the cutoff scale Λ_{UV} and the dimensional deviation from four-space dimensions $\varepsilon(\mu) = 4 - d(\mu) \ll 1$ as in

$$\Lambda_{UV}^2 = \mu^2 \exp(1/\varepsilon) \gg \mu^2 \quad (8)$$

where μ is the running scale. The asymptotic limit $m = O(\mu) \ll \Lambda_{UV}$ leads to the raw estimate,

$$\varepsilon(\mu) = 4 - d(\mu) = O[m^2(\mu)/\Lambda_{UV}^2] \ll 1 \quad (9)$$

By (7) and (9), we arrive at the effective approximation,

$$\xi(\mu) \propto [\varepsilon(\mu)]^{-1/2} \quad (10)$$

As a diverging correlation length is a characteristic feature of critical phenomena, (10) indicates that removing the dimensional regulator in QFT (that is, taking the classical continuum limit

$\varepsilon \rightarrow 0$) is analogous to tuning the corresponding statistical system towards the critical point [8, 17, 20-21]. In this sense, (10) underlies the idea of criticality in continuous dimension $d(\varepsilon)$, conjectured to play a key role in the ultraviolet regime of field theory and primordial cosmology. Since, by definition, fractal structures are characterized by continuous dimensions and are the underlying geometry of both critical phenomena and chaotic behavior, (10) leads to the conclusion that taking the limit $\varepsilon \rightarrow 0$ in Dimensional Regularization turns the classical spacetime into a minimal fractal manifold (MFM).

3 Complex dynamics of evolving spacetime dimensions

Reaction-diffusion processes are a subset of complex phenomena defined within the framework of Non-equilibrium Statistical Physics [2-3, 16]. These models are typically formulated in $d + 1$ dimensions, where d is the dimension of the Euclidean manifold representing the physical space and t is the time coordinate. Reaction-Diffusion models on discrete manifolds (called lattices) are characterized by the following features,

- a) local variables reside at lattice sites,
- b) reaction chains are driven by probabilistic transition rules among sites.

We consider below a toy Reaction-Diffusion model acting on a two-dimensional lattice ($d = 2$), whose local variables are time-varying dimensional deviations $[\delta\varepsilon(t)]$, referred herein to as “dimensional pixels”. The model is built on four premises, namely,

A1) At any given moment “ t ”, a pixel consists of a pair of lattice sites that are either *occupied* (1) or *empty* (0) and are located horizontally adjacent to each other.

A2) The representative pixel states are listed as,

$$[\delta\varepsilon(t)] = \{[0, 1]; [1, 0]; [1, 1]; [0, 0]\}_t$$

A3) There are four transpositions among these binary states from time “ t ” to time “ $t + dt$ ”, that is,

$$[1] \rightarrow [0] \quad \text{self-annihilation} \tag{11a}$$

$$[1] \rightarrow [1] + [1] \quad \text{decay/percolation} \tag{11b}$$

$$[1] + [1] \rightarrow [0] \quad \text{pair annihilation} \tag{11c}$$

$$[1] + [1] \rightarrow [1] \quad \text{clustering} \tag{11d}$$

A4) Following ref. [1] and figs. 1 - 2 below, dimensional pixels undergo transition events between “ t ” and “ $t + dt$ ” as described by,

$$[1, 0] \xrightarrow{D} [0, 1] \tag{12a}$$

$$[1, 1] \xrightarrow{u} [1, 0], [0, 1] \tag{12b}$$

$$[1, 0], [0, 1] \xrightarrow{\kappa} [1, 1] \tag{12c}$$

Here, (12a) denotes a scattering event at rate D , (12b) a clustering event at rate u and (12c) a decay (or percolation) event at rate $\kappa = \lambda - \lambda_c$, with λ being a control parameter approaching its critical value λ_c .

Up to a leading order approximation, the macroscopic attributes of Reaction-Diffusion processes may be encoded in a mean-field (MF) equation [2], which quantifies the competition between losses and gains in density $\rho(t)$. In particular, the decay/percolation process occurs with a rate proportional to $\kappa\rho(t)$ and leads to a gain in density. By contrast, the clustering process (12b) drops the density with a rate proportional to $u\rho^2(t)$. Ignoring diffusion (12a), the resulting MF equation takes the form

$$\frac{\partial\rho(t)}{\partial t} = \kappa\rho(t) - u\rho^2(t) \quad (14)$$

In the context of our paper, the control parameter $\lambda(t) = \lambda[\delta\varepsilon(t)]$ represents the *density of dimensional pixels* $\delta\varepsilon(t) \ll 1$ while $\rho(t)$ denotes the *density of active (or unstable) lattice sites*.



Fig. 1: *Left panel*: Scattering, *Right panel*: Clustering ([2])



Fig. 2 *Left panel*: Decay/Percolation, *Right panel*: Annihilation ([2])

The MF equation (14) exhibits a two-phase configuration: an absorbing phase with a vanishing density of active sites ($\rho = 0$) below the critical point λ_c and an active phase with a steady-state density $\rho = \kappa/u \neq 0$ above the critical point λ_c .

If the starting point of the time evolution is a fully occupied lattice $\rho(0) = 1$, the solution of (14) reads,

$$\rho(t) = \frac{\kappa}{u - (u - \kappa) \exp(-\kappa t)} \quad (15)$$

Relation (15) shows that, when the percolation rate vanishes at the critical point $\kappa = \lambda - \lambda_c = 0$, the density of unstable states drops asymptotically as in

$$\lim_{\kappa \rightarrow 0} \rho(t) = (1 + ut)^{-1} \quad (16a)$$

or,

$$\rho(t) \sim t^{-1} \quad (16b)$$

By (16a) and (16b), the number of unstable/active sites eventually goes to zero and the dynamics of dimensional deviations slows down. It follows that, in the far infrared regime ($t \rightarrow \infty$), spacetime settles in a stationary state matching the classical limit $\delta\varepsilon = O(\rho) \rightarrow 0$.

Before proceeding further, a few cautionary remarks are in order:

- a) Clearly, by design, the scenario embodied in (A1) – (A4) is merely a convenient simplification. Any realistic model of pixels defined in continuous spatial dimensions must necessarily include infinite sets of non-integer pairs having the form,

$$[\delta\varepsilon(t)] = \{[\alpha_1, \beta_1]; [\alpha_2, \beta_2]; \dots [\alpha_N, \beta_N]\}_t \quad (17)$$

in which $N \rightarrow \infty$ and α_i, β_i are arbitrary numbers with

$$0 \leq \alpha_i \leq 1; \quad 0 \leq \beta_i \leq 1; \quad \alpha_i \neq \beta_i \quad (18)$$

- b) It can be argued that, on Euclidean spacetime endowed with continuous dimensions, the deviations $\delta\varepsilon$ and their fluctuations $\delta\varepsilon$ play an identical role with the coefficients $g_{\mu\nu}$ of a corresponding non-Euclidean metric [see e.g. 5]. Stated differently, a flat spacetime endowed with a fractal structure may be considered as dual to a curved manifold. On this basis, a reasonable assumption is that classical gravitation is implicitly accounted for in the Reaction-Diffusion model detailed above.
- c) The stability of the MF solution with respect to perturbations can be studied in a variety of ways. In a general scenario, for example, one accounts for the combined effects of diffusion (D), Gaussian noise $\varsigma(t)$ and random fluctuations $\eta(t)$ on the percolation parameter,

$$\kappa \rightarrow \kappa + \eta(t) \quad (19)$$

In this scenario, (14) gets upgraded to a Langevin type equation, namely,

$$\frac{\partial \rho(t)}{\partial t} = \kappa \rho(t) - u \rho^2(t) + D \Delta \rho(t) + \varsigma(t) + \rho(t) \eta(t) \quad (20)$$

As discussed in [2–3, 16] and the complex dynamics literature, the generic model (20) has applications across a wide range of topics, including (but not limited to) percolation phenomena, epidemics spreading, forest fires, earthquake propagation, lattice dynamics with long-range correlations, spin glasses, spatiotemporal patterns in condensed matter physics, random graph theory, galaxy clustering, large network dynamics and so on.

4 From Reaction-Diffusion processes to Self-organized Criticality (SOC)

Of particular interest is the relationship between the Reaction-Diffusion model previously outlined and Self-organized Criticality (SOC). Among the many ways to unveil this connection a straightforward approach is to supplement (14) with a *driving source* whose function is to boost dimensional instability and prevent relaxation. Adding to the percolation rate a time-independent source term E turns (14) into [4]

$$\frac{\partial \rho(t)}{\partial t} = (\kappa + E) \rho(t) - u \rho^2(t) \quad (21)$$

According to [4], since E is a conserved quantity ($\dot{E} = 0$), it can be conveniently used as control parameter instead of κ . Following this scenario, the critical point is reached at $E_c = -\kappa$ and (21) replicates the *fixed sandpile model* of SOC, whose steady state is determined by E . The impact of SOC on the non-integrable sector of particle physics and early Universe cosmology is covered in Part 2 of this report.

In closing Part 1, we again emphasize that the hypotheses detailed here, despite offering encouraging leads forward, are strictly provisional thoughts. Much remains to be done to debunk, confirm, or place these ideas on a more robust foundation.

APPENDIX

Feigenbaum's Universality and the dynamic structure of the SM

The Reaction-Diffusion model (14) with $u = 1$ may be analyzed as an iterated map operating in discrete time $t_n = n\Delta t$, $n = 1, 2, \dots N$. Setting

$$\frac{\partial \rho}{\partial t} \Rightarrow \frac{\rho_{n+1} - \rho_n}{\Delta t} \quad (22)$$

turns (14) into

$$\rho_{n+1} = (1 + \kappa\Delta t)\rho_n \left[1 - \frac{\Delta t}{1 + \kappa\Delta t}\rho_n \right] \quad (23)$$

Under the change of variable

$$v_n = \frac{\Delta t}{1 + \kappa\Delta t}\rho_n \quad (24)$$

(23) takes the form of the *logistic map*

$$v_{n+1} = \gamma v_n (1 - v_n) \quad (25)$$

in which the critical parameter is

$$\gamma = (1 + \kappa\Delta t) \quad (26)$$

In doing so, one recovers the universal route to chaos of unimodal maps via the well-known scenario of period-doubling bifurcations [31-32]. One is led to the Feigenbaum's sequence of critical parameters as function of the sequence index $m = 1, 2, \dots M$

$$\gamma_m - \gamma_0 \Rightarrow \kappa_m - \kappa_0 \Rightarrow \lambda_m - \lambda_0 \propto \delta^{-m} \quad (27)$$

where δ stands for the Feigenbaum constant.

According to [31-32], iterated maps of the unit interval are generic models of dynamical systems in discrete time. The standard representation of these models is based on first order difference equations having the form

$$x_{n+1} = f(x_n, \gamma) \quad (28)$$

The dynamics of iterated maps can be either *conservative* or *dissipative*. In the former case, the function (28) is monotonic and describes a one-to-one mapping, whereas in the latter case is non-monotonic and describes a two-to-one mapping. Typical examples of dissipative systems include the *quadratic map* and *unimodal maps* of the type (25). In 1978, Feigenbaum discovered that the onset of chaos in quadratic maps occurs through *period-doubling bifurcations* driven by changes of the critical parameter γ . Unimodal maps exhibit the following behavior: for small values of γ , (28) has a single stable fixed point and all nearby points converge to it under multiple iterations of (22)-(23). Ramping up γ to a critical value γ_1 makes the fixed point unstable and produces a new stable pair of points of period 2. Further increasing γ to another value (γ_2) bifurcates this cycle into a cycle of period 4. The bifurcation process continues with a new sequence of cycles of period 2^j , $j \geq 3$, eventually leading to a Cantor set structure that attracts almost all the points of the interval $[-1, 1]$. On letting γ increase beyond an endpoint value γ_∞ , stable periodic orbits surface again and split up in a similar way. In the new sequence, γ scans another series of critical values corresponding to cycles of period $3 \cdot 2^j$, $j = 0, 1, 2, \dots$ and so on. When applied to the flavor composition of SM, this bifurcation scenario leads to the content of Tab. 1 below. The first branch of the fermion sector represents the set of 3 left-handed neutrinos occurring at $j = 0$, a finding which naturally accounts for both triplication of fermion families and the fermion chirality in the SM [see e.g. 24]. It is instructive to note that, according to this scenario, Dark Matter appears to echo the properties of anyons in three-dimensional space [33].

Parameter	Flavor Content	Bifurcation Pattern	Spin
λ_1	Higgs scalar	$2^j, j = 0$	0
λ_m	DM Gauge Bosons	2^j	1
λ_∞	Dark Matter	$2^j, j = \infty$	undefined
λ_m	Fermions	$3 \cdot 2^j$	$\frac{1}{2}$

Tab. 1: The flavor structure of visible and Dark Matter sectors

References (Part 1)

1. Goldfain, E., (2024), Fractional Dynamics and Physics Beyond Effective Field Theory (Part 1), Qeios, <https://doi.org/10.32388/MHWR52>
2. Hinrichsen, H., (2000), Non-equilibrium critical phenomena and phase transitions into absorbing states, *Advances in Physics*, 49(7), pp. 815–958. <https://doi.org/10.1080/00018730050198152>
3. Lübeck, S., (2004), Universal scaling behavior of non-equilibrium phase transitions, *International Journal of Modern Physics B* 2004 18:31, pp. 3977-4118, <https://doi.org/10.1142/S0217979204027748>
4. Buendía, V., et al., (2020), Feedback mechanisms for self-organization to the edge of a phase transition, *Front. Phys.*, vol. 8, <https://doi.org/10.3389/fphy.2020.00333>
5. Goldfain, E., (2023), Cantor Dust and the Gravitational Wave Background, Research Gate preprint, 10.13140/RG.2.2.19323.49447
6. Nicolis, G., (2012), *Foundations of Complex Systems; Emergence, Information and Prediction*, 2nd Edition, <https://doi.org/10.1142/8260>
7. Hébert-Dufresne, L., et al., (2024), The path of complexity., <https://doi.org/10.1038/s44260-024-00004-0>
8. Maggiore, M., (2004), *A Modern Introduction to Quantum Field Theory*, Oxford University Press, <https://doi.org/10.1093/oso/9780198520733.003.0001>.
9. Goldfain, E., (2008), Fractional dynamics and the Standard Model for particle physics, *Communications in Nonlinear Science and Numerical Simulation*, Volume 13, Issue 7, pp. 1397-1404, ISSN 1007-5704, <https://doi.org/10.1016/j.cnsns.2006.12.007>.
10. Goldfain, E., (2008), Feigenbaum attractor and the generation structure of particle physics, *Intl. Journal of Bifurcation and Chaos*, Vol. 18, No. 03, pp. 891-896, <https://www.worldscientific.com/toc/ijbc/18/03>
11. Goldfain, E., (2012), Fractional Field Theory and High-Energy Physics: New Developments, *Intl. Journal of Theoretical Physics, Group Theory, and Nonlinear Optics*, vol. 16, issue 3/4, pp. 239-262.
12. Goldfain, E., (2008), Bifurcations and pattern formation in particle physics: An introductory study, *Europhysics Letters (EPL)*, vol. 82, number 1, <https://doi.org/10.1209/0295-5075/82/11001>

13. Zaslavsky G. M., (2004), Hamiltonian Chaos and Fractional Dynamics, Oxford Univ. Press, <https://doi.org/10.1093/oso/9780198526049.001.0001>
14. Tarasov, V. E., (2021), General Fractional Dynamics, Mathematics 9, no. 13: p. 1464., <https://doi.org/10.3390/math9131464>
15. Christensen, K. and Moloney, N. R., (2005), Complexity and Criticality, Imperial College Press, <https://doi.org/10.1142/p365>
16. Täuber, U., (2014), Critical Dynamics: A Field Theory Approach to Equilibrium and Non-Equilibrium Scaling Behavior, Cambridge University Press 978-0-521-84223-5.
17. Ballhausen, J. et al., (2004), Critical phenomena in continuous dimension, Physics Letters B, Volume 582, Issues 1–2, pp 144-150, ISSN 0370-2693, <https://doi.org/10.1016/j.physletb.2003.12.033>
18. Goldfain, E., (2023), Hamiltonian Chaos and the Fractal Topology of Spacetime (Part 1). Qeios, <https://doi.org/10.32388/7ZYVDB>
19. Goldfain, E. (2023), Hamiltonian Chaos and the Fractal Topology of Spacetime (Part 2). Qeios, <https://doi.org/10.32388/OKXQIL>
20. Goldfain, E., (2023), Dimensional Regularization as Mass Generating Mechanism, Qeios, <https://doi.org/10.32388/DW6ZZS>
21. Goldfain, E., (2023), Dimensional Regularization and Fractal Spacetime, preprint <http://dx.doi.org/10.13140/RG.2.2.21554.99524/2>
22. Goldfain, E., (2021), Mapping Effective Field Theory to Multifractal Geometry, Preprints <https://doi.org/10.20944/preprints202105.0502.v2>
23. Goldfain, E., (2023). Complex Dynamics and the Age of the Universe. Qeios, <https://doi.org/10.32388/T9Z0VK>
24. Goldfain, E., (2020), Self-Organized Criticality and Standard Model Parameters, Preprints <https://doi.org/10.20944/preprints202011.0702.v1>
25. Goldfain, E., (2021), Bifurcations and the Gauge Structure of the Standard Model, preprint <http://dx.doi.org/10.13140/RG.2.2.34888.85769/1>
26. Goldfain, E., (2021), Bifurcations of the Higgs Potential and the Top Quark Mass, preprint <http://dx.doi.org/10.13140/RG.2.2.23144.80644>
27. Goldfain, E., (2024), Hamiltonian Chaos and Gravitational Physics, Qeios, <https://doi.org/10.32388/PGLOUE>
28. Afshordi, N., and Stojkovic, D., (2014), Emergent spacetime in stochastically evolving dimensions, Physics Letters B, 739, pp. 117-124, <https://doi.org/10.1016/j.physletb.2014.10.048>
29. Savvidy, G., (2020), Maximally chaotic dynamical systems, Annals of Physics, Volume 421, 168274, <https://doi.org/10.1016/j.aop.2020.168274>

30. Moreira, T. H., et al, (2024), Entropy production due to spacetime fluctuations, preprint <https://doi.org/10.48550/arXiv.2407.21186>
31. Collet, P. and Eckmann, J. P., (1980), Iterated Maps on the Interval as Dynamical Systems, Birkhäuser Boston, ISBN 0-8176-3510-6.
32. Falconer, K. J., (1988), The geometry of fractal sets, Cambridge University Press, ISBN 0-521-25694 1.
33. Goldfain, E., (2023), Dark Matter as Dimensional Condensate, Qeios <https://doi.org/10.32388/8A8579>
34. Goldfain, E., (2024), Cosmological Structure Formation and Fractal Spacetime, Qeios <https://doi.org/10.32388/WUQZE8>

On Complex Dynamics and Classical Field Theory

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Abstract

Complex Ginzburg-Landau equation (CGLE) is a paradigm for the onset of chaos and turbulence in nonlinear dynamics of extended systems. Here we point out that the underlying connection between CGLE and the Navier-Stokes (NS) equation bridges the divide between fluid flows, on the one hand, and the mathematics of General Relativity (GR) and classical gauge theory, on the other. The analogy hints to a possible link between the transition from laminar to turbulent flows and the mass generation mechanism of quantum field theory (QFT).

Key words: chaos and complexity, complex Ginzburg-Landau equation, turbulence, Navier-Stokes equation, classical gauge theory, General Relativity.

1. Introduction

Building on the assertion that *complex dynamics* plays a critical role in foundational physics [1-3], the object of this brief report is to show that the roots of both classical gauge theory and GR may be traced back to the CGLE.

The report contains a couple of paragraphs and an Appendix section. First paragraph delves into the derivation of the NS equation from CGLE and the role of kinematic viscosity in the transition from laminar flows to turbulence. The second paragraph points out to research studies looking into the analogy between NS and classical field theory, namely GR and Maxwell's electrodynamics. The analogy suggests an unforeseen parallel between the onset of fluid turbulence via the reduction of kinematic viscosity and the mass generation mechanism of QFT.

We caution from the outset that the sole intent of this report is to lay the groundwork for further exploration of the topic. Exclusively presented in a draft form, our analysis is far from completion and far from meeting the quality standards of a full blown research project. Independent work is needed to develop, validate, or reject the ideas presented here.

2. CGLE and the NS equation

We start by recalling that the CGLE encodes many key properties of out-of-equilibrium nonlinear dynamical systems with space-time dependence. As paradigm for the emergence of complex behavior, CGLE describes the generic onset of chaos, turbulence, and spatiotemporal patterns in extended systems [4-6]. It assumes the standard form

$$\partial_t z = \varepsilon z + (1 + ic_1)\Delta z - (1 - ic_3)z|z|^{2\sigma} \quad (1)$$

in which z is a complex-valued field, the parameters ε and σ are positive and the coefficients c_1 and c_3 are real. The *nonlinear Schrödinger equation* (NSE) is a particular embodiment of the CGLE in the limit $\varepsilon \rightarrow 0$, namely [11],

$$-i\partial_t z = c_1\Delta z + c_3|z|^{2\sigma}z \quad (2a)$$

To streamline the presentation, we work in 1+1 space-time and assume $\sigma = 1$. In its original formulation and natural units ($\hbar = 1$), the quantum-mechanical version of (2a) reads

$$i \frac{\partial}{\partial t} z(x, t) = \left[-\frac{1}{2m} \nabla^2 + V(x, t) \right] z(x, t) \quad (2b)$$

where $V(x, t)$ is the potential function. The so-called *Madelung transformation* enables one to turn (2b) into the quantum Euler equation for compressible potential flows [7]. In particular, taking the complex-valued field in the canonical form,

$$z(x, t) = \sqrt{\frac{\rho(x, t)}{m}} \exp[iS(x, t)] \quad (3)$$

and substituting it into (1)-(2) leads to

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad (4)$$

$$\frac{d\vec{u}}{dt} = \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{m} \nabla(Q + V) \quad (5)$$

Here, $\vec{u}(x, t)$ denotes the flow velocity, $\rho = m|z|^2$ stands for the mass density and

$$Q = -\frac{1}{2m} \frac{\nabla^2(\sqrt{\rho})}{\sqrt{\rho}} \quad (6)$$

is the so-called Bohm potential. The flow velocity and its associated probability current are given by

$$\vec{u}(x, t) = \frac{1}{m} \nabla S = -\frac{i}{m} \frac{\nabla z}{z} \quad (7)$$

$$\vec{j} = \rho \vec{u} = \frac{1}{2mi} [z^*(\nabla z) - z(\nabla z^*)] \quad (8)$$

Since the Schrödinger equation is conservative, the Madelung transformation naturally leads to the Euler equation, which is exclusively valid for *inviscid flows*. To account for fluid viscosity and arrive at the NS equation, one needs to either appeal to an extended version of the NSE containing non-conservative terms [7-8] or bring up the concept of *kinematic viscosity* – a concept linked to the mass of quantum particles [9] as in

$$\nu = \frac{1}{2m} \quad (9)$$

On account of (9) and for incompressible flows, the NS equation that mirrors (5) is given by

$$\frac{d\vec{u}}{dt} = \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u} \quad (10)$$

where p is the pressure. An alternative expression for the NS equation (10) may be obtained using the identity

$$\vec{u} \cdot \nabla \vec{u} = \frac{1}{2} \nabla u^2 - \vec{u} \times \vec{\omega} \quad (11)$$

where $\vec{\omega} = \nabla \times \vec{u}$ represents the vorticity vector [10]. The corresponding NS equation reads

$$\frac{\partial \vec{u}}{\partial t} = -\vec{\omega} \times \vec{u} - \nabla \left(\frac{p}{\rho} + \frac{u^2}{2} \right) + \nu \nabla^2 \vec{u} \quad (12)$$

Referring to (3), the phase of the field amounts to

$$S = m \int u \, dx \quad (13)$$

such that

$$z(x, t) = \sqrt{\frac{\rho(x, t)}{m}} \exp \left(-im \int u \, dx \right) \quad (14)$$

3. From CGLE to abelian gauge theory and GR

It is instructive to note that there is a wealth of literature dealing with the connection between the NS equation, on the one hand, and the formalism of classical electrodynamics and GR, on the other. The interested reader is invited to consult a cross-section of representative references listed under [12 - 19].

4. Concluding remarks

The brief analysis presented here falls in line with our previous findings where, under general boundary conditions, the long-run evolution of Renormalization Group flows is conjectured to converge on *strange attractors* [2, 21-22]. Supported by an underlying *multifractal structure*, these attractors provide realistic models for the onset of chaos in nonlinear dynamics, the transition to turbulence as well as for the phenomenology of the Standard Model near or above the electroweak scale [1, 3].

Relation (9) confirms that highly viscous fluids produce a hydrodynamic regime close to laminar flows. Interpreted in the context of the *minimal fractal manifold* [3], the kinematic viscosity assumes the role of a mass generation mechanism and corresponds to the inverse of the dimensional parameter

$$\varepsilon = 4 - D = O(m^2/\Lambda_{UV}^2) \ll 1$$

Stated differently,

$$\boxed{\nu = \frac{1}{2m} = O(\varepsilon^{-1/2})} \quad (15)$$

Extrapolating (15) to QFT, hints that the kinematic viscosity may justify the mechanism of mass generation in the Standard Model of particle physics.

APPENDIX

Our analysis suggests that the geometry of strange attractors underlies the equations of classical field theory in an inconspicuous way. The aim of this Appendix section is to reinforce this conclusion. Here we show that seemingly disparate concepts of quantum physics and classical field theory – namely, the Berry phase, gauge potentials and the connection coefficients of GR - share a common geometric foundation.

A) Berry phase in quantum physics

A quantum system adiabatically transported around a closed path C in the space of external parameters acquires a non-vanishing phase (*Berry phase*, BP in short). Since BP is exclusively

path-dependent, it provides key insights into the geometric structure of quantum mechanics and QFT. The BP concept is closely tied to *holonomy*, that is, the extent to which some of variables change as other variables or parameters defining a system return to their initial values.

Consider a quantum system described by the time-dependent Hamiltonian $H(t)$, whose associated eigenstate is $|\psi(t)\rangle$ and which is embedded in a slowly changing environment. After a periodic evolution of the environmental parameters ($t \rightarrow t + T$), the eigenstate returns to itself, apart from a phase angle,

$$|\psi(t)\rangle = e^{i\alpha} |\psi(0)\rangle \quad (\text{A1})$$

If ω denotes the eigenvalue of $|\psi(t)\rangle$, a generalization of the phase angle $\alpha = \omega T$ in units of $\hbar = 1$ is given by the “dynamical phase”

$$\gamma_d = \int_0^T \omega(t) dt = \int_0^T \langle \psi(t) | H(t) | \psi(t) \rangle dt \quad (\text{A2})$$

Berry has shown that there is a time-independent (but contour dependent) supplemental “geometric phase” entering the phase angle, namely,

$$\alpha = \gamma_d + \gamma(C) \quad (\text{A3})$$

where

$$\gamma(C) = \int_C \langle \psi | i \nabla \psi \rangle dr \quad (\text{A4})$$

The dynamical phase γ_d encodes information about the duration associated with the cyclic evolution of the complex vector $|\psi(t)\rangle$. By contrast, (A4) encodes information about the geometry of the environment where the transport takes place.

B) The geometry of gauge and gravitational fields

The gauge field concept may be built from a straightforward geometric interpretation, according to [20]. Consider the parallel transport of a complex vector $|\psi\rangle$ round a closed rectangular loop. The difference between the value of $|\psi\rangle$ at the starting point ($|\psi\rangle_0$) and at the end point $|\psi\rangle_0 \rightarrow |\psi\rangle_f$ is given by

$$\Delta\psi = \psi_f - \psi_0 = -ig \Delta S^{\mu\nu} F_{\mu\nu} \psi \quad (\text{B1})$$

in which $\Delta S^{\mu\nu}$ denotes the area enclosed by the rectangle and the strength of the gauge field is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu] \quad (\text{B2})$$

Echoing the formation of the Berry phase, the effect of parallel transport is to induce a non-vanishing rotation of $|\psi\rangle$ in internal space proportional to the strength of the gauge field. Likewise, the curvature tensor of GR may be motivated through similar arguments. Taking a vector V^κ on a round trip by parallel transport, the difference between the initial and final components of the vector amounts to

$$\Delta V^\kappa = \frac{1}{2} R^\kappa_{\lambda\mu\nu} V^\lambda \Delta S^{\mu\nu} \quad (\text{B3})$$

This equation faithfully replicates (B1) and signals the presence of a gravitational field, via the curvature tensor $R^\kappa_{\lambda\mu\nu}$. The geometric analogy between gauge theory and General Relativity is captured in the table below.

Gauge Theory	General Relativity
Gauge transformation	Coordinate transformation
Gauge group	Group of coordinate transformations
Gauge potential A_μ	Connection coefficient $\Gamma_{\mu\nu}^\kappa$
Field strength $F_{\mu\nu}$	Curvature tensor $R_{\lambda\mu\nu}^\kappa$

Comparison between the geometry of gauge and gravitational fields.

References

1. Available at the following site: https://www.researchgate.net/publication/344227402_Complex_Dynamics_and_Foundational_Physics
2. Available at the following site: https://www.researchgate.net/publication/339041409_From_Chaos_and_Strange_Attractors_to_Quantum_Theory
3. Available at the following sites: <http://www.aracneeditrice.it/aracneweb/index.php/publicazione.html?item=9788854889972>
https://www.researchgate.net/publication/278849474_Introduction_to_Fractional_Field_Theory_consolidated_version
4. Available at the following site: https://www.researchgate.net/publication/247813449_The_complex_Ginzburg-Landau_equation_as_a_model_problem#fullTextFileContent
5. <https://arxiv.org/pdf/cond-mat/0106115.pdf>
6. <https://iopscience.iop.org/article/10.1209/0295-5075/82/11001/meta>
7. <https://www.mdpi.com/2311-5521/1/2/18>
8. <https://arxiv.org/pdf/1409.7036.pdf>
9. <https://arxiv.org/ftp/arxiv/papers/1301/1301.3586.pdf>
10. <https://my.mech.utah.edu/~pardyjak/me6700/vorticity.pdf>
11. Available at the following site: <https://pdfs.semanticscholar.org/8867/0e1eb822da0246899c74ab71ebcbac3aed59.pdf>
12. <http://cds.cern.ch/record/435570/files/0004169.pdf>
13. <http://cds.cern.ch/record/517274/files/0109088.pdf>
14. Available at the following site: https://www.brown.edu/research/projects/crunch/sites/brown.edu/research/projects/crunch/files/uploads/Haralabos%20Marmanis%20thesis_0.pdf
15. <https://arxiv.org/pdf/1003.0070.pdf>
16. <https://link.springer.com/article/10.1007%2Fs00006-005-0004-1>
17. <https://arxiv.org/abs/1104.5502>
18. <https://arxiv.org/pdf/1101.2451.pdf>
19. <https://arxiv.org/pdf/1004.2632.pdf>
20. Ryder, L. H. "Quantum Field Theory", Cambridge Univ. Press, 1989.
21. Available at the following site: https://www.researchgate.net/publication/331315283_Multifractal_Foundation_of_Effective_Field_Theory#fullTextFileContent
22. Available at the following site <https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.212.9798&rep=rep1&type=pdf>

On Complex Dynamics and Primordial Structure Formation

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Abstract

Conventional wisdom says that the formation of large-scale structures in cosmology follows from the evolution of density perturbations under the combined effect of gravitation and cosmic expansion. We survey here the reasons why this framework fails to hold in primordial cosmology, due to the severe limitations placed on the Friedmann-Robertson-Walker (FRW) metric, the continuity hypothesis and the standard equations of fluid flows. We insist that understanding primordial cosmology must rely instead on a complex dynamics model of evolving dimensional fluctuations, conjectured to come into play far above the electroweak scale. A key outcome of this model is the universal generation of topological defects and condensates emerging from the complex Ginzburg-Landau equation.

Key words: scalar density perturbations, primordial Universe, FRW cosmology, structure formation, complex dynamics, dimensional fluctuations.

1 Introduction

Relativistic cosmology models the Universe as a nearly homogeneous and isotropic expanding spacetime, as described by the FRW metric. In this framework, the observed large-scale structure of the Universe (galaxies, clusters, cosmic web including voids, filaments, halos etc.) develops from small perturbations of the smooth overall background. Perturbations are studied through cosmological perturbation theory (CPT), which tracks how primordial inhomogeneities evolve into large-scale structures from fluctuations in energy density, pressure, and spacetime curvature. Despite its undisputed successes, CPT remains however inadequate for the study of early Universe formation, on spacetime-scales approaching the Planck regime.

The goal of this report is to survey the reasons why CPT fails to hold in primordial cosmology, due to the severe limitations placed on the FRW metric, the continuity hypothesis and the standard equations of fluid flows. In our view, understanding of primordial cosmology must rely instead on a complex dynamics model of evolving dimensional fluctuations, conjectured to come into play far above the electroweak scale. A key outcome of this model is the universal generation of topological defects and topological condensates emerging from the complex Ginzburg-Landau equation (CGLE).

The report is organized as follows: next section is a condensed review of how scalar density perturbations are dealt with in standard cosmology. Section 3 goes over the limitations of the FRW model and CPT in explaining structure formation in primordial cosmology. Section 4 details our perspective on the complex dynamics of primordial dimensional fluctuations and their impact on structure formation via CGLE and self-organization.

For the sake of clarity and accessibility, the report is designed in a “user-friendly” format, with emphasis on pedagogical exposition rather than formal derivations.

2 Scalar Density Perturbations in Standard Cosmology

We start by reviewing the evolution equation of scalar density perturbations in the standard model of cosmology (the Lambda-CDM model) [1 - 4]. Let $\rho(\vec{x}, t)$ denote the local matter or energy density. The main parameter of interest in CPT is the *density perturbation*, which quantifies the local density deviation normalized to the average density of the Universe,

$$\delta(\vec{x}, t) = \frac{\Delta\rho(\vec{x}, t)}{\bar{\rho}} = \frac{\rho(\vec{x}, t) - \bar{\rho}}{\bar{\rho}} \quad (1)$$

The density perturbation (1) can be studied in Fourier space using the decomposition in modes of momenta $\vec{k}$,

$$\delta(\vec{x}, t) = \int d^3k \delta_k(t) \exp(i\vec{k} \cdot \vec{x}) \quad (2)$$

Statistical distribution of modes is described using the concept of *power spectrum* $P(k)$. Observations indicate that the amplitude of density perturbations is *nearly scale invariant*, meaning that it stays roughly the same across all length scales. As a result, primordial fluctuations carry about the same strength and large- and small-scale structures start off with nearly similar density perturbations. In terms of the power spectrum, one has

$$P(k) \propto k^{n_s-1} \quad (3)$$

in which n_s denotes the *scalar spectral index*. While $n_s = 1$ corresponds to exact scale invariance, observations hint that $n_s \approx 0.965$, which shows an actual slight red tilt (meaning more power allocated to large scales).

The linear scalar perturbations to a flat FRW metric takes the form

$$ds^2 = a^2(\eta) [-(1 + 2\Psi)d\eta^2 + (1 - 2\Phi)\delta_{ij} dx^i dx^j] \quad (4)$$

where η is the conformal time and Ψ, Φ are scalar potentials (with $\Psi = \Phi$, if there are no anisotropic stresses). Relation (2) defines the FRW metric in the so-called *conformal Newtonian gauge*. There are two key equations of CPT in Fourier space, namely,

a) The *continuity equation* reflecting energy conservation and given by,

$$\dot{\delta} + (1 + w)(\theta - 3\dot{\Phi}) + 3H(c_s^2 - w)\delta = 0 \quad (5)$$

b) *Euler equation* reflecting momentum conservation and written as,

$$\dot{\theta} + H(1 - 3w)\theta + \frac{\dot{w}}{1 + w}\theta - \frac{k^2 c_s^2}{1 + w}\delta - k^2 \Psi = 0 \quad (6)$$

Here, $H = a'/a$ is the conformal Hubble parameter, $a' = da/d\eta$, $w = \bar{p}/\bar{\rho}$, $c_s^2 = \delta p/\delta \rho$ is the square of the sound speed and $\theta = ik^j v_j = i\vec{k} \cdot \vec{v}$ stands for the velocity potential with wavevector $\vec{k}$.

Using (5) – (6), the equation for the evolution of scalar perturbations in the Lambda-CDM Universe assumes the form,

$$\boxed{\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\delta = 0} \quad (\text{Newtonian limit}) \quad (7)$$

The full relativistic equation beyond the Newtonian limit (7) reads

$$\ddot{\delta} + A(H, w)\dot{\delta} + B(c_s^2, k^2, a^2, w, \bar{\rho})\delta = S(\Phi) \quad (8)$$

in which $A(\dots)$, $B(\dots)$ are functions of variables indicated in (8) and $S(\Phi)$ is the source term induced by Φ .

3 Limitations of FRW Cosmology in the Primordial Universe

While being adequate in describing the late dynamics of the Universe, FRW cosmology faces crucial limitations when applied to the evolution of the primordial Universe:

- 1) Along with General Relativity (GR), FRW breaks down near the Big Bang singularity where large and highly unstable curvature fluctuations ruin the *smooth topology of ordinary four-dimensional spacetime*. It is also conceivable that, in line with the Dimensional Reduction conjecture [11], spacetime dimensionality drops down in some continuous but unpredictable fashion.
- 2) The cosmological principle posits a smooth (differentiable) and uniform Universe, yet these assumptions are bound to be violated near the Big-Bang singularity, as hinted by the Sakharov conditions for baryogenesis and the far-from-equilibrium properties of Planck physics. In the same context, strong nonlinearities associated with (1), that is, $\delta = O(1)$ are prone to ruin the basis of the entire CPT model.
- 3) FRW does not naturally explain the horizon and flatness puzzles of cosmology without fine-tuned initial conditions—issues typically resolved by inflation, which is still hypothetical and external to the Lambda-CDM paradigm.
- 4) FRW lacks a mechanism of structure formation from primordial fluctuations as it serves only as a background for inflationary cosmology.
- 5) The relevance of quantum gravity effects near the Planck scale remains highly controversial [14].
- 6) The continuity equation (5) fails when mass/energy/momenta are not conserved due to the presence of sources or sinks. Likewise, Bianchi identities of GR break down under strong curvature fluctuations (the local spacetime manifold becomes non-smooth and singular) or when the effects of deterministic or stochastic chaos cannot be discarded (as implied from the generic nonintegrability of Hamiltonian systems [15]).
- 7) Fluid equations of the type (6) become invalid in the presence of boundary layers, unpredictable dissipation or turbulence, shocks, non-Newtonian stress-strain relationship or rarefied flow conditions.

4 Beyond CPT with Complex Dynamics of Dimensional Fluctuations

It is generally believed that, if the ongoing cosmological observations continue to diverge from predictions, our current formulation of Lambda-CDM may require a fundamental paradigm shift.

Parallel with the current situation in cosmology, independent research indicates that nonlinear dynamics of far-from-equilibrium systems has universal features linking the complex Ginzburg-Landau equation (CGLE) with the tenets of Quantum Field Theory (QFT), Standard Model of Particle Physics (SM) and the high-energy regime of primordial gravitation [16 – 18, Appendix]. Here we consolidate this view by arguing that CGLE lies behind structure formation in cosmology and the emergence of topological defects and condensates in high-energy physics. From this standpoint, complex dynamics appears to provide the most sensible way out of the current crisis in cosmology. Building upon the ideas of [12], the next paragraph elaborates on the emergence of CGLE from the dynamics of dimensional fluctuations.

4.1 CGLE from Dimensional Fluctuations above the Electroweak Scale

Reaction-Diffusion (RD) processes are a subset of complex phenomena defined within the framework of Nonequilibrium Statistical Physics. These models are typically formulated in $d + 1$ dimensions, where d is the dimension of the Euclidean manifold representing the physical space and t is the time coordinate. Ref. [12] develops a toy RD model acting on a two-dimensional lattice ($d = 2$), whose local variables are time-varying dimensional fluctuations $\delta\varepsilon(t) = \delta[2 - d(t)]$. The model includes a scattering event at rate D , a clustering event at rate u and a decay (or percolation) event at rate $\kappa = \lambda - \lambda_c$, with λ being a control parameter nearing its critical value λ_c . Up to a leading order approximation, the macroscopic properties of RD processes may be encoded in a mean-field (MF) equation, which quantifies the competition between losses and gains in a generic density parameter $\rho(t)$. In particular, the decay/percolation process occurs with a rate proportional to $\kappa\rho(t)$ and leads to a gain in density. By contrast, the clustering process drops the density with a rate proportional to $u\rho^2(t)$. Ignoring diffusion, the resulting MF equation takes the form

$$\frac{\partial\rho(t)}{\partial t} = \kappa\rho(t) - u\rho^2(t) \quad (9)$$

In the context of [12] the control parameter $\lambda(t) = \lambda[\delta\varepsilon(t)]$ represents the density of dimensional fluctuations $\delta\varepsilon(t) \ll 1$ while $\rho(t)$ denotes the density of active (or unstable) lattice sites. A straightforward extrapolation of (9) is given by the system of coupled partial differential equations

$$\frac{\partial\rho_1(x,t)}{\partial t} = D_1\Delta\rho_1(x,t) + f(\rho_1, \rho_2, \kappa) \quad (10a)$$

$$\frac{\partial\rho_2(x,t)}{\partial t} = D_2\Delta\rho_2(x,t) + g(\rho_1, \rho_2, \kappa) \quad (10b)$$

According to [12] and references therein, an arbitrary solution of (10) lying near the bifurcation point at $\kappa > \kappa_0$ can be expressed through a complex-valued function $\psi(r, \tau)$ obeying the CGLE in one spatial dimension,

$$\boxed{\frac{\partial\psi}{\partial\tau} = \psi + (1 + i\alpha)\frac{\partial^2\psi}{\partial r^2} - (1 + i\beta)\psi|\psi|^2} \quad (11)$$

Here, the set of new coordinates is given by

$$r = \sigma x \quad (12a)$$

$$\tau = \sigma^2 t \quad (12b)$$

where,

$$\sigma = (\kappa - \kappa_0)^{1/2} \propto (\lambda - \lambda_c)^{1/2} \ll 1 \quad (13)$$

As it is known, CGLE represents a nonlinear partial differential equation which serves as a universal model for the onset of spatiotemporal dynamics in nonlinear systems. It is especially helpful in the study of critical behavior where order parameters emerge – such as superfluidity, superconductivity, turbulence, Bose-Einstein condensation, nonlinear waves and pattern formation. In (11) $\psi(r, \tau)$ acts as order parameter, α, β are real coefficients accounting for diffusion and nonlinear effects, respectively. Stated differently, the term $\psi(r, \tau)$ stands for the linear growth or decay, the Laplacian measures the contribution of diffusion and the nonlinear term $\psi|\psi|^2$ embodies the contribution of self-interaction.

4.2 Cosmological Structure Formation from CGLE

Pattern/structure formation and self-organization emerging from CGLE is a rich and active research topic, with applications across various disciplines. To avoid excessive information and to stimulate further independent analysis, here we limit the discussion to few representative examples.

a) *Topological defects* are localized regions where the order parameter is singular or undefined – like points, lines or surfaces around which the phase of ψ winds in some nontrivial manner. In two- and three dimensions, typical examples include vortices (points where $\psi = 0$ and the phase of $\psi = \arg(\psi)$ winds by $2\pi n$ around the core), domain walls are interfaces between regions with different phase or amplitude configurations, and dislocations/spirals which are phase defects of wavefronts in pattern forming systems.

In the context of CGLE, topological defects emerge from broken symmetry and solutions with nontrivial phase structures. They form spontaneously from instabilities, noise or as remnants of quenched phase transitions (Kibble-Zurek mechanism). The so-called winding number (or topological charge) quantifies the phase change around defects.

b) *Topological condensates* represent coherent and long-lived, often self-organized macroscopic structures formed by the aggregation of topological defects.

Fig. 1 below is a visualization of a topological vortex arising from CGLE. The left panel shows the amplitude of the order parameter ψ , which vanishes at the center (the vortex core). The right panel shows the phase, which winds by 2π around the core and defines the topological charge of the vortex.

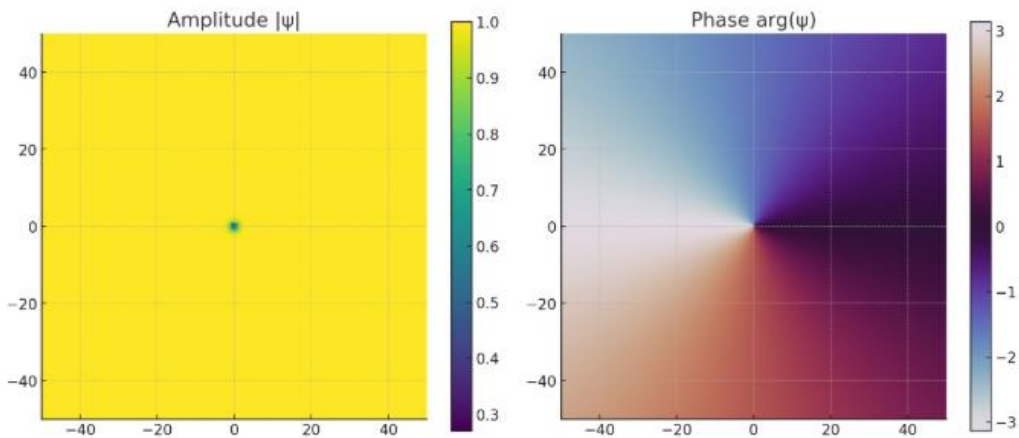
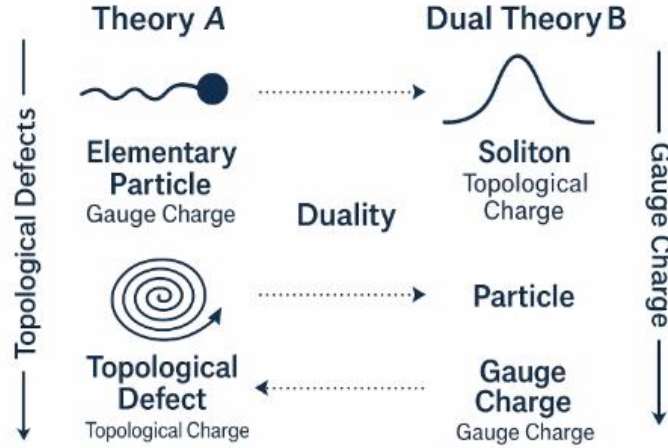


Fig. 1: Visualization of a topological vortex derived from CGLE.

It is instructive to note that topological defects can be regarded as *dual objects* to the elementary particles of high-energy physics, as highlighted by the table and symbolic diagram shown below.

According to this view, there is an intriguing analogy in how symmetries and topology lead to conserved quantities and defects [5 – 10].

Theory A	Dual Theory B
Particles with gauge charges	Topological defects with winding numbers
Electric field lines	Magnetic monopole worldlines
Quarks/gluons in QCD	Confining flux tubes or dual superconductor
Elementary bosons	Vortices or solitons in dual theory
Gauge theory with local symmetry	Topological theory with global symmetry



Vortex lattices derived from CGLE are periodic arrangements of vortices, forming structured patterns in the complex amplitude field. They represent a self-organized state of the underlying nonlinear system, often found in 2D simulations or real-world systems near oscillatory instabilities. The evolution of vortex lattices is a complex interplay of self-organization, turbulence, and external influences, leading to a variety of dynamic phases and structural transitions. Fig 2 displays snapshots of a two-dimensional order parameter $|\psi(x, y, t)|^2$ taken at various time intervals. It illustrates how structure changes dynamically over time, exhibiting features common in the evolution of vortex lattices and cosmic voids / halos.

Fig. 3 visualizes the evolution of vortex lines in the three-dimensional (3D) CGLE, which echoes the formation of helical strings in the cosmic web. This structure develops from the instability of vortex filaments in the high-dispersion limit of (11), namely $\alpha \gg 1$ [13].

5 Discussion

There are well motivated reasons why, due to their inherent assumptions and constraints, both FRW and CPT models are inadequate for understanding structure formation in primordial cosmology. By contrast, there are compelling hints that a framework based on complex dynamics of evolving dimensional fluctuations is essential for explaining primordial cosmology, as it connects to

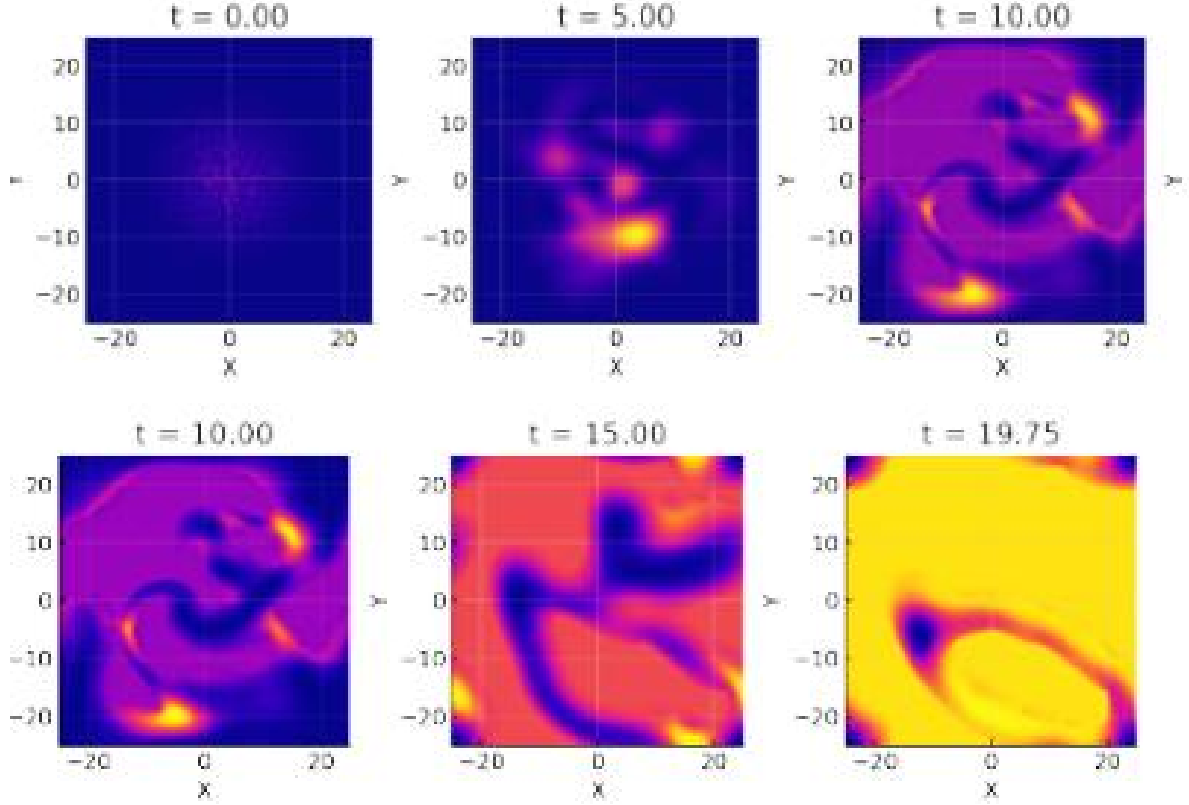


Fig. 2: Evolving 2D vortex lattices as analog of cosmic voids.

phenomena occurring far above the electroweak scale. The proposed framework builds upon the universal generation of topological defects and condensates associated with the complex Ginzburg-Landau equation (CGLE). The main benefit of this proposal is that the premises of CGLE are independent from any formulation of classical gravity, classical Thermodynamics or the theory of fluid flows. In addition, the duality of cosmological structure formation and topological solutions of Quantum Field Theory is likely to bring unforeseen insights into future model building efforts.

APPENDIX

From CGLE to the Standard Model of Particle Physics (SM)

CGLE is sometimes quoted in the form

$$\frac{\partial \psi}{\partial \tau} = b\psi + (1 + i\alpha)\frac{\partial^2 \psi}{\partial r^2} - (1 + i\beta)\psi|\psi|^2 \quad (\text{A1})$$

with $b \neq 1$. Under the assumption that there is no dissipation (spatial homogeneity), taking $\beta = 0$, rescaling the order parameter $\psi \rightarrow a\psi$, $\psi^* \rightarrow a\psi^*$ ($a \neq 1$) and substituting it in (A1) yields

$$\frac{\partial \psi}{\partial \tau} = b\psi - a^2\psi|\psi|^2 \quad (\text{A2})$$

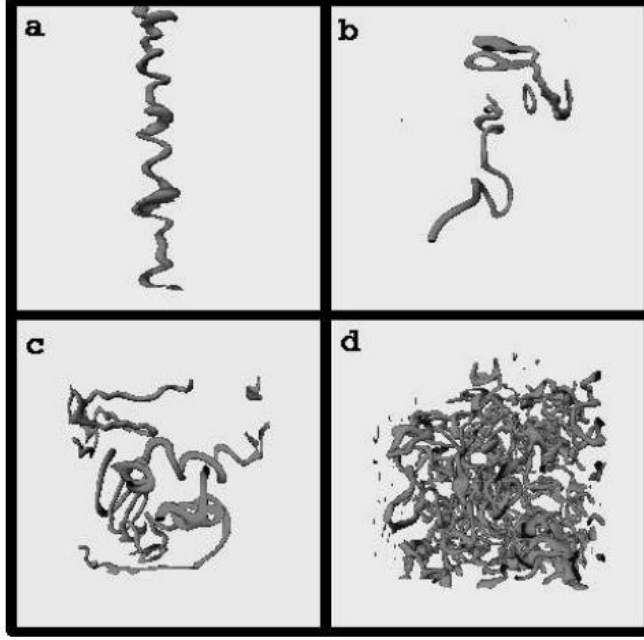


Fig. 3: Evolution of vortex lines in 3D CGLE.

Furthermore, if the order parameter reduces to a scalar field ($\psi \rightarrow \varphi$), side by side comparison of (A2) with the flow equation (12) of the Higgs potential in [19], gives the identification

$$\dot{\varphi} = -\frac{\partial V(\varphi)}{\partial \varphi} = 2\lambda\varphi(v^2 - 2\varphi^2) \quad (\text{A3})$$

$$b = v^2 \quad (\text{A4})$$

$$a^2 = 4\lambda \quad (\text{A5})$$

Here, λ, v denote the self-interaction coupling and vacuum expectation value of the Higgs scalar, respectively. By (A2) – (A5) and following [18 - 20], leads to the conclusion that CGLE recovers the entire flavor structure of the SM via successive bifurcations.

References

1. Peebles, P.J.E., *The Large-Scale Structure of the Universe* (1980).
2. Dodelson, S., *Modern Cosmology* (2003).
3. Bernardeau, F., et al., “Large-Scale Structure of the Universe and Cosmological Perturbation Theory” (Physics Reports, 2002).
4. Ma, C.-P. & Bertschinger, E., “Cosmological Perturbation Theory in the Synchronous and Conformal Newtonian Gauges” (Astrophysical Journal, 1995).
5. Aranson, I.S. & Kramer, L., “The World of the Complex Ginzburg-Landau Equation” (Reviews of Modern Physics, 2002).

6. David Tong, Lectures on Quantum Field Theory, <https://www.damtp.cam.ac.uk/user/tong/qft.html>
7. Michael E. Peskin, Duality in Supersymmetric Yang-Mills Theory, preprint <https://arxiv.org/abs/hep-th/9702094>
8. Seiberg & Witten, Electric-Magnetic Duality, Monopole Condensation, and Confinement in $N=2$ Supersymmetric Yang-Mills Theory, Nucl. Phys. B426 (1994) 19–52.
9. Montonen & Olive, Magnetic Monopoles as Gauge Particles? Phys. Lett. B72 (1977) 117–120.
10. Franz, Tesanovic, Duality and Quantum Criticality in 2+1 Dimensional Superconductors, Phys. Rev. Lett. 87, 257003 (2001)
11. Goldfain, E., (2023) Dimensional Reduction as Source of Cosmological Anomalies, preprint <http://dx.doi.org/10.32388/RX6BTE>.
12. Goldfain, E., (2024), From Complex Dynamics to Foundational Physics, preprint <http://dx.doi.org/10.32388/HC9S30>
13. Aranson I. S. et al., (1997), The dynamics of vortex lines in the three-dimensional complex Ginzburg-Landau equation: instability, stretching, entanglement, and helices, preprint <https://arxiv.org/abs/patt-sol/9712004>
14. Goldfain, E., Open Limitations of Quantum Gravity: a Brief Overview, (2024), preprint <http://dx.doi.org/10.13140/RG.2.2.36070.43841/3>
15. Goldfain, E., Hamiltonian Chaos and Fractal Topology of Spacetime, (2023), preprint <http://dx.doi.org/10.32388/7ZYVDB>
16. Goldfain, E. Complex Dynamics and the Schrödinger Equation, (2024), preprint <http://dx.doi.org/10.32388/AFBU2E.2>
17. Goldfain, E., Complex Dynamics and Primordial Gravity, (2024), preprint <http://dx.doi.org/10.13140/RG.2.2.20849.60007/1>
18. Goldfain, E., On the Bifurcation Structure of Particle Physics, (2024), preprint <http://dx.doi.org/10.13140/RG.2.2.14675.39209/1>
19. Goldfain, E., Bifurcations and the Gauge Structure of the Standard Model, (2021), preprint <http://dx.doi.org/10.13140/RG.2.2.34888.85769/1>
20. Goldfain, E., Bifurcations of the Higgs Potential and the Top Quark Mass, (2021), preprint <http://dx.doi.org/10.13140/RG.2.2.23144.80644>