On the model of electrodynamics consistent with the classical newtonian mechanics

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Abstract

We propose the electrodynamic model which is consistent with the classical newtonian mechanics including invariance under galilean transformations. In our model the aether is assumed to be a classical continuum medium like a fluid or gas and Maxwell equations in vacuum (aether) have the (usual) form:

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} \equiv \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \\ div_{\mathbf{x}}\mathbf{D} \equiv 4\pi\rho \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0 \\ div_{\mathbf{x}}\mathbf{B} \equiv 0, \end{cases}$$

together with the following constitutive relations:

$$\begin{cases} \mathbf{E} = \mathbf{D} - \frac{1}{c}\mathbf{v}\times\mathbf{B} \\ \mathbf{H} = \mathbf{B} + \frac{1}{c}\mathbf{v}\times\mathbf{D}, \end{cases}$$

where $\mathbf{v}$ is the field of the aether velocity. The presented model of Maxwell equations and the Lorentz force are invariant under galilean transformations:

$$\begin{cases} \mathbf{x}' = \mathbf{x} + t\mathbf{w} \\ t' = t, \end{cases}$$

together with the relations:

$$\begin{cases} \mathbf{D}' = \mathbf{D} \\ \mathbf{B}' = \mathbf{B} \\ \mathbf{E}' = \mathbf{E} - \frac{1}{c}\mathbf{w}\times\mathbf{B} \\ \mathbf{H}' = \mathbf{H} + \frac{1}{c}\mathbf{w}\times\mathbf{D}. \end{cases}$$

Moreover, the form of these equations is preserved in every non-inertial coordinate system. Further consequences of the model are also derived.

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1 Introduction

This paper is devoted to establish quantitative relations of Electrodynamics in the frames of Classical Newtonian Mechanics and the hypothesis of the aether, which is assumed to be a classical continuum medium. There are various classical aether theories including the theory of stationary aether, developed by Lorentz; the theory of complete aether drag, proposed by Stokes; the theory of partial aether drag of Fresnel; the theory of gravitational aether drag, proposed by des Coudres and Wien; more recent theory of local aether of C.C. Su (see [3]); et.al.

We assume here that the laws of Electrodynamics in the vacuum are invariant under the Galilean transformations. Furthermore, we suppose the existence of the physical luminiferous aether in the vacuum moving by the laws of the Newtonian Mechanics. Thus we assume that the velocity of the motion of the different regions of the aether can vary. So we assume the aether as a continuum which moves by the laws of the Classical Continuum Mechanics, as a fluid or gas. We also assume that in the microscopic scale the aether consists of discrete particles - photons. In this paper by the velocity of the aether in some point we will mean the average (macroscopic) velocity of the aether as a continuum. Furthermore, the Lorentz Force is assumed to be an electromagnetic force with which the environing aether acts on charged bodies. We also assume the validity of the Third Law of Newton. Therefore, we assume that the charged bodies acts on the environing aether with the electromagnetic force, opposite to the Lorentz Force. Moreover, we assume surface forces inside the aether, described by certain tensors. Since the aether is assumed to be a classical continuum, it is also reasonable to assume that the force of gravitation acts on particles of the aether.

In Section 3 of this paper we propose the simple and natural quantitative relations of Electrodynamics, substituting (with minor changes) the classical Maxwell equations in the case of an arbitrarily moving aether, and invariant under Galilean Transformations. For this propose we appeal to the Maxwell equations in a medium. It is well known that the classical Maxwell equations in a medium have the following form in the Gaussian unit system:

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} \equiv \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \\ div_{\mathbf{x}}\mathbf{D} \equiv 4\pi\rho \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0 \\ div_{\mathbf{x}}\mathbf{B} \equiv 0. \end{cases} \tag{1.1}$$

Here $\mathbf{x} \in \mathbb{R}^3$ and $t > 0$ are the place and the time, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, $\mathbf{D}$ is the electric displacement field, $\mathbf{H}$ is the $\mathbf{H}$-magnetic field, ρ is the charge density, $\mathbf{j}$ is the current density and c is the universal constant, called speed of light. It is assumed in the Classical Electrodynamics that for the vacuum we always have $\mathbf{D} = \mathbf{E}$ and $\mathbf{H} = \mathbf{B}$. We assume here that the Maxwell equations in the vacuum (pure aether) have the usual form (1.1), as in any other medium, i.e.

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} \equiv \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \\ div_{\mathbf{x}}\mathbf{D} \equiv 4\pi\rho \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0 \\ div_{\mathbf{x}}\mathbf{B} \equiv 0, \end{cases} \tag{1.2}$$

however, we assume the relations $\mathbf{D} = \mathbf{E}$ and $\mathbf{H} = \mathbf{B}$ in the vacuum to be valid only for the parts of the space, where the velocity of the motion of the aether is negligible. One can assume that the local aether is dragged by the Earth and other celestial bodies due to the force of gravitation. Then the Electrodynamics near the Earth described by the Maxwell Equations with the classical constitutive relations $\mathbf{D} = \mathbf{E}$ and $\mathbf{H} = \mathbf{B}$.

So we assume that if in some point and at some instant of time the velocity of the local aether vanishes, then in this point and at this time $\mathbf{D} = \mathbf{E}$ and $\mathbf{H} = \mathbf{B}$. In order to obtain the relations $\mathbf{D} \sim \mathbf{E}$ and $\mathbf{H} \sim \mathbf{B}$ in the general case we assume that the equations (1.2) and the Lorentz force $\mathbf{F} := \sigma\mathbf{E} + \frac{\sigma}{c}\,\mathbf{u} \times \mathbf{B}$ (where σ is the charge of the test particle and $\mathbf{u}$ is its velocity) are invariant under the Galilean Transformations of the Classical Mechanics:

$$\begin{cases} \mathbf{x}' = \mathbf{x} + t\mathbf{w}, \\ t' = t. \end{cases}$$

Then the analysis of our assumptions, presented in section 3, implies that the full system of Electrodynamics in the case of an arbitrarily moving aether has the following form:

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} \equiv \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \\ div_{\mathbf{x}}\mathbf{D} \equiv 4\pi\rho \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0 \\ div_{\mathbf{x}}\mathbf{B} \equiv 0 \\ \mathbf{E} = \mathbf{D} - \frac{1}{c}\,\mathbf{v} \times \mathbf{B} \\ \mathbf{H} = \mathbf{B} + \frac{1}{c}\,\mathbf{v} \times \mathbf{D}, \end{cases} \tag{1.3}$$

where $\mathbf{v}$ is the field of velocities of the motion of the aether. It can be easily checked that system (1.3) and the Lorentz force $\mathbf{F} := \sigma(\mathbf{E} + \frac{\mathbf{u}}{c} \times \mathbf{B})$ are invariant under the following transformations

$$\begin{cases} \mathbf{x}' = \mathbf{x} + t\mathbf{w}, \\ t' = t. \end{cases} \tag{1.4}$$

and

$$\begin{cases} \mathbf{D}' = \mathbf{D}, \\ \mathbf{B}' = \mathbf{B}, \\ \mathbf{E}' = \mathbf{E} - \frac{1}{c}\mathbf{w} \times \mathbf{B}, \\ \mathbf{H}' = \mathbf{H} + \frac{1}{c}\mathbf{w} \times \mathbf{D}. \end{cases} \tag{1.5}$$

Note here that $\mathbf{D}$ and $\mathbf{B}$ are invariant under the change of inertial coordinate system. Moreover, we can write the Lorentz force as $\mathbf{F} := \sigma(\mathbf{D} + \frac{\mathbf{u}-\mathbf{v}}{c} \times \mathbf{B})$, where $(\mathbf{u} - \mathbf{v})$ is the relative velocity of the test particle with respect to the aether. Note also that in the particular case of the complete aether drag hypothesis, the model of equations (1.3) coincides with the model, proposed by Hertz ([2]), with the only difference that the vector field $\mathbf{E}$ of Hertz's notations corresponds to our vector field $\mathbf{D}$. However, the complete aether drag hypothesis contradicts with the well known Sagnac effect.

In section 4 we derive that the laws of Electrodynamics have an invariant form not only in inertial but also in non-inertial coordinate systems. I.e. the system of Maxwell Equations has the same form:

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} \equiv \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \\ div_{\mathbf{x}}\mathbf{D} \equiv 4\pi\rho, \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0, \\ div_{\mathbf{x}}\mathbf{B} \equiv 0, \\ \mathbf{E} = \mathbf{D} - \frac{1}{c}\mathbf{v} \times \mathbf{B}, \\ \mathbf{H} = \mathbf{B} + \frac{1}{c}\mathbf{v} \times \mathbf{D} \end{cases} \tag{1.6}$$

in every non-inertial coordinate system. Moreover, the expression of the Lorentz force

$$\mathbf{F} = \sigma\mathbf{E} + \frac{\sigma}{c}\mathbf{u} \times \mathbf{B}$$

is valid in every such coordinate system. Furthermore, if we consider the change of non-inertial coordinate system of the form:

$$\begin{cases} \mathbf{x}' = A(t) \cdot \mathbf{x} + \mathbf{z}(t), \\ t' = t, \end{cases} \tag{1.7}$$

where $A(t) \in SO(3)$ is a rotation, i.e. $A(t) \in \mathbb{R}^{3\times 3}$, $detA(t) > 0$ and $A(t) \cdot A^T(t) = Id$, then we have the following transformations of the electromagnetic fields:

$$\begin{cases} \mathbf{D}' = A(t) \cdot \mathbf{D} \\ \mathbf{B}' = A(t) \cdot \mathbf{B} \\ \mathbf{E}' = A(t) \cdot \mathbf{E} - \frac{1}{c}\left(\frac{dA}{dt}(t) \cdot \mathbf{x} + \mathbf{w}(t)\right) \times \left(A(t) \cdot \mathbf{B}\right) \\ \mathbf{H}' = A(t) \cdot \mathbf{H} + \frac{1}{c}\left(\frac{dA}{dt}(t) \cdot \mathbf{x} + \mathbf{w}(t)\right) \times \left(A(t) \cdot \mathbf{D}\right), \end{cases} \tag{1.8}$$

where $\mathbf{w}(t) = \frac{d\mathbf{z}}{dt}(t)$.

As an example, if we consider that the local aether rests with respect to some inertial coordinate system $(*)$ and the second non-inertial coordinate system $(**)$ rotates with respect to the system $(*)$, then the laws of electrodynamics in the system $(**)$ are given by (1.6) with the rotating aether. This explains the Sagnac effect in the system $(**)$.

In section 5, in the frame of the proposed model we treat the case of electromagnetic fields in the dielectric and/or magnetic medium. Furthermore, in section 6 we investigate quasistationary fields in a slowly moving aether.

In section 7 we propose the models of the motion of the aether as a continuum, based on the second and the third laws of Newton. In particular, due to the third law of Newton, it is assumed that charged bodies acts on the environing aether with the force, which is opposite to the Lorenz force. Moreover, tensors of surface electromagnetic forces inside the aether are also assumed. In the Classical Electrodynamics the Maxwell tensor in the vacuum has the form

$$\frac{1}{4\pi}\left\{\mathbf{E}\otimes\mathbf{E}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\Big(|\mathbf{E}|^2+|\mathbf{B}|^2\Big)I\right\}\in\mathbb{R}^{3\times 3}, \tag{1.9}$$

where I is the identity matrix. We assume this formula to be valid only for the point where the velocity of the aether vanishes. Since $\mathbf{D}$ and $\mathbf{B}$ are invariant under the change of inertial system of coordinates and the forces must be also invariant, it is assumed that the tensor of surface electromagnetic forces inside the aether has the form

$$\frac{1}{4\pi}\left\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\Big(|\mathbf{D}|^2+|\mathbf{B}|^2\Big)I\right\}. \tag{1.10}$$

Furthermore, we assume that additional forces of non-electromagnetic nature can act on the particles of the aether. In particular, the gravitational interaction of the Earth with particles of the environing aether is assumed. This can be one of explanations why the velocity of the aether near the Earth is close to zero as was obtained by Michelson-Morley experiment. Note here that due to the Michelson-Gale-Pearson experiment, the aether cannot rotate together with the Earth. I.e. the velocity of the aether near the Earth is close to zero only with respect to the non-rotating coordinate system, related to the Earth. This is still consistent with the hypothesis of the gravitational interaction of the aether with the Earth.

In subsection 7.1 we derive that the quantity

$$W := \int_{\mathbb{R}^3}\frac{|\mathbf{D}|^2+|\mathbf{B}|^2}{8\pi} \tag{1.11}$$

is the total potential energy of all electromagnetic interactions in the vacuum.

Finally, in subsection 7.3, under some additional assumptions on the nature of non-electromagnetic forces, acting on the particles of the aether, we derive an estimation of the curl of the aether velocity field in an inertial coordinate system.

2 Notations

- By $\mathbb{R}^{p\times q}$ we denote the set of $p \times q$-matrixes with real coefficients.

- For a $p \times q$ matrix A with ij-th entry a_{ij} and for a $q \times d$ matrix B with ij-th entry b_{ij} we denote by $AB := A \cdot B$ their product, i.e. the $p \times d$ matrix, with ij-th entry $\sum_{k=1}^{q} a_{ik}b_{kj}$.

- We identify a vector $\mathbf{u} = (u_1, \ldots, u_q) \in \mathbb{R}^q$ with the $q \times 1$ matrix having $i1$-th entry u_i, so that for the $p \times q$ matrix A with ij-th entry a_{ij} and for $\mathbf{v} = (v_1, v_2, \ldots, v_q) \in \mathbb{R}^q$ we denote by $A\,\mathbf{v} := A \cdot \mathbf{v}$ the p-dimensional vector $\mathbf{u} = (u_1, \ldots, u_p) \in \mathbb{R}^p$, given by $u_i = \sum_{k=1}^{q} a_{ik}v_k$ for every $1 \leq i \leq p$.

- For a $p \times q$ matrix A with ij-th entry a_{ij} denote by A^T the transpose $q \times p$ matrix with ij-th entry a_{ji}.

- For a $p \times p$ matrix A with ij-th entry a_{ij} denote $tr(A) := \sum_{k=1}^{p} a_{kk}$ (the trace of the matrix A).

- For $\mathbf{u} = (u_1, \ldots, u_p) \in \mathbb{R}^p$ and $\mathbf{v} = (v_1, \ldots, v_p) \in \mathbb{R}^p$ we denote by $\mathbf{uv} := \mathbf{u} \cdot \mathbf{v} := \sum_{k=1}^{p} u_k v_k$ the standard scalar product. We also note that $\mathbf{uv} = \mathbf{u}^T\mathbf{v} = \mathbf{v}^T\mathbf{u}$ as products of matrices.

- For $\mathbf{u} = (u_1, u_2, u_3) \in \mathbb{R}^3$ and $\mathbf{v} = (v_1, v_2, v_3) \in \mathbb{R}^3$ we denote

$$\mathbf{u} \times \mathbf{v} := \Big(u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1\Big) \in \mathbb{R}^3.$$

- For $\mathbf{u} = (u_1, \ldots, u_p) \in \mathbb{R}^p$ and $\mathbf{v} = (v_1, \ldots, v_q) \in \mathbb{R}^q$ we denote by $\mathbf{u} \otimes \mathbf{v}$ the $p \times q$ matrix with ij-th entry $u_i v_j$ (i.e. $\mathbf{u} \otimes \mathbf{v} = \mathbf{u}\,\mathbf{v}^T$ as a product of matrices).

- Given a vector valued function $\mathbf{f}(x) = \big(f_1(x), \ldots, f_k(x)\big) : \Omega \to \mathbb{R}^k$ ($\Omega \subset \mathbb{R}^N$) we denote by $D\mathbf{f}$ the $k \times N$ matrix with ij-th entry $\frac{\partial f_i}{\partial x_j}$. In the case of a scalar valued function $\psi(x) : \Omega \subset \mathbb{R}^N \to \mathbb{R}$ we associate with $D\psi$ (which, by definition, belongs to $\mathbb{R}^{1\times N}$) the corresponding vector $\nabla\psi := \left(\frac{\partial\psi}{\partial x_1}, \ldots, \frac{\partial\psi}{\partial x_N}\right)$.

- Given a matrix valued function $F(x) := \{F_{ij}(x)\} : \Omega \subset \mathbb{R}^N \to \mathbb{R}^{k\times N}$, we denote by $\operatorname{div} F$ the $\mathbb{R}^k$-valued vector field defined by $div\, F(x) := (l_1, \ldots, l_k)(x)$ where $l_i(x) = \sum_{j=1}^{N} \frac{\partial F_{ij}}{\partial x_j}(x)$. Given a vector valued function $\mathbf{f}(x) := \big(f_1(x), \ldots, f_N(x)\big) : \Omega \subset \mathbb{R}^N \to \mathbb{R}^N$ we denote $\operatorname{div} \mathbf{f} := \sum_{j=1}^{N} \frac{\partial f_j}{\partial x_j}$.

- Given a vector valued function $\mathbf{f}(x) = \big(f_1(x), f_2(x), f_3(x)\big) : G \subset \mathbb{R}^3 \to \mathbb{R}^3$ we denote

$$curl\, \mathbf{f}(x) := \left(\frac{\partial f_3}{\partial x_2} - \frac{\partial f_2}{\partial x_3}, \frac{\partial f_1}{\partial x_3} - \frac{\partial f_3}{\partial x_1}, \frac{\partial f_2}{\partial x_1} - \frac{\partial f_1}{\partial x_2}\right)(x).$$

We have the following trivial identities:

$$\mathbf{a}\times\mathbf{b}=-\mathbf{b}\times\mathbf{a} \quad \text{and} \quad \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} \qquad \forall\,\mathbf{a},\mathbf{b},\mathbf{c}\in\mathbb{R}^3, \tag{2.1}$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=(\mathbf{a}\cdot\mathbf{c})\,\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\,\mathbf{c} \qquad \forall\,\mathbf{a},\mathbf{b},\mathbf{c}\in\mathbb{R}^3, \tag{2.2}$$

$$(A\cdot\mathbf{b})\times\mathbf{c}-(A\cdot\mathbf{c})\times\mathbf{b}=tr(A)\,(\mathbf{b}\times\mathbf{c})-A^T\cdot(\mathbf{b}\times\mathbf{c}) \qquad \forall A\in\mathbb{R}^{3\times3},\ \forall\,\mathbf{b},\mathbf{c}\in\mathbb{R}^3, \tag{2.3}$$

$$\mathrm{div}(\mathbf{f}\times\mathbf{g})=\mathbf{g}\cdot curl\,\mathbf{f}-\mathbf{f}\cdot curl\,\mathbf{g} \qquad \forall\,\mathbf{f},\mathbf{g}:G\subset\mathbb{R}^3\to\mathbb{R}^3, \tag{2.4}$$

$$\mathrm{div}(\psi\mathbf{f})=\psi\;\mathrm{div}\,\mathbf{f}+\nabla\psi\cdot\mathbf{f} \qquad \forall\,\psi:G\subset\mathbb{R}^3\to\mathbb{R},\ \forall\,\mathbf{f}:G\subset\mathbb{R}^3\to\mathbb{R}^3, \tag{2.5}$$

$$curl\,(\psi\mathbf{f})=\psi\,curl\,\mathbf{f}+\nabla\psi\times\mathbf{f} \qquad \forall\,\psi:G\subset\mathbb{R}^3\to\mathbb{R},\ \forall\,\mathbf{f}:G\subset\mathbb{R}^3\to\mathbb{R}^3, \tag{2.6}$$

$$\mathrm{div}\,\big(curl\,\mathbf{f}\big)=0 \qquad \forall\,\mathbf{f}:G\subset\mathbb{R}^3\to\mathbb{R}^3, \tag{2.7}$$

$$curl\,\big(curl\,\mathbf{f}\big)=\nabla\big(\,\mathrm{div}\,\mathbf{f}\big)-\Delta\mathbf{f} \qquad \forall\,\mathbf{f}:G\subset\mathbb{R}^3\to\mathbb{R}^3, \tag{2.8}$$

$$curl\,(\mathbf{f}\times\mathbf{g})=(\mathrm{div}\,\mathbf{g})\,\mathbf{f}-(\mathrm{div}\,\mathbf{f})\,\mathbf{g}+(D\mathbf{f})\cdot\mathbf{g}-(D\mathbf{g})\cdot\mathbf{f} \qquad \forall\,\mathbf{f},\mathbf{g}:G\subset\mathbb{R}^3\to\mathbb{R}^3, \tag{2.9}$$

$$\nabla(\mathbf{f}\cdot\mathbf{g})=(D\mathbf{f})^T\cdot\mathbf{g}+(D\mathbf{g})^T\cdot\mathbf{f} \qquad \forall\,\mathbf{f},\mathbf{g}:G\subset\mathbb{R}^3\to\mathbb{R}^3, \tag{2.10}$$

$$\mathbf{f}\times(curl\,\mathbf{g})=(D\mathbf{g})^T\cdot\mathbf{f}-(D\mathbf{g})\cdot\mathbf{f} \qquad \forall\,\mathbf{f},\mathbf{g}:G\subset\mathbb{R}^3\to\mathbb{R}^3, \tag{2.11}$$

$$\nabla(\mathbf{f}\cdot\mathbf{g})=\mathbf{f}\times(curl\,\mathbf{g})+\mathbf{g}\times(curl\,\mathbf{f})+(D\mathbf{f})\cdot\mathbf{g}+(D\mathbf{g})\cdot\mathbf{f} \qquad \forall\,\mathbf{f},\mathbf{g}:G\subset\mathbb{R}^3\to\mathbb{R}^3, \tag{2.12}$$

where we mean by $A\cdot\mathbf{l}$ the usual product of matrix $A\in\mathbb{R}^{3\times3}$ and vector $\mathbf{l}\in\mathbb{R}^3$ and by A^T we mean the transpose of matrix A.

3 Maxwell equations revised

We would like to make the laws of Electrodynamics in the vacuum to be invariant under the Galilean transformations of the classical Newtonian Mechanics. For this purpose we refer to the analogy with the Maxwell equations in a medium. It is well known that the classical Maxwell equations in a medium are the following:

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H}\equiv\frac{4\pi}{c}\mathbf{j}+\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty),\\ div_{\mathbf{x}}\mathbf{D}\equiv4\pi\rho \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty),\\ curl_{\mathbf{x}}\mathbf{E}+\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\equiv0 \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty),\\ div_{\mathbf{x}}\mathbf{B}\equiv0 \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty).\end{cases} \tag{3.1}$$

Here $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, $\mathbf{D}$ is the electric displacement field, $\mathbf{H}$ is the $\mathbf{H}$-magnetic field, ρ is the charge density, $\mathbf{j}$ is the current density and c is the universal constant, called speed of light. It is assumed in the Classical Electrodynamics that for the vacuum we always have $\mathbf{D}\equiv\mathbf{E}$ and $\mathbf{H}\equiv\mathbf{B}$.

We assume here the existence of the physical luminiferous aether in the vacuum which can move. Furthermore we assume that the velocity of the motion of the different regions of the aether can vary. So we assume the aether as a continuum which moves by the laws of Classical Mechanics as a fluid or gas. We also assume that in the microscopic scale the aether consists of discrete particles - photons. In this paper by the velocity of the aether in the point $\mathbf{x}$ at the time t we will mean the average (macroscopic) velocity of the aether as a continuum.

Next we assume that the Maxwell equations in the vacuum (pure aether) have the usual form (3.1), as in any other medium, i.e.

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} \equiv \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \quad \text{for } (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty), \\ div_{\mathbf{x}}\mathbf{D} \equiv 4\pi\rho \quad \text{for } (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty), \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0 \quad \text{for } (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty), \\ div_{\mathbf{x}}\mathbf{B} \equiv 0 \quad \text{for } (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty), \end{cases} \tag{3.2}$$

however we assume that $\mathbf{D}(\mathbf{x}, t) = \mathbf{E}(\mathbf{x}, t)$ and $\mathbf{H}(\mathbf{x}, t) = \mathbf{B}(\mathbf{x}, t)$ in the vacuum only in the case where the velocity of the motion of the aether on the point $\mathbf{x}$ at the time t equals to zero i.e.

$$\text{If } \mathbf{v}(\mathbf{x}, t) = 0 \text{ for some } (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty) \text{ then } \mathbf{D}(\mathbf{x}, t) = \mathbf{E}(\mathbf{x}, t) \text{ and } \mathbf{H}(\mathbf{x}, t) = \mathbf{B}(\mathbf{x}, t), \tag{3.3}$$

where $\mathbf{v}(\mathbf{x}, t)$ is the velocity of the motion of the aether on the point $\mathbf{x}$ at the time t. In order to obtain the relations $\mathbf{D} \sim \mathbf{E}$ and $\mathbf{H} \sim \mathbf{B}$ in the general case we assume that the equations (3.2) and the Lorentz force $\mathbf{F} := \sigma\mathbf{E} + \frac{\sigma}{c}\mathbf{u} \times \mathbf{B}$ (where σ is the charge of the test particle and $\mathbf{u}$ is its velocity) are invariant under the Galilean Transformations of the Classical Mechanics:

$$\begin{cases} \mathbf{x}' = \mathbf{x} + t\mathbf{w}, \\ t' = t. \end{cases} \tag{3.4}$$

First observe that if u is a velocity of the test particle then $\mathbf{u}' = \mathbf{u} + \mathbf{w}$. Thus since we assumed that the Lorentz force $\mathbf{F} := \sigma\mathbf{E} + \frac{\sigma}{c}\mathbf{u} \times \mathbf{B}$ is invariant under Galilean transformation we infer

$$\sigma\mathbf{E}' + \frac{\sigma}{c}(\mathbf{u} + \mathbf{w}) \times \mathbf{B}' = \sigma\mathbf{E}' + \frac{\sigma}{c}\mathbf{u}' \times \mathbf{B}' = \mathbf{F}' = \mathbf{F} = \sigma\mathbf{E} + \frac{\sigma}{c}\mathbf{u} \times \mathbf{B}.$$

Therefore, we obtain the following identities:

$$\begin{cases} \mathbf{E}' = \mathbf{E} - \frac{1}{c}\mathbf{w} \times \mathbf{B}, \\ \mathbf{B}' = \mathbf{B}. \end{cases} \tag{3.5}$$

It is easy to check that, under transformations (3.4) and (3.5), the last two equations in (3.2) are invariant. Next observe that in the absents of currents and charges the first two equations in (3.2) for $\mathbf{H}$ and $\mathbf{D}$ will be the same as the last two for $\mathbf{E}$ and $\mathbf{B}$ if we will change the sign of the time there. Therefore, it can be assumed that the first two equations will stay invariant under the transformation:

$$\begin{cases} \mathbf{H}' = \mathbf{H} + \frac{1}{c}\mathbf{w} \times \mathbf{D}, \\ \mathbf{D}' = \mathbf{D}. \end{cases} \tag{3.6}$$

Indeed, since $\rho' = \rho$ and $\mathbf{j}' = \mathbf{j} + \rho\mathbf{w}$, it can be easily checked that under the transformations (3.4) and (3.6) the first two equations will stay invariant also in the case of charges and currents. Therefore, we obtained that all equations in (3.2) are invariant under the transformations

$$\begin{cases} \mathbf{x}' = \mathbf{x} + t\mathbf{w}, \\ t' = t. \end{cases} \tag{3.7}$$

and

$$\begin{cases} \mathbf{D}' = \mathbf{D}, \\ \mathbf{B}' = \mathbf{B}, \\ \mathbf{E}' = \mathbf{E} - \frac{1}{c}\mathbf{w} \times \mathbf{B}, \\ \mathbf{H}' = \mathbf{H} + \frac{1}{c}\mathbf{w} \times \mathbf{D}. \end{cases} \tag{3.8}$$

Next fix some point $(\mathbf{x}_0, t_0) \in \mathbb{R}^3 \times [0, +\infty)$ and consider $\mathbf{w} := -\mathbf{v}(\mathbf{x}_0, t_0)$, where $\mathbf{v}$ is a velocity of the aether. Then since $\mathbf{v}' = \mathbf{v} + \mathbf{w}$ we obtain that at the point $(\mathbf{x}'_0, t'_0)$ we have $\mathbf{v}' = 0$. Therefore, by the assumption (3.3) we must have $\mathbf{E}' = \mathbf{D}'$ and $\mathbf{H}' = \mathbf{B}'$ at this point. Plugging it into (3.8), for this point we obtain

$$\begin{aligned} \mathbf{E}(\mathbf{x}_0, t_0) + \frac{\mathbf{v}(\mathbf{x}_0, t_0)}{c} \times \mathbf{B}(\mathbf{x}_0, t_0) = \mathbf{E}(\mathbf{x}_0, t_0) - \frac{\mathbf{w}}{c} \times \mathbf{B}(\mathbf{x}_0, t_0) = \mathbf{E}'(\mathbf{x}'_0, t'_0) = \mathbf{D}'(\mathbf{x}'_0, t'_0) = \mathbf{D}(\mathbf{x}_0, t_0) \\ \mathbf{H}(\mathbf{x}_0, t_0) - \frac{\mathbf{v}(\mathbf{x}_0, t_0)}{c} \times \mathbf{D}(\mathbf{x}_0, t_0) = \mathbf{H}(\mathbf{x}_0, t_0) + \frac{\mathbf{w}}{c} \times \mathbf{D}(\mathbf{x}_0, t_0) = \mathbf{H}'(\mathbf{x}'_0, t'_0) = \mathbf{B}'(\mathbf{x}'_0, t'_0) = \mathbf{B}(\mathbf{x}_0, t_0). \end{aligned} \tag{3.9}$$

Thus since the point $(\mathbf{x}_0, t_0) \in \mathbb{R}^3 \times [0, +\infty)$ was arbitrarily chosen, by (3.9) we obtain the following relations

$$\begin{cases} \mathbf{E}(\mathbf{x}, t) = \mathbf{D}(\mathbf{x}, t) - \frac{1}{c}\mathbf{v}(\mathbf{x}, t) \times \mathbf{B}(\mathbf{x}, t) \qquad \forall (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty) \\ \mathbf{H}(\mathbf{x}, t) = \mathbf{B}(\mathbf{x}, t) + \frac{1}{c}\mathbf{v}(\mathbf{x}, t) \times \mathbf{D}(\mathbf{x}, t) \qquad \forall (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty). \end{cases} \tag{3.10}$$

Plugging (3.10) into (3.2) we obtain the full system of Electrodynamics in the case of an arbitrarily moving aether:

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} \equiv \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \quad \text{for } (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty), \\ div_{\mathbf{x}}\mathbf{D} \equiv 4\pi\rho \quad \text{for } (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty), \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0 \quad \text{for } (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty), \\ div_{\mathbf{x}}\mathbf{B} \equiv 0 \quad \text{for } (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty), \\ \mathbf{E} = \mathbf{D} - \frac{1}{c}\mathbf{v} \times \mathbf{B} \quad \text{for } (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty) \\ \mathbf{H} = \mathbf{B} + \frac{1}{c}\mathbf{v} \times \mathbf{D} \quad \text{for } (\mathbf{x}, t) \in \mathbb{R}^3 \times [0, +\infty), \end{cases} \tag{3.11}$$

where $\mathbf{v}$ is the aether velocity field. It can be easily checked that system (3.11) and the Lorentz force $\mathbf{F} := \sigma(\mathbf{E} + \frac{\mathbf{u}}{c} \times \mathbf{B})$ are invariant under the following transformations

$$\begin{cases} \mathbf{x}' = \mathbf{x} + t\mathbf{w}, \\ t' = t. \end{cases} \tag{3.12}$$

and

$$\begin{cases} \mathbf{D}' = \mathbf{D}, \\ \mathbf{B}' = \mathbf{B}, \\ \mathbf{E}' = \mathbf{E} - \frac{1}{c}\,\mathbf{w}\times\mathbf{B}, \\ \mathbf{H}' = \mathbf{H} + \frac{1}{c}\,\mathbf{w}\times\mathbf{D}. \end{cases} \tag{3.13}$$

Note here that $\mathbf{D}$ and $\mathbf{B}$ are invariant under the change of inertial coordinate system. Moreover, we can write the Lorentz force as $\mathbf{F} := \sigma(\mathbf{D} + \frac{\mathbf{u}-\mathbf{v}}{c}\times\mathbf{B})$, where $(\mathbf{u}-\mathbf{v})$ is the relative velocity of the test particle with respect to the aether.

4 Maxwell equations in non-inertial coordinate systems

Consider the change of certain non-inertial coordinate system $(*)$ to another coordinate system $(**)$:

$$\begin{cases} \mathbf{x}' = A(t)\cdot\mathbf{x} + \mathbf{z}(t), \\ t' = t, \end{cases} \tag{4.1}$$

where $A(t) \in SO(3)$ is a rotation i.e. $A(t) \in \mathbb{R}^{3\times 3}$, $det A(t) > 0$ and $A(t)\cdot A^T(t) = I$ (here A^T is the transpose matrix of A and I is the identity matrix). Next assume that in coordinate system $(**)$ we observe a validity of Maxwell Equations for the vacuum in the form:

$$\begin{cases} curl_{\mathbf{x}'}\mathbf{H}' \equiv \frac{4\pi}{c}\mathbf{j}' + \frac{1}{c}\frac{\partial \mathbf{D}'}{\partial t'}, \\ div_{\mathbf{x}'}\mathbf{D}' \equiv 4\pi\rho', \\ curl_{\mathbf{x}'}\mathbf{E}' + \frac{1}{c}\frac{\partial \mathbf{B}'}{\partial t'} \equiv 0, \\ div_{\mathbf{x}'}\mathbf{B}' \equiv 0, \\ \mathbf{E}' = \mathbf{D}' - \frac{1}{c}\,\mathbf{v}'\times\mathbf{B}', \\ \mathbf{H}' = \mathbf{B}' + \frac{1}{c}\,\mathbf{v}'\times\mathbf{D}' \end{cases} \tag{4.2}$$

Moreover, we assume that in coordinate system $(**)$ we observe a validity of expression for the Lorentz force

$$\mathbf{F}' := \sigma'\mathbf{E}' + \frac{\sigma'}{c}\,\mathbf{u}'\times\mathbf{B}' \tag{4.3}$$

(where σ' is the charge of the test particle and $\mathbf{u}'$ is its velocity in coordinate system $(**)$). All above happens, in particular, if coordinate system $(**)$ is inertial. Observe that if $\mathbf{F}$ is the force in coordinate system $(*)$ which corresponds to the Lorentz force $\mathbf{F}'$ in coordinate system $(**)$, then we must have $\mathbf{F}' = A(t)\cdot\mathbf{F}$. Moreover, denoting $\mathbf{w}(t) = \mathbf{z}'(t)$, we have the following obvious relations between the physical characteristics in coordinate systems $(*)$ and $(**)$:

$$\mathbf{F}' = A(t)\cdot\mathbf{F}, \tag{4.4}$$

$$\sigma' = \sigma, \tag{4.5}$$

$$\mathbf{u}' = A(t)\cdot\mathbf{u} + A'(t)\cdot\mathbf{x} + \mathbf{w}(t), \tag{4.6}$$

$$\rho' = \rho, \tag{4.7}$$

$$\mathbf{v}' = A(t)\cdot\mathbf{v} + A'(t)\cdot\mathbf{x} + \mathbf{w}(t), \tag{4.8}$$

$$\mathbf{j}' = A(t)\cdot\mathbf{j} + \rho A'(t)\cdot\mathbf{x} + \rho\mathbf{w}(t) \tag{4.9}$$

(where $A'(t)$ is a derivative of $A(t)$). We consider the fields $\mathbf{E}$ and $\mathbf{B}$ in the coordinate system $(*)$ to be defined by the expression of Lorentz force:

$$\mathbf{F} = \sigma\mathbf{E} + \frac{\sigma}{c}\,\mathbf{u}\times\mathbf{B}. \tag{4.10}$$

Plugging it into (4.3) and using (4.4), (4.5) and (4.6) we deduce

$$\sigma\left(\mathbf{E}' + \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\mathbf{B}'\right) + \frac{\sigma}{c}\big(A(t)\cdot\mathbf{u}\big)\times\mathbf{B}' = \sigma\mathbf{E}' + \frac{\sigma}{c}\Big(A(t)\cdot\mathbf{u} + A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\mathbf{B}'$$
$$= \sigma'\mathbf{E}' + \frac{\sigma'}{c}\,\mathbf{u}'\times\mathbf{B}' = \mathbf{F}' = A(t)\cdot\mathbf{F} = \sigma A(t)\cdot\mathbf{E} + \frac{\sigma}{c}\,A(t)\cdot\big(\mathbf{u}\times\mathbf{B}\big) \tag{4.11}$$

Thus using the trivial identity

$$A\cdot\big(\mathbf{a}\times\mathbf{b}\big) = \big(A\cdot\mathbf{a}\big)\times\big(A\cdot\mathbf{b}\big) \quad \forall\,\mathbf{a}\in\mathbb{R}^3,\ \ \forall\,\mathbf{b}\in\mathbb{R}^3,\ \ \forall\,A\in SO(3), \tag{4.12}$$

by (4.11) we deduce

$$\sigma\left(\mathbf{E}' + \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\mathbf{B}'\right) + \frac{\sigma}{c}\big(A(t)\cdot\mathbf{u}\big)\times\mathbf{B}'$$
$$= \sigma A(t)\cdot\mathbf{E} + \frac{\sigma}{c}\big(A(t)\cdot\mathbf{u}\big)\times\big(A(t)\cdot\mathbf{B}\big). \tag{4.13}$$

Therefore, since (4.13) must be valid for arbitrary choices of $\mathbf{u}$ we deduce

$$\begin{cases}\mathbf{B}' = A(t)\cdot\mathbf{B}\\ \mathbf{E}' + \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\mathbf{B}' = A(t)\cdot\mathbf{E}\end{cases}$$

Therefore,

$$\mathbf{E}' = A(t)\cdot\mathbf{E} - \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\mathbf{B}' = A(t)\cdot\mathbf{E} - \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\Big(A(t)\cdot\mathbf{B}\Big).$$

So we obtained the following relations linking the fields $\mathbf{E},\mathbf{B}$ in coordinate system $(*)$ and $\mathbf{E}',\mathbf{B}'$ in coordinate system $(**)$:

$$\begin{cases}\mathbf{E}' = A(t)\cdot\mathbf{E} - \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{B}\big)\\ \mathbf{B}' = A(t)\cdot\mathbf{B}\end{cases} \tag{4.14}$$

Next, by (4.2) in coordinate system $(**)$ we have the relations

$$\begin{cases} \mathbf{D}' = \mathbf{E}' + \frac{1}{c}\,\mathbf{v}' \times \mathbf{B}', \\ \mathbf{H}' = \mathbf{B}' + \frac{1}{c}\,\mathbf{v}' \times \mathbf{D}' \end{cases}$$

Analogously we define $\mathbf{D}$ and $\mathbf{H}$ in coordinate system $(*)$ by the formulas:

$$\begin{cases} \mathbf{D} = \mathbf{E} + \frac{1}{c}\,\mathbf{v} \times \mathbf{B}, \\ \mathbf{H} = \mathbf{B} + \frac{1}{c}\,\mathbf{v} \times \mathbf{D}. \end{cases} \tag{4.15}$$

Then with the help of (4.14), (4.8) and (4.12) we deduce:

$$\mathbf{D}' = \mathbf{E}' + \frac{1}{c}\mathbf{v}' \times \mathbf{B}' = A(t)\cdot\mathbf{E} - \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big) \times \Big(A(t)\cdot\mathbf{B}\Big) + \frac{1}{c}\mathbf{v}' \times \Big(A(t)\cdot\mathbf{B}\Big) =$$
$$A(t)\cdot\mathbf{E} - \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big) \times \Big(A(t)\cdot\mathbf{B}\Big) + \frac{1}{c}\Big(A(t)\cdot\mathbf{v} + A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big) \times \Big(A(t)\cdot\mathbf{B}\Big) =$$
$$A(t)\cdot\mathbf{E} + \frac{1}{c}\Big(A(t)\cdot\mathbf{v}\Big) \times \Big(A(t)\cdot\mathbf{B}\Big) = A(t)\cdot\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right) = A(t)\cdot\mathbf{D},$$

and thus

$$\mathbf{H}' = \mathbf{B}' + \frac{1}{c}\mathbf{v}' \times \mathbf{D}' = A(t)\cdot\mathbf{B} + \frac{1}{c}\Big(A(t)\cdot\mathbf{v} + A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big) \times \Big(A(t)\cdot\mathbf{D}\Big) =$$
$$A(t)\cdot\mathbf{B} + \frac{1}{c}\Big(A(t)\cdot\mathbf{v}\Big) \times \Big(A(t)\cdot\mathbf{D}\Big) + \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big) \times \Big(A(t)\cdot\mathbf{D}\Big) =$$
$$A(t)\cdot\left(\mathbf{B}+\frac{1}{c}\mathbf{v}\times\mathbf{D}\right)+\frac{1}{c}\Big(A'(t)\cdot\mathbf{x}+\mathbf{w}(t)\Big)\times\Big(A(t)\cdot\mathbf{D}\Big) = A(t)\cdot\mathbf{H}+\frac{1}{c}\Big(A'(t)\cdot\mathbf{x}+\mathbf{w}(t)\Big)\times\Big(A(t)\cdot\mathbf{D}\Big).$$

I.e. the following relations are valid:

$$\begin{cases} \mathbf{D}' = A(t)\cdot\mathbf{D} \\ \mathbf{B}' = A(t)\cdot\mathbf{B} \\ \mathbf{E}' = A(t)\cdot\mathbf{E} - \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big) \times \big(A(t)\cdot\mathbf{B}\big) \\ \mathbf{H}' = A(t)\cdot\mathbf{H} + \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big) \times \big(A(t)\cdot\mathbf{D}\big). \end{cases} \tag{4.16}$$

Next by (4.1) for every vector field $\mathbf{\Gamma} : \mathbb{R}^3 \times [0, +\infty) \to \mathbb{R}^3$ we have

$$\begin{cases} d_{\mathbf{x}'}\mathbf{\Gamma} = \big(d_{\mathbf{x}}\mathbf{\Gamma}\big)\cdot A^{-1}(t) \\ curl_{\mathbf{x}'}\big(A(t)\cdot\mathbf{\Gamma}\big) = A(t)\cdot curl_{\mathbf{x}}\mathbf{\Gamma} \\ div_{\mathbf{x}'}\big(A(t)\cdot\mathbf{\Gamma}\big) = div_{\mathbf{x}}\mathbf{\Gamma}. \end{cases} \tag{4.17}$$

Furthermore, by the chain rule we obtain

$$\frac{\partial\mathbf{\Gamma}}{\partial t} = \frac{\partial\mathbf{\Gamma}}{\partial t'} + \big(d_{\mathbf{x}'}\mathbf{\Gamma}\big)\cdot\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)$$

and therefore,

$$\frac{\partial\mathbf{\Gamma}}{\partial t'} = \frac{\partial\mathbf{\Gamma}}{\partial t} - \big(d_{\mathbf{x}}\mathbf{\Gamma}\big)\cdot\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big) \tag{4.18}$$

Moreover, by (4.17), (4.18), (4.8) and (4.12) for every vector field $\mathbf{\Gamma} : \mathbb{R}^3 \times [0, +\infty) \to \mathbb{R}^3$ we get:

$$\frac{\partial\big(A(t)\cdot\mathbf{\Gamma}\big)}{\partial t'} - curl_{\mathbf{x}'}\Big(\mathbf{v}' \times \big(A(t)\cdot\mathbf{\Gamma}\big)\Big) + \Big(\operatorname{div}_{\mathbf{x}'}\big(A(t)\cdot\mathbf{\Gamma}\big)\Big)\mathbf{v}' =$$
$$\left(A(t)\cdot\frac{\partial\mathbf{\Gamma}}{\partial t} + A'(t)\cdot\mathbf{\Gamma} - A(t)\cdot\big(d_{\mathbf{x}}\mathbf{\Gamma}\big)\cdot\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big)\right)$$
$$- A(t)\cdot curl_{\mathbf{x}}\bigg(\Big(\mathbf{v} + A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big)\times\mathbf{\Gamma}\bigg) + \big(\operatorname{div}_{\mathbf{x}}\mathbf{\Gamma}\big)\Big(A(t)\cdot\mathbf{v} + A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)$$
$$= A(t)\cdot\left(\frac{\partial\mathbf{\Gamma}}{\partial t} - curl_{\mathbf{x}}\big(\mathbf{v}\times\mathbf{\Gamma}\big) + \big(\operatorname{div}_{\mathbf{x}}\mathbf{\Gamma}\big)\mathbf{v}\right) + A(t)\cdot\bigg(d_x\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big)\bigg)\cdot\mathbf{\Gamma}$$
$$- A(t)\cdot\big(d_{\mathbf{x}}\mathbf{\Gamma}\big)\cdot\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big) + A(t)\cdot\bigg(\big(\operatorname{div}_{\mathbf{x}}\mathbf{\Gamma}\big)\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big)\bigg)$$
$$- A(t)\cdot curl_{\mathbf{x}}\bigg(\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big)\times\mathbf{\Gamma}\bigg). \tag{4.19}$$

On the other hand, by (2.9) we have,

$$\bigg(d_x\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big)\bigg)\cdot\mathbf{\Gamma} - \big(d_{\mathbf{x}}\mathbf{\Gamma}\big)\cdot\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big)$$
$$+ \big(\operatorname{div}_{\mathbf{x}}\mathbf{\Gamma}\big)\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big) - curl_{\mathbf{x}}\bigg(\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big)\times\mathbf{\Gamma}\bigg)$$
$$= \bigg(\operatorname{div}_{\mathbf{x}}\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big)\bigg)\mathbf{\Gamma} \tag{4.20}$$

Therefore, by (4.19) and (4.20) we deduce:

$$\frac{\partial\big(A(t)\cdot\mathbf{\Gamma}\big)}{\partial t'} - curl_{\mathbf{x}'}\Big(\mathbf{v}' \times \big(A(t)\cdot\mathbf{\Gamma}\big)\Big) + \Big(\operatorname{div}_{\mathbf{x}'}\big(A(t)\cdot\mathbf{\Gamma}\big)\Big)\mathbf{v}' =$$
$$A(t)\cdot\left(\frac{\partial\mathbf{\Gamma}}{\partial t} - curl_{\mathbf{x}}\big(\mathbf{v}\times\mathbf{\Gamma}\big) + \big(\operatorname{div}_{\mathbf{x}}\mathbf{\Gamma}\big)\mathbf{v}\right) + A(t)\cdot\left(\bigg(\operatorname{div}_{\mathbf{x}}\Big(A^{-1}(t)\cdot A'(t)\cdot\mathbf{x} + A^{-1}(t)\cdot\mathbf{w}(t)\Big)\bigg)\mathbf{\Gamma}\right)$$
$$= A(t)\cdot\left(\frac{\partial\mathbf{\Gamma}}{\partial t} - curl_{\mathbf{x}}\big(\mathbf{v}\times\mathbf{\Gamma}\big) + \big(\operatorname{div}_{\mathbf{x}}\mathbf{\Gamma}\big)\mathbf{v}\right) + \bigg(tr\Big(A^{-1}(t)\cdot A'(t)\Big)\bigg)A(t)\cdot\mathbf{\Gamma}, \tag{4.21}$$

where $tr\big(A^{-1}(t)\cdot A'(t)\big)$ is the trace of the matrix $A^{-1}(t)\cdot A'(t)$ (sum of diagonal elements). However, since $A^T(t)\cdot A(t) = I$ we have $A^{-1}(t) = A^T(t)$ and $A^{-1}(t)\cdot A'(t) = S(t)$, where $S^T(t) = -S(t)$. In particular $trS(t) = 0$ and thus $tr\,\big(A^{-1}(t)\cdot A'(t)\big) = 0$. Therefore, by (4.21) for every vector field $\mathbf{\Gamma} : \mathbb{R}^3 \times [0, +\infty) \to \mathbb{R}^3$ we have:

$$\frac{\partial\big(A(t)\cdot\mathbf{\Gamma}\big)}{\partial t'} - curl_{\mathbf{x}'}\Big(\mathbf{v}'\times\big(A(t)\cdot\mathbf{\Gamma}\big)\Big) + \Big(\operatorname{div}_{\mathbf{x}'}\big(A(t)\cdot\mathbf{\Gamma}\big)\Big)\mathbf{v}' = A(t)\cdot\left(\frac{\partial\mathbf{\Gamma}}{\partial t} - curl_{\mathbf{x}}\big(\mathbf{v}\times\mathbf{\Gamma}\big) + \big(\operatorname{div}_{\mathbf{x}}\mathbf{\Gamma}\big)\mathbf{v}\right). \tag{4.22}$$

Next by (4.2) we have

$$curl_{\mathbf{x}'}\mathbf{B}' - \frac{4\pi}{c}\Big(\mathbf{j}' - \rho'\mathbf{v}'\Big) - \frac{1}{c}\left(\frac{\partial \mathbf{D}'}{\partial t'} - curl_{\mathbf{x}'}\Big(\mathbf{v}' \times \mathbf{D}'\Big) + \Big(\operatorname{div}_{\mathbf{x}'} \mathbf{D}'\Big)\mathbf{v}'\right) = curl_{\mathbf{x}'}\mathbf{H}' - \frac{4\pi}{c}\mathbf{j}' - \frac{1}{c}\frac{\partial \mathbf{D}'}{\partial t'} = 0 \tag{4.23}$$

$$curl_{\mathbf{x}'}\mathbf{D}' + \frac{1}{c}\left(\frac{\partial \mathbf{B}'}{\partial t'} - curl_{\mathbf{x}'}\Big(\mathbf{v}' \times \mathbf{B}'\Big) + \Big(\operatorname{div}_{\mathbf{x}'} \mathbf{B}'\Big)\mathbf{v}'\right) = curl_{\mathbf{x}'}\mathbf{E}' + \frac{1}{c}\frac{\partial \mathbf{B}'}{\partial t'} = 0 \tag{4.24}$$

Thus plugging (4.23) and (4.24) into (4.22) and using (4.15), (4.7), (4.8), (4.9) and (4.17) gives

$$\begin{aligned} A(t)\cdot\left(curl_{\mathbf{x}}\mathbf{H} - \frac{4\pi}{c}\mathbf{j} - \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{1}{c}\big(4\pi\rho - \operatorname{div}_{\mathbf{x}} \mathbf{D}\big)\mathbf{v}\right) &= \\ A(t)\cdot\left(curl_{\mathbf{x}}\mathbf{B} - \frac{4\pi}{c}\big(\mathbf{j} - \rho\mathbf{v}\big) - \frac{1}{c}\left(\frac{\partial \mathbf{D}}{\partial t} - curl_{\mathbf{x}}\big(\mathbf{v} \times \mathbf{D}\big) + \big(\operatorname{div}_{\mathbf{x}} \mathbf{D}\big)\mathbf{v}\right)\right) &= \\ curl_{\mathbf{x}'}\mathbf{B}' - \frac{4\pi}{c}\Big(\mathbf{j}' - \rho'\mathbf{v}'\Big) - \frac{1}{c}\left(\frac{\partial \mathbf{D}'}{\partial t'} - curl_{\mathbf{x}'}\Big(\mathbf{v}' \times \mathbf{D}'\Big) + \Big(\operatorname{div}_{\mathbf{x}'} \mathbf{D}\Big)\mathbf{v}'\right) &= 0 \end{aligned} \tag{4.25}$$

Similarly

$$\begin{aligned} A(t)\left(curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + \frac{1}{c}\big(\operatorname{div}_{\mathbf{x}} \mathbf{B}\big)\mathbf{v}\right) &= A(t)\left(curl_{\mathbf{x}}\mathbf{D} + \frac{1}{c}\left(\frac{\partial \mathbf{B}}{\partial t} - curl_{\mathbf{x}}\Big(\mathbf{v} \times \mathbf{B}\Big) + \Big(\operatorname{div}_{\mathbf{x}} \mathbf{B}\Big)\mathbf{v}\right)\right) \\ &= curl_{\mathbf{x}'}\mathbf{D}' + \frac{1}{c}\left(\frac{\partial \mathbf{B}'}{\partial t'} - curl_{\mathbf{x}'}\Big(\mathbf{v}' \times \mathbf{B}'\Big) + \Big(\operatorname{div}_{\mathbf{x}'} \mathbf{B}\Big)\mathbf{v}'\right) = 0 \end{aligned} \tag{4.26}$$

On the other hand, by (4.2), (4.17) and (4.7) we obtain:

$$4\pi\rho = 4\pi\rho' = div_{\mathbf{x}'}\mathbf{D}' = div_{\mathbf{x}}\mathbf{D} \quad \text{and} \quad 0 = div_{\mathbf{x}'}\mathbf{B}' = div_{\mathbf{x}}\mathbf{B}. \tag{4.27}$$

Thus plugging (4.25), (4.26) and (4.27) we obtain

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \\ div_{\mathbf{x}}\mathbf{D} = 4\pi\rho \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0 \\ div_{\mathbf{x}}\mathbf{B} = 0. \end{cases} \tag{4.28}$$

Then, plugging (4.28) into (4.15), we finally obtain that in coordinate system $(*)$ the Maxwell equations have the same form as in system $(**)$ i.e.

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} \equiv \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \\ div_{\mathbf{x}}\mathbf{D} \equiv 4\pi\rho, \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0, \\ div_{\mathbf{x}}\mathbf{B} \equiv 0, \\ \mathbf{E} = \mathbf{D} - \frac{1}{c}\,\mathbf{v} \times \mathbf{B}, \\ \mathbf{H} = \mathbf{B} + \frac{1}{c}\,\mathbf{v} \times \mathbf{D} \end{cases} \tag{4.29}$$

Therefore, since the assumption, that coordinate system (∗∗) is inertial, implies the relations of (4.2), we deduce that the Maxwell equations in the form (4.29) and the expression of the Lorentz force

$$\mathbf{F} := \sigma\mathbf{E} + \frac{\sigma}{c}\,\mathbf{u}\times\mathbf{B} \tag{4.30}$$

are valid in every non-inertial coordinate system. Moreover, under the change of the coordinate system of the form

$$\begin{cases}\mathbf{x}' = A(t)\cdot\mathbf{x} + \mathbf{z}(t),\\ t' = t,\end{cases} \tag{4.31}$$

we have the following transformations of the electromagnetic fields:

$$\begin{cases}\mathbf{D}' = A(t)\cdot\mathbf{D}\\ \mathbf{B}' = A(t)\cdot\mathbf{B}\\ \mathbf{E}' = A(t)\cdot\mathbf{E} - \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{B}\big)\\ \mathbf{H}' = A(t)\cdot\mathbf{H} + \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{D}\big),\end{cases} \tag{4.32}$$

where $\mathbf{w}(t) = \mathbf{z}'(t)$.

So the laws of Electrodynamics are invariant not only in inertial but also in non-inertial coordinate systems.

5 Presence of Dielectrics and Magnetics

5.1 General setting

Consider system (4.29) in some inertial or non-inertial coordinate system inside a dielectric and/or magnetic medium:

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H}_0 \equiv \frac{4\pi}{c}\big(\mathbf{j} + \mathbf{j}_m + \mathbf{j}_p\big) + \frac{1}{c}\frac{\partial \mathbf{D}_0}{\partial t} \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty),\\ div_{\mathbf{x}}\mathbf{D}_0 \equiv 4\pi\big(\rho + \rho_p\big) \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty),\\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0 \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty),\\ div_{\mathbf{x}}\mathbf{B} \equiv 0 \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty),\end{cases} \tag{5.1}$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, $\mathbf{v} := \mathbf{v}(\mathbf{x},t)$ is the aether velocity field, ρ is the average (macroscopic) charge density, ρ_p is the density of the charge of polarization, $\mathbf{j}$ is the average (macroscopic) current density, $\mathbf{j}_m$ is the density of the current of magnetization, $\mathbf{j}_p$ is the density of the current of polarization and

$$\mathbf{D}_0 := \mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\mathbf{B} \quad \text{and} \quad \mathbf{H}_0 := \mathbf{B} + \frac{1}{c}\,\mathbf{v}\times\mathbf{D}_0 \tag{5.2}$$

It is well known from the Lorentz theory that in the case of a moving dielectric/magnetic medium

$$\rho_p = -div_{\mathbf{x}}\mathbf{P} \quad \text{and} \quad \mathbf{j}_p = \frac{\partial \mathbf{P}}{\partial t} - curl_{\mathbf{x}}\big(\mathbf{u}\times\mathbf{P}\big), \tag{5.3}$$

where $\mathbf{P} : \mathbb{R}^3 \times [0,+\infty) \to \mathbb{R}^3$ is the field of polarization and $\mathbf{u} := \mathbf{u}(\mathbf{x},t)$ is the field of velocities of the dielectric medium (see also [1], page 610). Furthermore,

$$\mathbf{j}_m = c\, curl_{\mathbf{x}}\mathbf{M}, \tag{5.4}$$

where $\mathbf{M} : \mathbb{R}^3 \times [0,+\infty) \to \mathbb{R}^3$ is the field of magnetization. Thus if we consider

$$\mathbf{D} := \mathbf{D}_0 + 4\pi\mathbf{P} = \mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\mathbf{B} + 4\pi\mathbf{P}, \tag{5.5}$$

and

$$\begin{aligned}\mathbf{H} := \mathbf{H}_0 - 4\pi\mathbf{M} + \frac{4\pi}{c}\,\mathbf{u}\times\mathbf{P} = \mathbf{B} + \frac{1}{c}\,\mathbf{v}\times\mathbf{D}_0 + \frac{4\pi}{c}\,\mathbf{u}\times\mathbf{P} - 4\pi\mathbf{M}\\ = \mathbf{B} + \frac{4\pi}{c}\,\mathbf{u}\times\mathbf{P} + \frac{1}{c}\,\mathbf{v}\times\mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\Big(\frac{1}{c}\,\mathbf{v}\times\mathbf{B}\Big) - 4\pi\mathbf{M},\end{aligned} \tag{5.6}$$

we obtain

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} \equiv \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty),\\ div_{\mathbf{x}}\mathbf{D} \equiv 4\pi\rho \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty),\\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0 \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty),\\ div_{x}\mathbf{B} \equiv 0 \quad \text{for } (\mathbf{x},t)\in\mathbb{R}^3\times[0,+\infty),\end{cases} \tag{5.7}$$

We call $\mathbf{D}$ by the electric displacement field and $\mathbf{H}$ by the $\mathbf{H}$-magnetic field in a medium.

5.2 Change of inertial coordinate system

Consider the Galilean transformation

$$\begin{cases}\mathbf{x}' = \mathbf{x} + t\mathbf{w},\\ t' = t.\end{cases} \tag{5.8}$$

Then since

$$\begin{cases}\mathbf{D}_0' = \mathbf{D}_0,\\ \mathbf{B}' = \mathbf{B},\\ \mathbf{E}' = \mathbf{E} - \frac{1}{c}\,\mathbf{w}\times\mathbf{B},\\ \mathbf{H}_0' = \mathbf{H}_0 + \frac{1}{c}\,\mathbf{w}\times\mathbf{D}_0,\end{cases} \tag{5.9}$$

and since $\mathbf{P}' = \mathbf{P}$ and $\mathbf{M}' = \mathbf{M}$, by (5.5) and (5.6) we deduce

$$\begin{cases}\mathbf{D}' = \mathbf{D},\\ \mathbf{B}' = \mathbf{B},\\ \mathbf{E}' = \mathbf{E} - \frac{1}{c}\,\mathbf{w}\times\mathbf{B},\\ \mathbf{H}' = \mathbf{H} + \frac{1}{c}\,\mathbf{w}\times\mathbf{D}.\end{cases} \tag{5.10}$$

So we get exactly the same expressions of transformations in a dielectric/magnetic medium as in the vacuum.

5.3 Non-inertial coordinate systems

Consider the change of certain non-inertial coordinate system $(*)$ to another coordinate system $(**)$:

$$\begin{cases} \mathbf{x}' = A(t)\cdot\mathbf{x} + \mathbf{z}(t), \\ t' = t, \end{cases}$$

where $A(t) \in SO(3)$ is a rotation. Then, as before in (4.32), denoting $\mathbf{w}(t) = \mathbf{z}'(t)$, we have the following relations between the physical characteristics in coordinate systems $(*)$ and $(**)$:

$$\begin{cases} \mathbf{E}' = A(t)\cdot\mathbf{E} - \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{B}\big) \\ \mathbf{B}' = A(t)\cdot\mathbf{B} \\ \mathbf{D}_0' = A(t)\cdot\mathbf{D}_0 \\ \mathbf{H}_0' = A(t)\cdot\mathbf{H}_0 + \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{D}_0\big) \\ \mathbf{P}' = A(t)\cdot\mathbf{P} \\ \mathbf{M}' = A(t)\cdot\mathbf{M} \\ \mathbf{u}' = A(t)\cdot\mathbf{u} + A'(t)\cdot\mathbf{x} + \mathbf{w}(t). \end{cases} \tag{5.11}$$

Plugging it into (5.5) and (5.6) we deduce

$$\mathbf{D}' := \mathbf{D}_0' + 4\pi\mathbf{P}' = A(t)\cdot\big(\mathbf{D}_0 + 4\pi\mathbf{P}\big) = A(t)\cdot\mathbf{D}, \tag{5.12}$$

and

$$\begin{aligned} \mathbf{H}' &:= \mathbf{H}_0' - 4\pi\mathbf{M}' + \frac{4\pi}{c}\,\mathbf{u}'\times\mathbf{P}' = \\ A(t)\cdot\mathbf{H}_0 &+ \frac{1}{c}\Big(A'(t)\cdot\mathbf{x}+\mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{D}_0\big) - 4\pi A(t)\cdot\mathbf{M} + \frac{4\pi}{c}\Big(A(t)\cdot\mathbf{u} + A'(t)\cdot\mathbf{x}+\mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{P}\big) \\ &= A(t)\cdot\left(\mathbf{H}_0 - 4\pi\mathbf{M} + \frac{4\pi}{c}\,\mathbf{u}\times\mathbf{P}\right) + \frac{1}{c}\Big(A'(t)\cdot\mathbf{x}+\mathbf{w}(t)\Big)\times\Big(A(t)\cdot\big(\mathbf{D}_0 + 4\pi\mathbf{P}\big)\Big) \\ &= A(t)\cdot\mathbf{H} + \frac{1}{c}\Big(A'(t)\cdot\mathbf{x}+\mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{D}\big), \end{aligned} \tag{5.13}$$

So the expressions of transformations under the change of non-inertial coordinate system in a dielectric/magnetic medium exactly the same as in the vacuum:

$$\begin{cases} \mathbf{D}' = A(t)\cdot\mathbf{D} \\ \mathbf{B}' = A(t)\cdot\mathbf{B} \\ \mathbf{E}' = A(t)\cdot\mathbf{E} - \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{B}\big) \\ \mathbf{H}' = A(t)\cdot\mathbf{H} + \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{D}\big). \end{cases} \tag{5.14}$$

5.4 Case of simplest dielectrics/magnetics

It is well known that in the case of simplest homogenous isotropic dielectrics and/or magnetics we have

$$\begin{cases} \mathbf{P} = \gamma\Big(\mathbf{E} + \frac{1}{c}\,\mathbf{u}\times\mathbf{B}\Big), \\ \mathbf{M} = \kappa\mathbf{B}, \end{cases} \tag{5.15}$$

where γ and κ are material coefficients. Using (5.11), it can be easily seen that the laws in (5.15) are invariant under the changes of inertial or non inertial coordinate system. Next, plugging (5.15) into (5.5) and (5.6) gives,

$$\mathbf{D} = \mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\mathbf{B} + 4\pi\gamma\Big(\mathbf{E} + \frac{1}{c}\,\mathbf{u}\times\mathbf{B}\Big), \tag{5.16}$$

and

$$\mathbf{H} = \big(1 - 4\pi\kappa\big)\mathbf{B} + \frac{4\pi\gamma}{c}\,\mathbf{u}\times\Big(\mathbf{E} + \frac{1}{c}\,\mathbf{u}\times\mathbf{B}\Big) + \frac{1}{c}\,\mathbf{v}\times\Big(\mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\mathbf{B}\Big). \tag{5.17}$$

These equations take much simpler forms in the case $\mathbf{u} = \mathbf{v}$ i.e. in the case when the velocity of the aether equals to the velocity of the dielectric/magnetic medium (Conjecture of complete aether drag supposes $\mathbf{u} = \mathbf{v}$ everywhere). Indeed in this case

$$\mathbf{D} = \big(1 + 4\pi\gamma\big)\Big(\mathbf{E} + \frac{1}{c}\,\mathbf{u}\times\mathbf{B}\Big),$$

and

$$\mathbf{H} = \big(1 - 4\pi\kappa\big)\mathbf{B} + \frac{1 + 4\pi\gamma}{c}\,\mathbf{u}\times\Big(\mathbf{E} + \frac{1}{c}\,\mathbf{u}\times\mathbf{B}\Big) = \big(1 - 4\pi\kappa\big)\mathbf{B} + \frac{1}{c}\,\mathbf{u}\times\mathbf{D}.$$

Thus denoting $\gamma_0 = \frac{1}{1+4\pi\gamma}$ and $\kappa_0 = 1 - 4\pi\kappa$, in the later case we obtain the following relations:

$$\mathbf{E} = \gamma_0\mathbf{D} - \frac{1}{c}\,\mathbf{u}\times\mathbf{B}, \tag{5.18}$$

$$\mathbf{H} = \kappa_0\mathbf{B} + \frac{1}{c}\,\mathbf{u}\times\mathbf{D}. \tag{5.19}$$

5.5 Ohm's Law in a conducting medium

It is well known that the Ohm's Law in a conducting medium has the form

$$\mathbf{j} - \rho\,\mathbf{u} = \varepsilon\Big(\mathbf{E} + \frac{1}{c}\,\mathbf{u}\times\mathbf{B}\Big), \tag{5.20}$$

where $\mathbf{u}$ is the velocity of the medium and ε is a material coefficient. As before, using (5.11), it can be easily seen that the Ohm's Law is invariant under the changes of inertial or non inertial coordinate system.

6 Quasistationary Electromagnetic fields in a slowly moving aether

6.1 Some consequences of Maxwell Equations

Consider the system of Maxwell equations in the moving aether (vacuum):

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} \equiv \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \\ div_{\mathbf{x}}\mathbf{D} \equiv 4\pi\rho, \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0, \\ div_{\mathbf{x}}\mathbf{B} \equiv 0, \\ \mathbf{E} = \mathbf{D} - \frac{1}{c}\,\mathbf{v}\times\mathbf{B}, \\ \mathbf{H} = \mathbf{B} + \frac{1}{c}\,\mathbf{v}\times\mathbf{D}. \end{cases} \tag{6.1}$$

Next let $\psi_0(\mathbf{x}, t)$ be the Coulomb's potential which satisfies

$$-\Delta_{\mathbf{x}}\psi_0 \equiv 4\pi\rho. \tag{6.2}$$

Then defining

$$\widetilde{\mathbf{D}} := \mathbf{D} + \nabla_{\mathbf{x}}\psi_0, \tag{6.3}$$

we rewrite (6.1) as

$$\begin{cases} curl_{\mathbf{x}}\mathbf{B} \equiv \frac{4\pi}{c}\widetilde{\mathbf{j}} + \frac{1}{c}\frac{\partial \widetilde{\mathbf{D}}}{\partial t} - \frac{1}{c}curl_{\mathbf{x}}(\mathbf{v}\times\widetilde{\mathbf{D}}), \\ div_{\mathbf{x}}\widetilde{\mathbf{D}} \equiv 0, \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0, \\ div_{\mathbf{x}}\mathbf{B} \equiv 0, \\ \widetilde{\mathbf{D}} \equiv \nabla_{\mathbf{x}}\psi_0 + \mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\mathbf{B}, \end{cases} \tag{6.4}$$

where we set the reduced current:

$$\widetilde{\mathbf{j}} := \mathbf{j} - \frac{1}{4\pi}\frac{\partial}{\partial t}\Big(\nabla_{\mathbf{x}}\psi_0\Big) + \frac{1}{4\pi}curl_{\mathbf{x}}\Big(\mathbf{v}\times\nabla_{\mathbf{x}}\psi_0\Big). \tag{6.5}$$

Note that by the Continuum Equation of the Conservation of Charges

$$\frac{\partial\rho}{\partial t} + div_{\mathbf{x}}\mathbf{j} \equiv 0, \tag{6.6}$$

the reduced current clearly satisfies:

$$div_{\mathbf{x}}\widetilde{\mathbf{j}} \equiv 0. \tag{6.7}$$

Next by the third and the fourth equations in (6.4) we can write:

$$\begin{cases} \mathbf{B} \equiv curl_{\mathbf{x}}\mathbf{A}, \\ \mathbf{E} \equiv -\nabla_{\mathbf{x}}\Psi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \\ div_{\mathbf{x}}\mathbf{A} \equiv 0, \end{cases} \tag{6.8}$$

where Ψ and $\mathbf{A}$ are the scalar and the vectorial electromagnetic potentials. Therefore, we rewrite (6.4) as

$$\begin{cases} curl_{\mathbf{x}}\big(curl_{\mathbf{x}}\mathbf{A}\big) \equiv \frac{4\pi}{c}\widetilde{\mathbf{j}} + \frac{1}{c}\frac{\partial\widetilde{\mathbf{D}}}{\partial t} - \frac{1}{c}curl_{\mathbf{x}}(\mathbf{v}\times\widetilde{\mathbf{D}}),\\ div_{\mathbf{x}}\widetilde{\mathbf{D}} \equiv 0,\\ \widetilde{\mathbf{D}} \equiv \nabla_{\mathbf{x}}\psi_0 - \nabla_{\mathbf{x}}\Psi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} + \frac{1}{c}\mathbf{v}\times curl_{\mathbf{x}}\mathbf{A},\\ div_{\mathbf{x}}\mathbf{A} \equiv 0.\end{cases} \tag{6.9}$$

Then defining

$$\Phi := c\Psi - c\psi_0, \tag{6.10}$$

by (6.9) we deduce

$$\begin{cases} -\Delta_{\mathbf{x}}\Phi \equiv -div_{\mathbf{x}}\big(\mathbf{v}\times curl_{\mathbf{x}}\mathbf{A}\big),\\ div_{\mathbf{x}}\mathbf{A} \equiv 0.\end{cases} \tag{6.11}$$

and

$$\begin{split} -\Delta_{\mathbf{x}}\mathbf{A} \equiv \frac{4\pi}{c}\widetilde{\mathbf{j}} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \frac{1}{c^2}\frac{\partial}{\partial t}\Big(\nabla_{\mathbf{x}}\Phi\Big) + \frac{1}{c^2}\frac{\partial}{\partial t}\Big(\mathbf{v}\times curl_{\mathbf{x}}\mathbf{A}\Big)\\ + \frac{1}{c^2}curl_{\mathbf{x}}\Big(\mathbf{v}\times\frac{\partial\mathbf{A}}{\partial t}\Big) + \frac{1}{c^2}curl_{\mathbf{x}}\Big(\mathbf{v}\times\nabla_{\mathbf{x}}\Phi\Big) - \frac{1}{c^2}curl_{\mathbf{x}}\Big(\mathbf{v}\times\Big(\mathbf{v}\times curl_{\mathbf{x}}\mathbf{A}\Big)\Big), \end{split} \tag{6.12}$$

6.2 The case of quasistationary fields in a slow aether

Assume that we have a slow motion of the aether that means that at any instant on every point:

$$\frac{v_0^2}{c^2} \ll 1 \tag{6.13}$$

where

$$v_0 := \sup_{(x,t)}\big|\mathbf{v}(x,t)\big| \tag{6.14}$$

Furthermore, consider quasistationary fields. This means the following: Assume that the changes in time of the physical characteristics of the electromagnetic fields become essential only after certain interval of time T. Then we assume that

$$c^2T^2 \gg 1. \tag{6.15}$$

Furthermore, defining

$$\widetilde{\mathbf{v}} := \frac{1}{v_0}\mathbf{v}(x,t) \tag{6.16}$$

and

$$\widetilde{\Phi}(x,t) := \frac{1}{v_0}\,\Phi(x,t), \tag{6.17}$$

we rewrite (6.11) as

$$\begin{cases} -\Delta_{\mathbf{x}}\widetilde{\Phi} \equiv -div_{\mathbf{x}}\big(\widetilde{\mathbf{v}}\times curl_{\mathbf{x}}\mathbf{A}\big),\\ div_{\mathbf{x}}\mathbf{A} \equiv 0.\end{cases} \tag{6.18}$$

and (6.12) as

$$-\Delta_{\mathbf{x}}\mathbf{A}\equiv\frac{4\pi}{c}\widetilde{\mathbf{j}}-\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}-\frac{v_0}{c}\frac{1}{c}\frac{\partial}{\partial t}\Big(\nabla_{\mathbf{x}}\widetilde{\Phi}\Big)+\frac{v_0}{c}\frac{1}{c}\frac{\partial}{\partial t}\Big(\widetilde{\mathbf{v}}\times curl_{\mathbf{x}}\mathbf{A}\Big)$$
$$+\frac{v_0}{c}curl_{\mathbf{x}}\Big(\widetilde{\mathbf{v}}\times\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\Big)+\frac{v_0^2}{c^2}curl_{\mathbf{x}}\Big(\widetilde{\mathbf{v}}\times\nabla_{\mathbf{x}}\widetilde{\Phi}\Big)-\frac{v_0^2}{c^2}curl_{\mathbf{x}}\Big(\widetilde{\mathbf{v}}\times\Big(\widetilde{\mathbf{v}}\times curl_{\mathbf{x}}\mathbf{A}\Big)\Big),\quad(6.19)$$

Then using (6.13), (6.15) and the fact $|\widetilde{\mathbf{v}}|\leq 1$, by (6.18) and (6.19) we obtain

$$-\Delta_{\mathbf{x}}\mathbf{A}\approx\frac{4\pi}{c}\widetilde{\mathbf{j}}.\tag{6.20}$$

Plugging it into (6.11) and using (6.10) and (6.2) we deduce

$$\begin{cases}-\Delta_{\mathbf{x}}\mathbf{A}\approx\frac{4\pi}{c}\widetilde{\mathbf{j}},\\ -\Delta_{\mathbf{x}}\Psi=4\pi\rho-\frac{1}{c}\,div_{\mathbf{x}}\big(\mathbf{v}\times curl_{\mathbf{x}}\mathbf{A}\big).\end{cases}\tag{6.21}$$

where

$$\begin{cases}\widetilde{\mathbf{j}}:=\mathbf{j}-\frac{1}{4\pi}\frac{\partial}{\partial t}\big(\nabla_{\mathbf{x}}\psi_0\big)+\frac{1}{4\pi}curl_{\mathbf{x}}\big(\mathbf{v}\times\nabla_{\mathbf{x}}\psi_0\big),\\ -\Delta_{\mathbf{x}}\psi_0=4\pi\rho.\end{cases}\tag{6.22}$$

So in order to find the scalar and the vectorial potential we just need to solve Laplace equations. Knowing the potential we can find **E** and **B** by the formulas

$$\begin{cases}\mathbf{B}=curl_{\mathbf{x}}\mathbf{A},\\ \mathbf{E}=-\nabla_{\mathbf{x}}\Psi-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}.\end{cases}\tag{6.23}$$

Moreover, using the relations

$$\begin{cases}\mathbf{D}=\mathbf{E}+\frac{1}{c}\,\mathbf{v}\times\mathbf{B}\\ \mathbf{H}=\mathbf{B}+\frac{1}{c}\,\mathbf{v}\times\mathbf{D},\end{cases}\tag{6.24}$$

by (6.23) we obtain:

$$\begin{cases}\mathbf{D}=-\nabla_{\mathbf{x}}\Psi-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}+\frac{1}{c}\,\mathbf{v}\times\big(curl_{\mathbf{x}}\mathbf{A}\big)\\ \mathbf{B}=curl_{\mathbf{x}}\mathbf{A}\\ \mathbf{E}=-\nabla_{\mathbf{x}}\Psi-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\\ \mathbf{H}=curl_{\mathbf{x}}\mathbf{A}+\frac{1}{c}\,\mathbf{v}\times\Big(-\nabla_{\mathbf{x}}\Psi-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}+\frac{1}{c}\,\mathbf{v}\times\big(curl_{\mathbf{x}}\mathbf{A}\big)\Big).\end{cases}\tag{6.25}$$

Remark 6.1. The solutions of (6.21) and (6.25) satisfy the following equations:

$$\begin{cases}curl_{\mathbf{x}}\Big(\mathbf{B}+\frac{1}{c}\,\mathbf{v}\times\big(-\nabla_{\mathbf{x}}\psi_0\big)\Big)\equiv\frac{4\pi}{c}\mathbf{j}+\frac{1}{c}\frac{\partial(-\nabla_{\mathbf{x}}\psi_0)}{\partial t},\\ div_{\mathbf{x}}\mathbf{D}\equiv 4\pi\rho,\\ curl_{\mathbf{x}}\mathbf{E}+\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\equiv 0,\\ div_{\mathbf{x}}\mathbf{B}\equiv 0,\\ \mathbf{E}=\mathbf{D}-\frac{1}{c}\,\mathbf{v}\times\mathbf{B},\\ \mathbf{H}=\mathbf{B}+\frac{1}{c}\,\mathbf{v}\times\mathbf{D},\end{cases}\tag{6.26}$$

that differ from the original Maxwell equations (6.1) only by neglecting the divergence-free part of the vector field $\mathbf{D}$ on the first equation.

6.3 Change of inertial coordinate system for the approximate solutions

Under the Galilean Transformations:

$$\begin{cases} \mathbf{x}' = \mathbf{x} + t\mathbf{w}, \\ t' = t, \end{cases} \tag{6.27}$$

using (6.22) and the fact that

$$\rho' = \rho \quad \text{and} \quad \mathbf{j}' = \mathbf{j} + \rho\mathbf{w},$$

we obtain

$$\widetilde{\mathbf{j}}' = \widetilde{\mathbf{j}}. \tag{6.28}$$

Therefore, by (6.21) we obtain

$$\begin{cases} \mathbf{A}' \approx \mathbf{A}, \\ \Psi' \approx \Psi + \frac{1}{c}\mathbf{w} \cdot \mathbf{A}. \end{cases} \tag{6.29}$$

Thus by (6.25) we deduce that the approximations of $\mathbf{E}$ and $\mathbf{B}$ satisfy:

$$\begin{cases} \mathbf{E}' = \mathbf{E} - \frac{1}{c}\mathbf{w} \times \mathbf{B}, \\ \mathbf{B}' = \mathbf{B}. \end{cases} \tag{6.30}$$

Then plugging it into (6.24), we deduce that the approximate solutions in the case of quasistationary fields in a slow aether satisfy the same transformation (3.13) as the exact solutions i.e.

$$\begin{cases} \mathbf{D}' = \mathbf{D}, \\ \mathbf{B}' = \mathbf{B}, \\ \mathbf{E}' = \mathbf{E} - \frac{1}{c}\mathbf{w} \times \mathbf{B}, \\ \mathbf{H}' = \mathbf{H} + \frac{1}{c}\mathbf{w} \times \mathbf{D}. \end{cases} \tag{6.31}$$

6.4 Non-inertial coordinate systems

Consider the change of certain non-inertial coordinate system $(*)$ to another coordinate system $(**)$:

$$\begin{cases} \mathbf{x}' = A(t) \cdot \mathbf{x} + \mathbf{z}(t), \\ t' = t, \end{cases}$$

where $A(t) \in SO(3)$ is a rotation. Then, as before, denoting $\mathbf{w}(t) = \mathbf{z}'(t)$, we have:

$$\begin{cases} \rho' = \rho, \\ \mathbf{v}' = A(t) \cdot \mathbf{v} + A'(t) \cdot \mathbf{x} + \mathbf{w}(t), \\ \mathbf{j}' = A(t) \cdot \mathbf{j} + \rho A'(t) \cdot \mathbf{x} + \rho\mathbf{w}(t) \end{cases} \tag{6.32}$$

Next let ψ_0 and ψ_0' be the Coulomb's potentials in the systems $(**)$ and $(*)$, that satisfy

$$\begin{cases} -\Delta_{\mathbf{x}}\psi_0 \equiv 4\pi\rho, \\ -\Delta_{\mathbf{x}'}\psi_0' \equiv 4\pi\rho'. \end{cases} \tag{6.33}$$

Then, using (4.17) and the facts that $\rho' = \rho$ and $A^{-1}(t) = A^T(t)$, we deduce from (6.33) that,

$$\begin{cases} \psi_0' = \psi_0, \\ \nabla_{\mathbf{x}'}\psi_0' = A(t)\cdot\nabla_{\mathbf{x}}\psi_0. \end{cases} \tag{6.34}$$

Thus, plugging (4.22), (6.34) and (6.32) and using the fact that $-\Delta_{\mathbf{x}}\psi_0 = 4\pi\rho$, we deduce

$$\widetilde{\mathbf{j}}' = A(t)\cdot\widetilde{\mathbf{j}}. \tag{6.35}$$

where

$$\begin{cases} \widetilde{\mathbf{j}} := \mathbf{j} - \frac{1}{4\pi}\frac{\partial}{\partial t}\big(\nabla_{\mathbf{x}}\psi_0\big) + \frac{1}{4\pi}curl_{\mathbf{x}}\big(\mathbf{v}\times\nabla_{\mathbf{x}}\psi_0\big), \\ \widetilde{\mathbf{j}}' := \mathbf{j}' - \frac{1}{4\pi}\frac{\partial}{\partial t'}\big(\nabla_{\mathbf{x}'}\psi_0'\big) + \frac{1}{4\pi}curl_{\mathbf{x}'}\big(\mathbf{v}'\times\nabla_{\mathbf{x}'}\psi_0'\big). \end{cases} \tag{6.36}$$

Moreover, $div_{\mathbf{x}'}\widetilde{\mathbf{j}}' = div_{\mathbf{x}}\widetilde{\mathbf{j}} = 0$. Next let $\mathbf{A}, \Psi$ be solutions of

$$\begin{cases} -\Delta_{\mathbf{x}}\mathbf{A} = \frac{4\pi}{c}\widetilde{\mathbf{j}}, \\ -\Delta_{\mathbf{x}}\Psi = 4\pi\rho - \frac{1}{c}\,div_{\mathbf{x}}\big(\mathbf{v}\times curl_{\mathbf{x}}\mathbf{A}\big), \end{cases} \tag{6.37}$$

and

$$\begin{cases} \mathbf{E} = -\nabla_{\mathbf{x}}\Psi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} \\ \mathbf{B} = curl_{\mathbf{x}}\mathbf{A} \\ \mathbf{D} = \mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\mathbf{B} \\ \mathbf{H} = \mathbf{B} + \frac{1}{c}\,\mathbf{v}\times\mathbf{D}, \end{cases} \tag{6.38}$$

and let and $\mathbf{A}', \Psi'$ be solutions of

$$\begin{cases} -\Delta_{\mathbf{x}'}\mathbf{A}' = \frac{4\pi}{c}\widetilde{\mathbf{j}}', \\ -\Delta_{\mathbf{x}'}\Psi' = 4\pi\rho' - \frac{1}{c}\,div_{\mathbf{x}'}\big(\mathbf{v}'\times curl_{\mathbf{x}'}\mathbf{A}'\big). \end{cases} \tag{6.39}$$

and

$$\begin{cases} \mathbf{E}' = -\nabla_{\mathbf{x}'}\Psi' - \frac{1}{c}\frac{\partial\mathbf{A}'}{\partial t'} \\ \mathbf{B}' = curl_{\mathbf{x}'}\mathbf{A}' \\ \mathbf{D}' = \mathbf{E}' + \frac{1}{c}\,\mathbf{v}'\times\mathbf{B}' \\ \mathbf{H}' = \mathbf{B}' + \frac{1}{c}\,\mathbf{v}'\times\mathbf{D}'. \end{cases} \tag{6.40}$$

Then, plugging (6.35) and (4.17) into (6.37) and (6.39), we obtain

$$\mathbf{A}' = A(t)\cdot\mathbf{A}. \tag{6.41}$$

and therefore, by (6.38), (6.40) and (4.17) we have

$$\mathbf{B}' = A(t)\cdot\mathbf{B}. \tag{6.42}$$

Next by (6.37), (6.39) and (6.38), (6.40), using the facts that $div_{\mathbf{x}}\mathbf{A} = 0$ and $div_{\mathbf{x}'}\mathbf{A}' = 0$ we obtain

$$\begin{cases} div_{\mathbf{x}}\mathbf{D} = 4\pi\rho, \\ curl_{\mathbf{x}}\mathbf{D} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} + \frac{1}{c}curl_{\mathbf{x}}\big(\mathbf{v}\times\mathbf{B}\big), \\ div_{\mathbf{x}'}\mathbf{D}' = 4\pi\rho', \\ curl_{\mathbf{x}'}\mathbf{D}' = -\frac{1}{c}\frac{\partial\mathbf{B}'}{\partial t'} + \frac{1}{c}curl_{\mathbf{x}'}\big(\mathbf{v}'\times\mathbf{B}'\big), \\ div_{\mathbf{x}}\mathbf{B} = 0, \\ div_{\mathbf{x}'}\mathbf{B}' = 0. \end{cases} \tag{6.43}$$

Plugging (6.43) and (6.42) into (4.17) and (4.22) we deduce:

$$\begin{cases} div_{\mathbf{x}'}\mathbf{D}' = div_{\mathbf{x}'}\big(A(t)\cdot\mathbf{D}\big), \\ curl_{\mathbf{x}'}\mathbf{D}' = curl_{\mathbf{x}'}\big(A(t)\cdot\mathbf{D}\big). \end{cases} \tag{6.44}$$

Therefore,

$$\mathbf{D}' = A(t)\cdot\mathbf{D}. \tag{6.45}$$

Then using the relations

$$\begin{cases} \mathbf{E} = \mathbf{D} - \frac{1}{c}\,\mathbf{v}\times\mathbf{B} \\ \mathbf{H} = \mathbf{B} + \frac{1}{c}\,\mathbf{v}\times\mathbf{D} \\ \mathbf{E}' = \mathbf{D}' - \frac{1}{c}\,\mathbf{v}'\times\mathbf{B}' \\ \mathbf{H}' = \mathbf{B}' + \frac{1}{c}\,\mathbf{v}'\times\mathbf{D}', \end{cases} \tag{6.46}$$

and (6.45), (6.42) and (6.32), we finally deduce:

$$\begin{cases} \mathbf{D}' = A(t)\cdot\mathbf{D} \\ \mathbf{B}' = A(t)\cdot\mathbf{B} \\ \mathbf{E}' = A(t)\cdot\mathbf{E} - \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{B}\big) \\ \mathbf{H}' = A(t)\cdot\mathbf{H} + \frac{1}{c}\Big(A'(t)\cdot\mathbf{x} + \mathbf{w}(t)\Big)\times\big(A(t)\cdot\mathbf{D}\big), \end{cases} \tag{6.47}$$

So the approximate solutions in the case of quasistationary fields in a slow aether satisfy the same transformation as the exact solutions of Maxwell Equations (see (4.32)). Therefore, if in coordinate system $(**)$ we can use the quasistationary and slow aether approximation, given by (6.39) and (6.40), we can use the approximation, given by (6.37) and (6.38) also in coordinate system $(*)$, even in the case when in system $(*)$ the aether does not move slowly or/and electromagnetic fields are not quasistationary.

7 Aether dynamics

Consider the full system of Maxwell equations in the vacuum of the form:

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H} \equiv \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \\ div_{\mathbf{x}}\mathbf{D} \equiv 4\pi\rho, \\ curl_{\mathbf{x}}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \equiv 0, \\ div_{\mathbf{x}}\mathbf{B} \equiv 0, \\ \mathbf{E} = \mathbf{D} - \frac{1}{c}\mathbf{v}\times\mathbf{B}, \\ \mathbf{H} = \mathbf{B} + \frac{1}{c}\mathbf{v}\times\mathbf{D}. \end{cases} \tag{7.1}$$

We assume the validity of the third law of Newton. Therefore, we assume that the charged test particle acts on the environing aether with the electromagnetic force, opposite to the Lorentz force:

$$-\mathbf{F} = -\sigma\Big(\mathbf{E} + \frac{1}{c}\mathbf{u}\times\mathbf{B}\Big). \tag{7.2}$$

Furthermore, we assume tensions inside the aether i.e. the tensors of surface forces acting inside the aether.

In the Classical Electrodynamics the Maxwell tensor in the vacuum has the form

$$\mathcal{T}_0 := \frac{1}{4\pi}\Big\{\mathbf{E}\otimes\mathbf{E} + \mathbf{B}\otimes\mathbf{B} - \frac{1}{2}\Big(|\mathbf{E}|^2 + |\mathbf{B}|^2\Big)I\Big\} \in \mathbb{R}^{3\times 3}, \tag{7.3}$$

where I is the identity matrix. We assume this formula to be valid only for the point where the velocity of the aether vanishes. Thus since $\mathbf{D}$ and $\mathbf{B}$ are invariant under the change of inertial system of coordinates and the forces must be also invariant, we assume the interaction inside the aether with the tensor of the surface electromagnetic forces of the form

$$\mathcal{T} := \frac{1}{4\pi}\Big\{\mathbf{D}\otimes\mathbf{D} + \mathbf{B}\otimes\mathbf{B} - \frac{1}{2}\Big(|\mathbf{D}|^2 + |\mathbf{B}|^2\Big)I\Big\} \in \mathbb{R}^{3\times 3}. \tag{7.4}$$

Thus by (7.2) and (7.4) we have the electromagnetic force which acts on the aether with the following volume density

$$\mathbf{F} = \frac{1}{4\pi}div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D} + \mathbf{B}\otimes\mathbf{B} - \frac{1}{2}\Big(|\mathbf{D}|^2 + |\mathbf{B}|^2\Big)I\Big\} - \rho\mathbf{E} - \frac{1}{c}\mathbf{j}\times\mathbf{B}. \tag{7.5}$$

Finally one can assume additional non-electromagnetic tension $\mathcal{T}_+$ inside the aether and additional interaction of the aether with other matter which can produce additional non-electromagnetic external force acting on the aether with the volume density $\mathbf{G}_+$. Thus the equations of the motion of the aether in an inertial coordinate system will be

$$\frac{\partial(\mu\mathbf{v})}{\partial t} + div_{\mathbf{x}}\big(\mu\mathbf{v}\otimes\mathbf{v}\big) = \\ \frac{1}{4\pi}div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D} + \mathbf{B}\otimes\mathbf{B} - \frac{1}{2}\Big(|\mathbf{D}|^2 + |\mathbf{B}|^2\Big)I\Big\} - \rho\mathbf{E} - \frac{1}{c}\mathbf{j}\times\mathbf{B} + div_{\mathbf{x}}\mathcal{T}_+ + \mathbf{G}_+, \tag{7.6}$$

where μ is the volume density of the mass of the aether, which satisfies the continuum equation

$$\frac{\partial \mu}{\partial t} + div_{\mathbf{x}}(\mu \mathbf{v}) \equiv 0. \tag{7.7}$$

(in the case of incompressibility hypothesis μ is a constant and thus (7.7) becomes $div_{\mathbf{x}}\mathbf{v} \equiv 0$).

7.1 Conservation of the Energy

By Lemma 8.1 from Appendix we have

$$\frac{\partial}{\partial t}\left(\frac{|\mathbf{D}|^2+|\mathbf{B}|^2}{8\pi}\right)+div_{\mathbf{x}}\left\{\left(\frac{|\mathbf{D}|^2+|\mathbf{B}|^2}{8\pi}\right)\mathbf{v}\right\} = \frac{1}{4\pi}div_{\mathbf{x}}\left\{(\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B})\cdot\mathbf{v}-\frac{1}{2}\left(|\mathbf{D}|^2+|\mathbf{B}|^2\right)\mathbf{v}-c\mathbf{D}\times\mathbf{B}\right\}$$
$$-\left\{\frac{1}{4\pi}\left(div_{\mathbf{x}}\left\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\left(|\mathbf{D}|^2+|\mathbf{B}|^2\right)I\right\}\right)-\left(\rho\mathbf{E}+\frac{1}{c}\mathbf{j}\times\mathbf{B}\right)\right\}\cdot\mathbf{v}-\mathbf{j}\cdot\mathbf{E}. \tag{7.8}$$

Integrating this equality over $\mathbb{R}^3 \times (0, \tau)$ gives

$$\int_{\mathbb{R}^3}\frac{|\mathbf{D}(\mathbf{x},0)|^2+|\mathbf{B}(\mathbf{x},0)|^2}{8\pi}d\mathbf{x}-\int_{\mathbb{R}^3}\frac{|\mathbf{D}(\mathbf{x},\tau)|^2+|\mathbf{B}(\mathbf{x},\tau)|^2}{8\pi}d\mathbf{x}=$$
$$\int_{\mathbb{R}^3}\int_0^\tau \mathbf{j}\cdot\mathbf{E}\,d\mathbf{x}dt+\int_{\mathbb{R}^3}\int_0^\tau\left\{\frac{1}{4\pi}\left(div_{\mathbf{x}}\left\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\left(|\mathbf{D}|^2+|\mathbf{B}|^2\right)I\right\}\right)-\left(\rho\mathbf{E}+\frac{1}{c}\mathbf{j}\times\mathbf{B}\right)\right\}\cdot\mathbf{v}\,d\mathbf{x}dt. \tag{7.9}$$

Observe that the first integral in the right hand side of (7.9) is the total work till the instant τ of the Lorentz forces on all charged bodies and the second integral is the total work of the electromagnetic forces, opposite to the Lorentz forces, on the aether plus the work of the electromagnetic forces of the Maxwell tensions on the aether. Thus the quantity

$$W(t) := \int_{\mathbb{R}^3}\frac{|\mathbf{D}(\mathbf{x},t)|^2+|\mathbf{B}(\mathbf{x},t)|^2}{8\pi}d\mathbf{x} \tag{7.10}$$

is the total potential energy of the electromagnetic interaction.

7.2 Additional conjectures

One of the conjectures that could be made is that the celestial bodies acts on the aether with the forces of gravitation. In this case, consistently with (7.6), we assume that

- **Conjecture I**:

$$\mathbf{G}_+ = \mu(\nabla_{\mathbf{x}}\Psi), \tag{7.11}$$

where, in our case, Ψ is the gravitational potential. This can be one of explanations why the velocity of the aether near the Earth is close to zero as was obtained by Michelson-Morley experiment.

Another possible conjecture is that the aether is an incompressible medium i.e. it satisfies $div_{\mathbf{x}}\mathbf{v} \equiv 0$ and we have an additional pressure p in the aether as in an incompressible fluid. An alternative conjecture is that the aether is compressible and the additional pressure in the aether is a function of μ only i.e. $p := p(\mu)$, like in the case of isothermal compressible fluid. Then, since in the incompressible case $\mu \equiv const$ and in the case $p := p(\mu)$ we have $\nabla_{\mathbf{x}} p = \mu(p'(\mu)/\mu)\nabla_{\mathbf{x}}\mu = \mu(\nabla_{\mathbf{x}} Q)$, in both cases we obtain $div_{\mathbf{x}}\mathcal{T}_+ = \nabla_{\mathbf{x}} p = \mu(\nabla_{\mathbf{x}} Q)$. I.e. the following conjecture is valid,

- **Conjecture II**:

$$div_{\mathbf{x}}\mathcal{T}_+ = \mu(\nabla_{\mathbf{x}} Q). \tag{7.12}$$

In the case of Conjectures I-II the additional force acting on the aether has the form

$$\mathbf{F}_+ = \mu(\nabla_{\mathbf{x}}\Phi), \tag{7.13}$$

where

$$\mathbf{F}_+ = div_{\mathbf{x}}\mathcal{T}_+ + \mathbf{G}_+. \tag{7.14}$$

In the later case we can rewrite (7.6) as

$$\frac{\partial(\mu\mathbf{v})}{\partial t} + div_{\mathbf{x}}\big(\mu\mathbf{v}\otimes\mathbf{v}\big) = \frac{1}{4\pi} div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D} + \mathbf{B}\otimes\mathbf{B} - \frac{1}{2}\Big(|\mathbf{D}|^2 + |\mathbf{B}|^2\Big) I\Big\} - \rho\mathbf{E} - \frac{1}{c}\mathbf{j}\times\mathbf{B} + \mu(\nabla_{\mathbf{x}}\Phi) \tag{7.15}$$

or in another form as

$$\frac{\partial\mathbf{v}}{\partial t} + (d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{v} = \frac{1}{4\pi\mu} div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D} + \mathbf{B}\otimes\mathbf{B} - \frac{1}{2}\Big(|\mathbf{D}|^2 + |\mathbf{B}|^2\Big) I\Big\} - \frac{1}{\mu}\Big(\rho\mathbf{E} + \frac{1}{c}\mathbf{j}\times\mathbf{B}\Big) + \nabla_{\mathbf{x}}\Phi. \tag{7.16}$$

Remark 7.1. If we assume the hypothesis of the incompressible aether, then its motion cannot be affected by the force of gravitation and one can meet certain difficulties with the explanation of Michelson-Morley experiment. In the literature one can meet the incorrect statement that the hypothesis of the compressible aether will imply the essential dependence of the constant c in the Maxwell Equations on the density of the aether. However, we can consider the following analogy: as it is well known the speed of light in the static air vary insignificantly with the change of its density and it is very close to the constant c. Thus, one can assume that in the compressible aether the dependence on the density for the coefficient c of the Maxwell Equations in the vacuum can be also insignificant. One of the explanations of this could be in the assumption of the very heavy aether.

7.3 Estimation of $curl_{\mathbf{x}}\mathbf{v}$

Proposition 7.1. *Let* $\mathbf{D}, \mathbf{B}, \mathbf{E}, \mathbf{H}, \rho, \mathbf{j}, \mathbf{v}$ *be solutions of* (7.1) μ *is a mass density of the aether, satisfying* (7.7)*, and the velocity of the aether satisfies* (7.16)*. Consider the quantity*

$$\mathbf{L} := \frac{1}{4\pi c\mu}\mathbf{D}\times\mathbf{B} - \mathbf{v}. \tag{7.17}$$

Then

$$\frac{\partial\mathbf{L}}{\partial t} - \mathbf{v}\times(curl_{\mathbf{x}}\mathbf{L}) + \nabla_{\mathbf{x}}\Big(\frac{1}{2}|\mathbf{v}|^2 + \mathbf{v}\cdot\mathbf{L} + \Phi\Big) = 0. \tag{7.18}$$

In particular,

$$\text{the assumption that}\quad curl_{\mathbf{x}}\left(\frac{1}{4\pi c\mu(\mathbf{x},0)}\,\mathbf{D}(\mathbf{x},0)\times\mathbf{B}(\mathbf{x},0)\right)\equiv curl_{\mathbf{x}}\mathbf{v}(\mathbf{x},0)\quad\text{implies}$$

$$curl_{\mathbf{x}}\left(\frac{1}{4\pi c\mu(\mathbf{x},t)}\,\mathbf{D}(\mathbf{x},t)\times\mathbf{B}(\mathbf{x},t)\right)\equiv curl_{\mathbf{x}}\mathbf{v}(\mathbf{x},t)\quad\text{for every instant } t>0. \tag{7.19}$$

Moreover, in the latter case there exists $\Theta(\mathbf{x},t):\mathbb{R}^3\times[0,+\infty)\to\mathbb{R}$ *such that*

$$\mathbf{v}(\mathbf{x},t)=\frac{1}{4\pi c\mu(\mathbf{x},t)}\,\mathbf{D}(\mathbf{x},t)\times\mathbf{B}(\mathbf{x},t)+\nabla_{\mathbf{x}}\Theta(\mathbf{x},t)\qquad\forall\,(\mathbf{x},t)\in\mathbb{R}^3\times(0,+\infty), \tag{7.20}$$

and $\Theta(\mathbf{x},t)$ *is a solution of*

$$\frac{\partial\Theta}{\partial t}+\frac{1}{2}\big|\nabla_{\mathbf{x}}\Theta\big|^2=\frac{1}{2}\left|\frac{1}{4\pi c\mu}\,\mathbf{D}\times\mathbf{B}\right|^2+\Phi\qquad\forall\,(\mathbf{x},t)\in\mathbb{R}^3\times(0,+\infty). \tag{7.21}$$

Proof. By Lemma 8.2 we have

$$\frac{\partial}{\partial t}\Big(\frac{1}{4\pi c}\,\mathbf{D}\times\mathbf{B}\Big)+div_{\mathbf{x}}\Big\{\Big(\frac{1}{4\pi c}\mathbf{D}\times\mathbf{B}\Big)\otimes\mathbf{v}\Big\}=\\-(d_{\mathbf{x}}\mathbf{v})^T\cdot\Big(\frac{1}{4\pi c}\mathbf{D}\times\mathbf{B}\Big)+\frac{1}{4\pi}div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)I\Big\}-\Big(\rho\mathbf{E}+\frac{1}{c}\mathbf{j}\times\mathbf{B}\Big). \tag{7.22}$$

Therefore, by (7.7) and (7.22) we deduce

$$\frac{\partial}{\partial t}\Big(\frac{1}{4\pi c\mu}\,\mathbf{D}\times\mathbf{B}\Big)+\Big(d_{\mathbf{x}}\Big\{\frac{1}{4\pi c\mu}\mathbf{D}\times\mathbf{B}\Big\}\Big)\cdot\mathbf{v}=\\-(d_{\mathbf{x}}\mathbf{v})^T\cdot\Big(\frac{1}{4\pi c\mu}\mathbf{D}\times\mathbf{B}\Big)+\frac{1}{4\pi\mu}div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)I\Big\}-\frac{1}{\mu}\Big(\rho\mathbf{E}+\frac{1}{c}\mathbf{j}\times\mathbf{B}\Big). \tag{7.23}$$

Then subtracting (7.16) from (7.23) we deduce

$$\frac{\partial\mathbf{L}}{\partial t}+(d_{\mathbf{x}}\mathbf{L})\cdot\mathbf{v}=-(d_{\mathbf{x}}\mathbf{v})^T\cdot\mathbf{L}-\nabla_{\mathbf{x}}\Big(\frac{1}{2}|\mathbf{v}|^2+\Phi\Big)=(d_{\mathbf{x}}\mathbf{L})^T\cdot\mathbf{v}-\nabla_{\mathbf{x}}\Big(\frac{1}{2}|\mathbf{v}|^2+\mathbf{v}\cdot\mathbf{L}+\Phi\Big). \tag{7.24}$$

Thus using (2.11) we can rewrite (7.24) as (7.18) i.e.

$$\frac{\partial\mathbf{L}}{\partial t}-\mathbf{v}\times(curl_{\mathbf{x}}\mathbf{L})+\nabla_{\mathbf{x}}\Big(\frac{1}{2}|\mathbf{v}|^2+\mathbf{v}\cdot\mathbf{L}+\Phi\Big)=0. \tag{7.25}$$

Next assume that $curl_{\mathbf{x}}\mathbf{L}(\mathbf{x},0)\equiv 0$. Let $V(\mathbf{x},t)$ be a solution to $\Delta_{\mathbf{x}}V(x,t)=div_{\mathbf{x}}\mathbf{L}(x,t)$, which satisfies $V(\mathbf{x},t)\to 0$ as $|\mathbf{x}|\to+\infty$. Then set $\mathbf{R}(\mathbf{x},t)=\mathbf{L}(\mathbf{x},t)-\nabla_{\mathbf{x}}V(\mathbf{x},t)$. Thus $div_{\mathbf{x}}\mathbf{R}\equiv 0$ and $curl_{\mathbf{x}}\mathbf{R}=curl_{\mathbf{x}}\mathbf{L}$. In particular $\mathbf{R}(\mathbf{x},0)\equiv 0$. Moreover, by (7.25) $\mathbf{R}$ satisfies

$$\frac{\partial\mathbf{R}}{\partial t}-\mathbf{v}\times(curl_{\mathbf{x}}\mathbf{R})+\nabla_{\mathbf{x}}\Big(\frac{1}{2}|\mathbf{v}|^2+\mathbf{v}\cdot\mathbf{L}+\Phi+\partial_tV\Big)=0. \tag{7.26}$$

Thus since $div_{\mathbf{x}}\mathbf{R} \equiv 0$ and $\mathbf{R}(x,0) \equiv 0$, using (2.12) and (7.26) we deduce

$$\frac{1}{2}\int_{\mathbb{R}^3}\big|\mathbf{R}(\mathbf{x},\tau)\big|^2 d\mathbf{x} + \int_0^\tau\int_{\mathbb{R}^3}\left\{\mathbf{R}\cdot\big((d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{R}\big) - \frac{1}{2}|\mathbf{R}|^2 div_{\mathbf{x}}\mathbf{v}\right\} d\mathbf{x}dt =$$
$$\frac{1}{2}\int_{\mathbb{R}^3}\big|\mathbf{R}(\mathbf{x},\tau)\big|^2 d\mathbf{x} + \int_0^\tau\int_{\mathbb{R}^3} div_{\mathbf{x}}\Big(\frac{1}{2}|\mathbf{R}|^2 I - \mathbf{R}\otimes\mathbf{R}\Big)\cdot\mathbf{v}\, d\mathbf{x}dt =$$
$$\frac{1}{2}\int_{\mathbb{R}^3}\big|\mathbf{R}(\mathbf{x},\tau)\big|^2 d\mathbf{x} + \int_0^\tau\int_{\mathbb{R}^3}\Big(\mathbf{R}\times curl_{\mathbf{x}}\mathbf{R}\Big)\cdot\mathbf{v}\, d\mathbf{x}dt =$$
$$\int_0^\tau\int_{\mathbb{R}^3}\left\{\frac{\partial\mathbf{R}}{\partial t} - \mathbf{v}\times(curl_{\mathbf{x}}\mathbf{R})\right\}\cdot\mathbf{R}\, d\mathbf{x}dt =$$
$$-\int_0^\tau\int_{\mathbb{R}^3}\mathbf{R}\cdot\nabla_{\mathbf{x}}\Big(\frac{1}{2}|\mathbf{v}|^2 + \mathbf{v}\cdot\mathbf{L} + \Phi + \partial_t V\Big)d\mathbf{x}dt = \int_0^\tau\int_{\mathbb{R}^3}\Big(\frac{1}{2}|\mathbf{v}|^2 + \mathbf{v}\cdot\mathbf{L} + \Phi + \partial_t V\Big)div_{\mathbf{x}}\mathbf{R}\, d\mathbf{x}dt = 0. \tag{7.27}$$

Therefore, if we assume $\mathbf{v}$ to be a smooth vector field, by (7.27) we deduce that for every instant $\tau_0 > 0$ there exists a constant C such that

$$\int_{\mathbb{R}^3}\big|\mathbf{R}(\mathbf{x},\tau)\big|^2 d\mathbf{x} \le C\int_0^\tau\int_{\mathbb{R}^3}\big|\mathbf{R}(\mathbf{x},t)\big|^2 d\mathbf{x}dt \qquad \text{for every } \tau \le \tau_0. \tag{7.28}$$

Setting

$$f(\tau) := \int_0^\tau\int_{\mathbb{R}^3}\big|\mathbf{R}(\mathbf{x},t)\big|^2 d\mathbf{x}dt \tag{7.29}$$

we deduce $f(0) = 0$ and

$$\frac{df}{d\tau}(\tau) - Cf(\tau) \le 0 \qquad \text{for every } \tau \le \tau_0. \tag{7.30}$$

Thus

$$\frac{d}{d\tau}\Big\{f(\tau)e^{-C\tau}\Big\} \le 0 \qquad \text{for every } \tau \le \tau_0. \tag{7.31}$$

Thus $f(\tau)e^{-C\tau}$ is a nondecreasing function and therefore, $f(\tau)e^{-C\tau} \le f(0) = 0$. So

$$\int_0^\tau\int_{\mathbb{R}^3}\big|\mathbf{R}(\mathbf{x},t)\big|^2 d\mathbf{x}dt \le 0 \qquad \text{for every } \tau \le \tau_0 \tag{7.32}$$

and this is possible only if

$$\mathbf{R}(\mathbf{x},\tau) \equiv 0 \qquad \text{for every } \mathbf{x} \in \mathbb{R}^3 \text{ and every } \tau \le \tau_0. \tag{7.33}$$

Since by the definition $curl_{\mathbf{x}}\mathbf{L} \equiv curl_{\mathbf{x}}\mathbf{R} \equiv 0$, (7.19) follows for $t \le \tau_0$. Finally since τ_0 was chosen arbitrary (7.19) follows.

Next assume that for every $(\mathbf{x}, t)$ we have $curl_{\mathbf{x}}\mathbf{L} = 0$. Then there exists $\tilde{\Theta}(\mathbf{x}, t) : \mathbb{R}^3 \times [0, +\infty) \to \mathbb{R}$ such that $L(\mathbf{x}, t) \equiv -\nabla_{\mathbf{x}}\tilde{\Theta}(\mathbf{x}, t)$ i.e.

$$\mathbf{v} = \frac{1}{4\pi c\mu}\mathbf{D}\times\mathbf{B} + \nabla_{\mathbf{x}}\tilde{\Theta}, \tag{7.34}$$

and (7.18) reads

$$\frac{\partial(\nabla_{\mathbf{x}}\tilde{\Theta})}{\partial t} + \nabla_{\mathbf{x}}\Big(\mathbf{v}\cdot\nabla_{\mathbf{x}}\tilde{\Theta} - \frac{1}{2}|\mathbf{v}|^2 - \Phi\Big) = 0. \tag{7.35}$$

I.e. there exists a function $\sigma(t):\mathbb{R}\to\mathbb{R}$ of t only, such that

$$\frac{\partial\tilde{\Theta}}{\partial t}+\Big(\mathbf{v}\cdot\nabla_{\mathbf{x}}\tilde{\Theta}-\frac{1}{2}|\mathbf{v}|^2-\Phi\Big)=\sigma(t), \tag{7.36}$$

Then plugging (7.34) into (7.36) gives

$$\frac{\partial\tilde{\Theta}}{\partial t}+\left(\frac{1}{2}\big|\nabla_{\mathbf{x}}\tilde{\Theta}\big|^2-\frac{1}{2}\left|\frac{1}{4\pi c\mu}\mathbf{D}\times\mathbf{B}\right|^2-\Phi\right)=\sigma(t). \tag{7.37}$$

Next denote $\Theta(\mathbf{x},t):=\tilde{\Theta}(\mathbf{x},t)-\int_0^t\sigma(s)ds$. Then since $\nabla_{\mathbf{x}}\Theta\equiv\nabla_{\mathbf{x}}\tilde{\Theta}$ and $\frac{\partial\Theta}{\partial t}\equiv\frac{\partial\tilde{\Theta}}{\partial t}-\sigma(t)$, from (7.34) and (7.37) we obtain (7.20) and (7.21). □

8 Appendix

Consider the system:

$$\begin{cases} curl_{\mathbf{x}}\mathbf{H}\equiv\frac{4\pi}{c}\mathbf{j}+\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t},\\ div_{\mathbf{x}}\mathbf{D}\equiv 4\pi\rho,\\ curl_{\mathbf{x}}\mathbf{E}+\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\equiv 0,\\ div_{\mathbf{x}}\mathbf{B}\equiv 0,\\ \mathbf{E}=\mathbf{D}-\frac{1}{c}\mathbf{v}\times\mathbf{B},\\ \mathbf{H}=\mathbf{B}+\frac{1}{c}\mathbf{v}\times\mathbf{D}.\end{cases} \tag{8.1}$$

Lemma 8.1. *Let* $\mathbf{D},\mathbf{B},\mathbf{E},\mathbf{H},\rho,\mathbf{j},\mathbf{v}$ *be solutions of* (8.1). *Then*

$$\frac{\partial}{\partial t}\left(\frac{|\mathbf{D}|^2+|\mathbf{B}|^2}{8\pi}\right)+div_{\mathbf{x}}\left\{\left(\frac{|\mathbf{D}|^2+|\mathbf{B}|^2}{8\pi}\right)\mathbf{v}\right\}=\frac{1}{4\pi}div_{\mathbf{x}}\left\{(\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B})\cdot\mathbf{v}-\frac{1}{2}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)\mathbf{v}-c\mathbf{D}\times\mathbf{B}\right\}$$
$$-\left\{\frac{1}{4\pi}\left(div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)I\Big\}\right)-\Big(\rho\mathbf{E}+\frac{1}{c}\mathbf{j}\times\mathbf{B}\Big)\right\}\cdot\mathbf{v}-\mathbf{j}\cdot\mathbf{E}, \tag{8.2}$$

where I is the identity matrix.

Proof. By (8.1) and (2.4) we infer:

$$\frac{1}{2c}\frac{\partial}{\partial t}\Big(|\mathbf{D}|^2+|\mathbf{B}|^2\Big)=\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}\cdot\mathbf{D}+\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\cdot\mathbf{B}=\Big(curl_{\mathbf{x}}\mathbf{H}-\frac{4\pi}{c}\mathbf{j}\Big)\cdot\mathbf{D}-(curl_{\mathbf{x}}\mathbf{E})\cdot\mathbf{B}=$$
$$\Big\{curl_{\mathbf{x}}\Big(\mathbf{B}+\frac{1}{c}\mathbf{v}\times\mathbf{D}\Big)\Big\}\cdot\mathbf{D}-\Big\{curl_{\mathbf{x}}\Big(\mathbf{D}-\frac{1}{c}\mathbf{v}\times\mathbf{B}\Big)\Big\}\cdot\mathbf{B}-\frac{4\pi}{c}\mathbf{j}\cdot\mathbf{D}=$$
$$\frac{1}{c}\mathbf{D}\cdot curl_{\mathbf{x}}(\mathbf{v}\times\mathbf{D})+\frac{1}{c}\mathbf{B}\cdot curl_{\mathbf{x}}(\mathbf{v}\times\mathbf{B})+\mathbf{D}\cdot curl_{\mathbf{x}}\mathbf{B}-\mathbf{B}\cdot curl_{\mathbf{x}}\mathbf{D}-\frac{4\pi}{c}\mathbf{j}\cdot\mathbf{D}=$$
$$\frac{1}{c}\mathbf{D}\cdot curl_{\mathbf{x}}(\mathbf{v}\times\mathbf{D})+\frac{1}{c}\mathbf{B}\cdot curl_{\mathbf{x}}(\mathbf{v}\times\mathbf{B})-div_{\mathbf{x}}(\mathbf{D}\times\mathbf{B})-\frac{4\pi}{c}\mathbf{j}\cdot\mathbf{D}. \tag{8.3}$$

On the other hand, by (2.9) and (8.1) we obtain

$$
\begin{aligned}
&\frac{1}{c}\mathbf{D}\cdot curl_{\mathbf{x}}(\mathbf{v}\times\mathbf{D})+\frac{1}{c}\,\mathbf{B}\cdot curl_{\mathbf{x}}(\mathbf{v}\times\mathbf{B})=\\
&\qquad\frac{1}{c}(div_{\mathbf{x}}\mathbf{D})\,\mathbf{v}\cdot\mathbf{D}-\frac{1}{c}(div_{\mathbf{x}}\mathbf{v})|\mathbf{D}|^2+\frac{1}{c}\mathbf{D}\cdot\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{D}\Big\}-\frac{1}{2c}\mathbf{v}\cdot\nabla_{\mathbf{x}}|\mathbf{D}|^2\\
&\qquad+(div_{\mathbf{x}}\mathbf{B})\frac{1}{c}\,\mathbf{v}\cdot\mathbf{B}-(div_{\mathbf{x}}\mathbf{v})\frac{1}{c}|\mathbf{B}|^2+\frac{1}{c}\,\mathbf{B}\cdot\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{B}\Big\}-\frac{1}{2c}\mathbf{v}\cdot\nabla_{\mathbf{x}}|\mathbf{B}|^2=\\
&\frac{4\pi\rho}{c}\mathbf{v}\cdot\mathbf{D}-(div_{\mathbf{x}}\mathbf{v})\frac{1}{c}\Big(|\mathbf{D}|^2+|\mathbf{B}|^2\Big)+\frac{1}{c}\,\mathbf{B}\cdot\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{B}\Big\}+\frac{1}{c}\mathbf{D}\cdot\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{D}\Big\}-\frac{1}{2c}\Big\{\mathbf{v}\cdot\nabla_{\mathbf{x}}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)\Big\}\\
&=\frac{4\pi\rho}{c}\mathbf{v}\cdot\mathbf{D}-\frac{1}{c}\left(div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)I\Big\}\right)\cdot\mathbf{v}+\frac{1}{c}div_{\mathbf{x}}\Big\{(\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B})\cdot\mathbf{v}-\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)\mathbf{v}\Big\}.
\end{aligned}
\tag{8.4}
$$

Therefore,

$$
\begin{aligned}
\frac{1}{2c}\frac{\partial}{\partial t}\Big(|\mathbf{D}|^2+|\mathbf{B}|^2\Big)+\frac{1}{2c}div_{\mathbf{x}}\Big\{\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)\mathbf{v}\Big\}&=\frac{1}{c}div_{\mathbf{x}}\Big\{(\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B})\cdot\mathbf{v}-\frac{1}{2}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)\mathbf{v}-c\mathbf{D}\times\mathbf{B}\Big\}\\
&-\frac{1}{c}\left(div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)I\Big\}\right)\cdot\mathbf{v}-\frac{4\pi}{c}(\mathbf{j}-\rho\mathbf{v})\cdot\mathbf{D}.
\end{aligned}
\tag{8.5}
$$

Thus, since

$$
(\mathbf{j}-\rho\mathbf{v})\cdot\mathbf{D}=(\mathbf{j}-\rho\mathbf{v})\cdot\Big(\mathbf{E}+\frac{1}{c}\,\mathbf{v}\times\mathbf{B}\Big)=\mathbf{j}\cdot\mathbf{E}-\mathbf{v}\cdot\Big(\rho\mathbf{E}+\frac{1}{c}\,\mathbf{j}\times\mathbf{B}\Big),
\tag{8.6}
$$

we rewrite (8.5) in the form (8.2). □

Lemma 8.2. *Let* $\mathbf{D},\mathbf{B},\mathbf{E},\mathbf{H},\rho,\mathbf{j},\mathbf{v}$ *be solutions of* (8.1). *Then*

$$
\begin{aligned}
&\frac{\partial}{\partial t}\Big(\frac{1}{4\pi c}\,\mathbf{D}\times\mathbf{B}\Big)+div_{\mathbf{x}}\Big\{\Big(\frac{1}{4\pi c}\mathbf{D}\times\mathbf{B}\Big)\otimes\mathbf{v}\Big\}=\\
&\quad-(d_{\mathbf{x}}\mathbf{v})^T\cdot\Big(\frac{1}{4\pi c}\mathbf{D}\times\mathbf{B}\Big)+\frac{1}{4\pi}div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)I\Big\}-\Big(\rho\mathbf{E}+\frac{1}{c}\mathbf{j}\times\mathbf{B}\Big).
\end{aligned}
\tag{8.7}
$$

Proof. By (8.1) we have:

$$
\begin{aligned}
\frac{\partial}{\partial t}\Big(\frac{1}{c}\,\mathbf{D}\times\mathbf{B}\Big)&=\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}\times\mathbf{B}+\mathbf{D}\times\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}=\Big(curl_{\mathbf{x}}\mathbf{H}-\frac{4\pi}{c}\mathbf{j}\Big)\times\mathbf{B}-\mathbf{D}\times curl_{\mathbf{x}}\mathbf{E}=\\
&curl_{\mathbf{x}}\Big(\mathbf{B}+\frac{1}{c}\,\mathbf{v}\times\mathbf{D}\Big)\times\mathbf{B}-\mathbf{D}\times curl_{\mathbf{x}}\Big(\mathbf{D}-\frac{1}{c}\,\mathbf{v}\times\mathbf{B}\Big)-\frac{4\pi}{c}\mathbf{j}\times\mathbf{B}=\\
&\frac{1}{c}\,\mathbf{D}\times curl_{\mathbf{x}}(\mathbf{v}\times\mathbf{B})+\frac{1}{c}curl_{\mathbf{x}}(\mathbf{v}\times\mathbf{D})\times\mathbf{B}-\mathbf{D}\times curl_{\mathbf{x}}\mathbf{D}-\mathbf{B}\times curl_{\mathbf{x}}\mathbf{B}-\frac{4\pi}{c}\mathbf{j}\times\mathbf{B}.
\end{aligned}
\tag{8.8}
$$

On the other hand, by (2.9) and (8.1) we obtain

$$
\begin{aligned}
&\frac{1}{c}\,\mathbf{D}\times curl_{\mathbf{x}}(\mathbf{v}\times\mathbf{B})+\frac{1}{c}curl_{\mathbf{x}}(\mathbf{v}\times\mathbf{D})\times\mathbf{B}=\\
&\qquad(div_{\mathbf{x}}\mathbf{B})\frac{1}{c}\,\mathbf{D}\times\mathbf{v}-(div_{\mathbf{x}}\mathbf{v})\frac{1}{c}\,\mathbf{D}\times\mathbf{B}+\frac{1}{c}\,\mathbf{D}\times\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{B}\Big\}-\frac{1}{c}\,\mathbf{D}\times\Big\{(d_{\mathbf{x}}\mathbf{B})\cdot\mathbf{v}\Big\}\\
&\qquad+\frac{1}{c}(div_{\mathbf{x}}\mathbf{D})\,\mathbf{v}\times\mathbf{B}-\frac{1}{c}(div_{\mathbf{x}}\mathbf{v})\,\mathbf{D}\times\mathbf{B}+\frac{1}{c}\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{D}\Big\}\times\mathbf{B}-\frac{1}{c}\Big\{(d_{\mathbf{x}}\mathbf{D})\cdot\mathbf{v}\Big\}\times\mathbf{B}=\\
&\frac{1}{c}\,\mathbf{D}\times\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{B}\Big\}+\frac{1}{c}\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{D}\Big\}\times\mathbf{B}-2(div_{\mathbf{x}}\mathbf{v})\frac{1}{c}\,\mathbf{D}\times\mathbf{B}-\frac{1}{c}\big\{d_{\mathbf{x}}(\mathbf{D}\times\mathbf{B})\big\}\cdot\mathbf{v}+\frac{4\pi\rho}{c}\,\mathbf{v}\times\mathbf{B}=\\
&\frac{1}{c}\,\mathbf{D}\times\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{B}\Big\}+\frac{1}{c}\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{D}\Big\}\times\mathbf{B}-(div_{\mathbf{x}}\mathbf{v})\frac{1}{c}\,\mathbf{D}\times\mathbf{B}-\frac{1}{c}div_{\mathbf{x}}\Big\{(\mathbf{D}\times\mathbf{B})\otimes\mathbf{v}\Big\}+\frac{4\pi\rho}{c}\,\mathbf{v}\times\mathbf{B},
\end{aligned}
\tag{8.9}
$$

and by (2.12) and (8.1) we deduce

$$
\begin{aligned}
-\,\mathbf{D}\times curl_{\mathbf{x}}\mathbf{D}-\mathbf{B}\times curl_{\mathbf{x}}\mathbf{B}&=(d_{\mathbf{x}}\mathbf{D})\cdot\mathbf{D}-\frac{1}{2}\nabla_{\mathbf{x}}|\mathbf{D}|^2+(d_{\mathbf{x}}\mathbf{B})\cdot\mathbf{B}-\frac{1}{2}\nabla_{\mathbf{x}}|\mathbf{B}|^2\\
&=div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)I\Big\}-4\pi\rho\mathbf{D},
\end{aligned}
\tag{8.10}
$$

where $I\in\mathbb{R}^{3\times3}$ is the unit matrix (identity linear operator). Thus, plugging (8.9) and (8.10) into (8.8) and using (2.3), we obtain

$$
\begin{aligned}
&\frac{\partial}{\partial t}\Big(\frac{1}{c}\,\mathbf{D}\times\mathbf{B}\Big)+div_{\mathbf{x}}\Big\{\Big(\frac{1}{c}\mathbf{D}\times\mathbf{B}\Big)\otimes\mathbf{v}\Big\}=\frac{1}{c}\,\mathbf{D}\times\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{B}\Big\}+\frac{1}{c}\Big\{(d_{\mathbf{x}}\mathbf{v})\cdot\mathbf{D}\Big\}\times\mathbf{B}-(div_{\mathbf{x}}\mathbf{v})\frac{1}{c}\,\mathbf{D}\times\mathbf{B}\\
&\qquad+div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)I\Big\}-4\pi\rho\mathbf{D}-\frac{4\pi}{c}(\mathbf{j}-\rho\mathbf{v})\times\mathbf{B}\\
&\quad=-\frac{1}{c}(d_{\mathbf{x}}\mathbf{v})^T\cdot(\mathbf{D}\times\mathbf{B})+div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\big(|\mathbf{D}|^2+|\mathbf{B}|^2\big)I\Big\}-4\pi\rho\mathbf{E}-\frac{4\pi}{c}\mathbf{j}\times\mathbf{B}=\\
&\frac{1}{c}\big\{d_{\mathbf{x}}(\mathbf{D}\times\mathbf{B})\big\}^T\cdot\mathbf{v}+div_{\mathbf{x}}\Big\{\mathbf{D}\otimes\mathbf{D}+\mathbf{B}\otimes\mathbf{B}-\frac{1}{2}\Big(|\mathbf{D}|^2+|\mathbf{B}|^2+\frac{2}{c}\mathbf{v}\cdot(\mathbf{D}\times\mathbf{B})\Big)I\Big\}-4\pi\rho\mathbf{E}-\frac{4\pi}{c}\mathbf{j}\times\mathbf{B}.
\end{aligned}
\tag{8.11}
$$

□

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