Photon superluminal flow in the de Laval nozzle

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Abstract

We determine the velocity of the photon outflow from the blackbody in the de Laval nozzle. Derivation is based on the Saint-Venant-Wantzel equation for the thermodynamic of the blackbody photon gas and on the Einstein relation between energy and mass. The application of derived results in photon rockets for the interplanetary, interstellar and intergalactic missions is not excluded.

Key words: Euler equation, Bernoulli equation, thermodynamics, blackbody photon gas, de Laval nozzle.

1 Introduction

Some authors submitted ideas on the velocities of physical objects which are greater that the velocity of light. Let us remember particles called tachyons, existing only with superluminal velocities with imaginary mass. Or, the spot of laser light sweeping across a distant object which can easily be made to move across the object at a speed greater than c. Similarly, a shadow projected onto a distant object can be made to move across the object faster than c. However, in neither case does the light travel from the source to the object faster than c, nor does any information travel faster than light.

On the other hand the group velocity of a wave (e.g., a light beam) is exceeding c in some circumstances. In such cases, which typically at the same time involve rapid attenuation of the intensity, the maximum of the envelope of a pulse may travel with a velocity above c. However, this situation does not imply the propagation of signals with a velocity above c (Brillouin, 1960).

The further example is the Hartman tunneling effect through a barrier where the tunneling time tends to a constant for large barriers (Hartman, 1962; Winful, 2006).

In OPERA experiment in September 22, 2011, (Adam et al., 20011) neutrino anomaly was observed indicating detection of 17 and 28 GeV muon neutrinos, sent 730 kilometers (454 miles) from CERN to the Gran Sasso National Laboratory in Italy, traveling faster than light by a factor of 2.48105 (approximately 1 in 40,000). Later, the OPERA team reported two flaws in their equipment set-up that had caused errors far outside of their original confidence interval: a fiber optic cable attached improperly, which caused the apparently faster-than-light measurements, and a clock oscillator ticking too fast.

The surprising was the experiment, where Wallace Kantor (1962) reported experimental findings that "the relative speed of light is dependent on the uniform motion of the source", which contradicted the postulates of the special theory of relativity (STR). It was tested by Babcock and Bergman (1964), but with rotating glass plates placed in a vacuum. The result was different from the Kantor experiment but consistent with STR. A 0.02 fringe shift was still found. The recent theory by Jackson and Minkowski (2012) represents explanation of the fringe shift.

The Čerenkov effect,(including massive photons (Pardy, 2002)), is confirmation of the superluminal velocity. However, only for charged particle, moving in a dielectric medium, where the phase velocity of the electromagnetic signal is less than the velocity of light in vacuum and this fact enables the existence of the particle velocities greater that the velocities of light in such medium.

The supersonic velocities of gas flow in the de Laval nozzle leads to the idea to investigate an analogue situation with photon gas to consider the possibility of the superluminal motion of photon gas in the de Laval nozzle. So, we will consider here the analogue between the supersonic gas and the superluminal photons. We deal with the physics of the classical fluids and then elaborate the adequate theory of the photon gas.

The phenomena considered in fluid dynamics are macroscopic, and it means that fluid is regarded as a continuous medium. This means that any small volume element in the fluid is always supposed so large that it still contains a very great number of molecules. Accordingly, when we speak of infinitely small elements of volume, we shall always mean those which are "physically" infinitely small, i.e. very small compared with the volume of the body under consideration, but large compared with the distances between the molecules. If, for example, we speak of the displacement of some fluid particle, we mean not the displacement of an individual molecule, but that of a volume element containing many molecules, though still regarded as a point.

Gas of photons in a vessel is an analogue of the real molecular gas. We shall firstly calculate the velocity of outflow of the molecular gas from a vessel and then outflow of photon gas out from blackbody, which is a vessel with volume V, where inside of it are photons of different frequencies.

The spectral form of the blackbody photons was derived firstly by Planck (Planck, 1900; 1901) . His original derivation of the blackbody radiation was heuristic and based on the relation between the entropy of the system and the internal energy of the blackbody denoted by Planck as U. Later Einstein (1917) derived the Planck formula from the Bohr model of atom, where Bohr postulated the model of atom as follow. 1. every atom can exist in the discrete series of states in which electrons do not radiate even if they are moving at acceleration (the postulate of the stationary states), 2. transiting electron from the stationary state to other, emits the energy according to the law $\hbar\omega = E_m - E_n$, called the Bohr formula, where E_m is the energy of an electron in the initial state, and E_n is the energy of the final state of an electron to which the transition is made and $E_m > E_n$.

Einstein introduced coefficients of spontaneous and stimulated emission A_{mn}, B_{mn}, B_{nm} and derived the Planck power spectral formula:

$$P(\omega)d\omega = \hbar\omega G(\omega)\frac{d\omega}{\exp\frac{\hbar\omega}{k_BT} - 1}; \quad G(\omega) = \frac{\omega^2}{\pi^2 c^3}, \tag{1}$$

where $\hbar\omega$ is the energy of a blackbody photon and $G(\omega)$ is the number of electromagnetic modes inside of the blackbody, k_B is the Boltzmann constant, c is the velocity of light, T is the absolute temperature.

The internal density energy of the blackbody gas is given by integration of the last equation over all frequencies ω, or

$$u = \int_0^\infty P(\omega)d\omega = aT^4; \quad a = \frac{\pi^2 k_B^4}{15\hbar^3 c^3}, \tag{2}$$

where

$$a = 7,5657.10^{-16}\frac{\mathrm{J}}{\mathrm{K}^4\mathrm{m}^3}. \tag{3}$$

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In 1926 the chemist Gilbert N. Lewis coined the name photon for light quanta, and after 1927, when Arthur H. Compton won the Nobel Prize for his scattering studies, most scientists accepted the validity that quanta of light have an independent existence, and Lewis' term photon for light quanta was accepted. So, the notion photon gas was the natural consequence of particles named photons.

According to the theory of relativity, there is the equivalence between mass m and energy E. Namely, $m = E/c^2$. At the same time, there is the relation between pressure and the internal energy of the blackbody gas following from the electromagnetic theory of light, $p = u/3$. So, in our case

$$\varrho = u/c^2 = \frac{aT^4}{c^2}; \quad p = \frac{u}{3}. \tag{4}$$

So, in order to derive the outflow of photon gas from the blackbody (vessel of photons), we use the above formulas and the analogy with the ideal gas of fluid dynamics including laws of thermodynamics.

2 The adiabatic one-dimensional flow of gas without friction

It may be easy to see that the basic hydrodynamic and thermodynamic equations for the one-dimensional flow of gas including no friction is as follows (Shirokov, 1958):

$$\varrho v S = const \tag{5}$$

$$c_pT + \frac{v^2}{2} = const \tag{6}$$

$$\int \frac{dp}{\varrho(p)} + \frac{v^2}{2} = const \tag{7}$$

$$p = R\varrho T, \tag{8}$$

where ϱ is density of a gas, v is velocity of gas, and S is the cross-section of a tube, or nozzle where the one-dimensional gas flows, p is pressure, T is temperature, $R = c_p - c_v$ is the gas constant, with c_p being the specific heat at the constant pressure, c_v being the specific heat at the constant pressure.

Equation (5) is the equation of continuity expressing the law conservation of fluid mass. Quantity c_pT in eq. (6) is the thermal internal energy of gas and v is the velocity of the gas element. Equation (7) i the Bernoulli equation and the last equation (8) $p = R\varrho T$ is the equation of state of gas and it is the first approximation of the more general equation $p/\varrho RT = 1 + B(T) + C(T) + ...B(T)$, where $B(T), C(T), ...$ are some constant which depends on temperature. Van der Waals equation, or Kamerlingh Onnes equation can be obtained for the specification of constants $B, C, ...$ (Rumer et al., 1977). Internal energy of gas E depends only on temperature T in the proximate approach to thermodynamics. However, it depends also on the density of gas ϱ which was confirmed by so called Joule-Thomson effect.

After operation of logarithm and derivation of the eq. (5) we get

$$\frac{dS}{S} + \frac{dv}{v} + \frac{d\varrho}{\varrho} = 0. \tag{9}$$

After derivation of the next equation (6) we get

$$c_pdT + vdv = 0. \tag{10}$$

The derivation of the further equation is,

$$\frac{dp}{\varrho} + vdv = 0 \tag{11}$$

and from the final equation (8) we have after operation of logarithm and differentiation

$$\frac{dp}{p} = \frac{d\varrho}{\varrho} + \frac{dT}{T}, \tag{12}$$

or,

$$dp = p\frac{d\varrho}{\varrho} + p\frac{dT}{T} = p\frac{d\varrho}{\varrho} + (c_p - c_v)\varrho dT = p\frac{d\varrho}{\varrho} + c_p\varrho\frac{(\kappa - 1)}{\kappa}, \tag{13}$$

where $\kappa = c_p/c_v$.

We get from eqs. (10) and (11), that $c_pdT = dp/\varrho$, which we insert into eq. (13). We get the following equation:

$$\frac{dp}{p} = \frac{d\varrho}{\varrho}\kappa. \tag{14}$$

After integration, we get

$$\frac{p}{\varrho^\kappa} = const. \tag{15}$$

We get from the two equation (9), (10), the following equation:

$$\frac{dS}{S} = \frac{dv}{v}\left(\frac{v^2}{\frac{dp}{d\varrho}} - 1\right). \tag{16}$$

However, with regard to eq. (15), we get

$$\frac{dp}{d\varrho} = \kappa\varrho^{\kappa-1} \cdot const = \kappa\frac{P}{\varrho} = \kappa RT = c^2, \tag{17}$$

where c is the velocity of sound in the gas.

So, we get the final formula from eq. (16):

$$\frac{dS}{S} = \frac{dv}{v}\left(M^2 - 1\right), \tag{18}$$

where we introduce new number $M = v/c$, which is usually called the Mach number according to Austrian physicist Ernst Mach[1]. Mach determined the velocity of a moving body (bullet) by photographing the shock waves spreading from the nose and tail of a bullet in virtue of a fact that the compression of the air along their envelopes is sufficient to cast and optical shadow on the plate. (Richardson, 1950). Such experiments inspired him to introduced theoretically the so called Mach number and the so called Mach angle (The analogue of the Čerenkov angle, or, the Heaviside angle).

In case that $M < 1$, then after increasing of the area S, or, $dS > 0$, velocity v is decreasing, or, $dv < 0$. After decreasing of the area S, or, $dS < 0$, the velocity v is increasing, or, $dv > 0$.

In case that $M > 1$, then after increasing of the area S, or, $dS > 0$, velocity v is increasing, or, $dv > 0$. After decreasing of the area S, or, $dS < 0$, the velocity v is decreasing, or, $dv < 0$.

3 The speed of gas flow from the Bernoulli equation

In order to understand the flow of photon gas it is pedagogically useful to start with the hydrodynamics of the ideal fluid and gas.

The basic equations of the hydrodynamics are the Euler equation

$$\varrho\frac{d\mathbf{v}}{dt} = \mathbf{F} - \mathrm{grad}\, p, \tag{19}$$

the equation of continuity

[1] M is the Bairstow number in the western literature, the Mayevskiy number in the Russian literature. However, Leonhard Euler was the first scientist who introduced this number.

$$\frac{\partial \varrho}{\partial t} + \mathrm{div}\,(\varrho \mathbf{v}) = 0 \tag{20}$$

and the thermodynamical equation

$$p = f(\varrho), \tag{21}$$

where p is pressure, $\mathbf{v}$ is the velocity of the of the density element of fluid ϱ.

The structure of the derivative of $\mathbf{v}$ in eq. (19) is so called substantial derivative introduced and derived by Euler (Landau et al., 1987). The derivative $d\mathbf{v}/dt$ is not the rate of change of the fluid velocity at a fixed point in space, but the rate of change of the velocity of a given fluid particle as it moves in space. The change $d\mathbf{v}$ in the velocity of the given fluid particle during the time dt is composed of two parts, namely the change during dt in the velocity at a point fixed in space, and the difference between the velocities (at the same instant) at two points $d\mathbf{r}$ apart, where $d\mathbf{r}$ is the distance moved by the given $(\partial \mathbf{v}/\partial t)dt$ fluid particle during the time dt, where the derivative $(\partial \mathbf{v}/\partial t)$ is taken for constant x, y, z, i.e. at the given point in space. The second part is $(\partial \mathbf{v}/\partial x)dx + (\partial \mathbf{v}/\partial y)dy + (\partial \mathbf{v}/\partial z)dz = (d\mathbf{r} \cdot \mathrm{grad})\mathbf{v}$, or, dividing both sides by dt, we have the final relation:

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \mathrm{grad})\mathbf{v}. \tag{22}$$

The Euler equation with derivative (22) is of the form:

$$\varrho \frac{\partial \mathbf{v}}{\partial t} + \varrho(\mathbf{v} \cdot \mathrm{grad})\mathbf{v} = \mathbf{F} - \mathrm{grad}\, p. \tag{23}$$

It is well known, that for the stationary flow, the Bernoulli equation follows from the Euler equation:

$$\frac{1}{2}v^2 + \Phi(p) = \frac{1}{2}v_0^2 + \Phi(p_0), \tag{24}$$

where

$$\phi(p) = \int_0^p \frac{dp}{\varrho(p)}. \tag{25}$$

Supposing the existence of the adiabatic process during the outflow from the volume V, with the corresponding thermodynamic equation $pV^\kappa = const$, or,

$$\frac{1}{\varrho} = \frac{p_0^{\frac{1}{\kappa}}}{\varrho_0} \frac{1}{p^{\frac{1}{\kappa}}}, \tag{26}$$

we get

$$\Phi(p) = \frac{p_0^{\frac{1}{\kappa}}}{\varrho_0} \frac{\kappa}{\kappa - 1} p^{\frac{\kappa-1}{\kappa}}, \quad \Phi(p_0) = \frac{p_0^{\frac{1}{\kappa}}}{\varrho_0} \frac{\kappa}{\kappa - 1} p_0^{\frac{\kappa-1}{\kappa}}. \tag{27}$$

Then, supposing that the initial velocity of flowing gas is $v_0 = 0$, we get from the Bernoulli equation so called Saint-Venant[2]-Wantzel[3] equation:

$$v = \sqrt{2[\Phi(p_0) - \Phi(p)]} = \sqrt{\frac{2\kappa}{\kappa - 1}\frac{p_0}{\varrho_0}\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right]}. \tag{28}$$

The maximal velocity can be obtained for outflow of gas into vacuum. In other words, if we put $p = 0$, then

$$v_{max} = \sqrt{\frac{2\kappa}{\kappa - 1}\frac{p_0}{\varrho_0}}. \tag{29}$$

The same formula corresponds to the arbitrary maximal velocity for $p_0 \longrightarrow \infty$. However, the reality is, that it is not possible to get arbitrary maximal velocity. The obstacle was solved by Sweden inventor Gustav de Laval by his invention, which is used continually in the rocket technique in Baikonur cosmodrome, Canaveral space shuttle landing facility, and so on. The trick is to replace the cylindrical tube of flow by the so called de Laval nozzle with the conical cross section. However, the cone cannot be arbitrary, bur appropriate to get the supersonic outflow from the vessel of gas.

Gas flow in a vessel is supposing to be uniform over the cross-section at every point in the tube, and that the velocity is along the axis of the tube. For this to be so, the tube must not be too wide, and its cross-sectional area S must vary slowly along its length. Thus all quantities characterizing the flow will be functions only of the coordinate along the axis of the tube.

The volume dimensions of the vessel are supposed very large in comparison with the diameter of the tube. The velocity of the gas in the vessel may therefore be taken as zero. The more perfect theory of the de Naval nozzle is presented in the advanced theory of hydrodynamics of gases - especially in books on the rocket technique.

4 The speed of the photon gas out-flowing from the blackbody

The ideas of the last chapter can be applied also to the case of the blackbody gas. We know from the thermodynamics that the pressure p and the internal specific energy u are related by the equation $p = u/3$. According Einstein relativity, energy and mass are related by the equation $E = mc^2$, which follows from the formula of the mass dependence on velocity $m = m_0/\sqrt{1 - v^2/c^2}$, and which was confirmed by the nuclear reaction experiments with the so called mass defect, electron positron annihilation $e^+ + e^- \to 2\gamma$, and so on. The physical meaning of the Einstein formula in case of the electron positron annihilation is expressed by the statement that the energy of the rest mass of an electron-positron pair is the rest energy (the hidden energy, of the particles with the internal structure) and the energy of the resulting photons is the active energy. The discussion concerning the Einstein derivation of the energetic formula (including new aspects of it) was given by Sachs (1973).

[2] Adhémar Jean Claude Barré Saint Venant

[3] Pierre Laurent Wantzel

Now, if we identify $E = u$. Such identification was used by author for calculation of the velocity of sound in the relic photon sea (Pardy, 20013a).

Then,

$$m = \frac{u}{c^2} = \frac{3p}{c^2} = \varrho, \tag{30}$$

where ϱ in this case is the density of mass of photons in the blackbody gas.

Supposing the existence of the weak adiabatic process during the outflow of the photon energy from the volume V, with the corresponding thermodynamic equation $pV^\kappa = const$, ($\kappa \to 1$). We get for photons:

$$\frac{1}{\varrho} = \lim_{\kappa\to 1} \frac{p_0^{\frac{1}{\kappa}}}{\varrho_0} \frac{1}{p^{\frac{1}{\kappa}}}, \tag{31}$$

where

$$\lim_{\kappa\to 1} \frac{p_0^{\frac{1}{\kappa}}}{\varrho_0} = \frac{c^2}{3}, \tag{32}$$

Or,

$$\frac{1}{\varrho_0} = \lim_{\kappa\to 1} \frac{c^2}{3p_0^{\frac{1}{\kappa}}}, \tag{33}$$

Then,

$$\Phi(p) = \lim_{\kappa\to 1} \frac{p_0^{\frac{1}{\kappa}}}{\varrho_0} \frac{\kappa}{\kappa - 1} p^{\frac{\kappa-1}{\kappa}}, \quad \Phi(p_0) = \lim_{\kappa\to 1} \frac{p_0^{\frac{1}{\kappa}}}{\varrho_0} \frac{\kappa}{\kappa - 1} p_0^{\frac{\kappa-1}{\kappa}}. \tag{34}$$

Then, using eqs. (33) and (34), we get for the outflowing photon gas the photonic Saint-Venant-Wantzel equation:

$$v = \sqrt{2[\Phi(p_0) - \Phi(p)]} = \lim_{\kappa\to 1} \sqrt{\frac{2\kappa}{\kappa - 1} \frac{p_0}{\varrho_0} \left[1 - \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right]}$$
$$= \lim_{\kappa\to 1} c \sqrt{\frac{2\kappa}{\kappa - 1} \frac{p_0^{\frac{\kappa-1}{\kappa}}}{3} \left[1 - \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right]}. \tag{35}$$

The maximal velocity can be obtained for outflow of photon gas into vacuum. In other words, if we put $p = 0$, then

$$v_{max} = \frac{c}{3} \sqrt{\frac{6\kappa}{\kappa - 1} p_0^{\frac{\kappa-1}{\kappa}}}. \tag{36}$$

It is easy to see that for sufficient big pressure p_0, the velocity of photon energy outflow can be greater then the velocity of light and it means that such situation must be discussed and analyzed from the viewpoint of the special theory relativity and it is necessary to show that it is not in contradiction with the STR.

5 Discussion

We have considered steady outflow of a gas from large vessel through the de Laval nozzle. We have supposed that the gas flow was uniform over the cross-section at every point in the tube, and that the velocity is along the axis of the tube. The thermodynamics of the process was adiabatic. The impossibility of achieving supersonic velocities by flow through a continually narrowing nozzle is due to the fact that a velocity equal to the local velocity of sound can be reached only at the very end of such a tube. It is clear that a supersonic velocity can be achieved by means of a nozzle which first narrows and then widens again. This is called the de Laval nozzle. We have applied the classical thermodynamics to the photon gas in the blackbody. We derived that the velocity of outflow of photons can be superluminal. It can play the substantial role for the construction of the photon rackets. While Tsiolkovskii, Meščerskii, Korolyov, von Brown and others, considered only rockets with the chemical fuel generating gas with its dynamics, the photon rockets with the de Laval nozzle are also possible. The geometrical form of the de Laval nozzle have many volcano craters, and it is not excluded that eruption of Vesuvius, in year of 79 AD, was substantially influenced by the crater in the form of the de Laval nozzle. The same is valid for the volcano Krakatoa. Similarly, it is possible to expect, there are many volcanoes with the de Laval nozzles in planets, Sun, stars, supernovas and in many cosmological objects (including the black holes). It is not excluded that the modified form of the de Laval nozzle is in some cosmological situations identical with geometry of the wormholes, or, with the geometry of the Lobačevskii pseudospheres (Pardy, 2013). The cosmical rays can be hypothetically accelerated by the cosmical de Laval Nozzles.

Our derivation of the light velocity in the blackbody photon gas was based on the classical thermodynamical model with the adiabatic process ($\delta Q = 0$), controlling the spreading of sound in the gas. The problem was not presented by Einstein, because his special theory of relativity is based on the postulate that the velocity of light is the maximal velocity in universe.

After application of Boltzmann equation to the photon gas, we can expect the rigorous microscopical solution of the problem, however, with regard to the fact that the cross-section of the photon-photon interaction in the photon gas is very small. Namely (Berestetzkii et al., 1999):

$$\sigma_{\gamma\gamma} = \frac{973}{10125\pi}\alpha^2 r_e^2 \left(\frac{\hbar\omega}{mc^2}\right)^6; \quad \hbar\omega \gg mc^2, \tag{37}$$

where $r_e = e^2/mc^2 = 2,818 \times 10^{-13}$ cm is the classical radius of electron and $\alpha = e^2/\hbar c$ is the fine structure constant with numerical value $1/\alpha = 137,04$. No doubt, the solution of the Boltzmann equation can give the correct form of photon flow in the photon vessel and in the de Laval nozzle.

Serge Haroche (2012) and his research group in the Paris microwave laboratory used a small cavity between two mirrors about three centimeter apart. Photon bounced back and forth inside in this cavity. The mirrors were made from a superconductive material at temperature just above absolute zero. The reflectivity was so perfect that photon was confined for almost tenth of a second before it was lost, or, absorbed. During the long life-time of photons many quantum experiments were performed with the Ridberg atoms. We considered here as an analogue, the blackbody with the gas of photons (at temperature T) as the preamble for new experiments for the determination of the superluminal velocity of the photon gas in the de Laval nozzle. The verification of the superluminal velocity of the photon energy in the de Laval nozzle can be peformed immediately by the PALS giant laser in Prague. It is not excluded, that the experiments performed with the photons in the de Laval nozzle, including experiments with the superbig laser of ELI, will give new direction in the construction of the photon rockets in order to realize the interstellar and intergalactic missions in the near future.

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