

The relativistic Doppler effect with RLC circuit

Miroslav Pardy

Department of Physical Electronics

Masaryk University

Kotlářská 2, 611 37 Brno, Czech Republic

e-mail:pamir@physics.muni.cz

Abstract

The special theory of relativity formula for the photon Doppler effect is applied to the frequency of the RLC circuit moving with velocity v relatively to the rest system. The relativistic transformation of the RLC circuit components is derived in case the components are all in series with the voltage source ($R-L-C-v$). The generalization to the more complex situation was not considered. It is not excluded that the article is the preamble for the future investigation of electronic physics and will be integral part of such institutions as Bell Laboratories, NASA, CERN and so on.

1 Introduction

The Doppler effect (or, the Doppler shift), named after the Austrian physicist Christian Doppler, is the change in frequency, or, wavelength of a wave for an observer who is moving relative to the wave source (Doppler, 1842).

There is an elementary explanation for the acoustic Doppler effect using the stationary and moving sound source. Stationary source produces

sound waves at a constant frequency f , and the wave-fronts propagate symmetrically away from the source at a constant speed V_{sound} . The distance between wavefronts is the wavelength. All observers will hear the same frequency, which will be equal to the actual frequency of the source where f .

However, if now the sound source is moving with a speed v , the centre of each new wavefront is now slightly displaced to the right. As a result, the wavefronts begin to bunch up on the right side (in front of) and spread further apart on the left side (behind) of the source. An observer in front of the source will hear a higher frequency.

The relativistic Doppler effect is different from the non-relativistic Doppler effect because the equations include the time dilation effect of special relativity and do not involve the medium of propagation as a reference point. They describe the total difference in observed frequencies and possess the required Lorentz symmetry. We consider here the relativistic Doppler effect with RLC circuit.

An RLC circuit is an electrical circuit consisting of a resistor (R), an inductor (L), and a capacitor (C), connected in series or in parallel. The circuit forms a harmonic oscillator for current.

The three circuit elements, R , L and C can be combined in a number of different topologies. All three elements in series or all three elements in parallel are the simplest in concept and the most straightforward to analyse. There are, however, other arrangements, some with practical importance in real circuits. One issue often encountered is the need to take into account inductor resistance. Inductors are typically constructed from coils of wire, the resistance of which is not usually desirable, but it often has a significant effect on the circuit.

2 Series RLC circuit

In the situation where we consider series RLC circuit, the three components are all in series with the voltage source ($R-L-C-v$). The governing differential equation can be found by substituting into Kirchhoffs voltage law (KVL) the constitutive equation for each of the three elements. From KVL,

$$v_R + v_L + v_C = v(t), \tag{1}$$

where v_R, v_L, v_C are the voltages across R, L, C respectively and $v(t)$ is the time varying voltage from the source. Substituting the corresponding

physical term, in eq (1), we get the following inegral differential equation:

$$Ri(t) + L\frac{di}{dt} + \frac{1}{C} \int_{-\infty}^{\tau=t} i(\tau)d\tau = v(t). \quad (2)$$

To our goal it is sufficient to consider the more simple situation with $v = 0$. Then instead of equation (2) we write

$$L\ddot{Q} + R\dot{Q} + Q/C = 0 \quad (3)$$

with stationary solution

$$Q = Ae^{-\frac{R}{2L}t} \sin(\omega t + \alpha), \quad (4)$$

where

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}. \quad (5)$$

The Thomson formula for the period of oscillations is when $R = 0$

$$T = 2\pi/\omega = 2\pi\sqrt{LC}. \quad (6)$$

3 The relativistic transformation of R, L, C

The special principle of relativity states that the laws of physics should be the same in all inertial frames. It has been foundational to physical thinking since the time of Galileo, and gained renewed prominence as Einsteins first postulate of relativity theory (Einstein, 1905):

If a system of coordinates K is chosen so that, in relation to it, physical laws hold good in their simplest form, the same laws hold good in relation to any other system of coordinates K' moving in uniform translation relatively to K . This postulate we call the special principle of relativity.

The consequences of the Lorentz transformation of special theory of relativity is dilatation of time and contraction of length and the transformation of light frequency called the relativistic Doppler effect for photons. It is well known that when photons are measured in the frame that is moving toward the photon source with velocity v , then the measured frequency ω' of photon is given by the formula (Rohlf, 1994):

$$\omega' = \omega \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}. \quad (7)$$

When photons are measured in the frame that is moving away from the photon source with velocity v , then the measured frequency ω' of photon is given by the formula (Rohlf, 1994):

$$\omega' = \omega \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}. \quad (8)$$

Let us suppose that the same formulas are valid for the Thomson frequencies of the LC circuit in K' , or

$$\omega' = \sqrt{\frac{1}{L'C'}}. \quad (9)$$

After comparison of eq. (9) with eq. (7) we have (toward RLC source)

$$L'C' = LC \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}, \quad (10)$$

from which the symmetrical solution follows:

$$L' = L \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{1/4} \quad (11)$$

and

$$C' = C \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{1/4}. \quad (12)$$

In case taht we consider the situation when RLC frequency is measured in the frame that is moving away from the RLC source moving with velocity v , then the measured quantity L' and C' of the Thomson oscillator is as follows:

$$L' = L \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{1/4} \quad (13)$$

and

$$C' = C \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{1/4}. \quad (14)$$

The transformed resistance R follows form the equation (5) for the toward photon source:

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} = \sqrt{\frac{1}{L'C'} - \frac{R'^2}{4L'^2}}. \quad (15)$$

After insertion of the transformation (11) and (12) into eq. (15), we get after some elementary operations:

$$R' = 2L \sqrt{\frac{1}{LC} - \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right) \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}\right)^{3/2}}. \quad (16)$$

and in the second case with RLC circuit moving away from the observer we get

$$R' = 2L \sqrt{\frac{1}{LC} - \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right) \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}\right)^{3/2}}. \quad (17)$$

It is not excluded that the measurement of the quantities R, L, C can be the crucial problems for such famous institutions as Bell Laboratories, NASA, CERN and so on.

4 Two RLC circuits with the mutual induction

In this case the components R_1, L_1, C_1 are inductive bonded with R_2, L_2, C_2 configuration. The problem was published by Landau et al. (1989) and the frequency of the inductive system was calculated in the form (for $R_1 = R_2 = 0$).

$$\omega_{1,2}^2 = \frac{L_1 C_1 + L_2 L_2 \pm [(L_1 C_1 - L_2 C_2)^2 + 4C_1 C_2 L_{12}^2]^{1/2}}{2C_1 C_2 (L_1 L_2 - L_{12}^2)}. \quad (18)$$

After performing the transformations of all known components in the last formula, we can eliminate the term of the mutual induction L_{12} .

5 Discussion

We have applied the special theory of relativity formula for the photon Doppler effect to the problem of transformation of the RLC circuit moving with velocity v relatively to the rest system. We have supposed that the relativistic transformation of the light frequency is valid also for the frequency of the RLC circuit. We have considered only the situation where components are all in series with the voltage source ($R - L - C - v$). The generalization to the more complex situation was not considered. We have considered the symmetrical solution of the problem. The nonsymmetrical solution is also possible, however we have

supposed that the nature prefers the symmetrical solution. However, the spontaneously broken symmetry is not excluded. We here do not consider this Nobelian problem. The article forms the preamble of the future investigation of relativistic electronic systems (Nilsson et al. 2008) and it will be, no doubt, the integral part of such institutions as Bell Laboratories, NASA, CERN and so on.

In case, RLC circuit is inserted into gravitational field, the frequency of RLC is shifted due to gravity interaction with RLC and every component of RLC is modified by gravity field. If the gravitational field are gravity waves, then RLC detects gravity waves. It is not excluded that also small electronic laboratories working with RLC circuits can detect gravity waves in order to be in harmony with the LIGO famous experimental results.

References

Doppler, C. (1842). Über das farbige Licht der Doppelsterne und einiger anderer Gestirne des Himmels (About the coloured light of the binary stars and some other stars of the heavens.) Publisher: Abhandlungen der Knigl. Bhm. Gesellschaft der Wissenschaften (V. Folge, Bd. 2, S. 465-482) [Proceedings of the Royal Bohemian Society of Sciences (Part V, Vol 2)]; Prague: 1842 (Reissued 1903).

Einstein, A. (1905). Zur Elektrodynamik bewegter Körper, *Annalen der Physik* **17**: 891; English translation: "On the Electrodynamics of Moving Bodies" by George Barker Jeffery and Wilfrid Perrett (1923); Another English translation: "On the Electrodynamics of Moving Bodies" by Megh Nad Saha (1920).

Landau, L. D. and Lifshitz, E. M., *Electrodynamics of continuous media*, (Pergamon press, Oxford OX3 OBW, England, 1989).

Nilsson, J. W. and Riedel, S. A., *Electric circuits*, (Prentice Hall, 2008). ISBN 0-13-198925-1.

Rohlf, J. W., *Modern Physics from α to Z^0* , (John Willey & Sons, Inc., New York, 1994).