THE MAXWELL-HEAVISIDE EQUATIONS

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ABSTRACT

According to the gravitoelectromagnetic description of the gravitational phenomena, the Maxwell-Heaviside equations (GEM equations) govern the gravitational field. In this article these equations are mathematically deduced from the kinematics of the "informatons", that – according to the "theory of informatons" - are the constituent elements of that field. It is also shown that the GEM equations are mathematically consistent and that they imply the existence of gravitational waves.

CONTENTS

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1. THE GRAVITATIONAL FIELD IN VACUUM

In the gravitoelectromagnetic (GEM) description of gravitation[1],[2],[3] the gravitational field plays an intermediary role in the interactions between masses.

It is set up by a given distribution of - whether or not moving - masses and it is, just as the electromagnetic field, defined by a vector field with two components: the "*g-field*" characterized by the field vector $\vec{E}_g$ and the "*g-induction*" characterized by the field vector $\vec{B}_g$. These components each have a value defined at every point of space and time and are thus, relative to an inertial reference frame ***O***, regarded as functions of the space and time coordinates.

At a point *P* of a gravitational field where no matter is located - where ρ_G, the mass density, and $\vec{J}_G$, the density of the mass flow, are zero – $\vec{E}_g$ and $\vec{B}_g$ are the results of the superposition of the contributions of the various masses to respectively the g-field and the g-induction.

In the framework of the theory of informatons[4],[5] the gravitational field is understood as an expanding cloud of carriers of "gravitational information" or "g-information". According to that theory any material object manifests itself in space by emitting – at a rate proportional to its rest mass - mass and energy less particles that go away with the speed of light and that carry information about the position ("g-information") and about the state of movement ("β-information") of their emitter. These grains of g-information are called "informatons".

So, according to the theory of informatons, informatons are the constituent elements of the gravitational field of a certain mass distribution. At an arbitrary point *P* in that field there is a continuous flow of g-informaton, carried by informatons. The contribution of a certain mass *m* to that flow are informatons that pass near *P* in a specific direction with velocity $\vec{c}$. That flow is characterised by the flow density *N*: *N* is the rate per unit area at which these informatons cross an elementary surface perpendicular to the direction in which they move. And the cloud of informatons around *P* is characterized by the density *n*: *n* is the number of informatons per unit volume. *N* and *n* are linked by the relationship:

$$n = \frac{N}{c} \qquad (1)$$

The definition[4],[5] of an informaton implies that every informaton that passes near P is characterized by two attributes that refer to its emitter: its g-index $\vec{s}_g$ and its β-index $\vec{s}_\beta$. s_g, the magnitude of the g-index is the elementary quantity of g-information. It is a fundamental physical constant. $\vec{s}_\beta$ refers to the state of motion of the source of the informaton and is defined by the relationship

$$\vec{s}_\beta = \frac{\vec{c} \times \vec{s}_g}{c} \qquad (2)$$

The informatons emitted by m that pass near P with velocity $\vec{c}$ contribute there to the *density of the g-information flow* with an amount $(N.\vec{s}_g)$. $(N.\vec{s}_g)$ is the product of the density of the flow of informatons moving with velocity $\vec{c}$ near P, with $\vec{s}_g$, the quantity that characterizes the g-information per informaton at that point. So, that vectoral quantity is the rate per unit area at which g-information at P crosses an elementary surface perpendicular to the direction in which it moves. It is identified with $\vec{E}_g$, the g-field at P:

$$\vec{E}_g = N.\vec{s}_g$$

And the same informatons contribute there to the *density of the g-information cloud* with an amount $(n.\vec{s}_\beta)$. $(n.\vec{s}_\beta)$ is the product of the density of the cloud of informatons moving with velocity $\vec{c}$ near P, with $\vec{s}_\beta$, the quantity that characterizes the β -information per informaton at that point. So that vectoral quantity is at P the amount of β-information per volume unit. It is identified with $\vec{B}_g$, the g-induction at P:

$$\vec{B}_g = n.\vec{s}_\beta$$

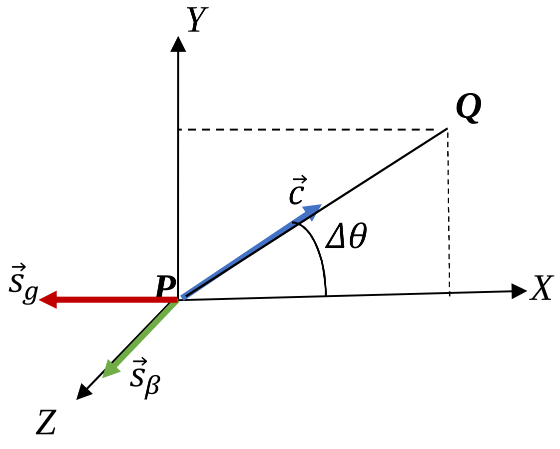

Fig 1

In fig 1, we consider the flow of informatons that - at the moment t - pass near P with velocity $\vec{c}$. They are completely defined by their attributes $\vec{s}_g$ and $\vec{s}_\beta$, respectively their g-index and their β-index. $\Delta\theta$ is their characteristic angle: the angle between the lines carrying $\vec{s}_g$ and $\vec{c}$ that it is characteristic for the movement of the emitter[4],[5].

An informaton that at the moment t passes at P is at the moment ($t + dt$) at Q, with $PQ = c.dt$. The infinitesimal change of the attributes of an informaton between the moments t and ($t + dt$) , is governed by the kinematics of that informaton

On the macroscopic level, this implies that there must be a relationship between the change in time of the gravitational field $(\vec{E}_g, \vec{B}_g)$ *at a point P and the spatial variation of that field in the vicinity of P.*

The intensity of the spatial variation of the components of the gravitational field at P is characterized by $div\vec{E}_g,\ div\vec{B}_g,\ rot\vec{E}_g$ and by $rot\vec{B}_g$ and the rate at which these components change in time by $\dfrac{\partial\vec{E}_g}{\partial t}$ and by $\dfrac{\partial\vec{B}_g}{\partial t}$.

From the above it can be concluded that it makes sense to investigate the relationships between the quantities that characterize the spatial variations of ($\vec{E}_g$,$\vec{B}_g$) and the rate's at which they change in time.

2 div$\vec{\mathbf{E}}_\mathbf{g}$ - THE FIRST EQUATION IN VACUUM

The physical fact that the rate at which g-information flows inside a closed empty space must be equal to the rate at which it flows out[4],[5], can be expressed as:

$$\oiint_S \vec{E}_g\,.\,\overrightarrow{dS} = 0$$

So (theorem of Ostrogradsky)[6]:

$$div\vec{E}_g = 0$$

In vacuum, the law of conservation of g-information can be expressed as followed:

(1) At a matter free point P of a gravitational field, the spatial variation of $\vec{E}_g$ *obeys the law:* $div\vec{E}_g = 0$

This is the first equation of Maxwell-Heaviside in vacuum.

Corollary: *At a matter free point P of a gravitational field*

$$\frac{\partial}{\partial t}[N.cos(\Delta\theta)] = 0$$

Because[6]

$$div\vec{E}_g = div\left(N.\vec{s}_g\right) = grad(N).\vec{s}_g + N.div\left(\vec{s}_g\right) \qquad (3)$$

it follows from the first equation that:

$$grad(N).\vec{s}_g + N.div(\vec{s}_g) = 0$$

1. First we calculate: $grad(N).\vec{s}_g$.

Referring to fig 1:

$$grad(N) = \frac{N_Q - N_P}{PQ}.\vec{e}_c = \frac{N_Q - N_P}{c.dt}.\vec{e}_c$$

Because an informaton that at the moment *t* passes at *P* is at the moment (*t* + *dt*) at *Q*, (with *PQ* = *c.dt*).

$$\frac{N_Q - N_P}{dt} = \frac{N(t - dt) - N(t)}{dt} = -\frac{\partial N}{\partial t}$$

So:

$$grad(N) = -\frac{1}{c}.\frac{\partial N}{\partial t}.\vec{e}_c = -\frac{1}{c}.\frac{\partial N}{\partial t}.\frac{\vec{c}}{c}$$

And:

$$grad(N).\vec{s}_g = -\frac{1}{c^2}.\frac{\partial N}{\partial t}.\ \vec{c}.\vec{s}_g = \frac{1}{c}.\frac{\partial N}{\partial t}.s_g.cos(\Delta\theta) \qquad (4)$$

2. Next, we calculate: $N.div(\vec{s}_g)$

$$div(\vec{s}_g) = \frac{\oiint \vec{s}_g.\overrightarrow{dS}}{dV}$$

For that purpose, we calculate the double integral over the closed surface *S* formed by the infinitesimal surfaces *dS* that are at *P* and *Q* perpendicular to the flow of informatons (perpendicular to $\vec{c}$) and by the tube that connects the edges of these surfaces (and that is parallel to $\vec{c}$). $dV = c.dt.dS$ is the infinitesimal volume enclosed by *S*:

$$div(\vec{s}_g) = \frac{\oiint \vec{s}_g.\overrightarrow{dS}}{dV} = \frac{s_g.dS.cos(\Delta\theta_P) - s_g.dS.cos(\Delta\theta_Q)}{dS.c.dt}$$

Because an informaton that at the moment *t* passes at *P* is at the moment ($t + dt$) at *Q*, (with $PQ = c.dt$):

$$\frac{cos(\Delta\theta_P) - cos(\Delta\theta_Q)}{dt} = \frac{\cos[\Delta\theta(t)] - \cos[\Delta\theta(t-dt)]}{dt} = \frac{\partial[\cos(\Delta\theta)]}{\partial t}$$

$$div(\vec{s}_g) = \frac{1}{c}.s_g.\frac{\partial\{cos(\Delta\theta)\}}{\partial t}$$

And:

$$N.div(\vec{s}_g) = \frac{N}{c}.s_g.\frac{\partial\{cos(\Delta\theta)\}}{\partial t} \qquad (5)$$

Substitution of (4) and (5) in (3) gives:

$$\frac{1}{c}.\frac{\partial N}{\partial t}.s_g.cos(\Delta\theta) + \frac{N}{c}.s_g.\frac{\partial\{cos(\Delta\theta)\}}{\partial t} = 0$$

Or:

$$\frac{\partial}{\partial t}[N.cos(\Delta\theta)] = 0 \qquad (\mathbf{6})$$

3 $\mathbf{div\vec{B}_g}$ – THE SECOND EQUATION IN VACUUM

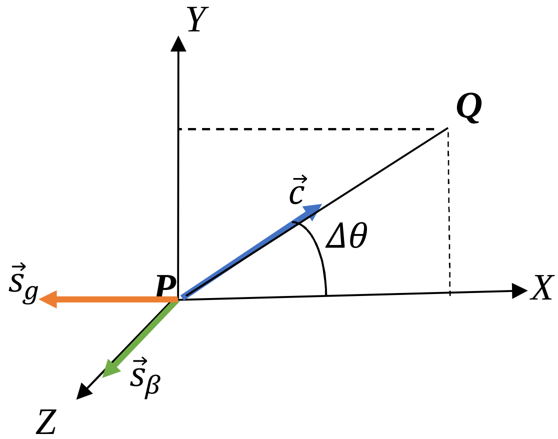

Fig 1

We refer again to fig 1 and notice that

$$\vec{s}_g = -s_g.\vec{e}_x \qquad \text{and} \qquad \vec{s}_\beta = \frac{\vec{c}\times\vec{s}_g}{c} = s_g.sin(\Delta\theta).\vec{e}_z$$

From mathematics[6] we know:

$$div\vec{B}_g = div\left(n.\vec{s}_\beta\right) = grad(n).\vec{s}_\beta + n.div\left(\vec{s}_\beta\right) \qquad (7)$$

1. First we calculate: $grad(n).\vec{s}_\beta$

$grad(n).\vec{s}_\beta = 0$ because *grad(n)* is perpendicular to $\vec{s}_\beta$. Indeed *n* changes only in the direction of the flow of informatons, so *grad(n)* has the same orientation as $\vec{c}$:

2. Next we calculate: $n.div(\vec{s}_\beta)$

$$div(\vec{s}_\beta) = \frac{\oiint \vec{s}_\beta.\overrightarrow{dS}}{dV}$$

We calculate the double integral over the closed surface *S* formed by the infinitesimal surfaces *dS* = *dz.dy* that are at *P* and at *Q* perpendicular to the *X*-axis and by the tube that connects the edges of these surfaces.

Because $\vec{s}_\beta$ is oriented along the *Z*-axis the flux of $\vec{s}_\beta$ through the planes *dz.dy* and *dx.dz* is zero while the fluxes through the planes *dx.dy* are equal and opposite::

$$div(\vec{s}_\beta) = \frac{\oiint \vec{s}_\beta . \overrightarrow{dS}}{dV} = 0$$

Both terms of the expression (7) of $div\vec{B}_g$ are zero, so $div\vec{B}_g = 0$, what implies (theorem of Ostrogradsky) that for every closed surface S in a gravitational field:

$$\oiint_S \vec{B}_g . \overrightarrow{dS} = 0$$

We conclude:

(2) *At a matter free point P of a gravitational field, the spatial variation of* $\vec{B}_g$ *obeys the law:* $div\vec{B}_g = 0$

This is the second equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that the β -index of an informaton is always perpendicular to both its g-index $\vec{s}_g$ and to its velocity $\vec{c}$.

4 $\mathbf{rot\vec{E}_g}$ - THE THIRD EQUATION IN VACUUM

The density of the flow of informatons that - at the moment *t* - passes near *P* with velocity $\vec{c}$ (fig 1) is defined as:

$$\vec{E}_g = N.\vec{s}_g = -N.s_g.\vec{e}_x$$

We know that[6]

$$rot\vec{E}_g = \{grad(N) \times \vec{s}_g\} + N.rot(\vec{s}_g) \qquad (8)$$

1. First we calculate: $\{grad(N) \times \vec{s}_g\}$

This expression describes the component of $rot\vec{E}_g$ caused by the spatial variation of *N* in the vicinity of *P* when $\Delta\theta$ remains constant.

N has the same value at all points of the infinitesimal surface that, at *P*, is perpendicular to the flow of informatons. So *grad(N)* is parallel to $\vec{c}$ and its magnitude is the increase of the magnitude of *N* per unit length. Thus, with *PQ = c.dt* , *grad(N)* is determined by:

$$grad(N) = \frac{N_Q - N_P}{PQ} \cdot \frac{\vec{c}}{c} = \frac{N_Q - N_P}{c.dt} \cdot \frac{\vec{c}}{c}$$

And:

$$grad(N) \times \vec{s}_g = \frac{N_Q - N_P}{c.dt} \cdot \frac{\vec{c}}{c} \times \vec{s}_g = \frac{N_Q - N_P}{c.dt} \cdot \vec{s}_\beta$$

The density of the flow of informatons at *Q* at the moment *t* is equal to the density of that flow at *P* at the moment (*t* - *dt*), so:

$$\frac{N_Q - N_P}{dt} = \frac{N(t-dt) - N(t)}{dt} = -\frac{\partial N}{\partial t}$$

And taking into account that :

$$\frac{N}{c} = n$$

we obtain:

$$grad(N) \times \vec{s}_g = -\frac{\partial n}{\partial t} \cdot \vec{s}_\beta \qquad (9)$$

2. Next we calculate: $\{N.rot(\vec{s}_g)\}$

This expression describes the component of $rot\vec{E}_g$ caused by the spatial variation of $\Delta\theta$ - the orientation of the g-index - in the vicinity of *P* - when *N* remains constant. $(\Delta\theta)_P$ is the characteristic angle of the informatons that pass near *P* and $(\Delta\theta)_Q$ is the characteristic angle of the informatons that at the same moment pass near *Q*. (fig 2)

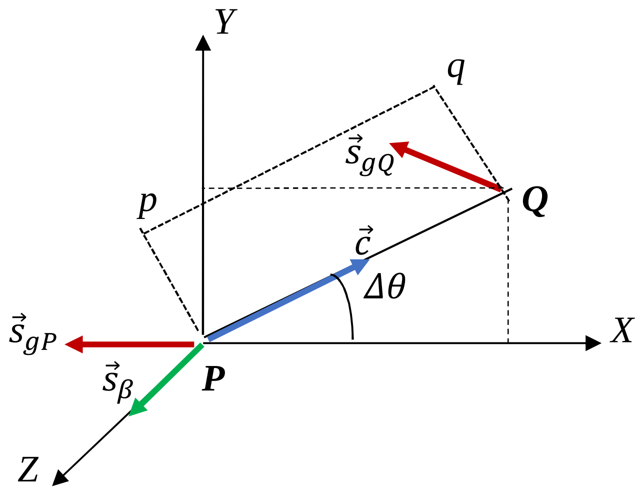

Fig 2

For the calculation of

$$rot(\vec{s}_g) = \frac{\oint \vec{s}_g.\overrightarrow{dl}}{dS}$$

with dS the encircled area, we calculate $\oint \vec{s}_g\,.\overrightarrow{dl}$ along the closed path $PQqpP$ that encircles dS: $dS = PQ.Pp = c.dt.Pp$. (PQ and qp are parallel to the flow of the informatons, Qq and pP are perpendicular to it).

$$N.rot(\vec{s}_g) = N.\frac{s_g.sin[(\Delta\theta)_Q].Qq - s_g.sin[(\Delta\theta)_P)].pP}{c.dt.Pp}.\vec{e}_z$$

The characteristic angle of the informatons at Q at the moment t is equal to the characteristic angle of the informatons at P at the moment (t - dt), so:

$$N.rot(\vec{s}_g) = N.\frac{s_g.\sin[\Delta\theta(t-dt)].Qq - s_g.\sin[\Delta\theta(t)].pP}{c.dt.Pp}.\vec{e}_z$$

The rate at which sin($\Delta\theta$) in P changes at the moment t, is:

$$\frac{\partial\{sin(\Delta\theta)\}}{\partial t} = \frac{sin\{[\Delta\theta](t)\} - \sin\{[\Delta\theta](t-dt)\}}{dt}$$

And taking into account that

$$n = \frac{N}{c}$$

we obtain:

$$N.rot\left(\vec{s}_g\right) = -\,n.s_g.\frac{\partial\{sin(\Delta\theta)\}}{\partial t}.\vec{e}_z = -\,n.\frac{\partial}{\partial t}\{s_g.sin(\Delta\theta).\vec{e}_z\}$$

or

$$N.rot\left(\vec{s}_g\right) = -n.\frac{\partial\vec{s}_\beta}{\partial t} \qquad (10)$$

Combining the results *(9)* and (10), we obtain:

$$rot\vec{E}_g = grad(N)\times\vec{s}_g + N.rot\left(\vec{s}_g\right)$$

$$= -\left(\frac{\partial n}{\partial t}.\vec{s}_\beta + n.\frac{\partial\vec{s}_\beta}{\partial t}\right)$$

$$= -\frac{\partial\left(n.\vec{s}_\beta\right)}{\partial t} = -\,\frac{\partial\vec{B}_g}{\partial t} \qquad (\mathbf{11})$$

We conclude:

(3) At a matter free point P of a gravitational field, the spatial variation of $\vec{E}_g$ *and the rate at which* $\vec{B}_g$ *is changing are connected by the relation:*

$$rot\vec{E}_g = -\frac{\partial\vec{B}_g}{\partial t}$$

This is the third equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that any change of the product $n.\vec{s}_\beta$ at a point of a gravitational field is related to a spatial variation of the product $N.\vec{s}_g$ in the vicinity of that point.

The relation

$$rot\vec{E}_g = -\frac{\partial\vec{B}_g}{\partial t}$$

implies (theorem of Stokes[6]):

$$\oint \vec{E}_g \,.\, \overrightarrow{dl} = -\iint_S \frac{\partial \vec{B}_g}{\partial t} . \overrightarrow{dS} = -\frac{\partial}{\partial t}\iint_S \vec{B}_g \,.\, \overrightarrow{dS} = -\frac{\partial \Phi_B}{\partial t}$$

The orientation of the surface vector $\overrightarrow{dS}$ is linked to the orientation of the path on L by the "rule of the corkscrew". $\Phi_B = \iint_S \vec{B}_g \,.\, \overrightarrow{dS}$ is called the "β -information-flux through S".

So, *in a gravitational field, the rate at which the surface integral of* $\vec{B}_g$ *over a surface S changes is equal and opposite to the line integral of* $\vec{E}_g$ *over its boundary L.*

5 rot$\vec{\mathbf{B}}_\mathbf{g}$ and $\frac{\partial \vec{\mathbf{E}}_\mathbf{g}}{\partial \mathbf{t}}$ - THE FOURTH EQUATION

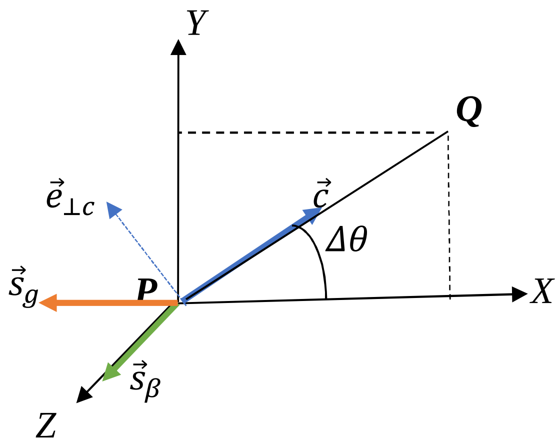

Fig 3

We consider again $\vec{E}_g$ and $\vec{B}_g$, the contributions of the informatons that - at the moment t - pass near P with velocity $\vec{c}$, to the g-field and to the g-induction at that point. (fig 3).

$$\vec{E}_g = N.\vec{s}_g = -N.s_g.\vec{e}_x$$

and

$$\vec{B}_g = n.\vec{s}_\beta = n.\frac{\vec{c} \times \vec{s}_g}{c} = n.s_g.sin(\Delta\theta).\vec{e}_z$$

A. Let us calculate $rot\vec{B}_g$.

We know that[6]

$$rot\vec{B}_g = \{grad(n) \times \vec{s}_\beta\} + n.rot(\vec{s}_\beta) \qquad (12)$$

1. First we calculate: $\{grad(n) \times \vec{s}_\beta\}$

This expression describes the component of $rot\vec{B}_g$ caused by the spatial variation of *n* in the vicinity of *P* when $\Delta\theta$ remains constant.

n has the same value at all points of the infinitesimal surface that, at *P*, is perpendicular to the flow of informatons. So *grad(n)* is parallel to $\vec{c}$ and its magnitude is the increase of the magnitude of *n* per unit length.

With *PQ = c.dt* , *grad(n)* is determined by:

$$grad(n) = \frac{n_Q - n_P}{PQ} . \frac{\vec{c}}{c} = \frac{n_Q - n_P}{c.dt} . \frac{\vec{c}}{c}$$

The density of the cloud of informatons at *Q* at the moment *t* is equal to the density of that flow at *P* at the moment (*t* - *dt*), so:

$$\frac{n_Q - n_P}{dt} = \frac{n(t-dt) - n(t)}{dt} = -\frac{\partial n}{\partial t}$$

And

$$grad(n) = -\frac{1}{c} . \frac{\partial n}{\partial t} . \frac{\vec{c}}{c} = -\frac{1}{c} . \frac{\partial n}{\partial t} . \vec{e}_c$$

The vector $\{grad(n) \times \vec{s}_\beta\}$ is perpendicular to het plane determined by $\vec{c}$ and $\vec{s}_\beta$. So, it lies in the *XY*-plane and is there perpendicular to $\vec{c}$ forming an angle $\Delta\theta$ with the axis *OY*. Taking into account the definition of vectoral product we obtain:

$$grad(n) \times \vec{s}_\beta = -\frac{1}{c} . \frac{\partial n}{\partial t} . s_g . sin(\Delta\theta) . (\vec{e}_c \times \vec{e}_z)$$

With

Typ hier uw vergelijking. $$\vec{e}_c \times \vec{e}_z = -\vec{e}_{\perp c}$$

$$grad(n) \times \vec{s}_\beta = \frac{1}{c} . \frac{\partial n}{\partial t} . s_g . sin(\Delta\theta) . \vec{e}_{\perp c}$$

And, taking into account that $n = \frac{N}{c}$, we obtain:

$$grad(n) \times \vec{s}_\beta = \frac{1}{c^2} . \frac{\partial N}{\partial t} . s_g . sin(\Delta\theta) . \vec{e}_{\perp c} \qquad (13)$$

2. Next we calculate $\{n. rot(\vec{s}_\beta)\}$

This expression is the component of $rot\vec{B}_g$ caused by the spatial variation of $\vec{s}_\beta$ in the vicinity of *P* when *n* remains constant. For the calculation of

$$rot(\vec{s}_\beta) = \frac{\oint \vec{s}_\beta . \overrightarrow{dl}}{dS}$$

with *dS* the encircled area, we calculate $\oint \vec{s}_\beta . \overrightarrow{dl}$ along the closed path *PpqQP* that encircles *dS*: *dS*= *PQ.Pp* = *c.dt.Pp* (fig 4). (*PQ* and *qp* are Parallel to the flow of the informatons, *Qq* and *pP* are perpendicular to it).

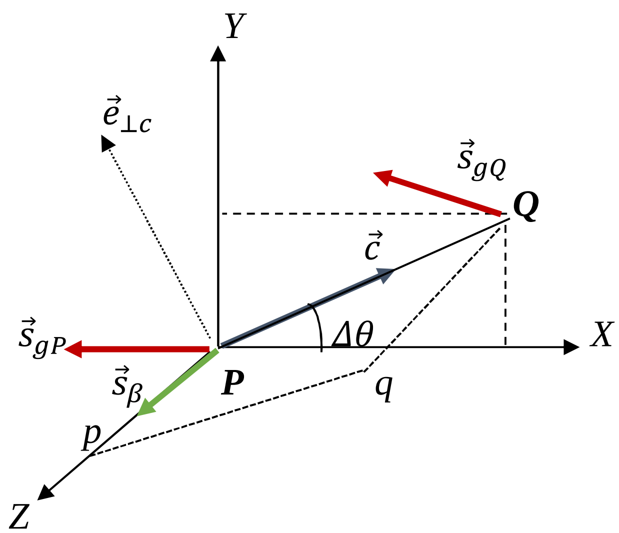

Fig 4

$$rot(\vec{s}_\beta) = \frac{\oint \vec{s}_\beta . \overrightarrow{dl}}{dS} . \vec{e}_{\perp c} = \frac{s_g . sin[(\Delta\theta)_P)] . Pp - s_g . \sin[(\Delta\theta)_Q] . qQ}{c . dt . Pp} \vec{e}_{\perp c}$$

The characteristic angle of the informatons at *Q* at the moment *t* is equal to the characteristic angle of the informatons at *P* at the moment (*t* – *dt*), so:

$$rot(\vec{s}_\beta) = \frac{\oint \vec{s}_\beta . \overrightarrow{dl}}{dS} . \vec{e}_{\perp c} = \frac{s_g . \{\sin[\,\Delta\theta(t)]. Pp - s_g . \sin[\Delta\theta(t-dt)]\}. qQ}{c. dt. Pp} \vec{e}_{\perp c}$$

The rate at which sin($\Delta\theta$) at *P* changes at the moment *t*, is:

$$\frac{\partial\{sin(\,\Delta\theta)\}}{\partial t} = \frac{sin\{(\,\Delta\theta)[t]\} \, - \sin\{(\Delta\theta)\,[t-dt]\}}{dt}$$

So:

$$rot\left(\vec{s}_\beta\right) = s_g . \frac{1}{c} . \frac{\partial[\sin(\Delta\theta)]}{\partial t} . \vec{e}_{\perp c}$$

And with

$$n = \frac{N}{c}$$

we finally obtain:

$$n. rot\left(\vec{s}_\beta\right) = s_g . \frac{1}{c^2} . N . \frac{\partial[\sin(\Delta\theta)]}{\partial t} . \vec{e}_{\perp c} \qquad (14)$$

Substituting the results (13) and (14) in (12) we obtain:

$$rot\vec{B}_g = \frac{1}{c^2} . s_g . \{\frac{\partial N}{\partial t} . sin(\,\Delta\theta) + N . \frac{\partial[\sin(\Delta\theta)]}{\partial t}\} . \vec{e}_{\perp c}$$

$$= \frac{1}{c^2} . s_g . \frac{\partial}{\partial t}[N . \sin(\Delta\theta)] . \vec{e}_{\perp c} \qquad (\mathbf{15})$$

B. Now we calculate $\frac{\partial \vec{E}_g}{\partial t}$

We know that[6]:

$$\frac{\partial \vec{E}_g}{\partial t} = \frac{\partial N}{\partial t} . \vec{s}_g + N . \frac{\partial \vec{s}_g}{\partial t}$$

And that:

$$\vec{s}_g = -s_g . \vec{e}_x \qquad \text{and} \qquad \frac{\partial \vec{s}_g}{\partial t} = s_g . \frac{\partial (\Delta\theta)}{\partial t} . \vec{e}_y$$

So:

$$\frac{\partial \vec{E}_g}{\partial t} = -\frac{\partial N}{\partial t} . s_g . \vec{e}_x + N . s_g . \frac{\partial (\Delta\theta)}{\partial t} . \vec{e}_y$$

Taking into account:

$$\vec{e}_x = cos(\Delta\theta) . \vec{e}_c - sin(\Delta\theta) . \vec{e}_{\perp c} \quad \text{and} \quad \vec{e}_y = sin(\Delta\theta) . \vec{e}_c + cos(\Delta\theta) . \vec{e}_{\perp c}$$

we obtain:

$$\frac{\partial \vec{E}_g}{\partial t} = \left[-\frac{\partial N}{\partial t} . s_g . cos(\Delta\theta) + N . s_g . \frac{\partial (\Delta\theta)}{\partial t} . sin(\Delta\theta)\right] . \vec{e}_c + \left[\frac{\partial N}{\partial t} . s_g . sin(\Delta\theta) + N . s_g . \frac{\partial (\Delta\theta)}{\partial t} . cos(\Delta\theta)\right] . \vec{e}_{\perp c}$$

or:

$$\frac{\partial \vec{E}_g}{\partial t} = s_g . \left\{-\frac{\partial}{\partial t}[N . cos(\Delta\theta)\right] . \vec{e}_c + \frac{\partial}{\partial t}[N . \sin(\Delta\theta)] . \vec{e}_{\perp c}\}$$

Taking into account (6), we find:

$$\frac{\partial \vec{E}_g}{\partial t} = s_g . \frac{\partial}{\partial t}[N . \sin(\Delta\theta)] . \vec{e}_{\perp c} \qquad (\mathbf{16})$$

C. From (15) an (16), we conclude:

$$rot\vec{B}_g = \frac{1}{c^2} \frac{\partial \vec{E}_g}{\partial t}$$

(4) *At a matter free point P of a gravitational field, the spatial variation of* $\vec{B}_g$ *and the rate at which* $\vec{E}_g$ *is changing are connected by the relation:*

$$rot\vec{B}_g = \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t}$$

This is the fourth equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that any change of the product $N.\vec{s}_g$ at a point of a gravitational field is related to a spatial variation of the product $n.\vec{s}_g$ in the vicinity of that point.

This relation implies (theorem of Stokes): *In a gravitational field, the rate at which the surface integral of* $\vec{E}_g$ *over a surface S changes is proportional to the line integral of* $\vec{B}_g$ *over its boundery L:*

$$\oint \vec{B}_g\,.\overrightarrow{dl} = \frac{1}{c^2}\iint_S \frac{\partial \vec{E}_g}{\partial t}.\overrightarrow{dS} = \frac{1}{c^2}\frac{\partial}{\partial t}\iint_S \vec{E}_g\,.\overrightarrow{dS} = \frac{1}{c^2}\frac{\partial \Phi_G}{\partial t}$$

The orientation of the surface vector $\overrightarrow{dS}$ is linked to the orientation of the path on *L* by the "rule of the corkscrew". $\Phi_G = \iint_S \vec{E}_g\,.\overrightarrow{dS}$ is called the "g-information-flux through *S*".

6 THE MAXWELL-HEAVISIDE EQUATIONS

The volume-element at a point *P* inside a mass continuum is in any case an emitter of g-information and, if the mass is moving, also a source of β -information. According to the theory of informatons[4],[5] , the instantaneous value of ρ_G - the mass density at *P* - contributes to the instantaneous value of $div\vec{E}_g$ at that point with an amount $-\frac{\rho_G}{\eta_0}$; and the instantaneous value of $\vec{J}_G$ - the mass flow density - contributes to the instantaneous value of $rot\vec{B}_g$ at *P* with an amount $-\nu_0.\vec{J}_G$.

Generally, at a point of a gravitational field - linked to an inertial reference frame ***O*** - one must take into account the contributions of the local values of $\rho_G(x, y, z; t)$ and of $\vec{J}_G(x, y, z; t)$. This results in the generalization and expansion of the laws in a mass free point. By superposition we obtain:

(1) *At a point P of a gravitational field, the spatial variation of* $\vec{E}_g$ *obeys the law:*

$$div\vec{E}_g = -\frac{\rho_G}{\eta_0}$$

In integral form:

$$\Phi_G = \oiint_S \vec{E}_g \,.\, \overrightarrow{dS} = -\frac{1}{\eta_0} . \iiint_G \rho_G . dV$$

(2) *At a point P of a gravitational field, the spatial variation of $\vec{B}_g$ obeys the law:*

$$div\vec{B}_g = 0$$

In integral form:

$$\Phi_B = \oiint_S \vec{B}_g \,.\, \overrightarrow{dS} = 0$$

(3) *At a point P of a gravitational field, the spatial variation of $\vec{E}_g$ and the rate at which $\vec{B}_g$ is changing are connected by the relation:*

$$rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

In integral form:

$$\oint \vec{E}_g \,.\, \overrightarrow{dl} = -\iint_S \frac{\partial \vec{B}_g}{\partial t} . \overrightarrow{dS} = -\frac{\partial}{\partial t} \iint_S \vec{B}_g \,.\, \overrightarrow{dS} = -\frac{\partial \Phi_B}{\partial t}$$

(4) *At a point P of a gravitational field, the spatial variation of $\vec{B}_g$ and the rate at which $\vec{E}_g$ is changing are connected by the relation:*

$$rot\vec{B}_g = \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t} - \nu_0 . \vec{J}_G$$

In integral form:

$$\oint \vec{B}_g \,.\, \overrightarrow{dl} = \frac{1}{c^2} \iint_S \frac{\partial \vec{E}_g}{\partial t} . \overrightarrow{dS} - \nu_0 . \iint_S \vec{J}_g \,.\, \overrightarrow{dS} = \frac{1}{c^2} . \frac{\partial}{\partial t} \iint_S \vec{E}_g \,.\, \overrightarrow{dS} - \nu_0 . \iint_S \vec{J}_G \,.\, \overrightarrow{dS}$$

These are the laws of Heaviside-Maxwell or the laws of GEM.

The mathematical deductions confirm that these equations nor their solutions indicate an existence of causal links between $\vec{\boldsymbol{E}}_{\boldsymbol{g}}$ and $\vec{\boldsymbol{B}}_{\boldsymbol{g}}$. Therefore, *we must conclude that a gravitational field is a dual entity always having a "field-" and* *an "induction-" component simultaneously created by their common sources: time-variable masses and mass flows*•.

GEM is consistent with special relativity. The GEM equations are analogue to Maxwell's equations in EM and it is proved[7] that these are consistent with special special relativity. Thus, *the Maxwell-Heaviside equations are invariant under a Lorentz transformation.* In this context it should be noted that the fact that the rate at which a material body emits informatons is independent of its velocity[4],[5] and completely defined by its rest mass m_0, implies that in equation (1) the value of $\rho_G = \frac{dm_0}{dV}$ depends on the state of motion – relative to the considered inertial reference system - of the mass element dm_0. Indeed in the case of a moving mass element, the Lorentz contraction must be taken into account in the determination of dV. Because a mass flow is made up of moving mass elements its density $\vec{J}_G$ also depends on the inertial reference frame in which it is considered. This implies that in equation (4) the expression of $\vec{J}_G$ depends on the inertial reference frame.

7 THE GEM EQUATIONS ARE MATHEMATICALLY CONSISTENT

At a point P of a gravitational field - where ρ_G is the mass density and $\vec{J}_G$ is the density of the mass flow - $\vec{E}_g$ and $\vec{B}_g$ must obey to the GEM equations (the Maxwell-Heaviside equations):

1. $div\vec{E}_g = -\frac{\rho_G}{\eta_0}$

2. $div\vec{B}_g = 0$

3. $rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$

4. $rot\vec{B}_g = \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t} - \nu_0.\vec{J}_G$

And: $\eta_0.\nu_0 = \frac{1}{c^2}$

• On the understanding that the induction-component equals zero if the source of the field is time independent.

We will prove that these equations are mathematically consistent.

7.1 The case of an object with invariable rest mass

Because $div(rot\vec{F}) = 0$, it follows from (4) that:

$$\frac{1}{c^2}\frac{\partial}{\partial t}\left(div\vec{E}_g\right) - \nu_0.div\vec{J}_G = 0 \qquad (4')$$

Substituting (1) in (4') gives:

$$-\frac{1}{c^2\eta_0}.\frac{\partial\rho_G}{\partial t} - \nu_0.div\vec{J}_G = 0$$

And with $\frac{1}{c^2\eta_0} = \nu_0$, we obtain from (4'):

$$\frac{\partial\rho_G}{\partial t} + div\vec{J}_G = 0 \qquad (4")$$

(4") is nothing else but the expression of the law of mass conservation. Indeed:

- The rate at which mass is flowing out form a closed surface S is:

$$\oiint_S \vec{J}_G.\overrightarrow{dS} \qquad (A)$$

- The rate of the decrease of the mass enclosed by S is (V is the volume enclosed by S):

$$-\frac{\partial}{\partial t}\iiint_V \rho_G dV = \iiint_V \left(-\frac{\partial\rho_G}{\partial t}\right).dV \qquad (B)$$

Because of the law of mass conservation (A) = (B), so

$$\oiint_S \vec{J}_G.\overrightarrow{dS} = \iiint_V \left(-\frac{\partial\rho_G}{\partial t}\right).dV \qquad (5)$$

Ostrogradsky's theorem (divergence theorem) states that

$$\oiint_S \vec{F}.\overrightarrow{dS} = \iiint_V div\vec{F}.dV$$

Substituting in (5) gives:

$$\iiint_V div\vec{J}_G.dV = \iiint_V (-\frac{\partial \rho_G}{\partial t}).dV$$

It follows:

$$div\vec{J}_G = -\frac{\partial \rho_G}{\partial t}$$

Or:

$$\frac{\partial \rho_G}{\partial t} + div\vec{J}_G = 0$$

We conclude that - in a system with invariable rest mass - the GEM equations of the gravitational field are in line with the law of mass conservation.

2. The case of an object with variable rest mass

Let us consider - relative to an inertial reference frame - an object with rest mass m_0 that - due to intern instability - during the period (0, Δt) emits EM radiation. This implies that that object during that time interval is emitting electromagnetic energy U_{EM} carried by photons (+ gravitomagnetic energy• U_{GEM} carried by gravitons) that propagate with the speed of light. Because of that event, from the moment $t = \Delta t$ the rest mass of the particle is decreased with an amount $\frac{U_{EM}+(U_{GEM})}{c^2}$ to the value m_0'.

Consider the surface S enclosing the object in whole or in part (V is the volume enclosed by S). At a moment $0 < t < \Delta t$:

- The rate of the decrease of the enclosed mass is:

• negligible in first approximation

$$-\frac{\partial}{\partial t}\iiint_V \rho_G dV = \iiint_V \left(-\frac{\partial \rho_G}{\partial t}\right).dV \qquad \text{(A)}$$

- $\vec{J}_G$, the density of the mass flow out from the enclosed volume at a point *P* of *S* has two components:

1. $\vec{J}_{G1}$ describing the outflow of massive mass;
2. $\vec{J}_{G2}$ describing the outflow of mass in the form of energy. If we represent the density of that energy flow by $\vec{S}$: $\vec{J}_{G2} = \frac{\vec{S}}{c^2}$

So:

$$\vec{J}_G = \vec{J}_{G1} + \vec{J}_{G2} = \vec{J}_{G1} + \frac{\vec{S}}{c^2}$$

,

and the rate at which mass-energy is flowing out from the closed surface *S* is:

$$\oiint_S \vec{J}_G.\overrightarrow{dS} \qquad \text{(B)}$$

(A) = (B) because of the law of mass-energy conservation, so

$$\oiint_S \vec{J}_G.\overrightarrow{dS} = \iiint_V \left(-\frac{\partial \rho_G}{\partial t}\right).dV$$

and

$$div\vec{J}_G = -\frac{\partial \rho_G}{\partial t} \qquad or \qquad \frac{\partial \rho_G}{\partial t} + div\vec{J}_G = 0$$

We conclude that in the case of a system with variable rest mass, the GEM equations of the gravitational field are in line with the law of mass-energy conservation.

8 GRAVITATIONAL WAVES

8.1 The wave equation

In free space - where $\rho_G = \vec{J}_G = 0$ - the GEM equations are:

1. $div\vec{E}_g = 0$

2. $div\vec{B}_g = 0$

3. $rot\vec{E}_g = -\frac{\partial\vec{B}_g}{\partial t}$

4. $rot\vec{B}_g = \frac{1}{c^2}\frac{\partial\vec{E}_g}{\partial t}$

To attempt a solution of a group of simultaneous equations, it is usually a good plan to separate the various functions of space to arrive at equations that give the distributions of each.

It follows from (3):

$$rot\left(rot\vec{E}_g\right) = -rot\left(\frac{\partial\vec{B}_g}{\partial t}\right) \qquad (3')$$

Because[6] $rot(rot\vec{F}) = grad(div\vec{F}) - \nabla^2\vec{F}$, where $\nabla^2$is the Laplacian,

(3') leads to:

$$grad(div\vec{E}_g) - \nabla^2\vec{E}_g = -rot(\frac{\partial\vec{B}_g}{\partial t}) = -\frac{\partial}{\partial t}(rot\vec{B}_g)$$

And taking into account (1) and (4):

$$\nabla^2\vec{E}_g = \frac{1}{c^2}.\frac{\partial^2\vec{E}_g}{\partial t^2} \qquad (5)$$

This is the general form of the wave equation. This form applies as well to the g-induction, as is readily shown by taking first the rotor of (4) and then substituting (2) and (3):

$$\nabla^2 \vec{B}_g = \frac{1}{c^2} \cdot \frac{\partial^2 \vec{B}_g}{\partial t^2} \qquad (5')$$

Solutions of this equation describe how disturbances of the gravitational field propagate as waves with speed c.

To illustrate this we consider the special case of space variation in one dimension only. If we take the *x*-component of (5) and have space variations only in the *z*-direction, the equation becomes simply:

$$\frac{\partial^2 E_{gx}}{\partial z^2} = \frac{1}{c^2} \cdot \frac{\partial^2 E_{gx}}{\partial t^2}$$

This equation has a general solution of the form

$$E_{gx} = f_1\left(t - \frac{z}{c}\right) + f_2\left(t + \frac{z}{c}\right) \qquad (6)$$

The first term of (6) represents the wave or function f_1 traveling with velocity *c* and unchanged form in the positive *z*-direction, the second term represents the wave or function f_2 traveling with velocity *c* and unchanging form in the negative *z*-direction.

8.2 Gravitational wave generated by an object with variable rest mass

Another phenomenon that is the source of a gravitational wave is the conversion of rest mass into energy (what per example happens in the case of radioactive processes). To illustrate this, let us - relative to an inertial reference frame - consider a particle with rest mass m_0 that - due to intern instability - during the period $(0, \Delta t)$ emits EM radiation.

This implies that that particle during that time interval is emitting electromagnetic energy U_{EM} carried by photons (and gravitational energy U_{GEM}• carried by gravitons) that propagate with the speed of light. Between the moment $t = 0$ and the moment $t = \Delta t$, the rest mass of the particle is, because of this event, decreasing *with* an amount $\frac{U_{EM}(+U_{GEM})}{c^2}$ from the value m_0 to the value m_0'. Because the gravitational field is determined by the rest mass, this implies that if $t < 0$ the source of the gravitational field of the particle is m_0 and for $t > \Delta t$ it is m_0'. It follows that at the moment t the gravitational field at a point P at a distance $r>c.t$ is proportional to m_0, and at a point at a distance $r < c.(t - \Delta t)$ to m_0'.

During the period (t, $t+\Delta t$) the gravitational field at a point at a distance $r = c.t$ changes from the situation where it is determined by m_0 to the situation where it is determined by m_0'. So, the conversion of rest mass of an object into radiation is the cause of a kink in the gravitational field of that object, a kink that with the speed of light - together with the emitted radiation - propagates out of the object.

We can conclude that the conversion of (a part of) the rest mass of an object into radiation goes along with the emission by that object of a gravitational wave.

The effect of the decrease - during the time interval (0, Δt) - of the rest mass of a point mass on the magnitude of its g-field E_g at the point P at a distance r is shown in the plot of fig. 5.

1. Until the moment $t = \frac{r}{c}$, the effect of the conversion of rest mass into radiation has not yet reached P. So, during the period $(0, \frac{r}{c})$ the quantity of mass-energy enclosed by an hypothetical sphere with radius r centered on the particle is still m_0 (the remaining part of the rest mass + all the radiation that during the mentioned period has arisen from the conversion of rest mass). From the first GEM equation it follows:

$$E_g = \frac{m_0}{4\pi\eta_0.r^2}$$

2. From the moment $t = \frac{r}{c} + \Delta t$, the radiation generated by the conversion of rest mass has left the space enclosed by the hypothetical sphere with radius r, that from that moment only contains the remaining rest mass m_0'. From the first GEM equation it follows:

• negligible in first approximation

$$E_g = \frac{m_0'}{4\pi\eta_0 . r^2}$$

3. During the time interval $(\frac{r}{c}, \frac{r}{c} + \Delta t)$, the mass-energy enclosed by the hypothetical sphere with radius r is decreasing (not necessary linearly) because mass-energy flows out in the form of radiation. So, during that period E_g at P is decreasing.

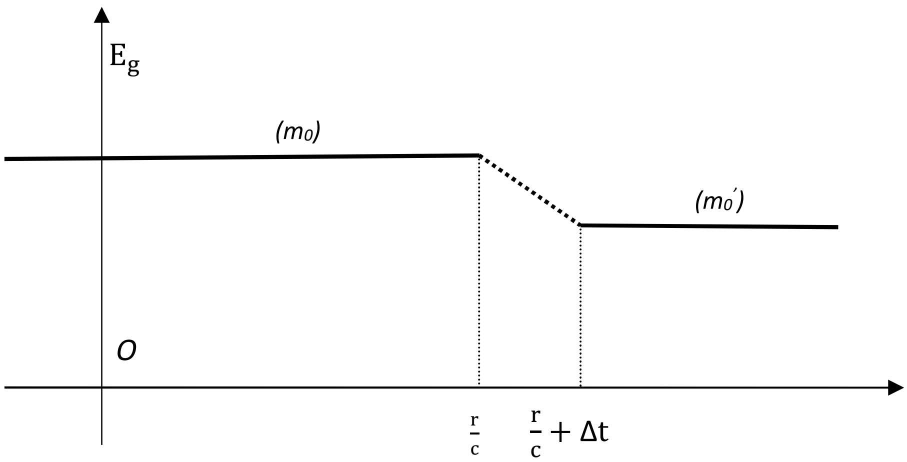

Fig 5

8.3 Gravitational wave generated by a harmonically oscillating particle

In fig 6 we consider a point mass *m* that harmonically oscillates, with frequency $\nu = \frac{\omega}{2.\pi}$, around the origin of the inertial reference frame ***O***. At the moment *t* it passes at *P₁*. We suppose that the speed of the charge is always much smaller than the speed of light and that it is described by:

$$v(t) = V.\cos\omega\, t$$

The elongation *z(t)* and the acceleration *a(t)* are then expressed as:

$$z(t) = \frac{V}{\omega}.\cos(\omega t - \frac{\pi}{2}) \qquad \text{and} \qquad a(t) = \omega.V.\cos(\omega t + \frac{\pi}{2})$$

We restrict our considerations about the gravitational field of *m* to points *P* that are sufficiently far away from the origin *O*. Under that condition we can posit that the fluctuation of the length of the vector $\overrightarrow{P_1P} = \vec{r}_1$ is very small relative to the length of the time-independent position vector $\vec{r}$, that defines the position of P relative to the origin O. In other words: we assume that the amplitude of the oscillation is very small relative to the distances between the origin and the points P on which we focus.

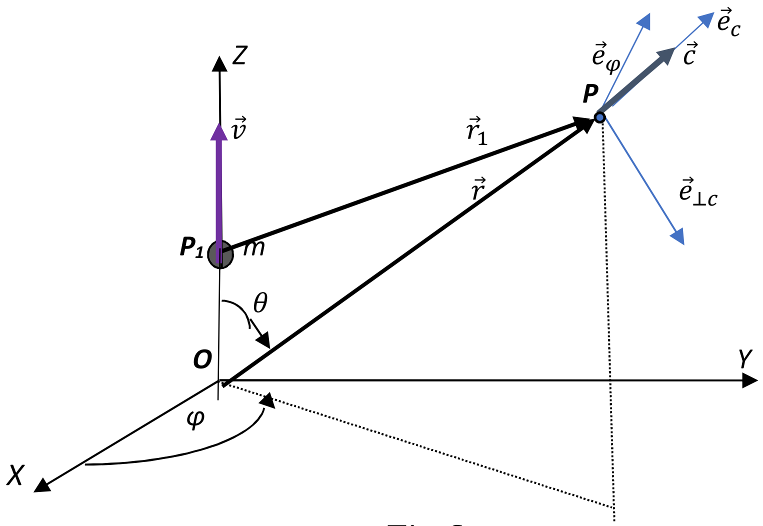

Fig 6

In the framework of the theory of informatons[4],[5] it is shown that, starting from the complex quantity $\bar{V} = V.e^{j.0}$ - representing $v(t)$ - $\bar{E}_{g\perp c}$, the complex representation of the time dependent part of the transversal component of $\vec{E}_g$ and $\bar{B}_{g\varphi}$, the complex representation of $\vec{B}_g$, at P are:

$$\bar{E}_{g\perp c} = -\frac{m.\bar{V}}{4\pi}.e^{-j.k.r}.(\frac{1}{\eta_0.c.r^2} + \frac{j.\omega.\nu_0}{r}).sin\,\theta$$

$$\bar{B}_{g\varphi} = -\frac{\nu_0.m.\bar{V}}{4\pi}.e^{-j.k.r}.(\frac{1}{r^2} + \frac{j.k}{r}).sin\,\theta$$

where $k = \frac{\omega}{c}$ the phase constant. Note that $\bar{B}_{g\varphi} = \frac{\bar{E}_{g\perp c}}{c}$.

Thus, relative to ***O***, $B_{g\varphi}$ and the time dependent part of $E_{g\perp c}$ are expressed as functions of the space and time coordinates as:

$$B_{g\varphi}(r,\theta;t) = \frac{E_{g\perp c}(r,\theta;t)}{c}$$
$$= \frac{\nu_0.m.V.sin\,\theta\,.\sqrt{1+k^2r^2}}{4\pi r^2}.cos(\,\omega t - kr + \Phi + \pi)$$

with $tg\Phi = kr$.

So, an harmonically oscillating particle emits a transversal "gravitomagnetic" wave that propagates out of the mass with the speed of light:

In points at a great distance from the oscillating mass, specifically there where $r >> \frac{1}{k} = \frac{c}{\omega}$, this expression asymptotically equals:

$$B_{g\varphi} = \frac{E_{g\perp c}}{c} = \frac{\nu_0.k.m.V.sin\,\theta}{4\pi r}.sin(\,\omega t - kr)$$
$$= \frac{\nu_0.m.\omega.V.sin\,\theta}{4\pi cr}.sin(\,\omega t - kr)$$

$$= -\frac{\nu_0.m.a\left(t - \frac{r}{c}\right).sin\,\theta}{4\pi cr}$$

The intensity of the "far gravitational field" is inversely proportional to *r*, and is determined by the component of the acceleration of *m*, that is perpendicular to the direction of $\vec{e}_c$.

We can conclude that the existence of gravitational waves is embedded in the GEM description of gravity. According to the theory of informatons a gravitational wave is the macroscopic manifestation of the fact that the "train" of informatons emitted by an oscillating source and travelling with the speed of light in a certain direction is a spatial sequence of informatons whose characteristic angle is harmonically fluctuating along the "train" what implies that the component of their g-index perpendicular to their velocity $\vec{c}$ and their β-index fluctuate harmonically in space. Gravitational waves transport gravitational energy because some of the informatons that constitute the "train" are carriers of energy. They are called gravitons. However, the energy quantum carried by a graviton is small in such a way that it is very difficult to give experimental evidence of its existence.

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