Primordial Black Holes in the Pre-inflationary Period, the Probabilities of Their Occurrence Taking Into Account Quantum-Gravitational Corrections, and Some Cosmological Implications

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Abstract

In the last decades the primordial black holes (**pbhs**) have attracted much attention of cosmologists and astrophysicists. This is associated with origination of such black holes in the early Universe as a result of a gravitational collapse of the high-density matter, making them natural "detectors" of the processes involved. In inflationary cosmology of particular importance are **pbhs** originated during the pre-inflationary period. And, since they are small and generated at the energies close to the Planck energies, for them we should take into consideration the quantum-gravitational corrections (**qgcs**). In turn, these corrections change (shift) the inflationary parameters. The paper presents a study of the above-mentioned shifts with regard to these corrections for different scenarios. It is shown that probabilities of occurrence of the **pbhs** under study with due regard for the given **qgcs** are rising as compared to the semiclassical consideration. Besides, high-energy deformations of Friedmann Equations created on the basis of these corrections have been derived for different patterns. Conclusion contains the general remarks concerning the above-mentioned **qgcs** for cosmological parameters and perturbations due to inflation; the steps for their investigation are outlined and the key problems of such a study are formulated.

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1 Introduction

In this paper the inclusion of the quantum-gravitational corrections (**qgcs**) for primordial black holes (**pbhs**) in the early Universe during the preinflationary era is studied. In [1] a semiclassical approximation was used to study the problem of scalar perturbations due to such **pbhs**. But, considering that all the processes in this case are proceeding at very high energies E close to the Planckian $E \simeq E_p$, the inclusion of **qgcs** for these black holes in this pattern is necessary. The paper presents an explicit solution of this problem; specifically, it is shown how in this pattern the inclusion of **qgcs** changes ("shifts") the basic inflationary parameters.
The explicit and effective formulae for these "shifts" have been derived in Section 3. Section 2 presents the instruments used to obtain the principal results. In Section 4 it is demonstrated that inclusion of **qgcs** increases the occurrence probability for such (**pbhs**). In Section 5 the high-energy deformations of Friedmann Equations on the basis of these **qgcs** are derived for different cases.
Finally in Section 6 (Conclusion) the general remarks are given for calculations of the indicated **qgcs** in the case of perturbations on inflation; the steps for investigation of the cosmological parameters corrections and cosmological perturbations due to these **qgcs** are enumerated; the problems of further studies are formulated.
In what follows the normalization $c = \hbar = 1$ is used, for which we have $G = l_p^2$.
As is known, the most common formation mechanism of primordial black holes (**pbhs**) in the early Universe [2]–[4], is a gravitational collapse of the high-density matter [5]. In several works it has been shown that (**pbhs**) in the early Universe may be responsible for its shifted cosmological parameters. We know a sufficiently accurate estimate of the mass **pbh** $M(t_M)$ formed in the period of time t since the Big Bang [6]–[8]

$$M(t_M) \approx \frac{c^3 t_M}{G} \approx 10^{15} \left(\frac{t}{10^{-23}\ \text{s}} \right) g. \tag{1}$$

As seen, for small times close to the Planckian time $t_M = t_p \approx 10^{-43}$s, the mass of **pbhs** is close to the Planck mass $M(t_M) \approx 10^{-5} g$, necessitating in this case the inclusion of the quantum-gravitational corrections **qgcs**. Though in the majority of works **pbhs** in the early Universe are studied by a semiclassical approach. To illustrate, in [1] the scalar cosmological perturbations associated with small-radius **pbhs** in the pre-inflationary era are studied precisely in the semiclassical approximation. This paper is devoted to inclusion of **qgcs** in such cases.

Despite the fact that presently there is no self-consistent theory of quantum gravity, a consensus is reached on correctness of some approaches to the theory, specifically, replacement of the Heisenberg Uncertainty Principle **(HUP)** by the Generalized Uncertainty Principle **(GUP)** on going to high (Planck's)energies, used in this paper.

2 PBH with the Schwarzschild-de Sitter Metric in the Early Universe

It should be noted that Schwarzschild black holes in real physics (cosmology, astrophysics) are idealized objects. As noted in (p.324,[10]): "Spherically symmetric accretion onto a Schwarzschild black hole is probably only of academic interest as a testing for theoretical ideas. It is of little relevance for interpretations of the observations data. More realistic is the situation where a black hole moves with respect to the interstellar gas..."

Nevertheless, black holes just of this type may arise and may be realistic in the early Universe. In this case they are primordial black holes (**pbhs**). Most common mechanism for the formation of **pbhs** is the high-density gravitation matter collapse generated by cosmological perturbations arising, e.g., in the process of inflation (not necessarily) in the early Universe [5]. But the idea about the formation of **pbhs** has been suggested much earlier than the first inflation models, specifically in [2] and independently in [3] or [4].

During studies of the early Universe the Schwarzschild metric [11],[10]

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \qquad (2)$$

for **pbhs** is replaced by the Schwarzschild-de Sitter (SdS) metric [1] that is associated with Schwarzschild black holes with small mass M in the early Universe, in particular in pre-inflation epoch

$$ds^2 = -f(\tilde{r})dt^2 + \frac{d\tilde{r}^2}{f(\tilde{r})} + \tilde{r}^2 d\Omega^2 \tag{3}$$

where $f(\tilde{r}) = 1 - 2GM/\tilde{r} - \Lambda\tilde{r}^2/3 = 1 - 2GM/\tilde{r} - \tilde{r}^2/L^2, L = \sqrt{3/\Lambda} = H_0^{-1}$, M - black hole mass, Λ – cosmological constant, and $L = H_0^{-1}$ is the Hubble radius.

In general, such a black hole may have two different horizons corresponding to two different zeros $f(\tilde{r})$: event horizon of a black hole and cosmological horizon. This is just so in the case under study when a value of M is small [12],[13]. In the general case of $L \gg GM$, for the event horizon radius of a black hole having the metric (3), r_{H} takes the following form (formula (9) in [14]):

$$r_{\mathrm{H}} \simeq 2GM\left[1 + \left(\frac{r_M}{L}\right)^2\right], where\ r_M = 2MG. \tag{4}$$

Then, due to the assumption concerning the initial smallness of Λ, we have $L \gg r_M$. In this case, to a high accuracy, the condition $r_{\mathrm{H}} = r_M$ is fulfilled, i.e. for the considered (SdS) BH we can use the formulae, given in the previous section for a Schwarzschild BH, to a great accuracy.

Remark 2.1.

Note that, because Λ is very small, the condition $L \gg GM$ and hence the formula of (4) are obviously valid not only for black hole with the mass $M \propto m_p$ but also for a much greater range of masses, i.e. for black holes with the mass $M \gg m_p$, taking into account the condition $L \gg GM$. In fact we obtain ordinary Schwarzschild black holes (2) with small radius.

Specifically, for the energies on the order of Plank energies (quantum gravity scales) $E \simeq E_p$, the Heisenberg Uncertainty Principle (**HUP**) [15]

$$(\delta X)(\delta P) \geq \frac{\hbar}{2}, \tag{5}$$

may be replaced by the Generalized Uncertainty Principle (**GUP**) [16]

$$(\delta X)(\delta P) \geq \frac{\hbar}{2}\left\langle \exp\left(\frac{\alpha^2 l_p^2}{\hbar^2}P^2\right)\right\rangle. \tag{6}$$

Then there is a possibility for existence of Planck Schwarzschild black hole, and accordingly of a Schwarzschild sphere (further referred to as ”minimal”) with the minimal mass M_0 and the minimal radius r_{min} (formula (20) in [16]) that is a theoretical minimal length r_{min}:

$$r_{min} = l_{min} = (\delta X)_0 = \sqrt{\frac{e}{2}}\alpha l_p, \quad M_0 = \frac{\alpha\sqrt{e}}{2\sqrt{2}}m_p, \tag{7}$$

where α - model-dependent parameters on the order of 1, e - base of natural logarithms, and $r_{min} \propto l_p, M_0 \propto m_p$.
In this case, due to GUP (6), the physics becomes nonlocal and the position of any point is determined accurate to l_{min}. It is impossible to ignore this nonlocality at the energies close to the Planck energy $E \approx E_p$, i.e. at the scales $l \propto l_p$ (equivalently we have $l \propto r_{min} = l_{min}$).
Actually, [16] presents calculated values of the mass $\mathcal{M}$ and the radius $\mathcal{R}$ for Schwarzschild BH with regard to the quantum-gravitational corrections within the scope of GUP (6).
With the use of the normalization $G = l_p^2$ adopted in [16], temperature of a Schwarzschild black hole having the mass $\mathcal{M}$ (the radius $\mathcal{R}$) [10] in a semi-classical approximation takes the form

$$T_{\rm H} = \frac{1}{8\pi G\mathcal{M}}. \tag{8}$$

Within the scope of GUP (6),the temperature $T_{\rm H}$ with regard to (**qgc**) is of the form ((23) in [16]))

$$T_{{\rm H},q} = \frac{1}{8\pi\mathcal{M}G}\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right) = \frac{1}{8\pi\mathcal{M}G}\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{A_0}{\mathcal{A}}\right)\right)\right), \tag{9}$$

where $\mathcal{A}$ is the black hole horizon area of the given black hole,$A_0 = 4\pi\left(\delta X\right)_0^2$ is the black hole horizon area of a minimal quantum black hole from formula (7) and $W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right) = W\left(-\frac{1}{e}\left(\frac{A_0}{\mathcal{A}}\right)\right)$ – value at the corresponding point of the Lambert W-function $W(u)$ satisfying the equation (formulae (1.5) in [17] and (9) in [16])

$$W\left(u\right)e^{W(u)} = u. \tag{10}$$

$W(u)$ is the multifunction for complex variable $u = x+yi$. However, for real $u = x, -1/e \leq u < 0$,$W(u)$ is the single-valued continuous function having two branches denoted by $W_0(u)$ and $W_{-1}(u)$, and for real $u = x, u \geq 0$ there is only one branch $W_0(u)$ [17].
Obviously, the quantum-gravitational correction **qgc** (9) presents a *deformation* (or more exactly, the *quantum deformation* of a classical black-holes theory from the viewpoint of the paper [18] with the deformation parameter $A_0/\mathcal{A}$):

$$\frac{A_0}{\mathcal{A}} = \frac{4\pi r_h^2}{4\pi R^2(\mathcal{A})} = \frac{l_{min}^2}{R^2(\mathcal{A})}, \tag{11}$$

where $r_h = l_{min}$ is the horizon radius of minimal **pbh** from formula (7) and $R(A)$ is the horizon radius of the given black hole with the black hole horizon area A.
It should be noted that this deformation parameter

$$l_{min}^2/R^2(\mathcal{A}) \doteq \alpha_{R(\mathcal{A})} \tag{12}$$

has been introduced by the author in his earlier works [19]–[22], where he studied deformation of quantum mechanics at Planck scales in terms of the deformed quantum mechanical density matrix. In the Schwarzschild black hole case $\alpha_{R(\mathcal{A})} = l_{min}^2\mathcal{K}$ – product of the minimal surface area l_{min}^2 by the Gaussian curvature $\mathcal{K} = 1/R^2(\mathcal{A})$ of the black-hole horizon surface [23] as indicated in [24],[25].
It is clear that, for a great black hole having large mass $\mathcal{M}$ and great event horizon area $\mathcal{A}$, the deformation parameter $\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2$ is vanishingly small and close to zero. Then a value of $W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)$ Is also close to $W(0)$. As seen, $W(0) = 0$ is an obvious solution for the equation (10). We have

$$\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right) \approx 1. \tag{13}$$

So, a black hole with great mass $\mathcal{M} \gg m_p$ necessitates no consideration of **qgcs**.
But in the case of small black holes we have

$$\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right) > 1. \tag{14}$$

In formulae above it is assumed that $\mathcal{M} > M_0$, i.e. the black hole under study is not minimal (7).
We can rewrite the formula of (9) as follows:

$$T_{\mathrm{H},q} = \frac{1}{8\pi \mathcal{M}_q G}, \mathcal{M}_q = \mathcal{M} \exp\left(\frac{1}{2} W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right);$$
$$\mathcal{R}_q = 2\mathcal{M}_q G = \mathcal{R} \exp\left(\frac{1}{2} W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right), \quad (15)$$

where $\mathcal{M}_q$ and $\mathcal{R}_q$ are respectively the initial black-hole mass and event horizon radius considering **qgcs** caused by GUP (6).
Remark 2.2
It is clear that the formula (15) with the substitution of $\mathcal{M} \mapsto \mathcal{M}_q$ is of the same form as formula (8), in fact representing (9),i.e. in the formula for temperature of a black hole the inclusion of **qgcs** may be realized in two ways with the same result: (a)the initial mass $\mathcal{M}$ remains unaltered and **qgcs** are involved only in the formula for temperature, in this case (9); (b)**qgcs** are involved in the mass – the above-mentioned substitution takes place $\mathcal{M} \mapsto \mathcal{M}_q$ (formula(15)). Such "duality" is absolutely right in this case if a black hole is considered in the stationary state in the absence of accretion and radiation processes. Just this case is also studied in the paper. A recent preprint [26] in the case (b) for the space-time dimension $D \geq 4$, using approaches to quantum gravity of the alternative GUP, gives a formula for the mass M_Q of a black hole with a due regard to **qgc**

$$M_Q = \left[1 - \eta \exp\left(-\frac{\pi r_0^{D-2}}{G_D}\right)\right]^{D-3} M. \quad (16)$$

Here in terms of [26] r_0 is the Schwarzschild radius of the primordial black hole with the mass M,G_D-gravitational constant in the dimension D, and $\eta = [0, 1]$ is a parameter. In case under study this parameter, as distinct from cosmology, has no relation to conformal time. Obviously, for $\eta = 0$ we have a semiclassical approximation and, as noted in [26], the case when $\eta = 1$ corresponds to **qgc** as predicted by a string theory.

3 Inflation Parameters Shifts Generated by QGC

To this end in cosmology, in particular inflationary, the metric (3) is conveniently described in terms of the conformal time η [1]:

$$ds^2 = a^2(\eta)\Bigg\{ - d\eta^2 + \left(1 + \frac{\mu^3\eta^3}{r^3}\right)^{4/3} \left[\left(\frac{1 - \mu^3\eta^3/r^3}{1 + \mu^3\eta^3/r^3}\right)^2 dr^2 + r^2 d\Omega^2\right] \Bigg\}, \tag{17}$$

where $\mu = (GMH_0/2)^{1/3}$, H_0 – de Sitter-Hubble parameter and scale factor, a – conformal time function η:

$$a(\eta) = -1/(H_0\eta), \eta < 0, \tag{18}$$

where with the preceding notation $\mathcal{M} = M, \mathcal{A} = A, ...$
Here r satisfies the condition $r_0 < r < \infty$ and a value of $r_0 = -\mu\eta$ in the reference frame of (17) conforms to singularity of the back hole.
Due to (4), μ may be given as

$$\mu = (r_M H_0/4)^{1/3}, \tag{19}$$

where r_M is the radius of a black hole with the SdS Schwarzschild-de Sitter metric (3).
Remark 3.1.
In [1] in general only the case $\mu = const$ is considered and, as noted in [1],we can exclude only the pattern with regard for radiation processes of **pbh**. Let us consider a much more general case: it is supposed that, as the mass M of **pbh** may be changed due to absorption and radiation processes, the corresponding change takes place for μ – in the general case we have ($\mu \neq const$) but μ is unaltered with regard to **qgcs**, i.e. in formula (19) we have $\mu = (r_M H_0/4)^{1/3} = (r_{M,q} H_{0,q}/4)^{1/3}$, where $r_{M,q}, H_{0,q}$ - values of r_M, H_0, respectively, with due regard for **qgcs**.
Let us consider several scenarios.
3.1. *The Stationary picture.* From the start the primordial black hole with the mass M and the event horizon area A is considered in the absence of absorption and radiation processes.
As $\mu = const$ and **pbh** is considered in the stationary state, then due to

Remark 2.2 with regard for **qgcs**, replacement $r_M \mapsto r_{M_q}$ in this formula leads to replacement of $H_0 \to H_{0,q}$, due to **Remark 3.1.** meeting the condition

$$\mu = (r_M H_0/4)^{1/3} = (r_{M,q} H_{0,q}/4)^{1/3}. \quad (20)$$

Here $r_{M,q} \doteq r_{M_q} = \mathcal{R}_q = \mathcal{R}_{\mathcal{M}_q}$ from the general formula (15).
Based on the last formula and formulae (9),(12),(15), for $\mathcal{M} = M, \mathcal{A} = A$ it directly follows that

$$H_{0,q} = H_0 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right). \quad (21)$$

Because the potential energy of inflation $V(\phi_0)$ is related to the initial Hubble parameter H_0 by the Friedmann equation $H_0^2 = V(\phi_0)/(3M_p^2) = \Lambda/3$,from (21) we can derive a shift for $V(\phi_0)$ that is due to quantum-gravitational corrections for the primordial Schwarzschild black hole with the mass M as follows:

$$\begin{aligned} V(\phi_0) \to V(\phi_0)_q = \Lambda_q M_p^2 = 3M_p^2 H_{0,q}^2 = \\ = 3\exp\left(-W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right) M_p^2 H_0^2, \end{aligned} \quad (22)$$

where Λ–effective cosmological constant and Λ_q is the same constant with regard to the above-mentioned **qgcs**. Here we have used the normalization differing from that used in [27],where $H_0^2 = 8\pi\Lambda/3$.
In a similar way we can find **qgcs** for all the remaining inflationary parameters, specifically for the scale factor $a(\eta)$ (18)

$$\begin{aligned} a(\eta) \to a(\eta)_q \doteq -1/(H_{0,q}\eta) = -1/(H_0 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right)\eta) = \\ = a(\eta)\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right), \eta < 0, \end{aligned} \quad (23)$$

for the Hubble parameter $H = a'(\eta)/a^2(\eta) \mapsto H_q(\eta) = a'(\eta)_q/a^2(\eta)_q$ as well as for the parameters in the mode of slow roll, e.g., for ϵ [27]:

$$(\epsilon = -\frac{\dot{H}}{H^2}) \mapsto (\epsilon_q = -\frac{\dot{H}_{0,q}}{H_{0,q}^2}), \quad (24)$$

where, as usual, a prime in the next to last formula means differentiation with respect to η, while a point in the last formula means differentiation with respect to t.
The condition $\epsilon \ll 1$ for slow roll in the inflationary scenario [27] due to (24) is transformed to the condition $\epsilon_q \ll 1$ from the last formula that should be additionally established for estimation of the boundary r_{M_q}.

3.2 *The case of "minimal" particle absorption by a black hole.*

Let M be the initial mass of a black hole with the event horizon area A. In [28],[29] a minimal increment of the event horizon area for the black hole absorbing a particle with the energy E and with the size R:$(\Delta A)_0 \simeq 4l_p^2\,(\ln 2)\,ER$ has been estimated. In quantum consideration we have $R \sim 2\delta X$ and $E \sim \delta P$.
However, in [28],[29] the consideration is based on a semiclassical pattern, i.e. for small δP, when GUP (6) goes to the well-known Heisenberg Uncertainty Principle **HUP**

$$(\delta X)\,(\delta P) \geq \frac{\hbar}{2},, \tag{25}$$

which, on equality of the left and the right sides of the last formula, gives $(\Delta A)_0 \simeq 4l_p^2\,(\ln 2)$.
Such absorption leads to the increased mass of a black hole $M \mapsto \tilde{M} = M + (\Delta M)_0$ and hence to its increased event horizon area A and radius $R(A)$:

$$\begin{aligned} M \mapsto \tilde{M} &= M + (\Delta M)_0, \\ A \mapsto \tilde{A} = A + (\Delta A)_0 &\simeq 4\pi R^2(A) + 4l_p^2 \ln 2, \\ R(A) \mapsto R(\tilde{A}) &\simeq \sqrt{R^2(A) + l_p^2 \pi^{-1} \ln 2}. \end{aligned} \tag{26}$$

It should be emphasized that the last formula of the pattern **3.2** has been obtained only for a semiclassical approximation [28],[29],i.e. at low energies $E \ll E_p$. The boundaries of its correctness at high energies $E \simeq E_p$ are questionable.
Using the result from [28],[29] in [16], an explicit expression has been obtained for **qgcs** at the energies $E \simeq E_p$, represented in terms of $(\Delta A)_{0,q}$,

to the event horizon area of any Schwarzschild black hole, provided GUP is valid (6), as follows: (formula (27) in [16]):

$$(\Delta A)_{0,q} \approx 4l_p^2 \ln 2 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\frac{A_0}{A}\right)\right), \qquad (27)$$

where A – event horizon area of the given Schwarzschild black hole.
Using the last formula for (26), we can derive its "quantum" analog

$$\begin{aligned} M \mapsto \tilde{M}_q \doteq M + (\Delta M)_{0,q}, \\ A \mapsto \tilde{A}_q = A + (\Delta A)_{0,q} \simeq 4\pi R^2(A) + 4l_p^2 \ln 2 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\frac{A_0}{A}\right)\right), \\ \mapsto R(\tilde{A}_q) \simeq \sqrt{R^2(A) + l_p^2\pi^{-1} \ln 2 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\frac{A_0}{A}\right)\right)}. \end{aligned} \qquad (28)$$

Here $(\Delta M)_{0,q} = (R(\tilde{A}_q) - R(A))/(2G)$.

Remark 3.2 *It should be noted that in the presented "minimal" variant of the absorption process the cardinal difference of a semiclassical consideration from consideration with due regard for* **qgcs** *resides in the fact that in the first case changes in all of the parameters of a black hole (its mass, event horizon area, radius, etc.) are independent of its sizes, whereas in the second case they are dependent on its sizes.*
Next it is assumed that the above-mentioned absorption of a particle by a black hole with the mass M takes place before the beginning of inflation, and by the beginning of inflation the mass and the radius of this black hole in a semiclassical pattern are given by $\tilde{M}$ and $R(\tilde{A})$ from the formula (26):

$$\begin{aligned} M \mapsto \tilde{M}, R(A) \mapsto R(\tilde{A}), \\ R(\tilde{A}) = 2G\tilde{M}. \end{aligned} \qquad (29)$$

And consequently, with due regard for **qgcs**, they are given by $\tilde{M}_q, R(\tilde{A}_q)$ form the formula (28):

$$\begin{aligned} M \mapsto \tilde{M}_q, R(A) \mapsto R(\tilde{A}_q), \\ R(\tilde{A}_q) = 2G\tilde{M}_q. \end{aligned} \qquad (30)$$

Then, according to **Remark 3.1**, by the substitution at $\mu = const$ in the formula (20) for $r_M \mapsto R(\tilde{A}), r_{M_q} \mapsto R(\tilde{A}_q)$ we obtain a shift of the inflatrionary parameters with regard to **qgcs** in the minimal absorption process. In particular, due to formulae (20) and (12), for H_0 we have

$$H_{0,q} = H_0 \frac{R(\tilde{A})}{R(\tilde{A}_q)} = H_0 \frac{\sqrt{R^2(A) + l_p^2 \pi^{-1} \ln 2}}{\sqrt{R^2(A) + l_p^2 \pi^{-1} \ln 2 \exp\left(-\frac{1}{2} W\left(-\frac{1}{e}\frac{A_0}{A}\right)\right)}} ==$$

$$= H_0 \frac{\sqrt{R^2(A) + l_p^2 \pi^{-1} \ln 2}}{\sqrt{R^2(A) + l_p^2 \pi^{-1} \ln 2 \exp\left(-\frac{1}{2} W\left(-\frac{1}{e}\frac{A_0}{A}\right)\right)}}. \quad (31)$$

By the substitution of $H_{0,q}$ from the last formula into (22),(23),(24),... , we can find in the pattern of **3.2** "quantum" shifts for all the inflationary parameters, specifically for $V(\phi_0)$

$$V(\phi_0)_q = \Lambda_q M_p^2 = 3M_p^2 H_{0,q}^2 = 3M_p^2 H_0^2 \frac{R^2(\tilde{A})}{R^2(\tilde{A}_q)}. \quad (32)$$

As seen from the foregoing formulae and from (13), for massive black holes with a large area of event horizon $A \gg l_p^2$, we have $R(\tilde{A})/R(\tilde{A}_q) \approx 1$ that is not surprising. But when considering small black holes in this pattern this quantity may be significant and should not be ignored.

Remark 3.3 *It is obvious that, for massive black holes,"minimal" absorption considered in point* **3.2** *is not a real physical process because a mass of the absorbed matter for them is always sufficiently great. At the same time, for small* **pbhs** *in the preinflationary period this process is quite real. Besides, in this case any absorption, in principle, may be represented as a chain of "minimal" absorptions (may be "expended into minimal absorptions").*

3.3 *Black Hole Evaporation and* **qgcs**

Also, black holes are associated with the process of Hawking radiation (evaporation). The primordial black holes are no exception. In the general case

this process is considered only within the scope of a semiclassical approximation (without consideration of the quantum-gravitational effects). Because of this, it is assumed that a primordial black hole may be completely evaporated [10].
Still, in this pattern the situation is impossible due to the validity of GUP (6) and due to the formation of a minimal (nonvanishing) Planckian remnant as a result of evaporation (7) [30].
We can compare the mass loss for a black hole in this process when using a semiclassical approximation and with due regard for **qgcs**.
Let M be the mass of a primordial black hole. Then a loss of mass as a result of evaporation, according to the general formulae, takes the following form ([10],p.356):

$$\frac{dM}{dt} \sim \sigma T_{\rm H}^4 A_M, \qquad (33)$$

where $T_{\rm H}$ - temperature of a black hole with the mass M,A_M - surface area of the event horizon of this hole $A_M = 4\pi r_M^2$, and $\sigma = \pi^2 k^4/(60\hbar^3 c^2)$ is the Stefan-Boltzmann constant.
Using this formula for the same black hole but with regard to **qgcs**, we can get the mass loss $[dM/dt]_q$ in this case

$$[\frac{dM}{dt}]_q \sim \sigma T_{{\rm H},q}^4 A_M, \qquad (34)$$

where $T_{{\rm H},q}$ - temperature of a black hole with the same mass M, when taking into consideration **qgcs** (9).
For all the foregoing formulae associated with a random black hole having the mass $\mathcal{M}$, the following estimate is correct ((10.1.19) in [10]):

$$-\frac{d\mathcal{M}}{dt} \sim b(\frac{M_p}{\mathcal{M}})^2(\frac{M_p}{t_p})^2 N, \qquad (35)$$

where $b \approx 2.59 \times 10^{-6}$, and N is the number of the states and species of particles that are radiated. The minus sign in the left part of the last formula denotes that the mass of a black hole diminishes as a result of evaporation, i.e. we have $d\mathcal{M}/dt < 0$.
Unfortunately, the last formula is hardly constructive as it is difficult to estimate the number N, especially at high energies $E \simeq E_p$.

Nevertheless, using the terminology and symbols of this paper, and also the results from [16], the formula (35) for the mass loss by a black hole with regard to **qgcs** may be written in a more precise and constructive form. If we take $\mathcal{M} = M$ (35), then, according to formula (45) in [16], within the scope of GUP (6) we will have

$$\frac{dM}{dt} = -\frac{\gamma_1}{M^2 l_p^4} \exp\left(-2W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) \times$$
$$\times \left(1 - \frac{8\gamma_2}{e\gamma_1}\left(\frac{M_0}{M}\right)^2 \exp\left(-W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)\right), \qquad (36)$$

where $\gamma_1 = \frac{\pi^2}{480}$, $\gamma_2 = \frac{\pi^2}{16128}$.
The minus sign in the right side of the last formula means the same as the minus sign in the left side of formula (35).
Due to (12), formula (36) is of the following form:

$$\frac{dM}{dt} = -\frac{\gamma_1}{M^2 l_p^4} \exp(-2W\left(-\frac{1}{e}\alpha_{r(M)}\right)) \times$$
$$\times \left(1 - \frac{8\gamma_2}{e\gamma_1}\alpha_{r(M)} \exp\left(-W\left(-\frac{1}{e}\alpha_{r(M)}\right)\right)\right). \qquad (37)$$

We can expand the right sides of formulae (36) and (37) into a series in terms of the small parameter $e^{-1}(M_0/M)^2 = e^{-1}\alpha_{r(M)}$ (formula (46) in [16]) that, proceeding from the deformation parameter $\alpha_{r(M)}$, takes the form

$$\frac{dM}{dt} = -\frac{\gamma_1}{M^2 l_p^4}\left(1 + \frac{2}{e}\alpha_{r(M)} + \frac{4}{e^2}\left(1 - \frac{2\gamma_2}{e\gamma_1}\right)\alpha^2_{r(M)} + \frac{25}{3e^3}\left(1 - \frac{72\gamma_2}{25e\gamma_1}\right)\alpha^3_{r(M)} + \ldots\right). \qquad (38)$$

Neglecting the last equation due to the time interval chosen, e.g., due to $\triangle t = t_{infl} - t_M$,where t_{infl}–time of the inflation onset and t_M– time during which the black hole under study has been formed, formula (1), the mass loss for a black hole with regard to **qgcs** by the inflation onset time may be given as

$$\Delta_{Evap,q}M(t_M,t_{infl}) \doteq \int\limits_{t_M}^{t_{infl}} \frac{dM}{dt} =$$

$$= -\int\limits_{t_M}^{t_{infl}} \frac{\gamma_1}{M^2 l_p^4}\left(1+\frac{2}{e}\alpha_{r(M)}+\frac{4}{e^2}\left(1-\frac{2\gamma_2}{e\gamma_1}\right)\alpha_{r(M)}^2+\frac{25}{3e^3}\left(1-\frac{72\gamma_2}{25e\gamma_1}\right)\alpha_{r(M)}^3+\ldots\right). \quad (39)$$

With the use of formulae $r_M = R(A), M = R(A)/2G$, the last formula may be written as

$$\Delta_{Evap,q}M(t_M,t_{infl}) = \int\limits_{t_M}^{t_{infl}} (2G)^{-1}\frac{dR(A)}{dt} =$$

$$= -\int\limits_{t_M}^{t_{infl}} \frac{4G^2\gamma_1}{R(A)^2 l_p^4}\left(1+\frac{2}{e}\alpha_{R(A)}+\frac{4}{e^2}\left(1-\frac{2\gamma_2}{e\gamma_1}\right)\alpha_{R(A)}^2+\frac{25}{3e^3}\left(1-\frac{72\gamma_2}{25e\gamma_1}\right)\alpha_{R(A)}^3+...\right). \quad (40)$$

Since, according to the chosen normalization, $G = l_p^2$, the last expression may take the form

$$\Delta_{Evap,q}M(t_M,t_{infl}) = \int\limits_{t_{M_q}}^{t_{infl}} (2G)^{-1}\frac{dR(A)}{dt} =$$

$$= -\int\limits_{t_M}^{t_{infl}} \frac{4\gamma_1}{R(A)^2} \times$$

$$\times \left(1+\frac{2}{e}\alpha_{R(A)}+\frac{4}{e^2}\left(1-\frac{2\gamma_2}{e\gamma_1}\right)\alpha_{R(A)}^2+\frac{25}{3e^3}\left(1-\frac{72\gamma_2}{25e\gamma_1}\right)\alpha_{R(A)}^3+...\right). \quad (41)$$

Next, we can determine the mass of a black hole after its evaporation until the inflation onset with regard to **qgcs**

$$M_{Evap,q}(t_{M_q},t_{infl}) \doteq M + \Delta_{Evap,q}M_q(t_{M_q},t_{infl}). \quad (42)$$

In the pattern of a semiclassical approximation the above-mentioned formulae are greatly simplified because in this case $\alpha_{R(A)} = 0$ due to the absence of a minimal black hole.
Then in a semiclassical pattern formula (42), with the use of the suggested formalism, takes the following form:

$$M_{Evap}(t_M, t_{infl}) \doteq M + \Delta_{Evap}M(t_M, t_{infl}), \tag{43}$$

where

$$\Delta_{Evap}M(t_M, t_{infl}) = \int\limits_{t_M}^{t_{infl}} \frac{dM}{dt} = -\int\limits_{t_M}^{t_{infl}} \frac{\gamma_1}{M^2 l_p^4}. \tag{44}$$

Accordingly, for the radii $M_{Evap}(t_M, t_{infl}), M_{Evap,q}$ we can get

$$\begin{aligned} r(M_{Evap}) &= 2GM_{Evap}(t_M, t_{infl}), \\ r(M_{Evap,q}) &= 2GM_{Evap,q}(t_M, t_{infl}). \end{aligned} \tag{45}$$

In accordance with **Remark 3.3**, we have

$$\begin{aligned} \mu_{Evap} \doteq (r_{M_{Evap}} H_{0,Evap}/4)^{1/3} = (r_{M_{Evap},q} H_{0,Evap,q}/4)^{1/3}; \\ H_{0,Evap,q} = \frac{r_{M_{Evap}}}{r_{M_{Evap},q}} H_{0,Evap}. \end{aligned} \tag{46}$$

The right side of the last line in formula (46) gives the **"quantum-gravitational shifts"** (abbreviated as **qgs**) of the de Sitter Hubble parameter H_0 for black holes evaporation process.
Substituting $H_{0,Evap,q}$ from (46) into formulae (22)–(24) and so on, we can obtain **qgsc** for all cosmological parameters in the inflationary scenario when a primordial black hole evaporates before the inflation onset.

4 Quantum-gravity Corrections for Appearance Probabilities PBHs in the Pre-Inflationary Era

For **pbh** with Schwarzschild-de Sitter **SdS** metric (3) in the pre-inflation epoch The problem of estimating the probability of occurrence of these **pbh**.

This problem has been studied in [1] without due regard for **qgc**. Let us demonstrate that consideration of **qgc** in this case makes the probability of arising **pbh** higher.
Similar to [1], it is assumed that in pre-inflation period non-relativistic particles with the mass $m < M_p$ are dominant (Section 3 in [1]). For convenience, let us denote the Schwarzschild radius r_M by R_S.
When denoting, in analogy with [1], by $N(R,t)$ the number of particles in a *comoving* ball with the physical radius $R = R(t)$ and the volume V_R at time t, in the case under study this number (formula (3.9) in [1]) will have **qgc** $N(R,t) \mapsto N(R,t)_q$

$$(\langle N(R,t)\rangle = \frac{m_p^2 H^2 R^3}{2m}) \mapsto (\langle N(R,t)_q\rangle = \frac{m_p^2 H_q^2 R^3}{2m}). \qquad (47)$$

Here the first part of the last formula agrees with formula (3.9) in [1], whereas H, H_q in this case are in agreement with $H_0, H_{0,q}$. And from (21) it follows that

$$\langle N(R,t)_q\rangle = \langle N(R,t)\rangle \exp\left(-W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right). \qquad (48)$$

According to (15), it is necessary to replace the Schwarzschild radius R_S by $R_{S,q} = R_S \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)$.
Then from the general formula $N(R_S,t) = \langle N(R_S,t)\rangle + \delta N(R_S,t)$, used because of the replacement of $R_S \mapsto R_{S,q}$, we obtain an analog of (3.12) from [1]

$$\delta N > \delta N_{\rm cr,q} \doteq \frac{m_p^2 R_{S,q}}{2m} - \langle N(R_S,t)_q\rangle = \frac{m_p^2 R_{S,q}}{2m}[1-(HR_S)^2] =$$
$$= \frac{m_p^2 R_S}{2m}[1-(HR_S)^2] \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) = \delta N_{\rm cr} \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right). \qquad (49)$$

In the last formula in square brackets we should have $(H_q R_{S,q})^2$ instead of $(HR_S)^2$ but, as we consider the case $\mu = const$, these quantities are coincident.
It should be noted that here the following condition is used:

$$HR_S < 1, \qquad (50)$$

i.e. Schwarzschild radius R_S less than Hubble radius, $R_S < R_H = 1/H$.
As we have $\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) < 1$, then

$$\delta N_{\rm cr,q} < \delta N_{\rm cr}. \tag{51}$$

Considering that for the formation of a Schwarzschild black hole with the radius R_S it is required that, due to statistical fluctuations, the number of particles $N(R_S, t)$ with the mass m within the black hole volume $V_{R_S} = 4/3\pi R_S^3$ be in agreement with the condition [1]

$$N(R_S, t) > R_S M_p^2/(2m), \tag{52}$$

which, according to **qgs** in the formula of (15), may be replaced by

$$N(R_{S,q}, t) > R_{S,q} M_p^2/(2m) = \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) R_S M_p^2/(2m). \tag{53}$$

As follows from these expressions, with regard to **qgc** for the formation of **pbh** in the pre-inflation period, the number of the corresponding particles may be lower than for a black hole without such regard, leading to a higher probability of the formation.
Such a conclusion may be made by comparison of this probability in a semi-classical consideration (formula (3.13) in [1])

$$P\big(\delta N(R_S, t) > \delta N_{\rm cr}(R_S, t)\big) = \int_{\delta N_{\rm cr}}^{\infty} d(\delta N) P(\delta N) \tag{54}$$

and with due regard for **qgc**

$$P\big(\delta N(R_{S,q}, t) > \delta N_{\rm cr}(R_{S,q}, t)\big) = \int_{\delta N_{\rm cr,q}}^{\infty} d(\delta N) P(\delta N). \tag{55}$$

Considering that in the last two integrals the integrands take positive values and are the same, whereas the integration domain in the second integral is wider due to (51), we have

$$\int_{\delta N_{\rm cr,q}}^{\infty} d(\delta N) P(\delta N) =$$
$$= \int_{\delta N_{\rm cr,q}}^{\delta N_{\rm cr}} d(\delta N) P(\delta N) + \int_{\delta N_{\rm cr}}^{\infty} d(\delta N) P(\delta N) > \int_{\delta N_{\rm cr}}^{\infty} d(\delta N) P(\delta N). \tag{56}$$

As follows from the last three formulae, in the case under study the probability that the above-mentioned **pbh** will be formed is higher with due regard for **qgc**.
It is interesting to find which changes should be expected in the pattern studied if the parameter μ ceases to be constant and is shifted with regard to **qgc** of the black hole mass $M \mapsto M_q$ (15): $(\mu = (GMH_0/2)^{1/3}) \mapsto (\mu_q = (GM_qH_0/2)^{1/3})$.
Note that in this case the general formula form Section 3 in [1] are also valid but for this pattern in formula (49) there is substitution of $HR_S \mapsto HR_{S,q}$:

$$\delta N > \delta N_{\rm cr,q} \doteq \frac{m_p^2 R_{S,q}}{2m} - \langle N(R_S,t)_q\rangle = \frac{m_p^2 R_{S,q}}{2m}[1-(HR_{S,q})^2] =$$
$$= \frac{m_p^2 R_S \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)}{2m}[1 - H^2R_S^2 \exp\left(W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)]. \quad (57)$$

To understand variations in the probability of **pbh** arising as compared to the case when **qgc** are neglected in the consideration, we compare the last expression with the corresponding quantity $\delta N_{\rm cr} = \frac{m_p^2 R_S}{2m}[1-(HR_S)^2]$.
Dividing the last expression and the right side (57) by the same positive number $\frac{m_p^2 R_S}{2m}$ and subtracting the second number from the first, we can obtain

$$\delta N_{\rm cr} - \delta N_{\rm cr,q} \sim [1 - H^2R_S^2 + H^2R_S^2 \exp\left(\frac{3}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) -$$
$$- \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)] \quad (58)$$

with a positive proportionality factor.
To have a positive quantity in the right side (58), fulfillment of the following inequality is required:

$$1 - \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) > R_S^2H^2[1 - \exp\left(\frac{3}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)]. \quad (59)$$

As from formula (10) it follows that $W(u) < 0$ for $u < 0$, we have $1 - \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) > 0, 1 - \exp\left(\frac{3}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) > 0$, from where it follows that (59) is equivalent to the inequality

$$(HR_S)^2 < \frac{1 - \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)}{1 - \exp\left(\frac{3}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)} =$$

$$= \frac{1}{1 + \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) + \exp\left(W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)} \tag{60}$$

or

$$HR_S < \frac{1}{\sqrt{1 + \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) + \exp\left(W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)}}. \tag{61}$$

We need that in the case under study $\mu \neq const$ the probability of **pbh** arising with regard to **qgc** be higher than the same probability but without due regard for **qgc**. It is sufficient to replace the condition $HR_S < 1$ in formula (50) by the condition in formula (61).
Note that, due to smallness of R_S, $\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right), \exp\left(W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)$ are also small and in the right side (61) the quantity is close to 1, i.e. the shorter the Schwarzschild radius of **pbh**,the greater consideration of **qgc** increases the probability of **pbh** arising.

5 High Energy Deformations of Friedmann Equations

Based on the obtained results, it is inferred that there is the deformation (having a quantum-gravitational character) of the Schwarzschild-de Sitter metric and Friedmann Equations due to **qgsc**. Indeed, for example, for **3.1**.

(*the stationary pattern*) from formulae (15),(19),(18) we can derive

$$H_{0,q} = H_0 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\alpha_{r_M}\right)\right),$$
$$a(\eta)_q = a(\eta)\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\alpha_{r_M}\right)\right). \qquad (62)$$

Substituting the expression $a(\eta)_q$ from the last formula for a into the Friedmann Equation ((2.4) in [27])

$$\frac{a'^2}{a^4} = \frac{8\pi}{3}G\rho, \qquad (63)$$

we can obtain the Quantum Deformation (**QD**) [18] of the Friedmann Equation due to **qgcs** for **pbh** in the early Universe

$$\frac{a_q'^2}{a_q^4} = \frac{a'^2}{\exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right)a^4} = \frac{8\pi}{3}G\rho \qquad (64)$$

or

$$\frac{a'^2}{a^4} = \frac{8\pi}{3}G\rho\exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) \doteq \frac{8\pi}{3}G\rho_q,$$
$$\rho_q \doteq \rho\exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) > \rho. \qquad (65)$$

The last line in (65) is associated with the fact that the Lambert W-function $W(u)$ is negative for $u < 0$.
Similarly, (ij)-components of the Einstein equations ((2.5) in [27])

$$2\frac{a''}{a^3} - \frac{a'^2}{a^4} = -\frac{8\pi}{3}Gp \qquad (66)$$

within the foregoing (**QD**) are replaced by

$$2\frac{a_q''}{a_q^3} - \frac{a_q'^2}{a_q^4} = -\frac{8\pi}{3}Gp \qquad (67)$$

or

$$2\frac{a''}{a^3} - \frac{a'^2}{a^4} = -\frac{8\pi}{3}Gp\exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) = -\frac{8\pi}{3}Gp_q,$$

$$p_q \doteq p\exp\left(-W\left(-\frac{1}{e}\alpha_{r_M}\right)\right) > p. \qquad (68)$$

It should be noted that the equation of the covariant energy conservation for the homogeneous background ((2.6) in [27])

$$\rho' = -3\frac{a'}{a}(\rho + p) \qquad (69)$$

remains unaltered with replacement of $\rho \mapsto \rho_q, p \mapsto p_q$.
So, in the pattern of **3.1** (*the stationary pattern*), taking into consideration of **qgcs** for **pbhs** in the pre-inflationary era increases the initial values of the density ρ and of the pressure p in Friedmann equations.
The above calculations are correct if, from the start, we assume that a black hole (i.e., its event-horizon radius) is invariable until the onset of inflation. But such a situation is idealized because this period is usually associated with the radiation and absorption processes
Then again for $\mu = const$ from formulae (19),(18) we have

$$H_{0,q} = H_0\frac{r_{M_{orig}}}{r_{M_{orig,q}}},$$

$$a(\eta)_q = a(\eta)\frac{r_{M_{orig,q}}}{r_{M_{orig}}}. \qquad (70)$$

Substituting the expression $a(\eta)_q$ from formula (70) in all formulae (64)–(69) we obtain analogues of these formulae in the general case. In particular, for formula (64) we have

$$\frac{a_q'^2}{a_q^4} = \frac{r^2_{M_{orig}}}{r^2_{M_{orig,q}}}\frac{a'^2}{a^4} = \frac{8\pi}{3}G\rho \qquad (71)$$

Or, equivalently,

$$\frac{a'^2}{a^4} = \frac{8\pi}{3} G\rho \frac{r^2_{M_{orig,q}}}{r^2_{M_{orig}}} = \frac{8\pi}{3} G\rho_q$$

$$\rho_q \doteq \frac{r^2_{M_{orig,q}}}{r^2_{M_{orig}}} \rho. \tag{72}$$

In the same way as for formula (67), in this pattern for the general quantum deformation (ij)-components of Einstein equations by substitution of the value for $a(\eta)_q$ from the formula (70) we obtain

$$2\frac{a''_q}{a^3_q} - \frac{a'^2_q}{a^4_q} = -\frac{8\pi}{3} Gp \tag{73}$$

or

$$2\frac{a''}{a^3} - \frac{a'^2}{a^4} = -\frac{8\pi}{3} Gp \frac{r^2_{M_{orig,q}}}{r^2_{M_{orig}}} = -\frac{8\pi}{3} Gp_q,$$

$$p_q \doteq \frac{r^2_{M_{orig,q}}}{r^2_{M_{orig}}} p. \tag{74}$$

It is clear that, in this most general pattern, the covariant energy conservation for the homogeneous background ((2.6) in [27])

$$\rho' = -3\frac{a'}{a}(\rho + p) \tag{75}$$

remains unaltered with replacement of $\rho \mapsto \rho_q, p \mapsto p_q$.

6 Some Implications, Final Comments and Further Research

This paper demonstrates the way to calculate in the explicit form the quantum-gravitational corrections for the basic cosmological parameters in the inflationary scenario which are due to **pbhs** originating during the pre-inflationary period of time. As follows from the above-mentioned formulae, for such black holes these corrections are especially great and may be significant for the basic parameters of inflation. Because of this, they are important for studies of the processes in the very early Universe. According to the results in [31],a local quantum field theory [32] has the upper applicability boundary $\widetilde{E}$ that is considerably lower than the Planck energy $\widetilde{E} \ll E_p$. It is clear that the quantum-gravitational corrections are most significant within the energy range $E, \widetilde{E} < E \leq E_p$.

6.1 The Cosmological Perturbation Corrections Generated by QGCS for PBHS.General Remarks

It is known that inflationary cosmology is characterized by *cosmological perturbations* of different nature (scalar, vector, tensor) [27],[33],[37], though vector perturbations are usually ignored as they die out fast.

It is clear that, as **qgcs** for **pbhs** in the early Universe cause shifts of the inflationary parameters, they inevitably lead to corrections of the cosmological perturbations on inflation.

Specifically, in the case of scalar cosmological perturbations consideration of the indicated **qgcs** for the rest of the Einstein equations (formulae (2.74)–(2.76) in [27]) in case **3.1** (*the stationary picture*) gives

$$\begin{aligned} \Delta\Phi - 3\frac{a'}{a}\Phi' - 3\frac{a'^2}{a^2}\Phi &= 4\pi G a^2 \exp\left(W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right)\cdot\delta\rho_{tot}; \\ \Phi' + \frac{a'}{a}\Phi &= -4\pi G a^2 \exp\left(W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right)\cdot[(\rho+p)v]_{tot}; \\ \Phi'' + 3\frac{a'}{a}\Phi' + (2\frac{a''}{a} - \frac{a'^2}{a^2})\Phi &= 4\pi G a^2 \exp\left(W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right)\cdot\delta p_{tot}. \end{aligned} \quad (76)$$

Here in the right sides of all lines in the last formula the scale factor a is taken with regard to **qgcs** from formula(23), i.e., $a = a(\eta)_q$. In the left sides of these lines additional factors of the type
$\exp\left(\pm\frac{1}{2}W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right), \exp\left(\pm W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right), ...$ are cancelled out because they are independent of η. This is so in the general case when taking in consideration **qgcs** for the **pbhs** formed in the pre-inflationary era (for all types of the cosmological perturbations, not only for those of the scalar

type).

Remark 6.1

These **qgcs** *are arising only in the expressions, where the total power of the scale factor $a(\eta)$ and of any its derivatives with respect to η, i.e. $a'(\eta), a''(\eta), \ldots$ is not equal to 0. In this case the corresponding* **qgcs** *is calculated from formula (23).*

According to this remark, under the linearized form of the gauge transformations (formulae (2.31) in [27]), spatial components of the metric perturbation transform are retained due to inclusion of **qgcs** ([27],p.30):

$$\widetilde{h}_{ij} = h_{ij} - 2\partial_i\partial_j\sigma - \frac{a'}{a}\delta_{ij}\sigma'. \tag{77}$$

And **qgcs** deform correspondingly the metric with scalar perturbations in the conformal Newtonian gauge (formulae (2.69) in [27]):

$$\{ds^2 = a^2(\eta)[(1+2\Phi)d\eta^2 - (1+2\Psi)d\mathbf{x}^2]\} \mapsto a^2(\eta)_q[(1+2\Phi)d\eta^2 - (1+2\Psi)d\mathbf{x}^2] = \\ = a^2(\eta)\exp\left(W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right)\left[(1+2\Phi)d\eta^2 - (1+2\Psi)d\mathbf{x}^2\right]. \tag{78}$$

6.2 Steps of Further Studies

Proceeding from the results of this paper, the following steps may be planned for further studies.

6.1 *Based on the results of this paper, the following steps may be planned to study the corrections of cosmological parameters and cosmological perturbations due to* **qgcs** *for* **pbhs** *in the pre-inflationary era:*

6.1.1 pbhs *having different masses M (different A), in fact for different values of* $\exp\left(\pm\frac{1}{2}W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right)$ *from formula (23);*

6.1.2 *different inflationary models (chaotic inflation, new inflation and so on [37]);*

6.1.3 *comparison of the results obtained in* **6.1.1** *and* **6.1.2** *with the experimental data accumulated by space observatories: (Planck Collaboration),* *(WMAP Collaboration) [34],[35],[36].*

6.2 Elucidation, how far these ”shifts” are involved in the general approaches to **qgcs** for cosmological perturbations on inflation (for example, see [38]).

6.3 Applicability estimation of the obtained results for other (noninflationary) cosmological models (e.g., bouncing cosmologies [39],[40]).

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this work.

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