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A Massive S-duality in Four Dimensions

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Abstract: We reduce the Type IIA supergravity theory with a generalized Scherk-Schwarz ansatz that exploits the scaling symmetry of the dilaton, the metric and the NS 2-form field. The resulting theory is a new massive, gauged supergravity theory in four dimensions with a massive 2-form field and a massive 1-form field. We show that this theory is S-dual to a theory with a massive vector field and a massive 2-form field, which are dual to the massive 2-form and 1-form fields in the original theory, respectively. The S-dual theory is shown to arise from a Scherk-Schwarz reduction of the heterotic theory. Hence we establish a massive, S-duality type relation between the IIA theory and the heterotic theory in four dimensions. We also show that the Lagrangian for the new four dimensional theory can be put in the most general form of a $D = 4,\ N = 4$ gauged Lagrangian found by Schön and Weidner, in which (part of) the $SL(2)$ group has been gauged.

Contents

1 Introduction

String compactifications in the presence of fluxes has been an important research area in recent years. Fluxes can be geometric (like p-form or metric fluxes, see [1] for a review) or non-geometric [2, 3]. The importance of introducing flux into the compactification scheme is that the lower dimensional theory is more realistic. The resulting theory is gauged and massive with mass parameters defining a scalar potential, which in turn gives rise to moduli stabilization.

An important question is the faith of string dualities, when fluxes are introduced. One of the oldest work, which explored this question is that of Kaloper and Myers [4], who considered flux compactifications of the heterotic string on the d-dimensional torus T^d. They showed that the perturbative $O(d, d+16)$ duality symmetry is still a symmetry of the resulting gauged, massive supergravity, provided that the mass parameters also transform under the duality group. On the other hand, flux compactifications of Type II theories on Calabi-Yau manifolds were studied in [5]. In the papers [5, 6] and later in [7] it was established that the mirror symmetry between IIA and IIB theories is still valid in the presence of fluxes. The U-duality symmetry of M-theory compactifications with flux was explored in [8].

Although much has been understood about the perturbative duality symmetries in flux compactifications, less is known about the non-perturbative ones. For example, the six-dimensional theory obtained from the compactification of IIA theory on $K3$ manifold is known to be S-dual to heterotic string theory compactified on T^4 [9–13]. Similarly, heterotic string theory compactified on $K3 \times T^2$ to four dimensions is S-dual to Type IIA theory compactified on a certain Calabi-Yau manifold [14–18]. It is natural to ask whether these duality relations continue to hold when fluxes are turned on. The main aim of the present paper is to contribute towards answering this question.

For the S-duality in four dimensions there were early attempts [19, 20], which identified the duals of some fluxes that can be introduced in the heterotic compactification. In more recent work [21], it was suggested that in order to find the duals of all heterotic fluxes, the IIA theory has to be lifted to M-theory.

For the six-dimensional S-duality symmetry with fluxes, earlier work was done in [22], where it is argued that the duality does not hold at the level of the action, when fluxes are introduced. However, they were able to establish a six dimensional massive S-duality by performing a Scherk-Schwarz reduction of seven dimensional IIA theory, obtained by a K3 compactification of M-theory, and the heterotic theory compactified on T^3. On the other hand, in [23] flux compactification of massive IIA theory was performed. The resulting theory was shown to possess the perturbative $O(4,20)$ symmetry. However, the S-duality symmetry which is to map the theory to heterotic theory could not be restored. The main problem is identified to be due to the fact that on the IIA side it is the NS-NS 2-form field which acquires mass, whereas on the heterotic side vector fields get massive. It might be possible to resolve this problem in four dimensions, since a four dimensional massive 2-form field has the same number of degrees of freedom as a massive vector field in four dimensions. So one can consider to perform a further T^2 reduction of both theories and seek the desired massive S-duality in four dimensions [24]. In the massless case, it is well known that both theories have an $O(6,22) \times SL(2)$ symmetry in four dimensions. On the IIA side, one has the perturbative $O(4,20)$ symmetry due to K3 compactification, combined with the $SL(2) \times SL(2)$ symmetry of the T^2 compactification. Under S-duality, the $SL(2)$ symmetry associated with the torus compactification gets mapped to the self-duality of the heterotic theory on T^6, whereas $O(6,22)$ is the T-duality symmetry of the heterotic theory associated with the compactification manifold T^6. When fluxes are introduced, the $O(4,20) \times SL(2) \times SL(2)$ symmetry of the IIA theory was shown to remain as the symmetry group, provided that the mass parameters also transform under this duality group [22–25]. On the heterotic side, although the $O(6,22)$ part is still a symmetry as was shown by Kaloper and Myers [4], the self-duality $SL(2)$ is problematic. Recall that in four dimensional heterotic theory, one dualizes the 2-form field coming from the reduction of the NS-NS 2-form field to a scalar, which then forms an $SL(2)$ doublet along with the dilaton. This dualization can no longer be performed in the presence of fluxes, as fluxes imply non-abelian gauge couplings for the 2-form field. However, if the massive S-duality is to hold in 4 dimensions, one expects that it maps the massive IIA theory (for which the $SL(2)$ is still a symmetry) to a massive heterotic theory, which still possesses the self-duality symmetry. One way to approach this problem is to start with the general $SL(2)$-gauged supergravity in 4 dimensions, seek for a string theory origin and see if this teaches us something about the (possible) massive S-duality between the IIA and the heterotic theory. This is the way we approach the problem of massive S-dualities in this paper.

The most general $O(6,22) \times SL(2)$ gauged supergravity was constructed by Schön and Weidner in [26]. The string/M-theory origin of the most general $SL(2)$ gauging is still not known. However, for certain types of gaugings, namely for those which correspond to scalings and shifts of axion and dilaton in four dimensions, a higher dimensional origin was found by Derendinger et al. [27]. They showed that the dimensional reduction of the ten dimensional heterotic string (to be more precise, the dimensional reduction of the NS sector of the heterotic string, without the Yang-Mills vectors) with a generalized Scherk-Schwarz ansatz gives in four dimensions, after certain dualizations, the Schön-Weidner Lagrangian with non-zero $SL(2)$ gaugings. In this paper, we utilize a similar Scherk-Schwarz ansatz for the reduction of the six dimensional type IIA theory (Such reductions were also considered by [28, 29]). We show that the resulting massive theory is S-dual to heterotic string theory reduced with the Scherk-Schwarz twist of Derendinger et al. [27]. Although we work with a restricted class of fluxes, our work is interesting, because it gives an explicit demonstration of how the duality between massive 2-forms and massive 1-forms work in the context of string theory. As a by-product we show that the inclusion of the Yang-Mills vectors to the heterotic string theory does not change the results of Derendinger et al. [27]. The resulting gauged supergravity is still of the Schön-Weidner type, characterized by the same embedding tensor.

The plan of our paper is as follows. In section 2, we introduce the aforementioned Scherk-Schwarz twist and perform the dimensional reduction of the six-dimensional Type IIA theory. In section 3 we dualize the resulting theory and show that the dual theory can be obtained from a dimensional reduction of the heterotic theory. In section 4, we discuss in more detail how the duality between the massive 2-form fields and the massive 1-form fields work. In section 5 we show that the dual massive theory can be put in the form of Schön-Weidner Lagrangian, and the gaugings are described by the same tensor as the one in Derendinger et al, although we also include the Yang-Mills vectors. We discuss our results in section 6.

2 Twisted Reduction of Type IIA theory from 6 to 4 Dimensions

In this section we perform a dimensional reduction of the six-dimensional Type IIA theory to four dimensions on a two-torus with a certain Scherk–Schwarz twist [30, 31]. The six-dimensional Type IIA Lagrangian is obtained by a standard Kaluza–Klein reduction of the ten-dimensional Type IIA supergravity on K3 [9–12]. The field content of the ten-dimensional Type IIA supergravity consists of a dilaton, a two-form Kalb-Ramond field, and a one- and a three-form Ramond-Ramond fields. The bosonic part of the six-dimensional Type IIA Lagrangian, given as

$$\mathcal{L}_6^{IIA} = e^{-\phi}\left(R*1 - d\phi\wedge *d\phi + \frac{1}{4}d\widetilde{M}_{IJ}\wedge *d\widetilde{M}^{IJ} - \frac{1}{2}H_{(3)}\wedge *H_{(3)} - \frac{1}{2}e^{\phi}\widetilde{M}_{IJ}F^I_{(2)}\wedge *F^J_{(2)}\right) \\ -\frac{1}{2}L_{IJ}B_{(2)}\wedge F^I_{(2)}\wedge F^J_{(2)}, \tag{2.1}$$

is $O(4,20)$-invariant and the full theory has $\mathcal{N}=2$ supersymmetry in six dimensions. Here $\widetilde{M}_{IJ}$ with $I=1,\dots,24$ is the scalar matrix that takes values in $O(4,20)/O(4)\times O(20)$ coset space; L_{IJ} is the invariant metric of $O(4,20)$; $H_{(3)}=dB_{(2)}$; and finally $F^I_{(2)}=dA^I_{(1)}$, where $A^I_{(1)}$ is the $O(4,20)$ vector, $A^I_{(1)}=(A^a_{(1)},B_{(1)a},A^A_{(1)})$, with $a=1,\dots,4$, and $A=1,\dots,16$.[1]

[1]How $A^I_{(1)}$ is related to ten-dimensional vector fields of Type IIA theory is explained, for example, in [38].

We reduce this Lagrangian on a 2-torus T^2 with the following twisted ansatz

$$\begin{aligned}
\phi(x,y) &= \widetilde{\phi}(x) - 2\lambda_m y^m, \\
G(x,y) &= e^{-\lambda_m y^m}\left(\widetilde{G} + \widetilde{G}_{mn}\eta^m \otimes \eta^n\right), \\
B_{(2)}(x,y) &= e^{-\lambda_m y^m}\left(\widetilde{B}_{(2)} + \widetilde{B}_{(1)m} \wedge \eta^m + \frac{1}{2}\widetilde{B}_{(0)mn}\eta^m \wedge \eta^n\right), \\
A^I_{(1)}(x,y) &= e^{1/2\lambda_m y^m}\left(\widetilde{A}^I_{(1)} + \widetilde{A}^I_{(0)m} \wedge \eta^m\right),
\end{aligned} \tag{2.2}$$

Here y^m with $m = 1,2$ are the coordinates on T^2, the parameters λ_m are arbitrary real numbers, $\eta^m = dy^m + \widetilde{\mathcal{A}}^m_{(1)}$, and $\widetilde{\mathcal{A}}^m_{(1)}$ is the graviphoton of the reduction. In this notation $\Omega_{(p)}$ is a p-form in six dimensions and $\widetilde{\Omega}_{(p)}$ is a p-form in four dimensions. This type of reduction is different from the Kaluza–Klein reduction in the sense that one takes into account not just the zeroth order term, but also the higher order terms in the harmonic expansion of fields on the compactification manifold, here T^2. However, dependence of the fields on the coordinates of the internal manifold cannot be arbitrary. The reduced Lagrangian should be independent of the coordinates of the compactification manifold. To attain to this requirement one has to choose the Scherk-Schwarz reduction ansatz according to some symmetry of the theory [32–38]. The reduction ansatz above is dictated by the $SL(2,R)$ scaling symmetry of the two-torus:

$$\phi \to \phi - 2\lambda, \quad G \to e^{-\lambda} G(x), \quad B_{(2)} \to e^{-\lambda} B_{(2)}(x) \tag{2.3}$$

This symmetry ensures that the ansatz (2.2) yields a consistent reduction. We first reduce the Einstein-Hilbert part together with the dilaton kinetic term of the six-dimensional Type IIA Lagrangian (2.1). We perform the reduction of the Ricci scalar by expressing it in the so called Palatini form [39]. By utilizing a standard ansatz for the vielbein we calculate the non-vanishing components of the anholonomy coefficients [39] and the corresponding spin connection components, in terms of which the Palatini form is given. Then the usual reduction of the metric is performed. To absorb the volume form of the compactification manifold it is also necessary to shift the dilaton and define the four-dimensional dilaton as $\widetilde{\phi} = \phi - \frac{1}{2}\log\det\widetilde{G}_{mn}$, where $\widetilde{G}_{mn}$ is a symmetric 2 by 2 metric on T^2. In order to write the action in the Einstein frame, we also perform a conformal rescaling of the four-dimensional metric, $\widetilde{G}_{\mu\nu} \to \frac{e^{\widetilde{\phi}}}{2}\widetilde{G}_{\mu\nu}$, with $\mu,\nu = 0,1,2,3$, and also a final rescaling $\widetilde{\phi} \to 2\widetilde{\phi}$ of the dilaton. The reduced form of the first two terms of (2.1) in the Einstein frame are then found to be

$$\begin{aligned}
\mathcal{L}^{IIA}_{4,\,gravity} &= \frac{1}{2}\widetilde{R} * 1 + \frac{1}{8}D\widetilde{G}_{mn} \wedge *D\widetilde{G}^{mn} \\
&\quad - D\widetilde{\phi} \wedge *D\widetilde{\phi} - \frac{1}{4}e^{-2\widetilde{\phi}}\widetilde{G}_{mn}\mathcal{F}^m_{(2)} \wedge *\mathcal{F}^n_{(2)} - \frac{1}{2}e^{2\widetilde{\phi}}\lambda_m\widetilde{G}^{mn}\lambda_n,
\end{aligned} \tag{2.4}$$

where $\mathcal{F}^m_{(2)} = d\mathcal{A}^m_{(1)}$ is the field strength of the graviphoton, $D\widetilde{\phi} = d\widetilde{\phi} - \frac{1}{2}\lambda_k\mathcal{A}^k_{(1)}$.

We now insert the reduction ansatz, (2.2), into the NS-NS part of the Lagrangian (2.1) and obtain in four dimensions an effective theory with the Lagrangian,

$$
\begin{aligned}
\mathcal{L}^{IIA}_{4,\ NS-NS} = -e^{-\widetilde{\phi}} \Big[& \frac{1}{2}\widetilde{H}_{(3)} \wedge *\widetilde{H}_{(3)} + \frac{1}{2}\widetilde{H}_{(2)m}\widetilde{G}^{mn} \wedge *\widetilde{H}_{(2)n} \\
& + \frac{1}{2}\widetilde{H}_{(1)mn}\widetilde{G}^{mp}\widetilde{G}^{nq} \wedge *\widetilde{H}_{(1)pq} + \frac{1}{2}\widetilde{M}_{IJ}\widetilde{F}^I_{(2)} \wedge *\widetilde{F}^J_{(2)} \\
& + \frac{1}{2}\widetilde{M}_{IJ}\widetilde{F}^I_{(1)m}\widetilde{G}^{mn} \wedge *\widetilde{F}^J_{(1)n} + \frac{1}{2}\widetilde{M}_{IJ}\widetilde{F}^I_{(0)mn}\widetilde{G}^{mp}\widetilde{G}^{nq} \wedge *\widetilde{F}^J_{(0)pq} \Big] + \mathcal{L}_{CS}.
\end{aligned}
\tag{2.5}
$$

where $\mathcal{L}_{CS}$ contains the Chern-Simons terms of the Lagrangian $\mathcal{L}^{IIA}_4$:[2]

$$
\begin{aligned}
\mathcal{L}_{CS} = & -L_{IJ}\epsilon^{mn}\widetilde{B}_{(2)} \wedge \widetilde{F}^I_{(1)[m} \wedge \widetilde{F}^J_{(1)n]} - \frac{1}{2}L_{IJ}\epsilon^{mn}\widetilde{B}_{(2)} \wedge \widetilde{F}^I_{(2)} \wedge \widetilde{F}^J_{(0)mn} \\
& -2L_{IJ}\epsilon^{mn}\widetilde{B}_{(1)[m} \wedge \widetilde{F}^I_{(2)} \wedge \widetilde{F}^J_{(1)n]} - \frac{1}{4}L_{IJ}\epsilon^{mn}\widetilde{B}_{(0)mn} \wedge \widetilde{F}^I_{(2)} \wedge \widetilde{F}^J_{(2)}.
\end{aligned}
\tag{2.6}
$$

The four dimensional fields that appear in the above Lagrangian are obtained from

$$
\begin{aligned}
F^I_{(2)}(x,y) &= e^{1/2\lambda_m y^m}\left(\widetilde{F}^I_{(2)} + \widetilde{F}^I_{(1)m} \wedge \eta^m + \frac{1}{2}\widetilde{F}^I_{(0)mn}\eta^m \wedge \eta^n\right), \\
H_{(3)}(x,y) &= e^{-\lambda_m y^m}\left(\widetilde{H}_{(3)} + \widetilde{H}_{(2)m} \wedge \eta^m + \frac{1}{2}\widetilde{H}_{(1)mn}\eta^m \wedge \eta^n\right).
\end{aligned}
\tag{2.7}
$$

Their explicit forms are

$$
\begin{aligned}
\widetilde{F}^I_{(2)} &= d\widetilde{A}^I_{(1)} + \widetilde{A}^I_{(0)m}\mathcal{F}^m_{(2)} + \frac{1}{2}\lambda_m\widetilde{A}^I_{(1)} \wedge \mathcal{A}^m_{(1)} \equiv D\widetilde{A}^I_{(1)} + \widetilde{A}^I_{(0)m}\mathcal{F}^m_{(2)}\ , \\
\widetilde{F}^I_{(1)m} &= d\widetilde{A}^I_{(0)m} - \frac{1}{2}\lambda_r\mathcal{A}^r_{(1)}\widetilde{A}^I_{(0)m} - \frac{1}{2}\lambda_m\widetilde{A}^I_{(1)} \equiv D\widetilde{A}^I_{(0)} - \frac{1}{2}\lambda_m\widetilde{A}^I_{(1)}\ , \\
\widetilde{F}^I_{(0)mn} &= \lambda_{[m}\widetilde{A}^I_{(0)n]}\ ,
\end{aligned}
\tag{2.8}
$$

and

$$
\begin{aligned}
\widetilde{H}_{(3)} &= d\widetilde{B}_{(2)} + \lambda_r\widetilde{B}_{(2)}\mathcal{A}^r_{(1)} - \widetilde{B}_{(1)m}\mathcal{F}^m_{(2)} \equiv \widetilde{D}\widetilde{B}_{(2)} - \widetilde{B}_{(1)m}\mathcal{F}^m_{(2)}\ , \\
\widetilde{H}_{(2)m} &= d\widetilde{B}_{(1)m} - \lambda_r\widetilde{B}_{(1)m}\mathcal{A}^r_{(1)} - \lambda_m\widetilde{B}_{(2)} - \widetilde{B}_{(0)mn}\mathcal{F}^n_{(2)} \\
&\equiv \widetilde{D}\widetilde{B}_{(1)m} - \lambda_m\widetilde{B}_{(2)} - \widetilde{B}_{(0)mn}\mathcal{F}^n_{(2)}\ , \\
\widetilde{H}_{(1)mn} &= d\widetilde{B}_{(0)mn} + \lambda_r\widetilde{B}_{(0)mn}\mathcal{A}^r_{(1)} + 2\lambda_{[m}\widetilde{B}_{(1)n]} \equiv \widetilde{D}\widetilde{B}_{(0)mn}\ .
\end{aligned}
\tag{2.9}
$$

The scalar matrix $\widetilde{M}_{IJ}$ is $O(4,20)/O(4)\times O(20)$ valued and it is given in terms of the geometric moduli on K3 and components of the $B-$field wrapping the harmonic cycles of K3. We do not need its explicit form here, which can be found in many sources, e.g. in [8].

The twisted reduction ansatz we employ here exploits the scaling symmetry of the dilaton, metric, the NS 2-form field and the vectors $A^I_{(1)}$. Therefore, these fields are charged under the gauge symmetry of the lower dimensional theory and their derivatives become covariant derivatives as above. The gauge field is the graviphoton $\mathcal{A}^m_{(1)}$, which is the vector field that comes from the reduction of the metric.

[2] To agree with the conventions of [27] we take $\psi_{[m}\chi_{n]} = \frac{1}{2}(\psi_m\chi_n - \psi_n\chi_m)$.

3 The Heterotic S-dual in 4 Dimensions

In this section, we dualize the 4d IIA theory (2.4, 2.5, 2.6) that we obtained in the previous section and show that the resulting massive/gauged theory can be obtained from a twisted reduction of the heterotic string theory. The dualization is nontrivial as the fields that will be dualized, namely $B_{(2)}, B_{(1)m}$ and $B_{(0)mn}$ appear through not only their field strengths, but also through their bare potentials. We overcome this difficulty in two steps. Firstly, we rewrite the Chern-Simons (CS) term (2.6) by adding total derivative terms that will not alter the field equations, such that the resulting CS term involves only the field strengths of the relevant fields. Secondly, we add to the Lagrangian several Lagrange multiplier terms, which couple the (field strengths of) the fields that will be dualized to not only the field strengths but also to the bare potentials of the "dual-to-be" fields. The duality is of the S-duality type, because under this duality the dilaton, whose expectation value determines the string coupling constant, changes sign.

The relevant Lagrange multiplier terms are:

$$
\begin{aligned}
&\left(d\widehat{B}_{(2)} - \lambda_r \widehat{B}_{(2)}\mathcal{A}^r_{(1)} - \widehat{B}_{(1)m}\mathcal{F}^m_{(2)}\right) \wedge \widetilde{H}_{(1)mn}\epsilon^{mn} \\
+&\left(d\widehat{B}_{(1)m} + \lambda_r \widehat{B}_{(1)m}\mathcal{A}^r_{(1)} + \lambda_m \widehat{B}_{(2)} - \widehat{B}_{(0)mr}\mathcal{F}^r_{(2)}\right) \wedge \widetilde{H}_{(2)n}\epsilon^{mn} \\
+&\left(d\widehat{B}_{(0)mn} - \lambda_r \widehat{B}_{(0)mn}\mathcal{A}^r_{(1)} - 2\lambda_{[m}\widehat{B}_{(1)n]}\right) \wedge \widetilde{H}_{(3)}\epsilon^{mn}.
\end{aligned} \tag{3.1}
$$

Variation of the Lagrangian with respect to the fields $\widehat{B}_{(2)}$, $\widehat{B}_{(1)m}$ and $\widehat{B}_{(0)mn}$ impose three different identities that the field strengths $\widetilde{H}_{(3)}$, $\widetilde{H}_{(2)m}$ and $\widetilde{H}_{(1)mn}$ should obey. These identities are respectively,

$$
\begin{aligned}
-\widetilde{D}\widetilde{H}_{(1)mn} - 2\lambda_{[m}\widetilde{H}_{(1)n]} &= 0, \\
\widetilde{D}\widetilde{H}_{(2)n} + \lambda_n \widetilde{H}_{(3)} - \mathcal{F}^m \widetilde{H}_{(1)mn} &= 0, \\
\widetilde{D}\widetilde{H}_{(3)} + \mathcal{F}^m \widetilde{H}_{(2)m} &= 0.
\end{aligned} \tag{3.2}
$$

These are precisely the Bianchi identities that should be satisfied by $\widetilde{H}_{(3)}, \widetilde{H}_{(2)n}$ and $\widetilde{H}_{(1)mn}$, as can be checked straightforwardly from (2.9).

To perform the variation of the Lagrangian with respect to the field strengths $\widetilde{H}_{(3)}$, $\widetilde{H}_{(2)m}$ and $\widetilde{H}_{(1)mn}$ we first need to write the Chern-Simons part of the four-dimensional Type IIA Lagrangian (2.6) in terms of these fields. After some work we find that $\mathcal{L}_{CS}$ can be written as

$$
\begin{aligned}
\mathcal{L}_{CS} =\ & \frac{1}{4} L_{IJ}\epsilon^{mn}\widetilde{H}_{(3)} \wedge \widetilde{A}^I_{(1)} \wedge \widetilde{F}^J_{(0)mn} - L_{IJ}\epsilon^{mn}\widetilde{H}_{(3)} \wedge \widetilde{A}^I_{(0)[m} \wedge \widetilde{F}^J_{(1)n]} \\
& + L_{IJ}\epsilon^{mn}\widetilde{H}_{(2)[m} \wedge \widetilde{A}^I_{(1)} \wedge \widetilde{F}^J_{(1)n]} + L_{IJ}\epsilon^{mn}\widetilde{H}_{(2)[m} \wedge \widetilde{A}^I_{(0)n]} \wedge \widetilde{F}^J_{(2)} \\
& + \frac{1}{4} L_{IJ}\epsilon^{mn}\widetilde{H}_{(1)mn} \wedge \widetilde{A}^I_{(1)} \wedge \widetilde{F}^J_{(2)},
\end{aligned} \tag{3.3}
$$

together with some complicated total derivative terms, which will not contribute to any equation obtained through variation of the action.

The variation of the Lagrangian (sum of eqs. (2.5), (3.3) and (3.1)) with respect to the field strengths $\widetilde{H}_{(3)}$, $\widetilde{H}_{(2)m}$ and $\widetilde{H}_{(1)mn}$ gives, respectively,

$$\begin{aligned}
e^{-\widetilde{\phi}}\epsilon^{mn} * \widetilde{H}_{(1)mn} &= \widehat{D}\widehat{B}_{(2)} - \widehat{B}_{(1)m}\mathcal{F}^m_{(2)} - \frac{1}{2}L_{IJ}\widehat{A}^I_{(1)} \wedge \widehat{F}^J_{(2)} \equiv \widehat{H}_{(3)} \qquad (3.4)\\
e^{-\widetilde{\phi}}\epsilon_m{}^n * \widetilde{H}_{(2)n} &= \widehat{D}\widehat{B}_{(1)m} + \lambda_m\widehat{B}_{(2)} - \widehat{B}_{(0)mn}\mathcal{F}^n_{(2)} - \frac{1}{2}L_{IJ}\widehat{A}^I_{(0)m}\widehat{F}^J_{(2)} - \frac{1}{2}L_{IJ}\widehat{A}^I_{(1)}\widehat{F}^J_{(1)m}\\
&\equiv \widehat{H}_{(2)m}\\
e^{-\widetilde{\phi}}\epsilon_{mn} * \widetilde{H}_{(3)} &= \widehat{D}\widehat{B}_{(0)mn} - \frac{1}{2}L_{IJ}\widehat{A}^I_{(1)}\widehat{F}^J_{(0)mn} + L_{IJ}\widehat{A}^I_{(0)[m}\widehat{F}^J_{(1)n]} \equiv \widehat{H}_{(1)mn},
\end{aligned}$$

where the covariant derivatives are defined as

$$\begin{aligned}
\widehat{D}\widehat{B}_{(2)} &= d\widehat{B}_{(2)} - \lambda_r\widehat{B}_{(2)}\mathcal{A}^r_{(1)}\\
\widehat{D}\widehat{B}_{(1)m} &= d\widehat{B}_{(1)m} + \lambda_r\widehat{B}_{(1)m}\mathcal{A}^r_{(1)}\\
\widehat{D}\widehat{B}_{(0)mn} &= d\widehat{B}_{(0)mn} - \lambda_r\widehat{B}_{(0)mn}\mathcal{A}^r_{(1)} - 2\lambda_{[m}\widehat{B}_{(1)n]}\ . \qquad (3.5)
\end{aligned}$$

Next we make the identifications

$$\widetilde{M}_{IJ} \to \widehat{M}_{IJ}\ , \quad \widetilde{A}^I_{(1)} \to \widehat{A}^I_{(1)}. \qquad (3.6)$$

The first identification here is understood as such that the scalar matrix of heterotic theory is constructed in terms of the geometric moduli of T^4 and the expectation value of the B-field on T^4. However, its form is the same as that of $\widetilde{M}_{IJ}$, for it still should be $O(4,20)/O(4) \times O(20)$ valued. This scalar matrix is given as

$$\widehat{M}^{IJ} = \begin{pmatrix} \widehat{G} + \widehat{C}^T\widehat{G}^{-1}\widehat{C} + \widehat{A}^T\widehat{A} & -\widehat{C}^T\widehat{G}^{-1} & \widehat{C}^T\widehat{G}^{-1}L\widehat{A} + \widehat{A}^T \\ -\widehat{G}^{-1}\widehat{C} & \widehat{G}^{-1} & -\widehat{G}^{-1}L\widehat{A} \\ \widehat{A}^TL\widehat{G}^{-1}\widehat{C} + \widehat{A} & -\widehat{A}^TL\widehat{G}^{-1} & 1 + \widehat{A}^TL\widehat{G}^{-1}L\widehat{A} \end{pmatrix}. \qquad (3.7)$$

Here $\widehat{G} \equiv \widehat{G}_{ab}$, with $a = 1, \ldots, 4$, is a symmetric 4 by 4 metric on T^4 and $\widehat{C} = \widehat{B} + \frac{1}{2}\widehat{A}^I L_{IJ}\widehat{A}^J$ with $\widehat{B} \equiv \widehat{B}_{(0)ab}$. For each I, $\widehat{A}^I$ is a 4-vector whose components are $\widehat{A}^I_{(0)a}$. L_{IJ} is the invariant metric of $O(4,20)$. Due to the second identification, the field strengths $\widetilde{F}_{(2)}$, $\widetilde{F}_{(1)m}$ and $\widetilde{F}_{(0)mn}$ are identified without any change in their expressions with the field strengths $\widehat{F}_{(2)}$, $\widehat{F}_{(1)m}$ and $\widehat{F}_{(0)mn}$, respectively.

Substituting expressions (3.4) back into (2.4) and (2.5), making the identifications (3.6), and changing the sign of the dilaton $\widetilde{\phi} \to -\widehat{\phi}$, one obtains the dual Lagrangian, which is

$$\begin{aligned}
\mathcal{L}^{Het}_4 = &\frac{1}{2}\widehat{R} * 1 + \frac{1}{8}D\widehat{G}_{mn} \wedge *D\widehat{G}^{mn} - D\widehat{\phi} \wedge *D\widehat{\phi} \qquad (3.8)\\
&-\frac{1}{4}e^{-2\widehat{\phi}}\widehat{G}_{mn}\mathcal{F}^m_{(2)} \wedge *\mathcal{F}^n_{(2)} - \frac{1}{2}e^{2\widehat{\phi}}\lambda_m\widehat{G}^{mn}\lambda_n\\
&-e^{-\widehat{\phi}}\Big[\frac{1}{2}\widehat{H}_{(3)} \wedge *\widehat{H}_{(3)} + \frac{1}{2}\widehat{H}_{(2)m}\widehat{G}^{mn} \wedge *\widehat{H}_{(2)n} + \frac{1}{2}\widehat{H}_{(1)mn}\widehat{G}^{mp}\widehat{G}^{nq} \wedge *\widehat{H}_{(1)pq}\\
&+\frac{1}{2}\widehat{M}_{IJ}\widehat{F}^I_{(2)} \wedge *\widehat{F}^J_{(2)} + \frac{1}{2}\widehat{M}_{IJ}\widehat{F}^I_{(1)m}\widehat{G}^{mn} \wedge *\widehat{F}^J_{(1)n} + \frac{1}{2}\widehat{M}_{IJ}\widehat{F}^I_{(0)mn}\widehat{G}^{mp}\widehat{G}^{nq} \wedge *\widehat{F}^J_{(0)pq}\Big].
\end{aligned}$$

The duality relation between the Lagrangians (2.4-2.5) and (3.8) is of the S-duality type, because it changes the sign of the dilaton. Since the string coupling constant is related to the dilaton with the relation $g = \exp\phi$, dilaton's sign change corresponds to going from strong coupling to weak coupling or vice versa.

Now we show that the Lagrangian (3.8) can be obtained from the six-dimensional Heterotic supergravity Lagrangian through a twisted reduction on T^2. The bosonic sector of Heterotic supergravity in ten dimensions consists of a scalar dilaton, a two-form NS-NS potential and gauge bosons $A^a_{(1)}$. It is often assumed that these vectors take values in the Lie algebra of $U(1)^{16}$, which is the Cartan subalgebra of either Heterotic string theory gauge groups, $E_8 \times E_8$ or $Spin(32)/Z_2$. The six-dimensional Heterotic supergravity Lagrangian is obtained by the standard Kaluza–Klein reduction of the ten-dimensional Heterotic supergravity on T^4. The details of this reduction can be found, for example, in [40]. Like the Type IIA theory in six dimensions, the six-dimensional Heterotic Lagrangian has rigid $O(4,20)$ symmetry. Combining fields into multiplets of $O(4,20)$ one can write the Lagrangian in a manifestly $O(4,20)$ invariant way as

$$\mathcal{L}_6^{Het} = e^{-\widehat{\phi}}\left(R*1 - d\phi\wedge *d\phi + \frac{1}{4}d\widehat{M}_{IJ}\wedge *d\widehat{M}^{IJ} - \frac{1}{2}H_{(3)}\wedge *H_{(3)} - \frac{1}{2}\widehat{M}_{IJ}F^I_{(2)}\wedge *F^J_{(2)}\right), \tag{3.9}$$

where M_{IJ} is the $O(4,20)/O(4)\times O(20)$ scalar matrix, $H_{(3)} = dB_{(2)} - \frac{1}{2}L_{IJ}A^I_{(1)}\wedge dA^J_{(1)}$, and finally $F^I_{(2)} = dA^I_{(1)}$ with $A^I_{(1)} = (A^a_{(1)}, B_{(1)a}, A^A_{(1)})$. We can again utilize the $SL(2,R)$ scaling symmetry of T^2 to write a twisted reduction ansatz as[3]

$$\begin{aligned}
\phi(x,y) &= \widehat{\phi}(x) + 2\lambda_m y^m, \quad m = 1,2\\
G(x,y) &= e^{\lambda_m y^m}\left(\widehat{G} + \widehat{G}_{mn}\eta^m\otimes\eta^n\right),\\
B_{(2)}(x,y) &= e^{\lambda_m y^m}\left(\widehat{B}_{(2)} + \widehat{B}_{(1)m}\wedge\eta^m + \frac{1}{2}\widehat{B}_{(0)mn}\eta^m\wedge\eta^n\right),\\
A^I_{(1)}(x,y) &= e^{1/2\lambda_m y^m}\left(\widehat{A}^I_{(1)} + \widehat{A}^I_{(0)m}\wedge\eta^m\right).
\end{aligned} \tag{3.10}$$

This reduction ansatz for the fields involved are the same as given in (2.2), except that the metric and three-form field strength scale differently under the $SL(2,R)$ scaling symmetry. After inserting the reduction ansatz (3.10) into the Lagrangian (3.9) we obtain in four dimensions an effective theory, whose Lagrangian is exactly the same as the dual Lagrangian given above in (3.8).

4 More on the Massive Duality

In this section, we examine the duality found in (3.4) further. We will see how it implies that the 2-form field on the IIA side, which becomes massive by "eating" one of the 1-form fields (more precisely a linear combination of two 1-form fields) is dual to a massive 1-form

[3] Note that sign of λ in (2.2) and (2.7) have been reversed in each expression except the ones in $A^I_{(1)}$ and $F^I_{(2)}$.

field on the heterotic side, which acquires its mass by absorbing the degree of freedom of the scalar field. Similarly, the remaining 1-form field on the IIA side becomes massive by eating the scalar field and is dual to the 2-form field on the heterotic side, which also becomes massive due to its Stückelberg coupling with the remaining 1-form field.

It is possible that some fields acquire masses by "eating" others due to the Stückelberg type couplings between various fields in equations (2.8) and (2.9). Before explaining how this mechanism works, let us count the number of physical degrees of freedom to see in advance how many fields we expect to become massive in the process. Recall that a massless p-form field in d dimensions has

$$C(d-2,p) = \begin{pmatrix} d-2 \\ p \end{pmatrix} = \frac{(d-2)!}{p!\ (d-2-p)!}$$

number of degrees of freedom, whereas the number of physical degrees of freedom of a massive p-form field in d dimensions is

$$C(d-1,p) = \begin{pmatrix} d-1 \\ p \end{pmatrix} = \frac{(d-1)!}{p!\ (d-1-p)!}.$$

Then in six dimensions a massless 2-form field has 6 degrees of freedom. If we reduced this 2-form to 4 dimensions with an ordinary Kaluza-Klein ansatz without the twist, we would obtain one 2-form field, two 1-form fields and a scalar field, all of which are massless with a total of $1+2\times 2+1 = 6$ degrees of freedom. However, the twist gives rise to Stückelberg type couplings as above, and upon examination one sees that the 2-form field eats one of the 1-form fields in 4 dimensions, whereas the other 1-form field eats the scalar field due to these coupling, as a result of which we end up with 3+3 = 6 degrees of freedom again. This will be possible by going to an appropriate gauge as we explain shortly. Note that a gauge can also be chosen such that the 1-form fields $A^I_{(1)}$ coming from the reduction of each $\widehat{A}^I_{(1)}$ eats one of the scalar fields $A^I_{(0)m}$. However, we prefer not to perform this gauge transformation and content ourselves with showing the mass gaining mechanism only for the 2-form field $B_{(2)}$. This is what we will essentially need when we discuss the duality between the two massive theories arising from the twisted heterotic and IIA reductions.

In order to explain how the mechanism works, let us first write down the gauge transformations of the relevant fields in 4 dimensions.

$$\begin{aligned} \delta B_{(2)} &= D\Lambda_{(1)} + \Lambda_{(0)1}\mathcal{F}^1 + \Lambda_{(0)2}\mathcal{F}^2 \\ \delta B_{(1)1} &= D\Lambda_{(0)1} - \lambda_1\Lambda_{(1)} \\ \delta B_{(1)2} &= D\Lambda_{(0)2} - \lambda_2\Lambda_{(1)} \\ \delta B_{(0)} &= -\lambda_1\Lambda_{(0)2} + \lambda_2\Lambda_{(0)1} \end{aligned} \tag{4.1}$$

It will be useful to define

$$\overline{B}_{(1)n} = \Omega^m{}_n B_{(1)m} \qquad \text{and} \qquad \overline{\Lambda}_{(0)\mathrm{n}} = \Omega^{\mathrm{m}}{}_{\mathrm{n}}\Lambda_{(0)\mathrm{m}},$$

where

$$\Omega = \frac{1}{\sqrt{\lambda_1^2+\lambda_2^2}} \begin{pmatrix} \lambda_2 & -\lambda_1 \\ \lambda_1 & \lambda_2 \end{pmatrix}. \tag{4.2}$$

Then we will have

$$\begin{aligned} \delta\overline{B}_{(1)1} &= D\overline{\Lambda}_{(0)1} \\ \delta\overline{B}_{(1)2} &= D\overline{\Lambda}_{(0)2} - \sqrt{\lambda_1^2+\lambda_2^2}\Lambda_{(1)} \\ \delta B_{(0)} &= \sqrt{\lambda_1^2+\lambda_2^2}\overline{\Lambda}_{(0)1} \end{aligned} \tag{4.3}$$

Using the last line in the above equation, we can go to a gauge in which we can set $B_{(0)} = 0$ and the field strengths in equation (2.9) become

$$\begin{aligned} \overline{H}_{(2)1} &= D\overline{B}_{(1)1} \\ \overline{H}_{(2)2} &= D\overline{B}_{(1)2} + \sqrt{\lambda_1^2+\lambda_2^2}B_{(2)} \\ H_{(1)} &= \overline{B}_{(1)1}. \end{aligned} \tag{4.4}$$

Here $\overline{B}_{(1)n} = \Omega^m{}_n B_{(1)m}$ and $\overline{H}_{(2)n} = \Omega^m{}_n H_{(2)m}$. On the other hand, using the gauge invariance

$$\delta\overline{B}_{(1)2} = -\sqrt{\lambda_1^2+\lambda_2^2}\Lambda_{(1)}, \quad \text{and} \quad \delta B_{(2)} = D\Lambda_{(1)}$$

we can perform the gauge transformation

$$B_{(2)} \longrightarrow B_{(2)} - \frac{1}{\sqrt{\lambda_1^2+\lambda_2^2}}\, D\overline{B}_{(1)2}, \tag{4.5}$$

as a result of which $\overline{B}_{(1)2}$ disappears and $H_{(3)}$ becomes

$$H_{(3)} = DB_{(2)} + \frac{1}{\sqrt{\lambda_1^2+\lambda_2^2}}(\lambda_1\overline{B}_{(1)1}\mathcal{F}^2 - \lambda_2\overline{B}_{(1)1}\mathcal{F}^1). \tag{4.6}$$

In summary we will have

$$\begin{aligned} H_{(3)} &= DB_{(2)} - \overline{B}_{(1)1}\wedge\overline{\mathcal{F}}^1 \\ \overline{H}_{(2)1} &= D\overline{B}_{(1)1} \\ \overline{H}_{(2)2} &= \sqrt{\lambda_1^2+\lambda_2^2}B_{(2)} \\ H_{(1)} &= \overline{B}_{(1)1}. \end{aligned} \tag{4.7}$$

Here we defined $\overline{\mathcal{F}}^m = \Omega^m{}_n\mathcal{F}^n$ Hence we see that the scalar field $B_{(0)}$ and the 1-form field $\overline{B}_{(1)2}$ have been eaten by the 1-form field $\overline{B}_{(1)1}$ and the the 2-form field $B_{(2)}$, respectively. Then, after these special choices of gauges, the four-dimensional duality relations (3.4) become

$$
\begin{aligned}
\widehat{D}\widehat{B}_{(2)} - \frac{1}{\sqrt{\lambda_1^2+\lambda_2^2}}\overline{\widehat{B}}_{(1)1}\wedge\overline{\mathcal{F}}^1 - \frac{1}{2}L_{IJ}A^I_{(1)}\wedge F^J_{(2)} &= e^{-\widetilde{\phi}} * \overline{\widetilde{B}}_{(1)1}, \\
\sqrt{\lambda_1^2+\lambda_2^2}\widehat{B}_{(2)} - L_{IJ}\overline{A}^I_{(0)2}\wedge F^J_{(2)} &= -e^{-\widetilde{\phi}} * D\overline{\widetilde{B}}_{(1)1}, \\
\widehat{D}\overline{\widehat{B}}_{(1)1} - L_{IJ}\overline{A}^I_{(0)1}\wedge F^J_{(2)} &= e^{-\widetilde{\phi}} * \sqrt{\lambda_1^2+\lambda_2^2}\widetilde{B}_{(2)}, \\
\overline{\widehat{B}}_{(1)1} + L_{IJ}A^I_{(0)1}\wedge\widehat{D}A^J_{(0)2} + \frac{1}{2}L_{IJ}\overline{A}^I_{(0)1}\wedge A^J_{(1)} &= e^{-\widetilde{\phi}} * (\widetilde{D}\widetilde{B}_{(2)} - \frac{1}{\sqrt{\lambda_1^2+\lambda_2^2}}\overline{\widetilde{B}}_{(1)1}\wedge\overline{\mathcal{F}}^1).
\end{aligned}
\tag{4.8}
$$

Here $\overline{A}^I_{(0)n} = \Omega^m{}_n A^I_{(0)m}$, and fields with $\widehat{}$ denote the Heterotic fields and fields with $\widetilde{}$ denote the Type IIA fields. So we see that the duality relation between the two-form fields in six dimension imply a massive duality relation between the massive one-form field $\overline{B}_{(1)1}$ and the massive two-form field $B_{(2)}$ in four dimensions. Note that both these fields have three degrees of freedom in four dimensions. This is an illustration in four dimensions of the general duality between massive p-forms and massive $(d-p-1)$-forms in d dimensions [41–43].

5 Relation with the gauged $N = 4$ supergravity in $D = 4$

In this section, we show that the 4 dimensional Lagrangian we obtained in the previous sections can be put in the form of a $N = 4$ gauged supergravity Lagrangian, whose most general form was found by Schön and Weidner [26]. This part of our work is an extension of the work of [27], where they reduce the NS-NS sector of the heterotic theory with a twisted ansatz utilizing the same scaling symmetry and show that the resulting gauged supergravity is of the Schön-Weidner type with nontrivial $SL(2)$ gaugings. Here we also include the vectors A^I and make the comparison for this more general case.

The $N = 4$ supergravity coupled to n vector multiplets has the global on-shell symmetry $SL(2,R)\times O(6,6+n)$. The bosonic sector of the pure $N = 4$ supergravity contains the graviton, six vectors and two scalars, whereas each vector multiplet contains a vector and six real scalars. The scalar fields of the theory constitute the coset space $SL(2,R)/SO(2)\times O(6,6+n)/O(6)\times O(6+n)$.

The gauged $N = 4$ supergravities are obtained by gauging a subalgebra of the global symmetry $SL(2,R)\times O(6,6+n)$. The generators of this subalgebra are the linear combinations of $SL(2,R)$ and $O(6,6+n)$ generators and the coefficients of this linear combinations are the components of the embedding tensor [26, 44–52]. However, there are several requirements which stem form the facts that the commutator of the generators of subalgebra should produce an adjoint action, the Jacobi identity for the subalgebra should be satisfied, and the supersymmetry of the theory should be preserved. Then one obtains a number of constraints that the components of the embedding tensor have to satisfy [26]. The components of the embedding tensor are group valued and usually denoted by $\xi_{\alpha M}$ and $f_{\alpha MNP} = f_{\alpha[MNP]}$. Here $\alpha = +,-$ is the $SL(2,R)$ index and $M = i, i', A$ with $i = 1,...,6$; $i' = 1,...,6$; and $A = 1,...,n$ is the $O(6,6+n)$ index.

The bosonic part of the gauged $N = 4$ supergravity action can be written as the sum of a kinetic term, a topological term and a scalar potential. The kinetic term has the form

$$
\begin{aligned}
e^{-1}\mathcal{L}_{kin} &= \frac{1}{2}R*1 + \frac{1}{16}(D\mathcal{M}_{MN})\wedge *(D\mathcal{M}^{MN}) + \frac{1}{8}(DM_{\alpha\beta})\wedge *(DM^{\alpha\beta}) \\
&\quad -\frac{1}{4}e^{-2\phi}\mathcal{M}_{MN}H^{M+}_{(2)}\wedge *H^{N+}_{(2)} + \frac{1}{8}a\eta_{MN}H^{M+}_{(2)}\wedge H^{N+}_{(2)} \ ,
\end{aligned}
\tag{5.1}
$$

the topological term has the form

$$
\begin{aligned}
e^{-1}\mathcal{L}_{top} &= -\frac{g}{2}\Big[\xi_{+M}\eta_{NP}A^{M-}_{(1)}\wedge A^{N+}_{(1)}\wedge dA^{P+}_{(1)} - (\hat{f}_{-MNP}+2\xi_{-N}\eta_{MP})A^{M-}_{(1)}\wedge A^{N+}_{(1)}\wedge dA^{P-}_{(1)} \\
&\quad -\frac{g}{4}\hat{f}_{\alpha MNR}\hat{f}_{\beta PQ}{}^{R}A^{M\alpha}_{(1)}\wedge A^{N+}_{(1)}\wedge A^{P\beta}_{(1)}\wedge A^{Q-}_{(1)} + \frac{g}{16}\Theta_{+MNP}\Theta_{-}{}^{M}{}_{QR}B^{NP}_{(2)}\wedge B^{QR}_{(2)} \\
&\quad -\frac{1}{4}(\Theta_{-MNP}B^{NP}_{(2)} + \xi_{-M}B^{+-}_{(2)} + \xi_{+M}B^{++}_{(2)})\wedge(2dA^{M-}_{(1)} - g\hat{f}_{\alpha QR}{}^{M}A^{Q\alpha}_{(1)}\wedge A^{R-}_{(1)})\Big] \ ,
\end{aligned}
\tag{5.2}
$$

and the scalar potential term has the form

$$
\begin{aligned}
e^{-1}\mathcal{L}_{pot} &= -\frac{g^2}{16}\Big[f_{\alpha MNP}f_{\beta QRS}M^{\alpha\beta}\left(\frac{1}{3}\mathcal{M}^{MQ}\mathcal{M}^{NR}\mathcal{M}^{PS} + \left(\frac{2}{3}\eta^{MQ} - \mathcal{M}^{MQ}\right)\eta^{NR}\eta^{PS}\right) \\
&\quad -\frac{4}{9}f_{\alpha MNP}f_{\beta QRS}\epsilon^{\alpha\beta}M^{MNPQRS} + 3\xi^M_\alpha\xi^N_\beta M^{\alpha\beta}\mathcal{M}_{MN}\Big] \ .
\end{aligned}
\tag{5.3}
$$

Now let us explain the terms that appear in the above action: η^{MN} is the $O(6, 6+n)$ metric, which can be written in blocks as

$$
\eta^{MN} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & L \end{pmatrix} \ , \tag{5.4}
$$

where L^{IJ} is the $O(4, 4+n)$ metric. $\mathcal{M}_{MN}$ is a symmetric positive definite scalar matrix that parametrize the coset manifold $O(6, 6+n)/O(6)\times O(6+n)$, likewise $M_{\alpha\beta}$ is a symmetric positive definite matrix that parametrizes the $SL(2, R)/SO(2)$ coset space. A suitable choice for $M_{\alpha\beta}$ is

$$
M_{\alpha\beta} = \frac{1}{Im(\tau)}\begin{pmatrix} |\tau|^2 & Re(\tau) \\ Re(\tau) & 1 \end{pmatrix} . \tag{5.5}
$$

where $\tau = a + ie^{-2\phi}$, a is the axion and ϕ is the dilaton field. The M_{MNPQRS} that appear in the scalar potential term of the Lagrangian is a scalar dependent, completely antisymmetric tensor, which is also defined in terms of $O(6, 6+n)/O(6)\times O(6+n)$ coset representatives [26].

The components of the embedding tensor, $\xi_{\alpha M}$ and $f_{\alpha MNP} = f_{\alpha[MNP]}$, frequently appear in the following combinations:

$$
\begin{aligned}
\Theta_{\alpha MNP} &= f_{\alpha MNP} - \xi_{\alpha[N}\eta_{P]M} \ , \\
\hat{f}_{\alpha MNP} &= f_{\alpha MNP} - \xi_{\alpha[M}\eta_{P]N} - \frac{3}{2}\xi_{\alpha N}\eta_{MP} \ .
\end{aligned}
\tag{5.6}
$$

The gauge coupling constant g will later be taken as $g = 1$. The covariant derivative is defined as

$$D = \nabla - gA^{M\alpha}_{(1)}\Theta_{\alpha M}{}^{NP}t_{NP} + gA^{M(\alpha}_{(1)}\epsilon^{\beta)\gamma}\xi_{\gamma M}t_{\alpha\beta}\ , \tag{5.7}$$

where t_{NP} and $t_{\alpha\beta}$ are generators of $O(6, 6 + n)$ and $SL(2, R)$, respectively and ∇ contains the spin connection. D acts on objects in an arbitrary representation of the global symmetry group.

$H^{M+}_{(2)}$ are the covariant field strengths of electric fields with forms given as [26]

$$H^{M+}_{(2)} = F^{M+}_{(2)} + \frac{g}{2}\Theta_-{}^M{}_{NP}B^{NP}_{(2)} + \frac{g}{2}\xi_+{}^M B^{++}_{(2)} + \frac{g}{2}\xi_-{}^M B^{+-}_{(2)}\,, \tag{5.8}$$

$$F^{M+}_{(2)} = dA^{M+}_{(1)} - g\hat{f}_{\alpha NP}{}^M A^{N\alpha}_{(1)} \wedge A_{(1)}{}^{P+}. \tag{5.9}$$

The covariant field strengths of magnetic fields, $H^{M-}_{(2)}$, are defined similarly by interchanging all $-$ and $+$ indices in the above expression. Note that there is no kinetic term for the magnetic fields $H^{M-}_{(2)}$.

To match the the four-dimensional Heterotic string Lagrangian (3.8) with the four-dimensional gauged supergravity Lagrangian (5.1–5.3) we have to make several field definitions and define a $O(6, 22)/O(6) \times O(22)$ valued scalar matrix which could be matched with $\mathcal{M}^{MN}$. We find that after following field definitions[4]

$$\widehat{C}_{(2)} = \widehat{B}_{(2)} + \frac{1}{2}\widehat{B}_{(1)m} \wedge \mathcal{A}^m_{(1)} + \frac{1}{4}L_{IJ}\widehat{A}^I_{(1)} \wedge \widehat{A}^J_{(0)m} \wedge \mathcal{A}^m_{(1)} \tag{5.10}$$

$$\widehat{C}_{(1)m} = \widehat{B}_{(1)m} + \frac{1}{2}L_{IJ}\widehat{A}^I_{(1)} \wedge \widehat{A}^J_{(0)m} \tag{5.11}$$

$$\widehat{C}_{(0)mn} = \widehat{B}_{(0)mn} + \frac{1}{2}L_{IJ}\widehat{A}^I_{(0)m} \wedge \widehat{A}^J_{(0)n} \tag{5.12}$$

the field strengths $\widehat{H}_{(3)}$, $\widehat{H}_{(2)m}$ and $\widehat{H}_{(1)mn}$ can be written as

$$\widehat{H}_{(3)} = \widehat{D}\widehat{C}_{(2)} - \frac{1}{2}\eta_{MN}\widehat{\mathcal{A}}^M_{(1)} \wedge \widehat{\mathcal{F}}^N_{(2)} - \frac{1}{4}\lambda_m\mathcal{A}^m_{(1)} \wedge \widehat{C}_{(1)p} \wedge \mathcal{A}^p_{(1)} \tag{5.13}$$

$$\widehat{H}_{(2)m} = \widehat{D}\widehat{C}_{(1)m} + \lambda_m\widehat{C}_{(2)} - \widehat{C}_{(0)mn} \wedge \mathcal{F}^n_{(2)} - L_{IJ}\widehat{A}^I_{(0)m} \wedge D\widehat{A}^J_{(1)} \tag{5.14}$$

$$\widehat{H}_{(1)mn} = \widehat{D}\widehat{C}_{(0)mn} - L_{IJ}\widehat{A}^I_{(0)m} \wedge D\widehat{A}^J_{(0)n}, \tag{5.15}$$

where $D\widehat{A}^I_{(p)}$ are as in (2.8), and we define $\widehat{\mathcal{A}}^M_{(1)} = (\mathcal{A}^m_{(1)}, \widehat{C}_{(1)m}, \widehat{A}^I_{(1)})$ and their field strengths,

$$\widehat{\mathcal{F}}^M_{(2)} = (\mathcal{F}^m_{(2)}, \widehat{D}\widehat{C}_{(1)m} + \lambda_m\widehat{C}_{(2)}, D\widehat{A}^I_{(1)}). \tag{5.16}$$

On the other hand the covariant derivatives have the following forms:

$$\widehat{D}\widehat{C}_{(2)} = d\widehat{C}_{(2)} - \lambda_r\widehat{C}_{(2)} \wedge \mathcal{A}^r_{(1)} \tag{5.17}$$

$$\widehat{D}\widehat{C}_{(1)m} = d\widehat{C}_{(1)m} + \lambda_r\widehat{C}_{(1)m} \wedge \mathcal{A}^r_{(1)} - \frac{1}{2}\lambda_m\widehat{C}_{(1)p} \wedge \mathcal{A}^p_{(1)} \tag{5.18}$$

$$\widehat{D}\widehat{C}_{(0)mn} = d\widehat{C}_{(0)mn} - \lambda_r\widehat{C}_{(0)mn} \wedge \mathcal{A}^r_{(1)} - 2\lambda_{[m}\widehat{C}_{(1)n]}\ . \tag{5.19}$$

[4]Note that $\widehat{C}_{(0)mn}$ is not antisymmetric in its indices. In fact, in matrix notation we have $\widehat{C} = \widehat{B} + \frac{1}{2}\widehat{A}^TL\widehat{A}$ so that $\widehat{C} + \widehat{C}^T = \widehat{A}^TL\widehat{A}$ rather than 0.

We also define the $O(6,22)/O(6)\times O(22)$ valued scalar matrix as

$$\widehat{\mathcal{N}}^{MN}=\begin{pmatrix}\widehat{G}+\widehat{C}^T\widehat{G}^{-1}\widehat{C}+\widehat{A}^T\widehat{M}\widehat{A}^J & -\widehat{C}^T\widehat{G}^{-1} & \widehat{C}^T\widehat{G}^{-1}L\widehat{A}+\widehat{A}^T\widehat{M}\\ -\widehat{G}^{-1}\widehat{C} & \widehat{G}^{-1} & -\widehat{G}^{-1}L\widehat{A}\\ \widehat{A}^TL\widehat{G}^{-1}\widehat{C}+\widehat{M}\widehat{A} & -\widehat{A}^TL\widehat{G}^{-1} & \widehat{M}+\widehat{A}^TL\widehat{G}^{-1}L\widehat{A}\end{pmatrix}. \tag{5.20}$$

where $\widehat{G}\equiv\widehat{G}_{mn}$, with $m=1,2$, is a symmetric 2 by 2 metric on T^2 and $\widehat{C}=\widehat{B}+\frac{1}{2}\widehat{A}^IL_{IJ}\widehat{A}^J$ with $\widehat{B}\equiv\widehat{B}_{(0)mn}$. For each I, $\widetilde{A}^I$ is a 2-vector whose components are $\widetilde{A}^I_{(0)m}$. L_{IJ} is the invariant metric of $O(4,20)$, and $O(4,20)/O(4)\times O(20)$ valued scalar matrix $\widehat{M}^{IJ}$ is given in (3.7).

We can now rewrite the four-dimensional Heterotic Lagrangian (3.8) in a form which is ready to be compared with the four-dimensional supergravity Lagrangian:

$$\begin{aligned}\mathcal{L}_4^{Het}&=\frac{1}{2}\widehat{R}*1-(D\widehat{\phi})\wedge*(D\widehat{\phi})-\frac{1}{2}e^{2\widehat{\phi}}\lambda_m\widehat{\mathcal{N}}^{mn}\lambda_n\\&\quad+e^{-\widehat{\phi}}\left[\frac{1}{4}D\widehat{\mathcal{N}}_{MN}\wedge*D\widehat{\mathcal{N}}^{MN}-\frac{1}{2}\widehat{H}_{(3)}\wedge*\widehat{H}_{(3)}-\frac{1}{2}\widehat{\mathcal{N}}_{MN}\widehat{\mathcal{F}}^M_{(2)}\wedge*\widehat{\mathcal{F}}^N_{(2)}\right].\end{aligned} \tag{5.21}$$

Now we need to solve for $\xi_{\alpha M}$ and $f_{\alpha MNP}$ in order to bring the Lagrangian of the gauged supergravity in four dimensions (5.1–5.3) to a form that is equivalent to the four-dimensional Heterotic Lagrangian (5.21). However, instead of solving for possible $\xi_{\alpha M}$ and $f_{\alpha MNP}$ from constraint equations (eq. (2.20) in [26]), we determine them by comparing the field strengths (5.8) in four-dimensional gauged supergravity Lagrangian with the field strengths $\widehat{\mathcal{F}}^M_{(2)}$ in the Heterotic supergravity Lagrangian. Then it can be shown that the solution we find indeed obeys the constraint equations of [26].

In (5.21) we have only the field strengths of electric fields. Therefore we first set $\xi_{-M}=0$ and $f_{-MNP}=0$. Now comparing the field strength $\widehat{\mathcal{F}}^M_{(2)}$ (5.16) with $H^{M+}_{(2)}$ (5.8) we firstly observe that to have an equivalence we need to identify $B^{++}_{(2)}$ with $2\widehat{C}_{(2)}$ and set the values of ξ_{+M} as

$$\xi_{+M}=(\xi_{+m},\xi_{+m'},\xi_{+I})=(\lambda_m,0,0). \tag{5.22}$$

The other observation is about the values of $\hat{f}_{+MNP}$, which we obtain as

$$\begin{aligned}\hat{f}_{+MNp'}-\hat{f}_{+NMp'}&=0\\ \hat{f}_{+m'np}-\hat{f}_{+nm'p}&=\lambda_p\eta_{nm'}-2\lambda_n\eta_{pm'}\\ \hat{f}_{+ImJ}-\hat{f}_{+mIJ}&=-\lambda_m\eta_{IJ}\end{aligned} \tag{5.23}$$

Using the definition (5.6) of $\hat{f}_{+MNP}$, values of ξ_{+M} (5.22) and the antisymmetry property $f_{+MNP}=f_{+[MNP]}$, we can now determine that

$$f_{+mnp'}=-\lambda_{[m}\eta_{n]p'}, \tag{5.24}$$

with all other components of f_{+MNP} vanishing. Here, λ_m has only two components λ_1 and λ_2, unlike [27]. This is because we put fluxes only on T^2, whereas in [27], a twisted reduction on the 6-torus T^6 is considered. Note that the components of the tensor f_{+MNP}

involving the indices I are zero, in spite of the non-Abelian field strengths of the vector fields $\widehat{A}^I$. Comparing $D\widehat{A}^I$ with (5.8), one finds the last equation in (5.23) above, yet the components f_{+mIJ} are computed to be zero. As a result, our embedding tensor contains no new nonvanishing components as compared to the one found in [27]. We refer the reader to [26] to check that the solution (5.24) satisfies the constraint equations that the embedding tensor should satisfy.

Plugging in the determined values for embedding tensor components (5.22, 5.24) and then integrating out the magnetic fields, $A^{M-}_{(1)}$, from the gauged supergravity Lagrangian (5.1–5.3) one obtains that the combination of the kinetic part and the topological part become [27]

$$\begin{aligned} e^{-1}\mathcal{L}_{kin} = &\frac{1}{2}R*1 + \frac{1}{16}(D\mathcal{M}_{MN})\wedge *(D\mathcal{M}^{MN}) - (D\phi)\wedge *(D\phi) \\ &-\frac{1}{4}e^{-2\phi}\mathcal{M}_{MN}\left(F^{M+}_{(2)} + \frac{1}{2}\xi^M_+ B^{++}_{(2)}\right)\wedge *\left(F^{N+}_{(2)} + \frac{1}{2}\xi^N_+ B^{++}_{(2)}\right) \\ &-\frac{1}{8}e^{-4\phi}\left(dB^{++}_{(2)} - \xi_{+M}A^{M+}_{(1)}\wedge B^{++}_{(2)} - \omega_{(3)}\right)^2 , \end{aligned} \tag{5.25}$$

and the scalar potential term become

$$\begin{aligned} e^{-1}\mathcal{L}_{pot} = -\frac{1}{16}e^{2\phi}\Big[&3\xi_{+M}\xi_{+N}\mathcal{M}^{MN} + \frac{1}{3}f_{+MNP}f_{+QRS}\mathcal{M}^{MQ}\mathcal{M}^{NR}\mathcal{M}^{PS} \\ &+ f_{+MNP}f_{+QRS}\left(\frac{2}{3}\eta^{MQ} - \mathcal{M}^{MQ}\right)\eta^{NR}\eta^{PS}\Big] , \end{aligned} \tag{5.26}$$

where $\omega_{(3)} = \eta_{MN}F^{M+}_{(2)}\wedge A^{N+}_{(1)} - \frac{1}{2}\lambda_m \mathcal{A}^{m+}_{(1)}\wedge A^{n+}_{(1)}\wedge A^{+}_{(1)n}$ and we set $g = 1$. Note that $\widehat{H}_{(3)} = \widehat{D}\widehat{C}_{(2)} - \frac{1}{2}\omega_{(3)}$.

We note that identifying $A^{m+}_{(1)}$ with the Kaluza-Klein gauge fields $\mathcal{A}^m_{(1)}$, $A^{m'+}_{(1)}$ with the field $\widehat{C}_{(1)m}$ (5.11), $A^{I+}_{(1)}$ with the vector fields of six dimensions, and $B^{++}_{(2)}$ with $2\widehat{C}_{(2)}$ one matches the kinetic terms of gauge fields in four-dimensional gauged supergravity Lagrangian (5.25) with the kinetic terms of gauge fields in the four-dimensional Heterotic string Lagrangian (5.21). One needs also to check whether the scalar potential term (5.26) of the four-dimensional gauged supergravity action matches with the scalar potential term in the above Lagrangian after the identification $\widehat{\mathcal{M}} \equiv 2\widehat{\mathcal{N}}$. Substituting in the scalar potential term (5.26) the matrix form of $\widehat{\mathcal{N}}$ one finds that scalar potential terms also match. This way we show that the compactification of heterotic string theory with the inclusion of the Yang-Mills vectors is equivalent to a gauged supergravity which is still of the Schön-Weidner type.

6 Conclusions

In this paper, we established a massive S-duality relation between the heterotic theory and type IIA theory in 4 dimensions. Both theories in four dimensions are obtained by a duality-twisted reduction, which exploits the scaling symmetry of various fields including the dilaton and the metric. This type of reduction ansatz was first used by Derendinger et al. [27] for the reduction of the NS-NS sector of the heterotic theory. Our ansatz for the reduction of heterotic and type IIA theories assign gauge coupling of the opposite sign to the NS-NS fields and couplings of the same sign to the 1-form fields. The massive duality between the two theories work in the following way. On the one side we have scalar, vector and 2-form fields ($p-$form fields with $p = 0, 1, 2$) with various Stückelberg type couplings. Such couplings allow a $p-$form field to become massive by absorbing the degrees of freedom of a $(p-1)$-form field after a certain gauge transformation. In the massless case, a $p-$form field is dual to a $(\tilde{p} = 2-p)$-form field in 4 dimensions. Similarly, a $(p-1)$-form field is dual to a $(3-p = \tilde{p}+1)$-form field. These dual fields also have Stückelberg type couplings among them. As a result, the $(\tilde{p}+1)$-form fields absorb the degrees of freedom of the $\tilde{p}$-form fields and hence become massive. This massive $(\tilde{p}+1 = 3-p)$-form field is the dual of the massive p-form field in the original theory. The duality between the two theories also changes the sign of the dilaton, and therefore it is of the S-duality type. So we see that the usual S-duality between the heterotic and IIA theories in 4 dimensions survive, even in the presence of (a certain class of) fluxes.

In the last section of our paper, we also showed that the Lagrangian for the massive theory we obtain in four dimensions can be put in the general form of $N = 4, D = 4, SL(2)\times O(6, 22)$ gauged Lagrangian, found by Schön and Weidner [26], where (part of) the $SL(2)$ group has been gauged. This had already been done by Derendinger et al. [27] for the NS-NS sector of the heterotic theory. Here, we also add the sector involving the vector fields coming from the reduction of the Yang-Mills vectors in 10 dimensions, and show that the resulting theory is still of the same type.

A natural generalization of our work would be to introduce a more general duality-twisted ansatz, which also gauges the $O(6, 22)$ part (and even more interestingly the whole of the $SL(2)$ part) of the symmetry group in 4 dimensions and explore the faith of S-duality in this more general case.

Another interesting direction is to analyze if the string-string-string triality in 4 dimensions [53] continues to hold in the presence of fluxes we consider here. It would be very interesting to find a duality-twisted ansatz for the reduction of type IIB theory, which gives in 4 dimensions a massive theory dual to the two massive theories we have found here.

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