Emergence of non-gravitational fields in dimensional reduction of 4d spin foam models

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Abstract

A Kaluza-Klein like approach for a 4d spin foam model is considered. By applying this approach to a model based on group field theory in 4d (TOCY model), and using the Peter-Weyl expansion of the gravitational field, reconstruction of new non gravitational fields and interactions in the action are found. The perturbative expansion of the partition function produces graphs colored with su(2) algebraic data, from which one can reconstruct a 3d simplicial complex representing space-time and its geometry; (like in the Ponzano-Regge formulation of pure 3d quantum gravity) , as well as the Feynman graph for typical matter fields. Thus a mechanism for generation of matter and construction of new dimensions are found from pure gravity.

I Introduction

Spin foam models [?, ?, ?], which represents a sum-over-histories approach of quantum gravity, can be defined in any space time dimension. They can be used in different approaches of quantum gravity, such as LQG[1] (as a path integral formulation for canonical formulation of quantum gravity), topological field theories, and simplicial gravity. Spin foam models are used lately as a general formulation for quantum gravity.

In spin foam models, one may use an abstract 2-complex for illustration of a discrete space time, and assigning algebraic data from the representation of Lorentz group, to it. This combination can assign an algebraic form of geometric data to the simplicial gravity.

On the other hand, one can consider spin foam models in terms of the so-called group field theories [?, ?], i.e. field theories over group manifolds. The GFT[2] formalism[?] represents a generalization of matrix models for 2-dimensional quantum gravity [?]. There is a group

[1] Loop Quantum Gravity

[2] Group Field Theory

field theory structure for any spin foam models [?, ?]. The GFT structure can be interpreted as a (discrete) third quantization of quantum gravity [?]. Moreover, they allow us to sum over different topologies [?, ?, ?, ?].
In this picture, spin foams (and thus space-time itself) appear as Feynman diagrams of a field theory which is defined on a group manifold. Thus, spin foam amplitudes are simply the Feynman amplitudes and the number of vertices of the spin foam complex correspond to the orders in perturbative expansion of the GFT.
There are some spin foam models, and group field theories, in 4-dimensions, that have been extensively studied [?, ?]; however their validities are still under investigation. In 4d, one can write a BF theory with some constraints to illustrate 4d gravity. The simplest model to illustrate (topological) 4d gravity without cosmological constant is TOCY[3] model [?], whose group field theory derivation was given by Ooguri [?]. In 3-dimentions there are spin foam models of gravity which provide a consistent quantization and are equivalent to those obtained from other approaches. However, each model has some distinctive advantages. Indeed, the first model of quantum gravity, the Ponzano-Regge model, was a spin foam model for Euclidean quantum gravity without cosmological constant [?]. This approach has been developed to a great extent in the 3-dimensional case. It is now clear that it provides a full quantization of pure gravity [?], whose relation with other approaches is well understood [?, ?]. The presentation of the Ponzano-Regge model as a discrete gauge theory, was given by Boulatov [?].
The above mentioned models only consider pure gravity.
There are some recent investigations which try to couple non-gravitational fields to gravitational field in framework of spin foam models, such as coupling of the Yang-Mills [?, ?] and matter fields [?, ?]. This has been done to GFT[?] as well.
In this research the focus is on a Kaluza-Klein like strategy, which can reduce interaction degree in group field theory approach of spin foam model of (topological) 4d-gravity. Also, the symmetry of the theory in 4d is reduced and then as a result a 3d spin foam model plus the non-gravitational fields is obtained.
In section II Ponzano-Regge model as a 3d, and TOCY model as a 4d spin foam models are reviewed in framework of GFT. In section III the Kaluza-Klein approach is applied to TOCY model and in the last section the possibility of how pure gravitational fields can be the source of non-gravitational fields is explained and concluded.

II GFT formulation in 3d and 4d

II.1 3d-spin foam model (Ponzano-Regge model)

Based on [?, ?] one can consider a real field $\phi(g_1, g_2, g_3)$ over a Cartesian product of three copies of $G = SU(2)$ as of the following:

$$\phi : SU(2) \otimes SU(2) \otimes SU(2) \longrightarrow R \tag{1}$$

It is convenient to require that ϕ be invariant under any permutation (π) of its arguments in the sence that:

$$\phi(g_1, g_2, g_3) \;\; = \;\; \phi(g_{\pi 1}, g_{\pi 2}, g_{\pi 3}) \tag{2}$$

[3] Turaev, Ooguri, Crane and Yetter

and invariant under the right diagonal action of SU(2)[4], in the sence that:

$$\forall g \in SU(2): \qquad \phi(g_1, g_2, g_3) \; = \; \phi(g_1 g, g_2 g, g_3 g). \tag{3}$$

Then it is straightforward to show that:

$$\phi(g_1, g_2, g_3) = \int_{SU(2)} dh\, \varphi(g_1 h, g_2 h, g_3 h), \tag{4}$$

where dh is the Haar measure.
Now consider a model defined by the following action:

$$\begin{aligned} S[\phi] \;&=\; \frac{1}{2} \int \prod_{i=1}^{3} dg_i \, |\phi(g_1, g_2, g_3)|^2 \\ &+\; \frac{\lambda}{4!} \int \prod_{i=1}^{6} dg_i \, \phi(g_1, g_2, g_3)\phi(g_3, g_4, g_5)\phi(g_5, g_2, g_6)\phi(g_6, g_4, g_1). \end{aligned} \tag{5}$$

Also, note that any quantum theory like this can be defined by a partition function Z, alternatively, which can be expanded in terms of the Feynman diagrams (Γ) as:

$$Z = \int D\phi \, e^{iS[\phi]} = \sum_{\Gamma} \frac{\lambda^v}{sym[\Gamma]} Z(\Gamma), \tag{6}$$

where v is the number of vertices of the graph Γ, $sym[\Gamma]$ is the symmetry factor, and $Z(\Gamma)$ is the Feynman amplitude corresponding to Γ.
It worth noting that in this case the Feynman diagrams are dual to 3d simplicial complexes, which are supposed to construct the geometrical space. For constructing the corresponding Feynman amplitudes $Z(\Gamma)$, one should consider the following steps: First going to the representation space, and expanding the field $\phi(g_1, g_2, g_3)$ over $SU(2) \otimes SU(2) \otimes SU(2)$. By using Peter-Weyl theorem, one can write:

$$\phi(g_1, g_2, g_3) = \sum_{j_1, j_2, j_3} \phi^{j_1, j_2, j_3}_{m_1, m_2, m_3} D^{j_1}_{m_1, l_1}(g_1) D^{j_2}_{m_2, l_2}(g_2) D^{j_3}_{m_3, l_3}(g_3) c^{j_1, j_2, j_3}_{l_1, l_2, l_3}, \tag{7}$$

where $c^{j_1, j_2, j_3}_{l_1, l_2, l_3}$ are 3j-symbols[5] and $D^{j}_{m,l}$ are the irreducible representation of g, and $\phi^{j_1, j_2, j_3}_{m_1, m_2, m_3}$ are Fourier like expansion coefficients for $\phi(g_1, g_2, g_3)$.
Then, by using (**??**) and (**??**), one can rewrite the action $S[\phi]$ in terms of the representation space variables in the form of:

$$\begin{aligned} S[\phi] = \frac{1}{2} \sum_{j_1, j_2, j_3} |\phi^{j_1, j_2, j_3}_{m_1, m_2, m_3}|^2 + \frac{\lambda}{4!} \sum_{j_1, \ldots, j_6} \phi^{j_1, j_2, j_3}_{m_1, m_2, m_3} \phi^{j_3, j_4, j_5}_{m_3, m_4, m_5} \\ \phi^{j_5, j_2, j_6}_{m_5, m_2, m_6} \phi^{j_6, j_4, j_1}_{m_6, m_4, m_1} \left\{ \begin{smallmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{smallmatrix} \right\}, \end{aligned} \tag{8}$$

[4]One can impose invariance under diagonal action of the group (G), to give a closure with d number of, (d-2)-faces to form a (d-1)-simplex (in d dimension). (d-1)-simplexes are needed for spin foams.

[5]Clebsh-Gordon coefficients

where $\left\{\begin{smallmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{smallmatrix}\right\}$ are 6j-symbols[6].
Finally, considering (**??**) and (**??**), one can show the Feynman amplitude of 3d GR[**?**] as:

$$Z(\Gamma) = \sum_{j_f} \prod_f dim(j_f) \prod_v \{6j\}_v, \tag{9}$$

which is the same as what one may obtain in Ponzano-Regge model.

II.2 4d-spin foam model (TOCY model)

In 4 dimensions, one can consider a real field $\phi(g_1, g_2, g_3, g_4)$ over a Cartesian product of four copies of $SO(4)$ which requires to be invariant under the right diagonal action of SO(4). Considering the same procedure as the above 3d GR, the TOCY model [**?**] can be obtained. Actually the 4d-action is

$$\begin{aligned} S[\phi] &= \frac{1}{2} \int \prod_{i=1}^{4} dg_i \, |\phi(g_1, g_2, g_3, g_4)|^2 \\ &+ \frac{\lambda}{5!} \int \prod_{i=1}^{10} dg_i \, \phi(g_1, g_2, g_3, g_4)\phi(g_4, g_5, g_6, g_7)\phi(g_7, g_3, g_8, g_9) \\ &\quad \phi(g_9, g_6, g_2, g_{10})\phi(g_{10}, g_8, g_5, g_1). \end{aligned} \tag{10}$$

In representation space, using Peter-Weyl decomposition of ϕ into unitary irreducible representations, one can write:

$$\phi(g_1, g_2, g_3, g_4) = \sum_{j_1,\ldots,j_4} \phi^{j_1,j_2,j_3,j_4,i}_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} R^{j_1}_{\alpha_1,\beta_1}(g_1) R^{j_2}_{\alpha_2,\beta_2}(g_2) R^{j_3}_{\alpha_3,\beta_3}(g_3) R^{j_4}_{\alpha_4,\beta_4}(g_4) v^i_{\beta_1,\beta_2,\beta_3,\beta_4}, \tag{11}$$

Where $R^j_{\alpha,\beta}$ are matrix elements of the unitary irreducible representations j[**?**], the indices α, β label basis vectors in the corresponding representation space, and the index i represents the orthonormal basis $v^i_{\beta_1,\beta_2,\beta_3,\beta_4}$ in the space of the intertwiners between the representations j_1, j_2, j_3, j_4. So, one can rewrite the action in terms of the representation variables as:

$$\begin{aligned} S[\phi] &= \frac{1}{2} \sum_{j_1,j_2,j_3,j_4} |\phi^{j_1,j_2,j_3,j_4}_{m_1,m_2,m_3,m_4}|^2 \\ &+ \frac{\lambda}{5!} \sum_{j_1,\ldots,j_{10}} \phi^{j_1,j_2,j_3,j_4}_{m_1,m_2,m_3,m_4} \phi^{j_4,j_5,j_6,j_7}_{m_4,m_5,m_6,m_7} \phi^{j_7,j_3,j_8,j_9}_{m_7,m_3,m_8,m_9} \\ &\quad \phi^{j_9,j_6,j_2,j_{10}}_{m_9,m_6,m_2,m_{10}} \phi^{j_{10},j_8,j_5,j_1}_{m_{10},m_8,m_5,m_1} \{15j\}. \end{aligned} \tag{12}$$

By considering (**??**) and (**??**), one can show the Feynman amplitude of (topological) 4d GR (TOCY model)[**?**] in representation space as of the following:

$$Z(\Gamma) = \sum_{j_f, i_e} \prod_f dim(j_f) \prod_v \{15j\}_v, \tag{13}$$

where $\{15j\}$ are the 15j symbols.

---

[6] $\{6j\} = \left\{\begin{smallmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{smallmatrix}\right\} = \sum_{l_1,\ldots,l_6} c^{j_1,j_2,j_3}_{l_1,l_2,l_3} c^{j_3,j_4,j_5}_{l_3,l_4,l_5} c^{j_5,j_1,j_6}_{l_5,l_1,l_6} c^{j_6,j_4,j_1}_{l_6,l_4,l_1}$

III Kaluza-Klein strategy

Returning to the TOCY model, and using SU(2) instead of SO(4)[7] for simplicity[8], and noting that the action is the same as equation (??), one can rewrite equation (??) as:

$$\phi(g_1,g_2,g_3,g_4)=\sum_{j_4}\Phi^{j_4}_{m_4,l_4}(g_1,g_2,g_3)D^{j_4}_{m_4,l_4}(g_4) \tag{14}$$

where

$$\Phi^{j_4}_{m_4,l_4}(g_1,g_2,g_3)\equiv\sum_{j_1,j_2,j_3,i}\phi^{j_1,j_2,j_3,j_4,i}_{m_1,m_2,m_3,m_4}D^{j_1}_{m_1,l_1}(g_1)D^{j_2}_{m_2,l_2}(g_2)D^{j_3}_{m_3,l_3}(g_3)v^{i}_{l_1,l_2,l_3,l_4}. \tag{15}$$

Equation (??) is similar to the Fourier expansion of a multi variable function in terms of one of its variables[9] (i.e. same as what is done in Kaluza-Klein approach).
Now by putting (??) in action formula (??) and integrating over g_4, one can obtain the following equation for the action[10]:

$$\begin{aligned} S[\Phi] &= \frac{1}{2}\int\prod_{i=1}^{3}dg_i\sum_j\frac{|\Phi^j_{m,l}(g_1,g_2,g_3)|^2}{2j+1} \\ &+ \frac{\lambda'}{4!}\int\prod_{i=1}^{6}dg_i\sum_{j_1,\ldots,j_4}\frac{\phi^{j_1,j_2,j_3,j_4}_{m_1,l_1,\ldots,m_4,l_4}}{(2j_1+1)\ldots(2j_4+1)}\Phi^{j_1}_{m_1,l_1}(g_1,g_2,g_3) \\ &\quad \Phi^{j_2}_{m_2,l_2}(g_3,g_4,g_5)\Phi^{j_3}_{m_3,l_3}(g_5,g_2,g_6)\Phi^{j_4}_{m_4,l_4}(g_6,g_4,g_1), \end{aligned} \tag{16}$$

It is known from LQG that, eigenvalues of area operator are related to j. Therefore, one can see denominators in (??) as something proportional to areas. Now one can consider the situation that for every 3−simplexes[11] assumes an area[12] of one of g_is is small[13]. By this requirement, the larger js which belongs to larger areas[?] are inaccessible, and produce smaller terms, and therefore can be regarded as perturbations.
In other words, looking at integrants in (??), one can observe the products of $(2j_i+1)s$ in

[7] BF Theory with gauge group SU(2)[?, ?]

[8] As it is shown in[?], any group element g of SO(n) can be written as $g=g^{(n-1)}h$, where $h\in SO(n-1)$ and $g^{(n-1)}$ can be expanded in terms of Euler angels in n dimensional space. If one use $g=g^{(3)}h$ in all calculation presented in this paper and integrate over $g^{(3)}$, it would be expected that, this strategy will work for SO(4) with some relatively complicated calculations![?]

[9] Here the selected variable for expansion is g_4; but it is clear that any other variable can be selected as well.

[10] It worth noting that, there are some different choices for interaction terms in expanded action. Since close simplexes are considered, the compatible interaction term should be the same as the interaction term in equation (??). This means that in action formula (??), one should expand g_1 in the first field, g_5 in the second field, g_8 in the third field, and g_{10} in the forth field. Then, if close simplexes are considered, one should expand all arguments in the fifth field as well.

[11] $(d-1)-$simplexes in d dimensions.

[12] $(d-2)-$volume in d dimensions.

[13] Since close simplexes are considered, the result of this assumption is that in one of 3−simplexes, all areas become small. This means that in equation (??) the areas belong to g_1(in the 3-simplex that belongs to the first field), g_5 in the second field, g_8 in the third field and g_{10} in the forth field are small, and as a result all areas belong to the fifth field will be small.

the denominators, and by assuming that $\phi^{j_1,j_2,j_3,j_4}_{m_1,l_1,\ldots,m_4,l_4}$ are finite, then the larger js produce the smaller terms. Following the strategy of Kaluza-Klein and keeping the $j=0$ term, the action (**??**) will be reduced to:

$$
\begin{aligned}
S[\phi] &= \frac{1}{2}\int\prod_{i=1}^{3} dg_i\,|\phi(g_1,g_2,g_3)|^2 \\
&+ \frac{\lambda}{4!}\int\prod_{i=1}^{6} dg_i\,\phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_2,g_6)\phi(g_6,g_4,g_1), \qquad (17)
\end{aligned}
$$

where the symbol $\phi(g_1,g_2,g_3)$ is substituted for $\Phi^0_{0,0}(g_1,g_2,g_3)$ and λ for $\lambda'\phi^{0,0,0,0}_{0,\ldots,0}$.
This action is the Ponzano-Regge action for 3d gravity with the correct vertex amplitude (i.e., $\{6j\}$ symbol). As SU(2) is the symmetry group in this study, it is straightforward to show $\phi(g_1,g_2,g_3)$ has those symmetries that the field in Ponzano-Regge action has.
But here, there is a very basic difference in comparison with the Kaluza-Klein approach. Contrary to Kaluza-Klein approach, where the zero mode corresponds to low energy (or large distance) regime, in our approach $j=0$ mode indicates the short distance regime. For long distance effects, if one let j become greater than zero, then the first choice is $j=\frac{1}{2}$. On the other hand since in all vertices, the angular momentum is conserved, there will be two choices for interaction terms, which are:
1: $j_1=j_2=\frac{1}{2}$, $j_3=j_4=0$
2. $j_1=j_2=j_3=j_4=\frac{1}{2}$.
Using these facts, then the action will be reduced to:

$$
S[\phi,\Phi]=S_{3d-pg}[\phi]+S[\phi,\Phi^{\frac{1}{2}}], \qquad (18)
$$

where $S_{3d-pg}[\phi]$ is pure gravity action in 3 dimensions, and

$$
\begin{aligned}
S[\phi,\Phi^{\frac{1}{2}}] &= \frac{1}{2}\int\prod_{i=1}^{3} dg_i\,\frac{|\Phi^{\frac{1}{2}}_{m,l}(g_1,g_2,g_3)|^2}{2} \\
&+ \frac{\lambda'}{4!}\int\prod_{i=1}^{6} dg_i\,\frac{\phi^{\frac{1}{2},\frac{1}{2}}_{m_1,l_1,m_2,l_2}}{4}\Phi^{\frac{1}{2}}_{m_1,l_1}(g_1,g_2,g_3)\Phi^{\frac{1}{2}}_{m_2,l_2}(g_3,g_4,g_5) \\
&\qquad \phi(g_5,g_2,g_6)\phi(g_6,g_4,g_1) \\
&+ \frac{\lambda'}{4!}\int\prod_{i=1}^{6} dg_i\,\frac{\phi^{\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}}_{m_1,l_1,\ldots,m_4,l_4}}{16}\Phi^{\frac{1}{2}}_{m_1,l_1}(g_1,g_2,g_3)\Phi^{\frac{1}{2}}_{m_2,l_2}(g_3,g_4,g_5) \\
&\qquad \Phi^{\frac{1}{2}}_{m_3,l_3}(g_5,g_2,g_6)\Phi^{\frac{1}{2}}_{m_4,l_4}(g_6,g_4,g_1), \qquad (19)
\end{aligned}
$$

As usual, sum over the repeated indices are assumed. The first term in $S[\phi,\Phi^{\frac{1}{2}}]$ is the kinetic term for $\Phi^{\frac{1}{2}}_{m,l}$ and the second term represents how the field $\Phi^{\frac{1}{2}}_{m,l}$ interacts with the gravitational field ϕ. The vertex contribution of this interaction comes from the coupling constant $\frac{\lambda'}{4!}\frac{\phi^{\frac{1}{2},\frac{1}{2}}_{m_1,l_1,m_2,l_2}}{4}$. The last term contains interactions among $\Phi^{\frac{1}{2}}_{m,l}$ fields.
Now one can interpret the fields in (**??**) and (**??**) as gravitational and matter like fields, and

take $\Phi^{\frac{1}{2}}_{m,l}$ as fermionic like fields[14]. By this interpretation, there are three interaction terms in (**??**). The first one belongs to self interaction of pure gravitational fields, the second one is interaction of two fermionic like fields with two pure gravitational fields, and the last one is the interaction of four fermionic like fields. If one let areas become larger, then she/he can include other terms with the js greater than $j = \frac{1}{2}$ in $S[\phi]$. For example, keeping the $j = 1$ terms, the action will become

$$S[\phi, \Phi] = S_{3d-pg}[\phi] + S[\phi, \Phi^{\frac{1}{2}}] + S[\phi, \Phi^{\frac{1}{2}}, \Phi^1], \tag{20}$$

where

$$\begin{aligned} S[\phi, \Phi^{\frac{1}{2}}, \Phi^1] &= \frac{1}{2}\int \prod_{i=1}^{3} dg_i \, \frac{|\Phi^1_{m,l}(g_1, g_2, g_3)|^2}{3} \\ &+ \frac{\lambda'}{4!}\int \prod_{i=1}^{6} dg_i \, \frac{\phi^{1,1}_{m_1,l_1,m_2,l_2}}{9} \Phi^1_{m_1,l_1}(g_1, g_2, g_3) \\ &\qquad \Phi^1_{m_2,l_2}(g_3, g_4, g_5)\phi(g_5, g_2, g_6)\phi(g_6, g_4, g_1) \\ &+ \frac{\lambda'}{4!}\int \prod_{i=1}^{6} dg_i \, \frac{\phi^{\frac{1}{2},\frac{1}{2},1}_{m_1,l_1,\ldots,m_3,l_3}}{12} \Phi^{\frac{1}{2}}_{m_1,l_1}(g_1, g_2, g_3) \\ &\qquad \Phi^{\frac{1}{2}}_{m_2,l_2}(g_3, g_4, g_5)\Phi^1_{m_3,l_3}(g_5, g_2, g_6)\phi(g_6, g_4, g_1) \\ &+ \frac{\lambda'}{4!}\int \prod_{i=1}^{6} dg_i \, \frac{\phi^{1,1,1}_{m_1,l_1,\ldots,m_3,l_3}}{27} \Phi^1_{m_1,l_1}(g_1, g_2, g_3) \\ &\qquad \Phi^1_{m_2,l_2}(g_3, g_4, g_5)\Phi^1_{m_3,l_3}(g_5, g_2, g_6)\phi(g_6, g_4, g_1) \\ &+ \frac{\lambda'}{4!}\int \prod_{i=1}^{6} dg_i \, \frac{\phi^{1,1,1,1}_{m_1,l_1,\ldots,m_4,l_4}}{81} \Phi^1_{m_1,l_1}(g_1, g_2, g_3) \\ &\qquad \Phi^1_{m_2,l_2}(g_3, g_4, g_5)\Phi^1_{m_3,l_3}(g_5, g_2, g_6)\Phi^1_{m_4,l_4}(g_6, g_4, g_1). \end{aligned} \tag{21}$$

This action contains the new fields with new interactions. According to [**?**] one may interpret the fields $\Phi^{\frac{1}{2}}, \Phi^1, ..., \Phi^j$ as different matter fields.
If one takes Φ^1 as a spin 1 like field , the third term in (**??**) is the interaction between two spin $\frac{1}{2}$ like fields and one spin 1 like field; which can be interpreted as the QED interaction in ordinary QFT. Now as an option, one may extend the lessons of this calculation to a general picture of higher-j matter fields and their interaction with the 3d gravitational field. If one starts with a pure gravity in 3 dimensional space and let these dimensions grow up -which is equivalent to appearing the new matter fields in appropriate manner- the new 4 dimensional space which contains a pure 4 dimensional gravity will emerge. In other words, by cooling the 3 dimensional space, many kinds of particles will appear in this space, and finally the appearance of the 4th dimension can be observed. In this new-born space, only pure 4 dimensional gravity exist.

[14] This interpretation can be found in details in [**?**]

III.1 Further achievements

As a toy model, one can easily show that by applying this strategy to any dimensions and use SU(2) instead of SO(d), similar results can be achieved. For example, one can start with GFT in 5 dimensions [?, ?], and assume similar symmetrries for the field; thus can write action as of the following:

$$\begin{aligned} S[\phi] &= \frac{1}{2}\int\prod_{i=1}^{5} dg_i\, |\phi(g_1,g_2,g_3,g_4,g_5)|^2 \\ &+ \frac{\lambda}{6!}\int\prod_{i=1}^{15} dg_i\, \phi(g_1,g_2,g_3,g_4,g_5)\phi(g_5,g_6,g_7,g_8,g_9) \\ &\qquad \phi(g_9,g_4,g_{10},g_{11},g_{12})\phi(g_{12},g_8,g_3,g_{13},g_{14}) \\ &\qquad \phi(g_{14},g_{11},g_7,g_2,g_{15})\phi(g_{15},g_{13},g_{10},g_6,g_1). \end{aligned} \tag{22}$$

By expanding the field ϕ similar to the expansion in (??) and rewriting the action, one can get the following statement for the action.

$$\begin{aligned} S[\Phi] &= \frac{1}{2}\int\prod_{i=1}^{4} dg_i \sum_j \frac{|\Phi^j_{m,l}(g_1,g_2,g_3,g_4)|^2}{2j+1} \\ &+ \frac{\lambda'}{5!}\int\prod_{i=1}^{10} dg_i \sum_{j_1,\ldots,j_5} \frac{\phi^{j_1,j_2,j_3,j_4,j_5}_{m_1,l_1,\ldots,m_5,l_5}}{(2j_1+1)\ldots(2j_5+1)}\Phi^{j_1}_{m_1,l_1}(g_1,g_2,g_3,g_4) \\ &\qquad \Phi^{j_2}_{m_2,l_2}(g_4,g_5,g_6,g_7)\Phi^{j_3}_{m_3,l_3}(g_7,g_3,g_8,g_9) \\ &\qquad \Phi^{j_4}_{m_4,l_4}(g_9,g_6,g_2,g_{10})\Phi^{j_5}_{m_5,l_5}(g_{10},g_8,g_5,g_1). \end{aligned} \tag{23}$$

Again, by applying the perturbation approach, and keeping the $j=0$ term, the TOCY model[15] will be obtained. Considering the next term, which is $j=\frac{1}{2}$, one can write the five-dimensional action as:

$$S[\phi,\Phi] = S_{4d-pg}[\phi] + S[\phi,\Phi^{\frac{1}{2}}], \tag{24}$$

where $S_{4d-pg}[\phi]$ is pure gravity action in 4 dimensions, and

$$\begin{aligned} S[\phi,\Phi^{\frac{1}{2}}] &= \frac{1}{2}\int\prod_{i=1}^{4} dg_i\, \frac{|\Phi^{\frac{1}{2}}_{m,l}(g_1,g_2,g_3,g_4)|^2}{2} \\ &+ \frac{\lambda'}{5!}\int\prod_{i=1}^{10} dg_i\, \frac{\phi^{\frac{1}{2},\frac{1}{2}}_{m_1,l_1,m_2,l_2}}{4}\Phi^{\frac{1}{2}}_{m_1,l_1}(g_1,g_2,g_3,g_4)\Phi^{\frac{1}{2}}_{m_2,l_2}(g_4,g_5,g_6,g_7) \\ &\qquad \phi(g_7,g_3,g_8,g_9)\phi(g_9,g_6,g_2,g_{10})\phi(g_{10},g_8,g_5,g_1) \\ &+ \frac{\lambda'}{5!}\int\prod_{i=1}^{10} dg_i\, \frac{\phi^{\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}}_{m_1,l_1,\ldots,m_4,l_4}}{16}\Phi^{\frac{1}{2}}_{m_1,l_1}(g_1,g_2,g_3,g_4)\Phi^{\frac{1}{2}}_{m_2,l_2}(g_4,g_5,g_6,g_7) \\ &\qquad \Phi^{\frac{1}{2}}_{m_3,l_3}(g_7,g_3,g_8,g_9)\Phi^{\frac{1}{2}}_{m_4,l_4}(g_9,g_6,g_2,g_{10})\phi(g_{10},g_8,g_5,g_1). \end{aligned} \tag{25}$$

[15] Since SU(2) is the symmetry group in this study, one can easily show $\Phi^0_{0,0}$ has those symmetries that the field in TOCY model has.

Finally, one can keep other fields, and add them order by order; threfore, these fields can be interpreted as new non-gravitational fields.

IV Conclusions

The lesson that can be learnt from the above calculations is that if one start with a pure gravity in 3 dimensional space and let these dimensions grow up, which is equivalent to appearing the new matter fields in appropriate manner, then what will emerge is the new 4 dimensional space which contains a pure 4 dimensional gravity. In other words by cooling the 3 dimensional space, many kinds of particles will appear in this space, and finally the appearance of the 4th dimension can be observed. In this new-born space there exist only pure 4 dimensional gravity. Actually this is a scenario to show how the new dimensions and matter (or even dark energy) will appear and extend from pure gravity.

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