Ohm's law: Fundamental mistakes made by authors and teachers

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Abstract. Ohm's law has been stated and re-stated by many a physicist and physics teachers in a variety of ways, the most usual being either of the following. *...keeping all physical conditions constant the* ***current flowing through a conductor is directly proportional to the potential difference applied across its end*** OR *.... keeping all physical conditions constant* ***potential difference across the conductor is directly proportional to the current flowing through it****.*
In this paper I will show that both are equivalent but require different ways to experiment with and understand.

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1. Introduction

George Simon Ohm gave the famous law which is named after him and is known as Ohm's law. The main essence of this law is that if we can keep all physical conditions constant then the current in a conductor has a direct relation with the voltage across the conductor. Famously $\frac{V}{I} = R$. Here V is the potential difference, I is current and R is resistance. The major problem that plagues Ohm's law is its inherent understanding and how teachers and authors put it down for students. Let us look at the two facets of the same coin.

2. Resistance: A brief introduction

In electricity resistance plays a very important role in the designing and operation of circuits. Generally resistance is an inherent property of all materials due to which they offer resistance to the flow of electrons. Current density of a conductor is given as

$$J = \sigma E \tag{1}$$

where J is current density, σ is conductivity and E is the electric field. Also we know that

$$V = E.l \tag{2}$$

where V = potential difference and l = length of the conductor. (The above equation is a simplified version.) We also know that under simple conditions

$$J = \frac{I}{A} \tag{3}$$

where I is current flowing in the conductor and A is the area of the conductor. Combining the above equations we get

$$V = \frac{Il}{\sigma A} \tag{4}$$

Now if we invoke the Ohm's law equation

$$\frac{V}{I} = R \tag{5}$$

we get

$$\frac{V}{I} = R = \frac{l}{\sigma A} = \frac{\rho l}{A} \tag{6}$$

where ρ is the inverse of σ and is known as resistivity.

3. The problem

The major problem is neither with the empirical treatment of Ohm's law neither into the equations related to the calculation of resistance. The main problem is the representation and interpretation of Ohm's law and the teaching process which gets flawed at times due to the underlying fundamental differences between the two generally used methods by authors and teachers.

3.1. ***Method - 1***

3.1.1. ***Definition :*** In this case the definition goes like this. *Keeping all the physical conditions constant, the* ***current flowing through a conductor is directly proportional to the potential difference applied across its ends***.

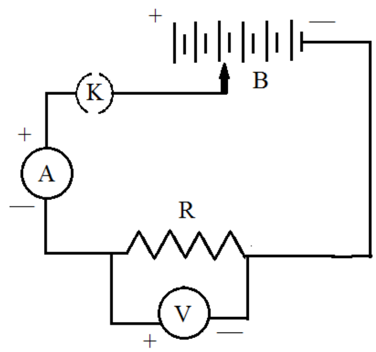

Figure 1

Let us now analyze the circuit and Ohm's law in detail related to the above circuit and also let us see how we should go ahead and measure resistance of the given conductor using the above circuit. In the above circuit there are a few cells arranged together to form a battery. Instead of this arrangement a variable DC power source can also be used like a battery eliminator. The range of a normal battery eliminator is between 0 V to 12 V which is sufficient for this experiment. The observation table for this experiment is given below.

Table 1: Ohm's law observation table

S.No.	Voltage(V)	Current (I)	Resistance $\frac{V}{I}$
1	2		
2	4		
3	6		
4	8		
5	10		

3.1.2. ***Procedure :***

(i) Keep the input voltage at 0 V and verify that the voltmeter and ammeter readings are at 0 V and 0 A each.

(ii) Turn the knob of the battery eliminator (B) to 2 V.

(iii) Take the reading of the ammeter (A) and note it down in the table under (I).

(iv) Increase the voltage of the battery eliminator by 2 V each and repeat step 3.

(v) Calculate the resistance in each case.

3.1.3. ***Analysis :*** Now let us analyze the above circuit. In the above circuit what we have done is that we have continuously changed the **potential difference** across the resistance (R). By doing this we have changed the current that was flowing through the resistance. In this case the point to be noted is ***that the voltage was changed and hence the current in the circuit changed. Hence, the current is dependent on the change of the voltage***. The mathematical formulation for this circuit and way of finding Ohm's law is

$$I \propto V \tag{7}$$

$$I = \frac{V}{R} \tag{8}$$

3.2. ***Method - 2***

3.2.1. ***Definition :*** In this case the definition goes like this.
Keeping all the physical conditions constant, the ***potential difference across a conductor is directly proportional to the current flowing through it****.*

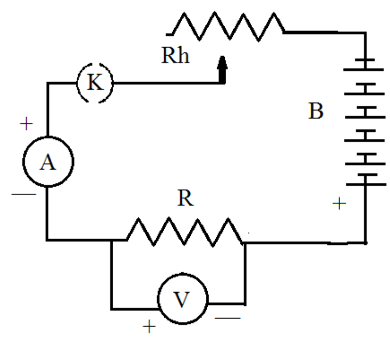

Figure 2

Table 2: Ohm's law observation table
Constant Voltage :

S.No.	Current (I)	Voltage (V)
1		
2		
3		
4		
5		

3.2.2. ***Procedure :***

(i) Keep the input voltage constant at 4 - 8 V.
(ii) Use the rheostat (Rh) to slowly increase the current to the desired value.
(iii) Take the reading of the ammeter (A) and the corresponding voltage reading in the voltmeter (V) and note it down in the table.
(iv) Increase the current in the circuit by a suitable step which will depend on the ammeter specs and repeat step 3.
(v) Make a graph of V vs I.
(vi) Take the slope of the graph and you will get the resistance (R).

3.2.3. ***Analysis :*** Let us analyse the circuit. The major difference in the circuit of Figure 2 is the addition of a rheostat. The rheostat is a device which ***changes the overall resistance of the circuit thereby changing the current in the circuit and hence the potential difference across the resistance***. This leads to a fundamental difference between Method 1 and Method 2. In Method-1 the voltage across the resistance R was being changed directly due to change in input voltage by the battery eliminator and hence the current was changing. In Method-2 the overall current in the circuit was being changed (remember the input voltage is constant) by the rheostat and hence the potential drop ($V = IR$) across the resistance was being changed.The mathematical formulation for this circuit and way of finding Ohm's law is

$$V \propto I \tag{9}$$

$$V = IR \tag{10}$$

4. A close look

To a naive looker the above mathematical equations 7 and 9 may look the same but there is a fundamental difference between the two. In the first case the current is dependent on the voltage whereas in the second case the voltage is dependent on the current. The main reason being that the underlying mechanism of electricity in the circuit is different. In both the cases the external resistance R remains the same. In the first case the input voltage keeps changing thereby changing the current in the circuit. In the second case the input voltage remains constant and the rheostat keeps changing the overall resistance of the circuit thereby changing the current in the circuit. This changing current changes the potential drop V across the resistance R by the value $V = IR$ which is measured by the voltmeter. Hence the two cases are fundamentally different. One of the major mistake most authors and teachers make is that they write $V \propto I$ and $I \propto V$ interchangeably. Let us look in detail why the two methods are fundamentally different and why caution is required while teaching or authoring.

4.1. ***Calculation based analysis***

Let us understand both the scenarios using some mathematical calculations.

4.1.1. ***Considering Method-1 :*** In Method-1 we know that the input voltage is changed from 0 to 12 V DC. Let us assume that the resistance coil has a resistance of 10Ω. Keeping the above in mind the calculations will be as follows.

Table 3: Calculation for Ohm's law based on Method-1

S.No.	Voltage (V)	Current (I)	Resistance ($\frac{V}{I}$)
1	2	0.2	10
2	4	0.4	10
3	6	0.6	10
4	8	0.8	10
5	10	1.0	10

What happened here is as follows. As the input voltage is increased the resistance of the circuit (10Ω) helps create a current in the circuit which is given as $I = \frac{V}{R}$. Since the potential drop across the resistance is $V = IR$ hence we get $V = IR = \frac{V}{R} \times R = V$. That is precisely what is measured by the voltmeter across the resistance shown in Figure 1.

4.1.2. ***Considering Method-2 :*** In Method-2 we know that the input voltage is constant, let us assume at 10 V(for sake of easier calculation). Let us assume that the resistance coil has a resistance of 10Ω. Now the new addition in this circuit is a Rheostat which normally has a resistance range of $0 - 100\Omega$. Keeping the above in mind the calculations will be as follows.

Table 4: Calculation for Ohm's law based on Method-2
Constant Voltage = 10 V

S.No.	Rheostat Value (Ω)	Current(I)	Voltage (V)	Resistance ($\frac{V}{I}$)
1	10	0.5	5.0	10
2	20	0.33	3.3	10
3	30	0.25	2.5	10
4	40	0.2	2.0	10
5	50	0.16	1.6	10

What happened here is as follows.
When rheostat is at 10Ω the total resistance of the circuit is $10 + 10 = 20\Omega$. hence the current drawn $I = \frac{V}{R} = \frac{10}{20} = 0.5A$. Since current in a series combination of resistance remains same hence the potential drop across the resistance is $V = IR = 0.5 \times 10 = 5V$. For rheostat resistance at 20Ω we get $R = 20 + 10 = 30\Omega$ which translates into $I = \frac{1}{3}A$ and $V = 3.3V$ which makes $R = 10\Omega$. In similar fashion the rest of the values can be calculated and each time the resistance comes out to be 10Ω which is exactly the same as the one obtained in Method-1 and also equal to the value of resistance that we have taken.

5. Conclusion

We can see from the previous section that both methods are equivalent but the difference lies in the fundamental aspect. In first method the current through the circuit is dependent on the voltage applied. In the second method the potential drop across the resistance is dependent on the change of current in the circuit which is brought about by the resistance change by the rheostat. Hence, though both the methods are correct it is better to explain the above procedure in advance to the students so as to avoid any confusion.