A Theory of Quantum Preparation

F Herbut

Serbian Academy of Sciences and Arts, Knez Mihajlova 35, 11000 Belgrade, Serbia

E-mail: fedorh@sanu.ac.rs

Abstract

Based on an analysis of two conventional preparators, the Stern-Gerlach and the hole-in-the-screen ones, it is argued that four entities can be taken as the basic ingredients of a rather general theory of a quantum preparator. These are the composite-system (object plus preparator) state coming about as a result of a suitable interaction between the subsystems, a suitable preparator projector called the triggering event, the conditional quantum state (density operator) of the quantum object coming about as a consequence of the occurrence of the triggering event on the preparator, and, finally, a unitary evolution operator of the object subsystem acting after preparation. The concepts of a general conditional state and of retrospective apparent ideal occurrence (which appears in the theory) are discussed in considerable detail. Ideal occurrence and the selective Lüders formula, which are made use of, are reviewed. Dynamical and geometrical preparators are distinguished in the general theory. They are described by the same entities in the same way, but in terms of different physical mechanisms from the point of view of standard interpretation with collapse.

PACS numbers: 03.65.-w, 03.65.Ca, 03.65.Ta

Key words: Experiment, conditional state, retrospective apparent ideal occurrence, puzzles

1 Introduction

Quantum physics is concerned with quantum experiments. These have three parts: preparation, (time) evolution, and measurement. The *first motivation* for this article is to present a quantum-mechanical understanding of the first part in a class of experiments because it is by far less elaborated in the literature than the measurement at the end of the experiment. The *second motivation* is an attempt to find out why has preparation received so little attention in the literature.

It will turn out that there are two kinds of preparations: dynamical and geometrical ones. It is intended to treat both kinds by the same formalism in order to keep preparation theory as simple as possible. *This goal* is achieved at the expense of introducing a fictitious event in preparation. It is fictitious as far as the standard quantum-mechanical concept of occurrence of an event is concerned. But it will be seen, as a rule, to be quite real concerning classical intuition.

In the present study a central role is played by an event (projector) Q on the preparator, called 'triggering' event. Hence, it must be clarified what *occurrence* of an event means in the standard Bohrian Copenhagen interpretation that pervades the textbooks of quantum mechanics. Let us be reminded of a known definition by Bohr himself [1]

> "As a more appropriate way of expression I advocate the application of the word *phenomenon* exclusively to refer to the observations obtained under specified circumstances, including an account of the whole experimental arrangement.(The italics are Bohr's.)

I think that Bohr means by "phenomenon" a *real phenomenon*, i. e., that this is where 'reality', i. e., real occurrence, enters the scene in the view of Bohr. In the textbooks one says that an event takes place in quantum mechanics if an event is measured by some kind of classical experimental arrangement.

2 Stern-Gerlach Preparators

To begin with, let us sum up some of the familiar aspects of Stern-Gerlach (SG) spin-projection measurement in order to single out those of its features that are relevant for a theory of a quantum preparator.

2.1 Stern-Gerlach measurement

We assume that it is the z-projection of the spin of a spin-one-half particle that is measured. The incoming particle is in the uncorrelated pure state given by a state vector of the form

$$\mid\Psi\rangle_{I,II} \equiv \Big(\alpha \mid +, z\rangle_I + \beta \mid -, z\rangle_I\Big) \mid \psi^{in}\rangle_{II}.$$

Here $\alpha, \beta \in \mathbf{C},\ |\alpha|^2 + |\beta|^2 = 1$. The first subsystem is that of spin one-half and the second one consists of the spatial degrees of freedom of the particle. (Note that quantum-mechanically subsystems need not be material ones; they can be degrees of freedom as in this case.) Further, $\mid \pm, z\rangle_I$ are the spin-up and spin-down (along z) state vectors. Finally, $\mid \psi^{in}_{II}\rangle$ is the incoming spatial state vector of the particle.

As well known, the magnetic field couples the z-projection of spin with the spatial degrees of freedom of the outgoing particle (leaving the field and reaching the plates) as follows. Let us denote by $|\psi^+\rangle_{II}$ the upward moving particle state (which reaches the upper plate), and by $|\psi^-\rangle_{II}$ the downward moving particle state (reaching the lower plate). Then

$$\mid\Phi\rangle_{I,II} \equiv \alpha \mid +, z\rangle_I \mid \psi^+\rangle_{II} + \beta \mid -, z\rangle_I \mid \psi^-\rangle_{II} \tag{1}$$

is the composite-system state after the coupling.

Let us introduce the projectors

$$Q^+_{II} \equiv \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{0}^{+\infty} \mid x, y, z\rangle_{II}\langle x, y, z\mid_{II} \, dxdydz, \tag{2a}$$

$$Q^-_{II} \equiv \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{0} \mid x, y, z\rangle_{II}\langle x, y, z\mid_{II} \, dxdydz \tag{2b}$$

projecting onto the upper and the lower half-spaces respectively.

One should note that

$$Q^-_{II} = I_{II} - Q^+_{II} \equiv (Q^+_{II})^\perp,$$

I_{II} being the identity operator for the spatial subsystem. Thus, Q^-_{II} is determined by (2a).

Ideal occurrence of the second-subsystem event $(I_I \otimes Q^+_{II})$ (usually written simply as Q^+_{II}) in the composite-system state $\mid \Phi\rangle_{I,II}$ given by (1) brings the entire system, as it is known, into the state

$$cQ^+_{II} \mid\Phi\rangle_{I,II} = \mid +, z\rangle_I \mid \psi^+\rangle_{II},$$

where c is the corresponding (positive) normalization constant. Namely, according to the selective Lüders formula [2], [3], [4], one applies the projector onto the state vector and one renormalizes the result. By this, the spin subsystem is brought into the state

$$|+, z\rangle_I. \tag{3}$$

The SG measuring apparatus in its standard form performs retrospective or second-kind measurement when the particle is stopped on one of the plates. This causes drastic change in the spatial state of the particle (spot on the plate) and a corresponding change in the state of the measuring instrument.

In spite of the above usual simple theory, the SG measurement is actually so-called distant measurement [5]: the spin projections and 'being in the upper or lower half-space' are so-called twin observables in (1). Interaction takes place only between the measuring instrument and the spatial degrees of freedom (subsystem II) of the particle leading to direct measurement of the latter; but this is then, by this very act, an interaction-free measurement, called distant measurement, of the spin projection on account of the strong correlations that give rise to the twin observables in (1). We do not need to go into the niceties of this conceptually intricate measuring process. (More details can be found in [5], [6], as well as in [7] and in the references in it.)

To obtain a preparator, modification is required.

2.2 The first modification for a SG preparator

Let us imagine the following modification of the SG measuring arrangement. The upper plate is replaced by a detector that detects the arrival of the particle via spatial interaction only; and it lets the particle continue its movement to the right out of the arrangement. The point is that the measurement affects only the spatial degree of freedom of the particle, and not at all its spin projection. Therefore, it can be considered a spin-projection measurement, in which the spin-up event occurs.

In the described measurement the particle is detected at the place of the upper plate, and the purely spatial interaction changes the corresponding component state $|\psi^+\rangle_{II}$ in (1) into an outgoing spatial state with the spin state unchanged:

$$|\Phi\rangle_{I,II} \quad \rightarrow \quad |+,z\rangle_I \, |\psi^{out}\rangle_{II}. \tag{4}$$

One can evaluate the spin state as a conditional state from the composite state (1) with the condition Q_{II}^+ (cf (2a)) by *the general conditional-state formula*

$$\rho_I \equiv \mathrm{tr}_{II}\Big(Q_{II}^+\rho_{I,II}\Big)\Big/\mathrm{tr}\Big(Q_{II}^+\rho_{I,II}\Big) = \mathrm{tr}_{II}\Big(Q_{II}^+ \, |\Phi\rangle_{I,II}\langle\Phi|_{I,II} \Big)\Big/(1/2), \tag{5}$$

where tr_{II} denotes the partial trace over the second subsystem (the spatial tensor factor space in this case), and the total trace ($\mathrm{tr}_{I,II}$ in this case) is written throughout without subscripts. Substituting the bipartite state vector from (1) then gives in a straightforward manner

$$\rho_I = |+,z\rangle_I\langle +,z|_I$$

in full agreement with (4).

The spin projection further does not change until it is measured at t_f , the final moment of the experiment.

One is again dealing with distant measurement in this preparation. (More about distant measurement is given in section 6.)

2.3 The second modification for a SG preparator

The upper plate is removed. The particle that would hit the upper plate in the standard SG instrument may now freely leave the instrument. The lower plate is in place or, perhaps, it is also removed and replaced by some other (more suitable) particle detector.

We want a so-called *negative measurement*: it consists in the anti-coincidence of arrival of the particle on the plates and of non-detection on the lower plate. This amounts to *ideal occurrence* of 'arrival of the particle in the upper half space', i. e., the occurrence of the event Q_{II}^+ (cf (2a)).

One should note that Q_{II}^+ really happens (in contrast to fictitious events introduced below). The conditional-state evaluation (5) is again applicable. (More about negative measurement can be found in section 6.)

The described anti-coincidence is hard to achieve in the laboratory because it is not easy to make certain when the particle is supposed to arrive at the plates. Nevertheless it is possible in principle. Next, we design a more realistic preparator.

2.4 The third modification for a SG preparator

We remove the upper plate, and leave the lower one in place. The *geometry* is such that it makes it possible to confine our interest to the upper half-space, where, further to the right, we put the measuring apparatus. If it measures anything on the particle (at the final moment t_f of the experiment), and one obtains a result, then, due to the geometry, the particle arriving at the detector must be in the upper half-space. Therefore, it must be in the state

$$|+,z\rangle_I \Big(U_{II}(t_f - t_i, t_i)\, |\psi^+\rangle_{II} \Big),$$

where $U_{II}(...)$ is a unitary evolution operator in the state space of the second (spatial) subsystem (the spin does not change), and t_i is throughout this study the initial moment of the experiment (and the final moment of preparation if it is not instantaneous).

This amounts to the same as if we had occurrence of the event Q_{II}^+ (cf (2a)) at t_i in the state $|\Phi\rangle_{I,II}$ (and subsequent evolution). This fictitious occurrence will be explained in detail in section 5.

We have obtained *three basic entities* for a *preparator theory*:

$$|\Phi\rangle_{I,II} \qquad Q_{II}^+ \qquad |+,z\rangle_I \qquad (6a,b,c)$$

(cf (1), (2a) and (3) respectively). We call Q_{II}^+ *the triggering event* as the event the occurrence of which brings about the (conditional) prepared subsystem state $|+,z\rangle_I$ (via (5)).

The SG composite-system state $|\Phi\rangle_{I,II}$ given by (1) is very simple. In particular, the action of the projector Q_{II}^+ on this state leaves the object in the pure subsystem state $|+,z\rangle_I$. To check if (6a,b,c) are the relevant entities for preparation also in other cases, we take another, a quite different and very well known example.

3 Preparation Through a Hole in the Screen

In one-hole preparation the first subsystem is the particle, the second is the screen. We think of the screen as of an infinite surface perpendicular to the motion of the incoming particle.

The screen is thought of as broken up into two non-overlapping segments: the hole is one of them (segment h) and the rest of the screen is the other (segment rs). Hitting the latter, i.e., transfer of linear momentum at this segment, corresponds to the occurrence of, say, the event (projector) Q_{II}^{rs} .

Let us think of a *negative measurement* consisting in the arrival of the particle at the screen and the non-occurrence of Q_{II}^{rs} . Then the particle passes the hole, and one has *ideal occurrence* of the opposite event

$$Q_{II}^{h} = I_{II} - Q_{II}^{rs} \qquad (7)$$

on the screen.

For simplicity, we assume that the composite-system at the end of the preparation interaction is in a pure state $\mid \Phi, t_i \rangle_{I,II}$. The event Q_{II}^{h} is *the triggering event*. The state of the particle when the preparation is completed, i. e., *the prepared state*, is due to the *ideal occurrence* of the triggering event. It is given by taking the reduced density operator (the subsystem state) after projection and renormalization (the selective Lüders formula in the pure-state case):

$$\rho_I^h(t_i) \equiv \mathrm{tr}_{II}\Big[\Big(c' Q_{II}^h \mid \Phi, t_i\rangle_{I,II}\Big)\Big(c'\langle \Phi, t_i \mid_{I,II} Q_{II}^h\Big)\Big], \qquad (8a)$$

where c' is the (positive) normalization constant. It is, of course, assumed that $Q_{II}^{h} \mid \Phi, t_i\rangle_{I,II} \neq 0$, i.e., that the process considered allows passage through the hole with positive probability. The reduced density operator given by (8a) is more often written in the simpler and more explicit form:

$$\rho_I^h(t_i) = \mathrm{tr}_{II}\Big((\mid \Phi, t_i\rangle_{I,II}\langle \Phi, t_i \mid_{I,II})Q_{II}^h\Big)\Big/\Big[\mathrm{tr}\Big((\mid \Phi, t_i\rangle_{I,II}\langle \Phi, t_i \mid_{I,II})Q_{II}^h\Big)\Big]. \qquad (8b)$$

(This is possible due to the commutation of the operator Q_{II}^{h} with the other operator $\mid \Phi, t_i \rangle_{I,II}\langle \Phi, t_i \mid_{I,II} Q_{II}^{h}$ under the partial trace in (8a), and due to idempotency of the former operator. It is easy to prove that the subsystem operator Q_{II}^{h} has the stated commutation property under the partial trace over the same subsystem II just like it is usual under a full trace.)

One can see by comparing (8b) with (5) that the prepared state $\rho_I^h(t_i)$ in this case is determined by the composite-system state and the triggering event *in the same way* as in the case of the SG device.

In analogy with the modifications of the SG device (in the preceding section), we can think of modifications of the hole-preparator. We give just one of them.

Let us take a *geometrical* modification, in which no real occurrence of event takes place in the preparation. The geometry is such that if anything is measured on the particle to the right of the screen at t_f , the former must have passed the hole, i.e., it is as if the triggering event had occurred at t_i . (This will be discussed in detail in section 5.)

We have now a good deal of inductive insight for a general quantum mechanical theory of preparation. Nevertheless, it is desirable to clarify several important points first.

4 General Conditional State

Let $\rho_{I,II}$ be an arbitrary given composite-system (mixed or pure) state (a density operator). Let, further, Q_{II} be a second-subsystem event (projector) and let it *occur in whatever way* in the state $\rho_{I,II}$. We want an answer to the question: In what state $\bar{\rho}_I$ leaves this occurrence the first subsystem? We had an answer for ideal occurrence via the Lüders formula in the preceding subsection (cf (8b)). Now we are interested in the answer in the case of a general occurrence of some event Q_{II}. The answer is known, but not well known.

The sought-for state (density operator) $\bar{\rho}_I$ gives probability prediction for an arbitrary first-subsystem event (projector) P_I through the quantum-mechanical probability formula in the trace-rule form $\mathrm{tr}\Big(\bar{\rho}_I P_I\Big)$, and, as it is known from the theorem of Gleason [8], $\bar{\rho}_I$ is, in its turn, *determined by the totality* of all possible projectors P_I via this same formula.

Since an *arbitrary* first-subsystem event P_I and the given event Q_{II} are compatible events (commuting projectors), their coincidence on the one hand and the occurrence of P_I *immediately after* that of Q_{II} on the other can be considered as one and the same thing. The coincidence probability can be written in a factorized form:

$$\mathrm{tr}\Big(\rho_{I,II}(P_I \otimes Q_{II})\Big) = \mathrm{tr}_I\Big[\Big(\mathrm{tr}_{II}(\rho_{I,II}Q_{II})\Big)P_I\Big] = \Big(\mathrm{tr}(\rho_{I,II}Q_{II})\Big)\Big(\mathrm{tr}(\bar{\rho}_I P_I)\Big), \tag{9}$$

where

$$\bar{\rho}_I \equiv \mathrm{tr}_{II}(\rho_{I,II}Q_{II})\Big/\Big(\mathrm{tr}(\rho_{I,II}Q_{II})\Big), \tag{10}$$

and $\mathrm{tr}\Big(\rho_{I,II}Q_{II}\Big)$ is the probability of the event Q_{II} in $\rho_{I,II}$. (Note that in the first equation in (9) use is made of the fact that P_I behaves under the opposite partial trace tr_{II} as a constant does under a full trace, i. e., it can be taken outside the partial trace. This is easily proved. Further, it is easily seen that $\bar{\rho}_I$ given by (10) is a density operator.)

Coincidence can be thought of as taking place in one measurement, hence (9) can be viewed classically, as the well-known conditional-probability formula. In particular, the second factor $\mathrm{tr}(\bar{\rho}_I P_I)$ in the third expression in (9) is then, by definition, the *conditional probability* of P_I in the state $\bar{\rho}_I$ in which the second subsystem is left (immediately) after Q_{II} (the condition) has occurred in $\rho_{I,II}$.

One should note that since P_I is an arbitrary event, one has $\bar{\rho}_I$ given by (10) is the sought-for expression for the *conditional state*. Thus, (10) *extends* the partial-trace evaluation in (8b) to the *general case of occurrence* of Q_{II} as a condition.

Now we can conclude, without discussion of more intricate examples of preparation, that we can abandon the above restrictions to pure states and ideal occurrence of the triggering event. In a *general theory of preparation* we have so far the following three crucial entities: the composite-system (object plus preparator) state $\rho_{I,II}(t_i)$ at the initial instant t_i of the experiment, a triggering event Q_{II} , and, finally, the (conditional) prepared state $\rho_I(t_i)$, which is determined by the preceding two entities (cf (10)).

5 Retroactive Apparent Ideal Occurrence

When there is no detector in the preparator, i.e., when it is no measurement at all (the third modification in the SG case and the modification in the hole preparator discussed above), the prepared state $\rho_I(t_i)$ given by (8b), e. g., has, nevertheless, the meaning of a conditional state, assuming validity under the fictitious condition that a triggering event Q_{II} occurs in the composite-system state $\rho_{I,II}$ at t_i.

In the cases at issue there is no actual occurrence of any event until t_f, when a measurement result is obtained. Then, owing to the *geometry* of the experiment, this amounts to the same, as it was claimed above, as if Q_{II} had occurred in $\rho_{I,II}$. More precise explanation is desirable.

Every measuring arrangement is located in some (spatial) region $\mathbf{R}$, and if a detection event, e. g. F, occurs in any kind of measurement with a positive probability, then one has $F \leq P(\mathbf{R})$ $\Big[\equiv \Big(F = FP(\mathbf{R}) \Big) \Big]$ meaning physically that, by *implication*, also $P(\mathbf{R})$ occurs, which, in turn, means physically that the measured quantum system is found in the region $\mathbf{R}$.

Lemma on Localization If the mentioned implication of events is valid, then

$$\mathrm{tr}(F\rho) = \Big[\mathrm{tr}\Big(P(\mathbf{R})\rho\Big)\Big]\Big[\mathrm{tr}\Big(F\rho'\Big)\Big],$$

where

$$\rho' \equiv P(\mathbf{R})\rho P(\mathbf{R}) \Big/ \mathrm{tr}\Big(P(\mathbf{R})\rho\Big).$$

In words, the probability of an event localized in $\mathbf{R}$ equals the product of the probability of localization and the probability of the same event in the state ρ', which is the collapsed state (evaluated via the selective Lüders formula) due to the occurrence of the localization event $P(\mathbf{R})$ in an ideal way.

Proof Utilizing the above implication, the projector idempotency and commutation under the trace, one has

$$\mathrm{tr}(F\rho) = \mathrm{tr}\Big(P(\mathbf{R}))F\rho\Big) = \mathrm{tr}\Big(FP(\mathbf{R})\rho P(\mathbf{R})\Big),$$

and from this the claimed relation immediately follows. □

We have in mind an experiment, in which an initial state $\rho(t_i)$ evolves, assuming the system is dynamically isolated from its environment in the interval from t_i till t_f, unitarily into the final state $\rho(t_f)$.

Now we introduce the notion of retroactive apparent ideal occurrence (RAIO).

Theorem on RAIO Let Q and P be two events (projectors) satisfying the following conditions:

(i) $0 < \mathrm{tr}\Big(Q\rho(t_i)\Big) < 1$, i. e., both Q and the opposite event $Q^\perp$ $\Big(\equiv (1-Q)\Big)$ can occur, i. e., have a positive probability, in $\rho(t_i)$,

(ii) the occurrence of Q in $\rho(t_i)$ in an ideal way makes P certain in $\rho(t_f)$, and also dually,

(iii) the occurrence of $Q^\perp$ in $\rho(t_i)$ in an ideal way makes $P^\perp$ certain in $\rho(t_f)$.

Then the following relation is satisfied.

$$P\rho(t_f)P/\Big[\mathrm{tr}\Big(P\rho(t_f)\Big)\Big] = U\Big\{Q\rho(t_i)Q/\Big[\mathrm{tr}\Big(Q\rho(t_i)\Big)\Big]\Big\}U^\dagger. \qquad (11)$$

In words, we obtain the same state if, on the one hand, the system evolves from t_i till t_f and then P occurs in an ideal way, and, on the other hand, when Q occurs in an ideal way in $\rho(t_i)$ and then the system evolves till t_f .

Proof is given in [9]. (The cited previous article of the present author is devoted to the, somewhat intricate, proof of this theorem, and to its applications among which also preparation is mentioned.)

If any measurement result is obtained *in whatever measurement* on the particle at t_f , this takes place in a certain spatial region $\mathbf{R}$, e. g., to the right of the screen in the hole-preparator example if the particle approaches the screen before t_i from the left. Hence, the mentioned result of measurement implies the occurrence of the event $P_I(\mathbf{R})$, by which it is meant that the particle is found in the mentioned region $\mathbf{R}$.

If the triggering event Q_{II} does occur in the composite-system state $\rho_{I,II}(t_i)$, then the event $P_I(\mathbf{R})$ is *certain* to occur in the state $\rho(t_f)$, the final moment of the experiment. This means that the particle must reach the region $\mathbf{R}$. Moreover, if the triggering event does not occur, i. e., if the opposite event $Q_{II}^\perp$ occurs, at t_i in $\rho_{I,II}$, then $P_I(\mathbf{R})$ will not occur, i.e., $[I_I - P_I(\mathbf{R})]$ will occur in $\rho(t_f)$ - the particle does not reach region $\mathbf{R}$.

Let us resort now to the special application of the, still general, formula (11).

The above Theorem on RAIO implies:

$$\rho_{I,II}(t_f)^{\mathbf{R}} = P_I(\mathbf{R})\Big[U_{I,II}\rho_{I,II}(t_i)U_{I,II}^\dagger\Big]P_I(\mathbf{R})\Big/\Big[\mathrm{tr}\Big(P_I(\mathbf{R})U_{I,II}\rho_{I,II}(t_i)U_{I,II}^\dagger\Big)\Big] =$$

$$U_{I,II}\Big\{Q_{II}\rho_{I,II}(t_i)Q_{II}\Big/\Big[\mathrm{tr}\Big(Q_{II}\rho_{I,II}(t_i)\Big)\Big]\Big\}U^\dagger_{I,II}. \tag{12}$$

This means that one would obtain *the same* localized final state $\rho_{I,II}(t_f)^{\mathbf{R}}$ if, on the one hand, the event $P_I(\mathbf{R})$ occurred ideally at t_f in the final state $\rho_{I,II}(t_f)$, i. e., if we restricted the final state to the spatial region $\mathbf{R}$, and, on the other hand, if the triggering event Q_{II} would occur ideally at t_i in the initial state $\rho_{I,II}(t_i)$ (which actually evolves into $\rho_{I,II}(t_f)$), and then the system evolved in the collapsed state till t_f . Naturally, the composite system system being at t_f , actually, in the state $\rho_{I,II}(t_f)$, it is found with probability $\mathrm{tr}\Big(\rho_{I,II}(t_f)P(\mathbf{R})\Big)$ in the spatial region $\mathbf{R}$.

If one utilizes the rhs of (12) instead of its lhs, then one says that one has *retroactive apparent ideal occurrence* (RAIO) of the event Q_{II} in $\rho_{I,II}(t_i)$. This is, according to (12), equivalent to the *actual ideal occurrence* of the event $P_I(\mathbf{R})$ in the final state $U_{I,II}\rho_{I,II}(t_i)U^\dagger_{I,II}$ at t_f . According to the above Lemma on Localization, this occurrence consists in restriction of the state to the region $\mathbf{R}$ with the corresponding probability.

We are interested in the first subsystem. At the initial moment t_i of the experiment it is, of course, in the state described by the reduced density operator:

$$\mathrm{tr}_{II}\Big(Q_{II}\rho_{I,II}(t_i)Q_{II}\Big)\Big/\mathrm{tr}\Big(Q_{II}\rho_{I,II}(t_i)\Big),$$

which equals $\rho_I(t_i)$ given by (8b) if one puts

$$\rho_{I,II}(t_i)\equiv\mid\Phi,t_i\rangle_{I,II}\langle\Phi,t_i\mid_{I,II}\ .$$

We make now another short digression.

6 Ideal Occurrence

Though the concept of *ideal occurrence* of an event (projector) F in a state (density operator) ρ , and the corresponding change of state obtained by application of the *selective Lüders formula*

$$\rho\quad\rightarrow\quad F\rho F/\Big(\mathrm{tr}(\rho F)\Big) \tag{13}$$

are known, but perhaps they are not sufficiently well known. Ideal occurrence plays an important role in this study. Therefore, it might be justified to make a few remarks about it.

In *direct interaction* between quantum system and measuring instrument ideal measurement of an event (projector) F *can never occur*. This is obvious from the fact that if the state is an 1-eigenstate of the event (the event has occurred, if it is a property, it is possessed), then $F\rho = \rho$ is valid. (This is the algebraic equivalent of the certainty formula $\mathrm{tr}(\rho F) = 1$. If unfamiliar with this equivalence, see proof in [9], Lemma A.4. in Appendix 2 there.) It has the consequence that, as (13) obviously implies, the state should *not change at all* in ideal occurrence. This is not possible, because direct interaction in quantum mechanics requires exchange of at least one quantum of action. Hence, it must result in change of state.

Ideal occurrence takes place in negative, in distant, and in implied measurement. In the above second modification of SG measurement we had an example of *negative measurement* and ideal occurrence (cf subsection 2.3).

An example of *distant measurement* appeared in the above first modification of the SG measurement (cf subsection 2.2). There direct measurement was performed on the spatial degree of freedom of the particle, and, by this very act, the spin projection was distantly measured. (The term "distantness", in analogy with two distant but entangled particles, is here used to emphasize that the measurement of spin projection is due exclusively to the suitable strong correlations between the spatial and the spin degrees of freedom, which are formally equal to the case of the two material subsystems.)

Finally, ideal occurrence in *implied measurement* we had in our, perhaps, most important formula (12) in the case of 'being found in the spatial region $\mathbf{R}$'.

7 Evolution After Preparation

Since the evolution operator used so far applies to the composite system, there is redundancy in it. We now eliminate this burden.

Utilizing the identity $I_{II} = Q_{II} + Q_{II}^{\perp}$, we can write

$$U_{I,II}\rho_{I,II}U_{I,II}^{\dagger} = U_{I,II}\Big(Q_{II} + Q_{II}^{\perp}\Big)\rho_{I,II}U_{I,II}^{\dagger}.$$

Further, the screen and the particle, e. g., do not interact any longer *if the latter has passed the hole* etc., hence the evolution operator $U_{I,II}$ *factorizes tensorically* into the evolution operator of the particle U_I and that of the screen U_{II} . More precisely,

$$U_{I,II}Q_{II} = \Big(U_I \otimes U_{II}\Big)Q_{II}, \tag{14}$$

and, owing to this, we can derive a simple form of the state of the particle at t_f in the region $\mathbf{R}$ (relations (16) and (17) below).

Some event (corresponding to the measurement result) occurs in the region $\mathbf{R}$. Since the measurement apparatus is in this region, the occurrence of this event implies, as it was explained above, the occurrence of the event $P_I(\mathbf{R})$. Since this event is not actually measured (only implied), we are justified, in accordance with the above Lemma on Localization, to assume that its occurrence takes place in the ideal way. Hence, we take the lhs of (12) as the relevant composite-system state, and we replace it by the rhs of (12) both in the case when the collapse (occurrence of Q_{II}) does take place at t_i and when it is only a retroactive apparent ideal occurrence. Then we have

$$\rho_I(t_f) = \mathrm{tr}_{II}\Big\{U_{I,II}\Big[Q_{II}\rho_{I,II}(t_i)Q_{II}\Big/\mathrm{tr}\Big(Q_{II}\rho_{I,II}(t_i)\Big)\Big]U_{I,II}^\dagger\Big\}. \tag{15}$$

Taking into account (14), one can write

$$Q_{II}U_{I,II}^\dagger = (U_{I,II}Q_{II})^\dagger = \Big((U_I \otimes U_{II})Q_{II}\Big)^\dagger.$$

Further, let us substitute the obtained expression in (15) to obtain

$$\rho_I(t_f) = \mathrm{tr}_{II}\Big\{\Big[\Big(U_I \otimes U_{II}\Big)Q_{II}\Big]\rho_{I,II}(t_i)\Big[\Big(U_I \otimes U_{II}\Big)Q_{II}\Big]^\dagger\Big/\mathrm{tr}\Big(Q_{II}\rho_{I,II}(t_i)\Big)\Big]\Big\}.$$

Finally, we can take U_I and $U_I^\dagger$ outside the partial trace and we can omit U_{II} and $U_{II}^\dagger$ under the partial trace (these are known partial-trace identities, which run parallel to those valid for full traces). Thus we obtain:

$$\rho_I(t_f) = U_I\rho_I(t_i)U_I^\dagger, \tag{16}$$

where

$$\rho_I(t_i) \equiv \mathrm{tr}_{II}\Big[Q_{II}\rho_{I,II}(t_i)Q_{II}\Big/\mathrm{tr}\Big(Q_{II}\rho_{I,II}(t_i)\Big)\Big]. \tag{17}$$

8 General Theory of Preparation

To begin with, it should be noted that actual occurrences of events (collapses) are not encompassed by the evolution operator. (This fact is known as the paradox of quantum measurement theory). Therefore, preparation cannot be described by unitary evolution all over. Occurrences of events must be separately included in the change of state in preparation.

In this section we use the same notation as in the preceding section. The two subsystems, subsystem I on which an experiment is performed and the preparator, subsystem II , interact and reach a *composite-system state* (density operator) $\rho_{I,II}(t_i)$. This is the first basic entity of the preparation theory. The second one is a *triggering event* (projector) Q_{II} on the preparator. The third basic entity is *the prepared state*, actually the conditional state $\rho_I(t_i)$ of the first subsystem to which the occurrence of the triggering event Q_{II} in the composite-system state $\rho_{I,II}(t_i)$ gives rise. The occurrence may take place in whatever way, i.e., it need not be ideal. The conditional state is given by (17).

Finally, there is an important fourth entity that belongs more to the rest of the experiment than to the preparation. But it is preparation that must provide the *separate evolution of the object subsystem*.

In this sense one has the fourth entity of preparation. It is the evolution operator $U_{I,II} \equiv U_{I,II}(t_f - t_i, t_i)$ with the important factorization property (14), which means lack of interaction between object and preparator in the interval from t_i till t_f after the triggering event has happened actually in whatever way or retroactively apparently in the ideal way.

As a matter of fact, for a given preparator it is only U_I , the evolution operator of the object, that must be known (cf (14)). As to U_{II} , the evolution operator of the preparator, it is sufficient to know that it enters the theory via (14). The concrete form of U_{II} is of no consequence for the experiment with subsystem I .

One should note that one assumes the validity of a unitary evolution (by $U_{I,II}(t_f - t_i, t_i)$) in the composite, quantum object plus preparator, system. This includes the object-preparator interaction, but excludes any interaction with the environment. The unitary operator part $U_I(t_f - t_i, t_i)$ (cf (14)) of $U_{I,II}$ implies that the object does not interact not only with the preparator, but also with the environment. Hence, if $\rho_I(t_i)$ contains coherence (the interference-creating property) it will be preserved up to the final moment in

$\rho_I(t_f) = U_I \rho_I(t_i) U_I^\dagger$. In other words, the everywhere lurking *decoherence*, the coherence-destroying interaction with the environment, is not operating because a quantum experiment is typically sufficiently dynamically isolated from the environment.

The proposed preparation theory includes only occurrence (any occurrence cf section 4) of events occurring on the preparator. One may wonder what if in some preparation events on the system or on the composite-system (system plus preparator) (or both) occur, and what if no event (actual or fictitious) occurs on the preparator.

Occurrence of possible events on the system or on the composite system in preparation have to be included in the first above preparation entity, the composite-system state $\rho_{I,II}(t_i)$. If no event (actual or fictitious) occurs on the preparator in preparation, then the third preparation entity, the prepared or conditional state is

$$\rho_I(t_i) \equiv \mathrm{tr}_{II}\Big(\rho_{I,II}(t_i)\Big) = \mathrm{tr}_{II}\Big(\rho_{I,II}(t_i) I_{II}\Big). \tag{18}$$

In words, the subsystem state (reduced density operator) is a special case of a conditional state with the condition being the certain event, expressed by the identity operator.

9 Puzzling Features

We must distinguish *two kinds of preparators*: the *dynamical* (or immediate-occurrence) ones, in which the triggering event does actually occur at t_i (due to a dynamical process, a measurement), and the *geometrical* (or delayed-occurrence) ones, in which a special geometry singles out a spatial region **R**, and some event (some measurement result) actually occurs on the object in **R** at the delayed moment t_f . (It is delayed as far as the preparation is concerned. It is the final moment of the experiment.) But, as explained in detail in section 5, this gives rise to the retroactive apparent ideal occurrence (RAIO) of a triggering event Q_{II} , and the entire theory has exactly the same form as for a dynamical preparator.

One should be aware of some *puzzling features* of the preparator theory presented.

(i) We have one and the same formalism, but two *different physical mechanisms*, i.e., the mentioned two kinds of preparators are equally described, but they are understood as different processes.

(ii) The concept of RAIO, which enables us to describe both kinds of preparators by the same formalism, is itself a puzzling one:

In our hole-in-the-screen example, intuitively we do feel that the particle must have passed the hole if it reaches region $\mathbf{R}$. But the conventional interpretation of quantum mechanics, with the idea of collapse that makes the events actually occur, seems to prove us wrong. Since there is no measurement at t_i , there is no collapse and no occurrence at that moment in actuality.

The composite system is decribed by $\rho_{I,II}(t_f)$ $\Big[\equiv U_{I,II}\rho_{I,II}(t_i)U_{I,II}^\dagger \Big]$ at t_f . This state may include, possibly in a coherent (i.e., interference-allowing) way, also the possibility that the hole is not passed in the described example. At the final moment t_f , and only then, something happens, some measurement result is obtained. From the very fact that this result is obtained in the region $\mathbf{R}$, we have the collapse described by the lhs of (12). It does imply the RAIO of the triggering event, the rhs of (12), but this is only formal (or apparent).

In the standard or conventional interpretation of quantum mechanics, which is utilized throughout, one does not search for the mechanism of the collapse that gives rise to the occurrence of some event. But, collapse is taken seriously: it is considered to be a *real, objectively happening physical process.* Hence, one is puzzled that lack of such occurrence, equally as occurrence, can lead to correct preparation of an experiment.

Besides the embarrassing appearance of the RAIO in preparation, there is also the well known conceptual collision of collapse with unitary evolution. In particular, if the latter would reign by itself, then the prepared state would always be $\mathrm{tr}_{II}\rho_{I,II}$, and not $\mathrm{tr}_{II}\Big\{Q_{II}\rho_{I,II}Q_{II}\Big/\Big[\mathrm{tr}\Big(\rho_{I,II}Q_{II}\Big)\Big]\Big\}$ (cf (12)) as in the expounded theory.

At the end of a quantum experiment one obtains a definite measurement result, though the theory predicts, as a rule, more than one result. It is believed to be due to collapse. This conceptual collision of collapse and unitary evolution is well known under the name of 'the paradox of quantum measurement theory'. Since we have such a conceptual collision also at the beginning of the experiment, in preparation, one should actually speak of *the* *paradox of the quantum experiment* with a double collision of the mentioned basic concepts.

It is important to note that the paradoxes are not due to the proposed formalism; they stem from the fact that there are two kinds of preparations that differ by occurrence and non-occurrence of the triggering event. (Perhaps unfortunately, the proposed theory may seem to sweep this distinction under the carpet.)

The above paradoxes can be regarded as symptoms of false interpretation. Many foundationally-minded physicists reject the conventional (textbook) interpretation of quantum mechanics precisely for such reasons.

Since most experiments are performed with ensembles, one sometimes tends to forget that ensembles are many copies of identical individual systems in one and the same individual-system state. (The ensemble and the individual-system state are described by the same density matrix.)

Though ensembles are indispensable because one cannot make practical sense of the probabilities in any other way than as relative frequencies, one cannot understand what is going on in the experiment unless one considers the experiment in the individual-system version as done in this article.

10 Conclusion

It has been shown that, applying the Theorem on RAIO to preparation, a consistently quantum-mechanical theory can be obtained. It is relevant for many experiments, possibly not for all. In this way the first aim (motivation from the Introduction) is achieved.

Also the second aim is achieved. Namely, in the geometrical cases discussed, the RAIO *gave full confirmation to the simple, classical intuitive pictorial representation suggested by the geometry of the experiment.* In this lies the answer why preparation has obtained so little attention in the literature: *one can lean on classical intuition.*

The goal to treat the dynamical and the geometrical preparations by the same theory has also been achieved.

The RAIO is fictitious as far as the standard quantum-mechanical concept of occurrence of an event is concerned. But it is often seen to be quite real *concerning classical intuition*. Both in the third example of the Stern-Gerlach and the second example of the hole-in-screen preparators one has the distinct feeling, based on classical intuition, that the geometry is such that if the particle arrives in the measuring instrument (in whatever way the measurement interaction takes place), then it must have passed the preparator in the upper half space or the hole respectively. This makes the RAIO plausible. The point is not plausibility, but the exact quantum-mechanical theorem on RAIO (in section 5), which makes possible to describe dynamical and geometrical preparations by the same formalism proposed in this article.

Finally, it is important to note that in the proposed theory preparation is almost completely *decoupled* from the measurement at the end of the experiment. Actually, the only connection consists in the application of the Theorem on RAIO in the localization formula (12). This is why no mention is made of the kind of measurement that is taking place at the end of the experiment.

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