Gravity on All the Energy Steps. The Contours of a Future Building

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Abstract

At the present time a theory of gravity is subdivided into two absolutely different parts: low-energy theory represented by the General Relativity (GR) and hypothetical high-energy theory – Quantum Gravity (QG) – that is still unresolved. In this way there is a certain dichotomy in gravity considered as a unified theory. This paper is an effort to reveal the main causes for such a dichotomy; the means for departure from this dichotomy are proposed. By one of the approaches gravity is considered at low and at high energies as a single whole dependent on the same parameters, which are discrete for the fundamental length if present.

1 Introduction. Gravity at Low and High Energies and Main Problems.

At the present time a theory of gravity is subdivided into two absolutely different parts: low-energy theory represented by the General Relativity (GR) that has been brilliantly verified by experiment and hypothetical high-energy theory – Quantum Gravity (QG) – that is still unresolved.
The General Relativity (GR) is one of the most basic and beautiful physical theories advanced and accepted during the whole history of physical science. It has been convincingly verified by the experiments (for example,

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[1]). Nevertheless, some problems associated with GR still must be tackled. The primary problem concerns extension of GR for high energies i.e. the gravity quantization problem [2]–[6]. According to the generally recognized viewpoint, the quantum effects in gravity are introduced at the Planck's length scales $l_P = (\hbar G/c^3)^{1/2} \approx 10^{-33} cm$ [5],[6].
The majority of researchers agree that in the high-energy (quantum) limit of gravity the minimal length l_{min} appears inevitably and this length most likely but (not necessarily!) is on the order of the Planck length $l_{min} \propto l_P$,[2]–[6].
Just with this point there is a discrepancy between the two above-mentioned parts of gravity or, more precisely, between the mathematical apparatus (instruments) of GR and of hypothetical QG. Starting from the highest (Planck) energies E_P and from a minimal length $l_{min} \propto l_P$ in QG and ≪coming down the energy steps≫, in GR we, in accordance with the Heisenberg Uncertainty Principle (HUP), should have arrived at the length scales considerably exceeding l_{min} but, quite the contrary, all the mathematical apparatus of GR is based on the notion of infinitesimal variations in the space-time quantities.
Really, the current mathematical formalism of the theory [8] is utterly incompatible with the HUP [9]:

$$\Delta x_i \Delta p_i \geq \frac{\hbar}{2}. \qquad (1)$$

Or for the pair energy - time we have

$$\Delta E \Delta t \geq \frac{\hbar}{2}. \qquad (2)$$

The explanation is that the mathematical formalism of GR is based on the concept of infinitesimal variations in the space-time quantities $ds, dx_\mu, ...$, which for a probe particle, in accordance with (1),(2), will inevitably result in the infinitely large momentum and energy fluctuations

$$\Delta p_i \rightarrow \infty; \Delta E \rightarrow \infty. \qquad (3)$$

But then, if measurable quantities are concerned, (3) is in conflict with GR because, as immediately follows from (3), when measuring the characteristics for variations in the probe particle positions within the scope of GR

and, actually, being at low energies, we can derive the momentum and energy characteristic for the scales of Quantum Gravity (QG)! In other words, by this procedure GR, in fact, exceeds its own limits. As a consequence, the theory becomes **nonclosed** (at least with respect to HUP) but it should not be. It is clear that this **nonclosure** arises from the
mathematical formalism of GR admitting the existence of infinitesimal variations in the space-time quantities .
The apparent incompatibility of the mathematical apparatus (instruments) used in GR and HUP is the main cause (problem) of *dichotomy* between GR and QG, provided we suppose that the fundamental quantities involved in the theory *are measurable*.
It is clear that gravity as a unified theory considered independent of the energies should be saved from this limitation. Then it is clear that in a future gravity theory, valid for any energy (low or high), GR should arise in the low-energy limit to a high degree of accuracy. In other words, the future Quantum Gravity that at low energies would become GR to a high accuracy should be built as a high-energy deformation of the latter.
The deformation is understood as an extension of a particular theory by inclusion of one or several additional parameters in such a way that the initial theory appears in the limiting transition [7].
What are the parameters of interest in the case under study? Proceeding from the above, the parameters must be inevitably associated with l_{min} (probably with $l_{min} \propto l_P$)and hence E_P.
This means that in a high-energy gravitation theory the energy- or, what is the same, measuring scales-dependent parameters should be necessarily introduced.
But, on the other hand, these parameters could hardly disappear totally at low energies, i.e. for GR too. However, since the well-known canonical (and in essence the classical) statement of GR has no such parameters [8], the inference is as follows: their influence at low energies is so small that it may be disregarded at the modern stage in evolution of the theory and of the experiment.
Still this does not imply that they should be ignored in future evolution of the theory, especially on going to its high-energy limit.
This is one more (implicit) problem of the above-mentioned dichotomy be-

tween GR and QG.
The problem in the explicit form may be stated as follows:
what (latent) parameters exist in GR considering the minimal length l_{min}?
This work is motivated by a search for the unifying principles for GR and QG aimed at elimination of the above-indicated dichotomy between them. More exactly, we look for the identical parameters, possibly unifying both theories but varying for each of them in different domains, which will allow for the solution of the above-stated problems.
Clearly, for solution of the first problem, the mathematical formalism presently used for the General Relativity should be revised if we want to derive a theory **measurable** from the viewpoint of Quantum Mechanics, (i.e., at least compatible with HUP).
All the infinitesimal space-time quantities $ds, dx_\mu, ...$ in General Relativity should be replaced by the finite quantities dependent on the measuring scales l (energies $E \sim 1/l$):

$$ds^2 \mapsto \tau s^2(l); dx_\mu \mapsto \tau x_\mu(l), ... \quad (4)$$

With such a problem stating, the deformation parameters mentioned in the first part of this Introduction will be introduced quite naturally both in the low-energy and high-energy, quantum, domain of this theory. And only on going from one domain to the other their values, perhaps their physical meaning, will be changed, **possibly leading to a solution of the second problem at hand**.

It seems expedient to emphasize once more that in this case the matter concerns modification of the mathematical formalism in accordance with the above arguments (appearing logical, consistent, and convincing) rather than GR itself. In what follows one of the approaches to solving of the problem at hand is be proposed.
This work is a continuation of the paper [10] and, because of this it is somewhat intertwined with it.

2 Quantum Fluctuations of Space-Time and New Parameters in Gravity

To solve the above-mentioned problems, **initially** we can use the Space-Time Quantum Fluctuations (STQF) imposing considerable constraints on HUP with regard to gravity. The definition (STQF) is closely connected to the notion of "space-time foam".

The notion "space-time foam", introduced by J. A. Wheeler about 60 years ago for the description and investigation of physics at Planck's scales (Early Universe) [11],[12], is fairly settled. Despite the fact that in the last decade numerous works have been devoted to physics at Planck's scales within the scope of this notion, for example [13]–[32], by this time still their no clear understanding of the "space-time foam" as it is.

On the other hand, it is undoubtful that a quantum theory of the Early Universe should be a deformation of the well-known quantum theory.

In my works with the colleagues [33]–[41] I has put forward one of the possible approaches to resolution of a quantum theory at Planck's scales on the basis of the density matrix deformation.

In accordance with the modern concepts, the space-time foam [12] notion forms the basis for space-time at Planck's scales (Big Bang). This object is associated with the quantum fluctuations generated by uncertainties in measurements of the fundamental quantities, inducing uncertainties in any distance measurement. A precise description of the space-time foam is still lacking along with an adequate quantum gravity theory. But for the description of quantum fluctuations we have a number of interesting methods (for example, [42],[22]-[32]).

In what follows, we use the terms and symbols from [24]. Then for the fluctuations $\widetilde{\delta l}$ of the distance l we have the following estimate:

$$\widetilde{\delta l} \gtrsim l_P^{\gamma} l^{1-\gamma} = l_P\left(\frac{l}{l_P}\right)^{1-\gamma} = l\left(\frac{l_P}{l}\right)^{\gamma}, \qquad (5)$$

where $0 \leq \gamma \leq 1$.

At the present time three principal models associated with different values of the parameter γ are considered:

A) $\gamma = 1$ that conforms to the initial (canonical) model from [11],[12]

$$\widetilde{\delta l} \gtrsim l_P; \tag{6}$$

B) $\gamma = 2/3$ that conforms to the Salecker - Wigner model (inequalities) [42],[24] compatible with the holographic principle [43]–[47]

$$\widetilde{\delta l} \gtrsim (l l_P^2)^{1/3} = l_P \left(\frac{l}{l_P}\right)^{1/3}; \tag{7}$$

C) $\gamma = 1/2$ - random-walk model [31] [32]

$$\widetilde{\delta l} \gtrsim (l l_P)^{1/2} = l_P \left(\frac{l}{l_P}\right)^{1/2}. \tag{8}$$

But, because of the experimental data obtained with the help of the Hubble Space Telescope [48], a random-walk model C) may be excluded from consideration (for example, see [29]) and is omitted in this work.

Moreover, in fact it is clear that **at Planck's scales, i.e. for**

$$l \rightarrow \propto l_P \tag{9}$$

models A) are B) are coincident.
Using (6)–(8), we can derive the quantum fluctuations for all the primary characteristics, specifically for the time $\widetilde{\delta t}$, energy $\widetilde{\delta E}$, and metrics $\widetilde{\delta g}_{\mu\nu}$. In particular, for $\widetilde{\delta g}_{\mu\nu}$ we can use formula (10) in [24]

$$\widetilde{\delta g}_{\mu\nu} \gtrsim (l_P/l)^{\gamma}. \tag{10}$$

Let us denote the parameter l_P/l in terms of λ as follows:

$$\lambda_l \equiv \frac{l_P}{l}. \tag{11}$$

What is the principal meaning of STQF? It is important that with allowance for the gravitational interactions they impose hard constraints on the quantities $\Delta x_i, \Delta p_i, \Delta E, \Delta t...$, ... and hence on HUP depending on the existing scales and energies. The energies will have nonzero minimal (and correspondingly maximal) values which depend on the existent energies (measuring scale) and Planck's energies.
As mentioned in the previous Section, there are no infinitesimal variations in space-time quantities and no infinitely large variations of momenta and energies.
The Generalized Uncertainty Principle (GUP) that is an extension of HUP for Planck's energies, where gravity must be taken into consideration [49]–[56], conveys the same, namely:

$$\triangle x \geq \frac{\hbar}{\triangle p} + \alpha' l_P^2 \frac{\triangle p}{\hbar}, \tag{12}$$

where α' is the model-dependent dimensionless numerical factor.
(12) leads to the minimal length $l_{min} \propto l_P$. But this minimum is **global** as it is dependent only on l_P and model parameters of GUP, being independent of the measuring scale l.
GUP (12) leads to the same minimal boundary $l_{min} \propto l_P$ for the measuring scale l as the heuristic model A)(6). However, even J. A. Wheeler has used (6) not over the whole energy range but at Planck's scales only. Because of this, the model for STQF governed by (6) is limited, in essence being a particular case of the Salecker - Wigner model B) but on going to the Planck scales.

In what follows we assume the presence of a minimal length on the order of the Planck length $l_{min} \propto l_P$. This fact may be inferred from GUP or may be taken independently without regard to GUP.
Then it is obvious that any quantity having the dimensions of length is quantized, i.e. is determined by a discrete set of values as follows from the obvious statement:

Provided some quantity has a minimal measuring unit, values of

this quantity are multiples of this unit.
Naturally, any quantity having a minimal measuring unit is uniformly discrete.
The latter property is not met, in particular, by the energy E.
As $E \sim 1/l$, where l – measurable scale,
the energy E is a discrete quantity but the nonuniformly discrete one.
It is clear that the difference between the adjacent values of E is the less the lower E. In other words, for

$$E \ll E_P \tag{13}$$

E becomes a practically continuous quantity.
Note that all the above-mentioned models of STQF A)–C) are actually dependent on one and the same dimensionless parameter λ_l and on the Planck quantities $l_P, E_P...$). Since $l_{min} \propto l_P$ with the constant proportionality factor (in a particular case of GUP this is the model-dependent quantity α' (12)), this parameter may be represented by l_{min}/l or (at least within the scope of HUP or of a linear variant of GUP [57],[58]) by $E/E_{max} \propto E/E_P$.

In the above notation (5) takes the form

$$(\widetilde{\delta l})_{min} = \beta l_P^{\gamma} l^{1-\gamma} = \beta l_P (\frac{l}{l_P})^{1-\gamma} = \beta l \lambda_l^{\gamma}, \tag{14}$$

where $1/2 < \gamma < 1$ and β is a numerical factor on the order of unity [59].
Obviously, we get

$$(\widetilde{\delta t})_{min} = c^{-1} (\widetilde{\delta l})_{min}. \tag{15}$$

And then in formula (4) the corresponding substitution may be as follows:

$$\begin{aligned} ds^2 \mapsto \tau s^2(l) \sim (\widetilde{\delta l})^2_{min} = \beta^2 l^2 \lambda_l^{2\gamma}, \\ dx_\mu \mapsto \tau x_\mu(l) \sim l. \end{aligned} \tag{16}$$

Now it is clear that for the metric $g_{\mu\nu}(x)$ there is a dependence on l (or on the corresponding energy E)

$$g_{\mu\nu}(x) \mapsto g_{\mu\nu}(x, l) = g_{\mu\nu}(x, \lambda_l^{\gamma}). \tag{17}$$

As l is quantized in units of $l_{min} \propto l_P$, we obtain (16) in terms of Planck's quantities and of the natural parameter k_l in the following way:

$$\tau s^2(l) \sim (\widetilde{\delta l})^2_{min} = \widetilde{\beta} k_l^{2-2\gamma} l_P^2, \qquad (18)$$

where $k_l = l/l_{min}$ and $\widetilde{\beta}$ has the factor β and the corresponding factor due to the ratio l_{min}/l_P.
Note that at low energies the flat metric $(1, -1, -1, -1)$, with a high degree of accuracy, gives the ansatz:

$$g_{\mu\nu}(x) = (\pm\delta_{\mu\nu} \exp(\pm\lambda_l^\gamma)),$$

where the sign $+$ before $\delta_{\mu\nu}$ corresponds to the case $\mu = \nu = 0$ only.
Thus, from the start, we obtain some discrete set of the initial data whose variations are governed by the nonuniformly discrete parameter λ_l. At low energies $E \ll E_P$, i.e. for $k_l \gg 1$, its variation is almost continuous because the intervals between the adjacent points λ_l will be constantly reducing as E is decreased(or similarly as l is increased) and a practically continuous theory arises in the limit $E \to 0$.
At high energies, with $E \approx E_P$ or $k_l \to 1$, it will be really discrete. Obviously, in this case it is responsible for great fluctuations of the metric (17).

3 Dimensionless Parameter λ_l and Thermodynamic Approach to Gravity. Some Illustrations.

Note also that, in fact, the nonuniformly discrete parameter λ_l (11) is introduced as a deformation parameter on going from the well-known quantum mechanics (QM) to a quantum mechanics with the fundamental length (QMFL), provided this length l_{min} is on the order of Planck's length $l_{min} \propto l_P$, as revealed by the author in the works written with his colleagues [33] –[41]. The main concept of these works is a high-energy (Planck's) deformation of the well-known quantum-mechanical density matrix ρ.

Yet it should be noted that actually in works [33] –[41] the author has used the parameter λ_l^2

$$\alpha_l = l_{min}^2/l^2 \propto \lambda_l^2. \tag{19}$$

This parameter up to a factor is variable within the interval

$$0 < \alpha_l \leq 1/4, \tag{20}$$

whereas the density matrix in QMFL becomes deformed and dependent on α_l: $\rho = \rho(\alpha_l)$, and we get

$$\lim_{\alpha_l \to 0} \rho(\alpha_l) \to \rho, \tag{21}$$

where ρ – known density matrix from QM.

In this way the condition (20) naturally resultant from the procedure giving rise to the deformed density matrix $\rho(\alpha_l)$ imposes a natural constraint on λ due to $l \geq 2l_{min}$.

In the last few years new very interesting approaches to gravity studies have been proposed, which may be divided into "thermodynamical" and "theoretical-informational" approaches. The approach suggested in the pioneer work by T. Jacobson [60] has been considerably extended in a series of remarkable papers by T.Padmanabhan [61]–[72].

As shown in [71], the Einstein Equation for several cases of horizon spaces may be written as a thermodynamic identity (the first principle of thermodynamics): ([71], formula (119))

$$\underbrace{\frac{\hbar c f'(a)}{4\pi}}_{k_B T} \underbrace{\frac{c^3}{G\hbar} d\left(\frac{1}{4} 4\pi a^2\right)}_{dS} \underbrace{- \frac{1}{2}\frac{c^4 da}{G}}_{-dE} = \underbrace{P d\left(\frac{4\pi}{3} a^3\right)}_{P\,dV}, \tag{22}$$

where a static, spherically symmetric horizon in space-time is described by the metric

$$ds^2 = -f(r)c^2dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2, \tag{23}$$

and the horizon location will be given by simple zero of the function $f(r)$ ($f(a) = 0$, $f'(a) \neq 0$) at $r = a$.(Here $r = a$ is the radius of a sphere.) And $P = T_r^r$ is the trace of the momentum-energy tensor and radial pressure.

In Sections 5 and 6 of [73] first the Einstein Equations on horizon (22) have

been written in terms of the parameter α_a, next the high-energy ($\alpha_a \to 1/4$), α_a – deformation of these equations has been derived in two different cases: equilibrium and nonequilibrium thermodynamics.
The latter case is distinguished from the first one by the dynamic cosmological term dependent on α_a, appearing with the corresponding factor in the right side of high-energy deformed (22) as follows:

$$\Lambda = \Lambda[\alpha_a]. \tag{24}$$

It is evident that by virtue of (19) the α_a – deformation of these equations is equivalent to their λ_a – deformation, and we have

$$\Lambda = \Lambda[\alpha_a] = \Lambda[\lambda_a]. \tag{25}$$

Also, it is noteworthy that the parameter λ_l (11) is implicitly introduced in the noted work by E. Verlinde [74], where gravity is considered as an "Entropic Force". But, since for $\mathcal{R} \gg l_P$, where $\mathcal{R}$ – radius of a holographic screen, its value is close to zero $\lambda_{\mathcal{R}} \approx 0$ and varies very smoothly, it is omitted in the final formulae at low energies [74].
However, at high (Planck's) energies on going to the GUP-correction and further on to the high-energy deformation of Einstein equations it is inevitable as shown in the latest works [75](evidently in formulae ((27)–(39),[76] (and in the implicit form in (32)–(36)).

So, at least for horizon spaces the parameter $\lambda_{\mathcal{R}}$ is just the latent parameter of GR mentioned in Section 1, infinitesimal at low energies $\lambda_{\mathcal{R}} \approx 0$ but turning to $\lambda_{\mathcal{R}} \to 1/2$ on going to high (Planck's) energies (i.e., to QG).

This parameter in the "energy representation" $\sim E/E_P$ is directly or indirectly introduced in all the formulae of the noted review on Quantum-Spacetime Phenomenology [2].

4 Conclusion

Thus, provided the main concept of this work is realized, at all energy levels gravity may be governed by **the same set of discrete parameters**which still have different variation rates and differing values in the low- and high-energy regions. The transfer from low energies (GR) to high (Planck) energies (QG) may be schematically represented as

$$GR[k_l \gg 1, \lambda \approx 0, ...] \rightarrow QG[k_l \rightarrow 1, \lambda \rightarrow 1, ...]. \qquad (26)$$

However, as noted in [7], in nature the direction was opposite – from high to low energies. Because of this, it seems more natural to consider the transition opposite to (26), in [7] referred to as the ≪**dequantization**≫:

$$QG[k_l \rightarrow 1, \lambda \rightarrow 1, ...] \rightarrow GR[k_l \gg 1, \lambda \approx 0, ...] \qquad (27)$$

The parameter set in the left and right sides of (26),(27) is the same. The dots in parenthesis are given for additional parameters which may arise in the process of a theory resolution.

It is important that in this case **gravity could be considered as a single whole without its subdivision into the Classical Gravity (GR) and Quantum Gravity (QG)**.

Of course, the dependence of the principal space-time quantities on the measuring scale l (existing energy E) that is based on STQF and suggested in (16)–(18) is very tentative and may vary in the process of the theory evolution. Still, by author's opinion, a set of the principal discrete parameters in (26) will be invariant with respect to these variations.

The primary criterion for resolution of a future theory must be the **Conformity Principle**:

on going to low energies GR must be reproduced to a high degree of accuracy, at least its experimentally verified part.

The title of this work is not chosen by chance as ***gravity by this approach for all the energy levels is treated as a single whole (one building), where the descent from the upper levels (steps) to the lower ones by the energy steps is governed by a single discrete parameter λ_l, the step height being steadily reduced as we descend***

lower and lower, whereas their length $\sim l$ ***will be ever increasing***.

The proposed concept is rather important for better understanding and investigation of the cosmological term Λ, especially in view of the Dark Energy Problem [77]–[81]. In principle, they may be used to answer the question whether $\Lambda = const$ or $\Lambda = \Lambda(t)$ is a time-variable quantity.
Despite the fact that the works taking Λ as $\Lambda(t)$, i.e. as a dynamic quantity, are numerous(for example, [82]– [85]) quite forceful arguments are given against this point of view (for example, [86]).
Indeed, according to the General Relativity, the cosmological term Λ has been considered constant $\Lambda = const$ as, due to the Bianchi identities [8],

$$\nabla^{\mu}G_{\mu\nu} = 0. \tag{28}$$

But in this work it has been demonstrated that, actually, Bianchi identities (28) are introduced at the low-energy limit only

$$\lim_{\lambda_l \to 0} \nabla^{\mu}G^{[\lambda_l]}_{\mu\nu} = \nabla^{\mu}G_{\mu\nu} = 0. \tag{29}$$

Because of this, the really measured cosmological term Λ in fact is dynamic $\Lambda = \Lambda[\lambda_l(t)]$, practically invariable in the modern epoch, i.e. at low energies, due to slow variations of the deformation parameter $\lambda_l(t)$ at low energies and due to its very small value.
In the works [87]–[89] a behavior of the term Λ has been studied reasoning from $\alpha_l(t)$ on the assumption that it is dynamic, similar to the case proven in [87] GUP for the pair of conjugate variables (Λ, V), where V is the space-time volume, as with the holographic principle applied to the whole Universe [90], where $\alpha_l \propto \lambda_l^2$ because of (19),(20) of the Section 3. And as noted in (25) α_l-deformation theory is equivalent to its λ_l-deformation, (in fact to its λ_l^2-deformation).
Then the main difference of these two different cases, examined in [87]–[89] is in the leading order of expansion $\Lambda[\lambda]$ in terms of λ. In the first case it is the second

$$\Lambda^{GUP}(\lambda) \propto (\lambda^4 + \eta_1\lambda^6 + ...)\Lambda_p, \tag{30}$$

whereas in the second case it is the first

$$\Lambda^{Hol}(\lambda) \propto (\lambda^2 + \xi_1\lambda^4 + ...)\Lambda_p, \tag{31}$$

where $\Lambda_p = \Lambda_{\lambda \to 1/2}$ – cosmological term at Planck's scales.
As Λ^{Hol} is practically coincident with the experimental value of the cosmological term Λ_{exper}, a holographic model is preferable – model B) of Section 2 developed for quantum fluctuations is supported experimentally.

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