Fermat's Theorem

$$X^n + Y^n = Z^n \quad [1.1]$$

n is a prime number, $n > 2$; X, Y, Z are integers.

Decisions can be X, Y-coprime numbers.

There are two options: X, Y-even and odd, Z-odd

or X, Y- is odd,Z-even.

1.Decomposition Z on multipliers.

2.Equivalent record of multiplier of Z22.

3.Expression for X + Y without the multiplier of n.

4.Impossibility of existence of $X + Y = Z1^n$ is in integers.

5.Equalization is n- degrees of Z, not having a decision in integers.

1.Decomposition [1.1] on multipliers.

If n is odd number, then $X^n + Y^n$ will on multipliers :

$$X^n + Y^n = (X + Y)(X^{n-1} - X^{n-2}*Y +...-X * Y^{n-2} + Y^{n-1})$$

where the second brace geometric progression

the first member of the $a1 = X^{n-1}$, and q multiplier $=-Y/X$.

The sum of which $S = a1(1-q^n)/(1-q)$:

$$S = (X^n +Y^n)/(X+Y)$$

$$Z^n = Z11 * Z22$$

where

$$Z11 = X + Y$$

$$Z22 = X^{n-1} - X^{n-2}*Y +...-X * Y^{n-2} + Y^{n-1}$$

2. Another view of the Z22.

If we summarize the regularly spaced members of environments

-the Member of the progression of the Z22 in pairs are:

$$X^{n-1} + Y^{n-1} = (X + Y)(X^{n-2} + Y^{n-2}) - X*Y^{n-2} - Y*X^{n-2} =$$

$$=... -X*Y(X^{n-3} + Y^{n-3})= ...-X*Y(X + Y)(X^{n-4} + Y^{n-4})+$$

$+X^2 * Y^{n-3} + Y^2 * X^{n-3} = ...+X^2 *Y^2 (X^{n-5} + Y^{n-5}) =$

$=... + X^{(n-3)/2} * Y^{(n-3)/2} *(X^2 + Y^2 + 2*X*Y - 2*X*Y)=$

$= ...(-1)^{(n-1)/2} *2 *X^{(n-1)/2} * Y^{(n-1)/2}$

$-X^{n-2} * Y - X * Y^{n-2} = -X*Y(X^{n-3} + Y^{n-3}) = ... +$

$+X^{(n-3)/2} * Y^{(n-3)/2} *(X + Y)^2 + (-1)^{(n-1)/2} *2 *$

$X^{(n-1)/2} * Y^{(n-1)/2}$

...

$X^{(n-3)/2} * Y^{(n+1)/2} + X^{(n+1)/2} * Y^{(n-3)/2} =$

$= X^{(n-3)/2} * Y^{(n-3)/2} *(X^2 + Y^2 + 2*X*Y - 2*X*Y)=$

$= X^{(n-3)/2} * Y^{(n-3)/2} *(X + Y)^2 -2*(-1)^{(n-1)/2} *$

$*X^{(n-1)/2} *Y^{(n-1)/2}$

and the sum of the average member of the progression

and it said this:

$(-1)^{(n-1)/2} *(n-1)/2 * 2*X^{(n-1)/2} *Y^{(n-1)/2} +$

$+ (-1)^{(n-1)/2} *X^{(n-1)/2} *Y^{(n-1)/2} = n*(-1)^{(n-1)/2} *$

$*X^{(n-1)/2} *Y^{(n-1)/2}$

According to the above Z22 representable as:

$Z22= (X + Y)^{n-1} - X * Y * n * L_1 * (X + Y)^{n-3} +$

$+ ... + L * n * (-1)^{(n-3)/2} X^{(n-3)/2} * Y^{(n-3)/2} *(X + Y)^2 +$

$+ n*(-1)^{(n-1)/2} *X^{(n-1)/2} *Y^{(n-1)/2}$

where

$L * n = 1^2 + 2^2 + ...+ (n+1)/4^2 + (n-1)/4^2$

Sum of squares of natural row N*(N+1)*(2*N+1)/6

after a substitution we have:

$$L * n = n * (n^2 - 1)/24 ;$$

$$Z11 = X + Y$$

If X + Y does not have a multiplier-n

$$Z11 = Z1^n$$

$$Z22 = Z2^n$$

where Z1 and Z2-inter simple, otherwise between X and

Z must be a common factor that is not valid.

Consider the case where X is divided into n;

Let X contains n. then let Z = Y + A :

$$X^n + Y^n = (Y + A)^n$$

After the conversion, we have:

$$X^n = n * Y^{n-1} * A + n * (n - 1)/2 * Y^{n-2} * A^2 +...+$$
$$+ n * Y * A^{n-1} + A^n \quad [2.1]$$

First cut the prime number n would lead to his presence in A^n

member and since he is the only one in [2.1] without

multiplier of n And it means that Z contains n

That is not permissible;Y and Z coprime numbers.

Similarly, these thoughts in the p2 we have:

$$Z-Y=X1^n \quad [2.2]$$

$$Z-X=Y1^n \quad [2.3]$$

3.take Z = X+Y-B, then:

$$X^n + Y^n =(X + Y - B)^n,$$

over the bracket and get:

$$X^n + Y^n = (X + Y)^n - n * (X + Y)^{n-1} * B + ...+$$

$$+ n * (X + Y) * B^{n-1} - B^n \quad [3.1]$$

Unveiling the $(X + Y)^n$ and leave out both parts of the X^n and Y^n

and then the only member of the B^n -factor n.

B has a factor of n.

Then, if X + Y is not included n:

$$B = X - X1^n = Y - Y1^n = Z1^n - Z \quad [3.2]$$

$$X = X1^n + n*X1*Y1*Z1*K \quad [3.3]$$

$$Y = Y1^n + n*X1*Y1*Z1*K \quad [3.4]$$

$$Z1^n = X1^n + n*X1*Y1*Z1*2*K + Y1^n \quad [3.5]$$

where is k- any integer

Z1 = X1 + Y1

The other view will cause the numerical K,X1,

Y1, change that does not affect the [3.5]

$$2*K = [(X1 + Y1)^n - X1^n - Y1^n]/n*X1*Y1*Z1 \quad [3.6]$$

4.Let X1 + Y1 = Z1

Right side [3.6]-cannot be even number.

According to Newton's binomial $Z1^n$ has the form:

$$X1^{n-3} + E2 * X1^{n-4} *Y1 + \ldots +ES * X1^{(n-3)/2} *Y1^{(n-3)/2} + \ldots +$$

$$+ E2*X1 *Y1^{n-4} + Y1^{n-3} \quad [4.1]$$

where E is the sum of the corresponding combinations

replication $X1^{\{\}} * Y1^{\{\}}$.

When X1,Y1 different parity obviously [4.1]-odd.

Cut on the Z1 the right piece [3.5]

$$[(X1 + Y1)^{n-1} - (X1^{n-1} - X1^{n-2} * Y1 + \ldots - X1 * Y1^{n-2} +Y1^{n-1})]/$$

$$/n*X1*Y1 \quad [4.2]$$

If X1 and Y1-odd, the second brace contained n-odd members,

and expression is odd.

We pass to the general case

$Z1 = X1 + Y1 + a$

where a-any integer with a sign + or -.

According to [3.4] we have:

$$(X1 + Y1 + a)^n = X1^n + n*X1*Y1*Z1*2*K + Y1^n$$

and further we will present:

$2K = K1 + K2$

where K1-odd number, at which $Z1 = X1 + Y1$

After transformation we have:

$$(X1 + Y1)^n + n*(X1 + Y1)^{n-1} * a + ... + n*(X1 + Y1) * a^{n-1} +$$

$$+ a^n = X1^n + n*X1*Y1*Z1*K1 + n*X1*Y1*Z1*K2 + Y1^n$$

From what follows:

$$n*X1*Y1*(X1 + Y1)*K2 = n*(X1 + Y1)^{n-1} * a + ... + n*(X1 + Y1) * a^{n-1} + a^n \quad [4.3]$$

a^n and X1 + Y1 has the general multiplier, we will call it (c).

Thus

(c) belonging to € (a) - c^i ;

(c) belonging to € (X1 + Y1) - c^j .

If $c^j > c^i$ that

$$K = [(X1 + Y1 + a)^n -(X1+Y1)(X1^{n-1} - ... +Y1^{n-1})]/2*n*X1*Y1*(X1 + Y1 + a)$$

that K-contains (c) that is impossible as between Z2 and Z1 there will

be the general multiplier that follows from [3.2]

$Z1^{n-1} - Z2 = n*X1*Y1*K$ and it isn't admissible.

We will note also that $(X1^{n-1} - ... +Y1^{n-1})$ - has no multiplier (c) of p2.

It agrees contains (a) all multipliers belonging above X1+Y1.

We will designate $a/(X1 + Y1)^j = L$.

$$Z1= X1 + Y1 + L(X1 + Y1)^j$$

$$K2 = 2*K - K1$$

$$K2 = [(X1 + Y1 + L(X1 + Y1)^{j}]^{n} - X1^{n} - Y1^{n})]/n*X1*Y1*[(X1 + Y1 + L(X1 + Y1)^{j}] -$$

$$- [(X1 + Y1)^{n} - X1^{n} - Y1^{n}]/n*X1*Y1*(X1 + Y1)$$

after transformation we have:

$$K2 = [(X1 + Y1 + L(X1 + Y1)^{J}]^{n} * (X1 + Y1) - (X1 + Y1)^{n} *[(X1 + Y1 + L(X1 + Y1)^{j}]+$$

$$+ L(X1 + Y1)^{j} *(X1^{n} + Y1^{n})/$$

$$/n*X1*Y1*(X1 + Y1)*[(X1 + Y1 + L(X1 + Y1)^{j}]$$

$$K2 = L* (X1 + Y1)^{n-1+j-1} /n*X1*Y1 + L*(X1 + Y1)^{j-1} * (X1^{n} + Y1^{n})/$$

$$/n*X1*Y1[X1 + Y1 + L(X1 + Y1)^{j}]$$

$$K2 = [X1 + Y1 + L(X1 + Y1)] * L* (X1 + Y1)^{n+j-2} + L*(X1 + Y1)^{j-1} * (X1^{n} + Y1^{n})/$$

$$/ n*X1*Y1[X1 + Y1 + L(X1 + Y1)^{j}]$$

$$K2 =(X1 + Y1)^{j-1} * \{[X1 + Y1 + L(X1 + Y1)^{j}] * L* (X1 + Y1)^{n-1} + L * (X1^{n} +Y1^{n})\}/$$

$$/ n*X1*Y1[X1 + Y1 + L(X1 + Y1)^{j}] \quad [4.4]$$

Brace {} contents after reduction on (X1 + Y1) in a denominator

doesn't may contain the specified multiplier as the first composed

him supports, and the second isn't presen $L*(X1^{n} + Y1^{n})/(X1 + Y1)$

p.2.So the possible multiplier in K2 is $(X1 + Y1)^{j-1}$ [4.4]

That the decision in integers for [4.3] needed to be had in K2

multiplier (X1 + Y1) in that minimum degree which contains

member on the right side

$$n* (X1 + Y1)^{n-1} * a = n* (X1 + Y1)^{n-1} * (X1 + Y1)^{j} =$$

$$= n * (X1 + Y1)^{n-1+J}$$

There is no coincidence of possible and necessary K2.

Thus a = 0.

In a consequence of that in case of X + Y doesn't contain n multiplier,

in integers there is no decision.

5. If X + Y is a factor of n, that the only

possible according to p4,that of [2.0] in Z22 can enter n

only in the first degree.Otherwise between X and Y

general multiplier, that not possibly.What it is necessary from

$$Z11 = n^{(m*n - 1)} * Z1^{n} \quad [5.1]$$

m > 1 concordantly [3.5]because $X1^{n} + Y1^{n}$ no less n^{2} and degree

n in $X1^{n} + Y1^{n} = 2*Z - (X + Y)$ coincides with a degree

n in Z.

$$Z22 = n*Z2^{n}$$

$$X = Z - Y1^{n}$$

$$Y = Z - X1^{n}$$

$$(Z - X1^{n})^{n} + (Z - Y1^{n})^{n} = Z^{n}$$

$$Z^{n} - n * Z^{n-1} * (X1^{n} + Y1^{n}) + ... + \underbrace{n*(n-1)*(n-2)/6 * Z^{3} * (X1^{n*(n-3)} + Y1^{n*(n-3)})}_{1} -$$

$$- \underbrace{n*(n-1)*/2*Z^{2} * (X1^{n*(n-2)} + Y1^{n*(n-2)})}_{2} + \underbrace{n*Z*(X1^{n*(n-1)} + Y1^{n*(n-1)})}_{3} -$$

$$\underbrace{-(X1^{n^{n}} + Y1^{n^{n}})}_{4} = 0 \quad [5.1]$$

We will define maintenance of multiplier of n in underline members [5.1]:

in {4} odd degree $n^{m} * n = n^{m+1}$

in {3}- odd degree $n*n^{m} = n^{m+1}$

in {2}- $n * n^{2m} * n^{m} * n = n^{3m+1}$

in {1}- $n * n^{3m} = n^{3m+1}$

{ $X1^{n} + Y1^{n}$ contains n one time anymore, what $X1^{n} + Y1^{n}$ in the first and in other odd number degree.

$X1^{n} + Y1^{n}$ in even n does not contain:

```
   2*K*n    2*K*n       2*K*n
 X1     - Y1     +  2*Y1  a difference at decomposition will give necessarily
```

X1 + Y1,and n has, when the last member can not have him}

Each of other members contains n in a degree higher what in {1} .

We will define maintenance of n in a sum 3 and 4 :

```
   n    n
 X1 + Y1 = 2*Z - (X + Y)

                         n*(n-1)      n*(n-1)       (n-1)/2        n*(n-1)/2  n*(n1)/2
{3}+{4} = n * Z * (X1        + Y1         - (-1)        *2 * X1        *Y1         )-
            ---------------------------------5--------------------------------

          (n-3)/2         n*(n-3)/2  n*(n-3)/2    n     n 2
- 2*Z *(-1)       * L1* X1        * Y1         * (X1  + Y1 )  +
---------------------------------6---------------------------

+2*Z * L2*(X1  + Y1)*

   (n-5)/2    (n-5)/2      (n-5)/2        n     n 4           n     n n-1
* X1       * Y1       * (-1)  2*Z * L2*(X1  + Y1 )  - ... - (X1  + Y1 ) * 2 *Z -

                n     n
               n     n     n     n
  - (X + Y)* (X1  + Y1 )/(X1  + Y1 )                  [5.2]
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                                      m 2m     3m+1
  {5} и {6} n has in a degree    n*n *n = n    .
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We summarize in [5.1] {1} и {2} :

```
                  3    n*(n-3)    n*(n-3)                2    n*(n-2)   n*(n-2)
n*(n-1)*(n-2)/6 *Z *(X1       + Y1       ) - n*(n-1)*/2*Z *(X1       +Y1      )=
  -------------------1-------------------   ----------------2------------------

                     3    n*(n-3)    n*(n-3)              2
 = n*(n-1)*(n-2)/6 *Z *(X1       + Y1       ) - n*(n-1)*/2*Z *(2*Z - X - Y) *
   -----------------------------------------
      n    n n-1            (n-5)/2        (n-5)/2    (n-5)/2    n    n 2
 * [(X1 + Y1 )   - ... + (-1)       *L1* X1       * Y1       * (X1 + Y1 ) +

     (n-3)/2          n*(n-3)/2  n*(n-3)/2
 +(-1)       *(n-2)* X1        * Y1         ]                          [5.3]
 --------------------------------------
```

Let (n - 3)/odd number, then (n - 1)/2-even. We will define

maintenance of n in a sum

underline members.All other we drop, because in each of them his maintenance

exceeds > 3m + 1.

```
                  3    n*(n-3)    n*(n-3)
n*(n-1)*(n-2)/6 *Z *(X1       + Y1       ) +
```

$$+ n*(n-1)*(n-2)*Z^3 * X1^{n*(n-3)/2} * Y1^{n*(n-3)/2}$$

then we will get expression:

$$n*Z^3 *[\underline{1/3}*(X1^{n*(n-3)} + Y1^{n*(n-3)} + 2* X1^{n*(n-3)/2} * Y1^{n*(n-3)/2} + \underline{X1^{n*(n-3)/2} * Y1^{n*(n-3)/2}})]$$

where underline member and I am remaining not divided by n.n contain two

underline member in [5.2]:

In {6} we will drop multipliers, having n,putting $2*Z - X - Y = X1^n + Y1^n$ и [2.a]

{6} will make $-1/3*X1^{n*(n-3)/2} * Y1^{n*(n-3)/2}$

Content $\{5\}= (X1^{n*(n-1)/2} + Y1^{n*(n-1)/2})^2$ if $(n - 1) /2$ odd number,

and casting aside members containing n in a remain we have :

$1/4* X1^{n*(n-3)/2} * Y1^{n*(n-3)/2}$ at $(n - 1)/2$ not even.

At $(X1^{n*(n-1)/2} - Y1^{n*(n-1)/2})^2$ where $(n - 1)/2$ even,

before to come to the odd number multiplier

it is necessary to take into account all even multipliers

and then contained {5}:

$$(n - 1)^2 /4^i *4^j *(X1^{(n-3-2k)/4} * Y1^{(n-3-2k)/4} * Y1^k)^2$$

where i- number 2 in the number of n - 1.

j- number 2 in a number (n - 1)/2

k- odd number number because sum of equal degree indexes

at X1 and Y1 and k is equal (n - 3)/2 - even.

We drop members with n and {5} after simplification will make

$$1/4* X1^{n*(n-3-2*k)/2} * Y1^{n*(n-3+2*k)/2}$$

at $(n - 1)/2$ even;

Then sum of all four members at $(n - 3)/2$ an odd -number

$$E = 1/4* X1^{n*(n-3-2*k)/2} * Y1^{n*(n-3+2*k)/2}$$

and at even :

$$E = 1/4 * X1^{n*(n-3)/2} * Y1^{n*(n-3)/2}$$

That in both cases means: content of sum of four members of containing n^{3m+1} there will be the same n^{3m+1} and decisions [5.1] in whole does not exist.

It is actual for n>5.

At n = 5 by a foregoing method we have:

$$n*Z*[- 1/2*(X1^{n*(n-3)} + Y1^{n*(n-3)}) - 2* X1^{n*(n-3)/2} * Y1^{n*(n-3)/2} + X1^{n*(n-3)/2} * Y1^{n*(n-3)/2}) = n * Z^{3m} * (...+X1^{n*(n-3)/2} * Y1^{n*(n-3)/2})$$

$$E = X1^{n*(n-3)/2} * Y1^{n*(n-3)/2} - 1/3 X1^{n*(n-3)/2} * Y1^{n*(n-3)/2} + 1/4 * X1^{n*(n-3-2*k)/2} * Y1^{n*(n-3+2*k)/2} = X1^{n*(n-3-2*k)/2} * Y1^{n*(n-3)/2} (2/3*X1^{k} + 1/4*Y1^{k})$$

$$E1 = \underline{2/3*X1^{k} + 2/3*Y1^{k}} - 2/3*Y1^{k} + 1/4*Y1^{k} = -7/12*Y1^{k} + ...$$

7 without a remain not divided by 5.

For n=3 it is well-proven by Euler.