Local functions given integral average

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Abstract. Purpose – to establish and tabulate the integral relationship between the average and the weights for the direct construction of serendipity elements (12 nodes) from the corresponding Lagrangian element (16 nodes). The proposed procedure eliminates the need for making condensation element matrix equation, and is suitable for the general case of any exception parameters of nodes located within the element.
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Key Terms. Model,FormalMethod,MathematicalModeling.

1 Introduction

With the development of high-speed computers has evolved the idea of sampling the computational domain and localization in small areas – media. Usually local functions satisfying certain interpolation hypothesis, for example, Lagrange or Hermite. It is so constructed finite element method (FEM). Of interest are local functions, satisfying not only the interpolation hypothesis, but also certain additional condition. Most often it is preassigned integral characteristic. For example, integral averages are used in the problem of localization of the final load carrier as well as the construction of approximate integration formulas, according to Newton - Cotes. From this point of view, we consider here the local (finite) function of two arguments.

1.1 Analysis of previous publications

Specific problems in the theory of approximation of functions with the conditions described in [1]. Key ideas FEM model and bicubic interpolation (model A) can be found in [2], [3]. In [3] discussed a rolling paradoxical distribution uniform mass force on serendipity elements, in particular, on the bicubic elements. At least, the statement looks strange Zienkiewicz that unnatural load distribution on the standard serendipity element ensures more accurate results. I wonder what could compare Zienkiewicz, if in 1971, when the book was published [3], was not yet open alternative models serendipity elements. Our calculations [4] show that Zienkiewicz mistaken. Provide higher accuracy serendipity elements physically adequate load distribution. Non-standard (*alternative*) model serendipity elements appeared in 1982. [5] Their appearance has stimulated the development of the theory of serendipity approximations. Over the past 30 years, this theory has been enriched with new constructive method serendipity elements. The following describes one such method, based on the decomposition of polynomials serendipity Lagrange polynomials of the same order. This procedure is very simple and convenient as Lagrangian polynomials have become classics in the interpolation theory.

The main part. 1 shows a Lagrangian element bicubic ($|x| \leq 1, |y| \leq 1$). Lagrangian model for transformation in serendipity enough to eliminate the internal nodes. This should be done in terms of rolling mass distributions of uniform strength. In other words, the proportion of internal nodes of the load to be distributed between the boundaries nodes. It is easy to understand that the "recipe" of the distribution is the set. The main thing is that this approach is practically impossible to get a negative load, as it happened with the standard model Zienkiewicz [3].

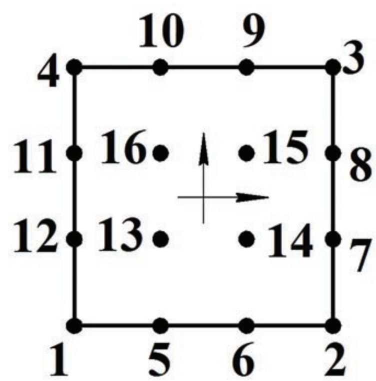

Fig. 1. The finite element of the third order

To find nodes share a single load on the element of Lagrange type 1, it is sufficient to use three local functions (Lagrange interpolation coefficients):

$$\begin{aligned} L_1(x,y) &= \frac{1}{256}(1-x)(1-y)(1-9x^2)(1-9y^2), \\ L_5(x,y) &= \frac{9}{256}(1-x^2)(1-y)(9y^2-1)(1-3x), \\ L_{13}(x,y) &= \frac{81}{256}(1-x^2)(1-y^2)(1-3x)(1-3y). \end{aligned} \tag{1}$$

Of L_1(x,y) is easy to get the other "corner" of the function of L_5(x,y) – the other "intermediate" functions of L_{13}(x,y) – the rest of the "internal" functions.

We denote γ_i share a single node element downloads Lagrange type. As usual, the nodal loads are determined by averaging the integral of local functions:

$$\gamma_i = \frac{1}{S} \iint_D L_i(x,y) dS, \tag{2}$$

This S is area of the integration domain D.

The result of integration is as follows:

$$\gamma_i = \frac{1}{64}, i = \overline{1;4}; \gamma_i = \frac{3}{64}, i = \overline{5;12}; \gamma_i = \frac{9}{64}, i = \overline{13;16}.$$

This result provides a model of the 8-jointed rods and the rule of "*three-eighths*". Local functions serendipity element (after eliminating internal nodes) are defined by the rule [6]:

$$N_i(x,y) = L_i(x,y) + \sum_{k=13}^{16} \alpha_k \cdot L_k(x,y), i = \overline{1;4},$$

$$N_i(x,y) = L_i(x,y) + \sum_{k=13}^{16} \beta_k \cdot L_k(x,y), i = \overline{5;12}. \tag{3}$$

Recall that our focus is nodes 1 and 5. Presented in Table nodal load γ_1 and γ_5, as well as the corresponding coefficients α_k and β_k give the opportunity to find N_1(x,y) and N_5(x,y). The remaining polynomials defined serendipity type of N_1(x,y) and N_5(x,y). We show how to use the table. Natural to start with the standard model [2], [3], which Ergatoudis, Irons and Zienkiewicz found the selection in 1968 α_k coefficients required for N_1(x,y) and β_k for N_5(x,y) are written in the first capital of the table. By 3 we obtain:

$$N_1(x,y) = \frac{1}{32}(1-x)(1-y)(-10+9(x^2+y^2)),$$

$$N_5(x,y) = \frac{9}{32}(1-x^2)(1-y)(1-3x). \tag{4}$$

4 Can be easily and quickly get the remaining 10 functions using the symmetry element 1. Local functions 4 have the necessary properties of the Lagrange interpolation coefficients. They can be obtained from a cubic polynomial:

$$u(x,y) = c_1 + c_2 x + c_3 y + c_4 x^2 + c_5 xy + c_6 y^2 + c_7 x^3 + c_8 x^2 y + c_9 xy^2 + c_{10} y^3 + \\ + c_{11} x^3 y + c_{12} xy^3, \tag{5}$$

Making and solving system of linear equations 12×12. In the scientific literature the model 4, referred to as standard. The main drawback of standard models is physical inadequacy of the uniform distribution of a rolling mass force. As a rule, the vertices are negative element of loading (see table). Asking table gives unlimited possibilities for modeling bicubic local functions, including the physically adequate range of nodal loads. Take, for example, the fourth column of the table with integral average $\gamma_1 = \frac{2}{48}$, $\gamma_5 = \frac{5}{48}$.

By 3 we obtain:

$$N_1(x,y) = \frac{1}{32}(1-x)(1-y)(1-3xy)(2+3x+3y),$$

$$N_5(x,y) = \frac{3}{32}(1-x^2)(1-y)(1-9x-2y). \tag{6}$$

Generally, for the nodes at the vertices of the element (i=1, 2, 3, 4):

$$N_i(x,y) = \frac{1}{32}(1+x_ix)(1+y_iy)(1-3x_ixy_iy)(2-3x_ix-3y_iy), x_iy_i = \pm 1.$$

For intermediate nodes (i=5, 6, 9, 10):

$$N_i(x,y) = \frac{3}{32}(1-x^2)(1+y_iy)(27x_ix+2y_iy+1), x_i = \pm\frac{1}{3}, y_i = \pm 1.$$

For intermediate nodes (i=7, 8, 11, 12):

$$N_i(x,y) = \frac{3}{32}(1-y^2)(1+x_ix)(2x_ix+27y_iy+1), x_i = \pm 1, y_i = \pm\frac{1}{3}.$$

Note that in this model are hidden additional degrees of freedom. In other words, the polynomial of the type 5 the number of parameters exceeds the number of nodes on element. This means that the inverse matrix is not applicable here. Until now, in such cases, we used probabilistic and geometric modeling [AJBAS]. Now offering table. The user has a choice.

Table 1. Nodal load γ_i and weights α_k, β_k

γ_1	-$\frac{6}{48}$	0	$\frac{1}{48}$	$\frac{2}{48}$	$\frac{3}{48}$	$\frac{4}{48}$	$\frac{5}{48}$	$\frac{6}{48}$	$\frac{7}{48}$	$\frac{8}{48}$	$\frac{9}{48}$	$\frac{10}{48}$	$\frac{11}{48}$	$\frac{12}{48}$
α_{13}	-$\frac{24}{54}$	-$\frac{6}{54}$	-$\frac{3}{54}$	0	$\frac{3}{54}$	$\frac{6}{54}$	$\frac{9}{54}$	$\frac{12}{54}$	$\frac{15}{54}$	$\frac{18}{54}$	$\frac{21}{54}$	$\frac{24}{54}$	$\frac{27}{54}$	$\frac{30}{54}$
α_{14}	-$\frac{6}{27}$	0	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{3}{27}$	$\frac{4}{27}$	$\frac{5}{27}$	$\frac{6}{27}$	$\frac{7}{27}$	$\frac{8}{27}$	$\frac{9}{27}$	$\frac{10}{27}$	$\frac{11}{27}$	$\frac{12}{27}$
α_{15}	-$\frac{6}{54}$	0	$\frac{1}{54}$	$\frac{2}{54}$	$\frac{3}{54}$	$\frac{4}{54}$	$\frac{5}{54}$	$\frac{6}{54}$	$\frac{7}{54}$	$\frac{8}{54}$	$\frac{9}{54}$	$\frac{10}{54}$	$\frac{11}{54}$	$\frac{12}{54}$
α_{16}	-$\frac{6}{27}$	0	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{3}{27}$	$\frac{4}{27}$	$\frac{5}{27}$	$\frac{6}{27}$	$\frac{7}{27}$	$\frac{8}{27}$	$\frac{9}{27}$	$\frac{10}{27}$	$\frac{11}{27}$	$\frac{12}{27}$
β_{13}	$\frac{36}{54}$	$\frac{30}{54}$	$\frac{29}{54}$	$\frac{28}{54}$	$\frac{27}{54}$	$\frac{26}{54}$	$\frac{25}{54}$	$\frac{24}{54}$	$\frac{23}{54}$	$\frac{22}{54}$	$\frac{21}{54}$	$\frac{20}{54}$	$\frac{19}{54}$	$\frac{18}{54}$
β_{14}	0	-$\frac{6}{54}$	-$\frac{7}{54}$	-$\frac{8}{54}$	-$\frac{9}{54}$	-$\frac{10}{54}$	-$\frac{11}{54}$	-$\frac{12}{54}$	-$\frac{13}{54}$	-$\frac{14}{54}$	-$\frac{15}{54}$	-$\frac{16}{54}$	-$\frac{17}{54}$	-$\frac{18}{54}$
β_{15}	0	-$\frac{6}{54}$	-$\frac{7}{54}$	-$\frac{8}{54}$	-$\frac{9}{54}$	-$\frac{10}{54}$	-$\frac{11}{54}$	-$\frac{12}{54}$	-$\frac{13}{54}$	-$\frac{14}{54}$	-$\frac{15}{54}$	-$\frac{16}{54}$	-$\frac{17}{54}$	-$\frac{18}{54}$
β_{16}	$\frac{18}{54}$	$\frac{12}{54}$	$\frac{11}{54}$	$\frac{10}{54}$	$\frac{9}{54}$	$\frac{8}{54}$	$\frac{7}{54}$	$\frac{6}{54}$	$\frac{5}{54}$	$\frac{4}{54}$	$\frac{3}{54}$	$\frac{2}{54}$	$\frac{1}{54}$	0
γ_5	$\frac{18}{96}$	$\frac{12}{96}$	$\frac{11}{96}$	$\frac{10}{96}$	$\frac{9}{96}$	$\frac{8}{96}$	$\frac{7}{96}$	$\frac{6}{96}$	$\frac{5}{96}$	$\frac{4}{96}$	$\frac{3}{96}$	$\frac{2}{96}$	$\frac{1}{96}$	0

From a physical standpoint α_k coefficient numerically equal share of the load to γ_k which gives internal node k (k=$\overline{13;16}$) edge node 1. Accordingly β_k is the proportion of the load of the internal node k, which is transmitted to the boundary node 5.

Negative coefficients indicate that the condensation can be "in transit" when one edge node transmits some of its load to another edge node via an internal node k. Moreover conveys more possible.

From a geometrical point of view α_k (k=$\overline{13;16}$) is the value of the surface N_1(x,y) at the corresponding point with coordinates x=$\pm\frac{1}{3}$; y=$\pm\frac{1}{3}$. The same applies to β_k and N_5(x,y). Changing α_k, β_k we manage shaping surfaces N_i(x,y), that allows you to receive preassigned integral means.

In the proposed table, we sought to eliminate the abnormal spectra nodal loads, considering the range: $0 \leq \gamma_i \leq \frac{1}{4}$. The first column is a tribute to our predecessors (Ergatoudis, Irons and Zienkiewicz), who discovered a wonderful serendipity family of standard polynomials. The table was made for N_1(x,y) and N_5(x,y). This is enough for a complete set of local functions without rebuilding the table. If it is not necessary to further table easily make the method of bisection. As expected, a linear relationship prevails (arithmetic progression).

Conclusion. The study of the ability to generate a local function with three arguments on the tetrahedron and hexahedron with internal nodes.

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