

Fermat's Theorem

$$X^n + Y^n = Z^n \quad [1.1]$$

n is a prime number, $n > 2$; X, Y, Z are integers.

Decisions can be X, Y -coprime numbers.

There are two options: X, Y -even and odd, Z -odd

or X, Y - is odd, Z -even.

1. Decomposition [1.1] on multipliers.

If n is odd number, then $X^n + Y^n$ will on multipliers :

$$X^n + Y^n = (X + Y) (X^{n-1} - X^{n-2} * Y + \dots - X * Y^{n-2} + Y^{n-1})$$

where the second brace geometric progression

the first member of the $a_1 = X$, and q multiplier $= -Y/X$.

The sum of which $S = a_1(1 - q^n) / (1 - q)$:

$$S = (X^n + Y^n) / (X + Y)$$

$$Z = Z_{11} * Z_{22}$$

where

$$Z_{11} = X + Y$$

$$Z_{22} = X^{n-1} - X^{n-2} * Y + \dots - X * Y^{n-2} + Y^{n-1}$$

2. Another view of the Z_{22} .

If we summarize the regularly spaced members of environments

-the Member of the progression of the Z_{22} in pairs are:

$$X^{n-1} + Y^{n-1} = (X + Y) (X^{n-2} + Y^{n-2}) - X * Y^{n-2} - Y * X^{n-2} =$$

$$= \dots - X * Y (X^{n-3} + Y^{n-3}) = \dots - X * Y (X + Y) (X^{n-4} + Y^{n-4}) +$$

$$+ X^2 * Y^2 + Y^2 * X^2 = \dots + X^2 * Y^2 (X^{n-5} + Y^{n-5}) =$$

$$= \dots + X^{(n-3)/2} * Y^{(n-3)/2} * (X + Y + 2 * X * Y - 2 * X * Y) =$$

$$= \dots (-1)^{n-2} * X^{(n-1)/2} * Y^{(n-1)/2} * (X + Y + 2 * X * Y - 2 * X * Y) =$$

$$- X^{(n-3)/2} * Y^{(n-3)/2} - X^{(n-3)/2} * Y^{(n-3)/2} = -X * Y (X^{n-2} + Y^{n-2}) = \dots +$$

$$+ X^{(n-1)/2} * Y^{(n-1)/2} * (X + Y) + (-1)^{n-2} * X^{(n-1)/2} * Y^{(n-1)/2}$$

...

$$\begin{aligned}
& X^{(n-3)/2} * Y^{(n+1)/2} + X^{(n+1)/2} * Y^{(n-3)/2} = \\
& = X^{(n-3)/2} * Y^{(n-3)/2} * (X^2 + Y^2 + 2*X*Y - 2*X*Y) = \\
& = X^{(n-3)/2} * Y^{(n-3)/2} * (X + Y)^2 - 2^{(n-1)/2} * X * Y \\
& * X^{(n-1)/2} * Y^{(n-1)/2}
\end{aligned}$$

and the sum of the average member of the progression

and it said this:

$$\begin{aligned}
& (-1)^{(n-1)/2} * X^{(n-1)/2} * Y^{(n-1)/2} + \\
& + (-1)^{(n-1)/2} * X^{(n-1)/2} * Y^{(n-1)/2} = n * (-1)^{(n-1)/2} * X^{(n-1)/2} * Y^{(n-1)/2}
\end{aligned}$$

According to the above Z22 representable as:

$$Z22 = k * (X + Y)^2 + n * (-1)^{(n-1)/2} * X^{(n-1)/2} * Y^{(n-1)/2}$$

$$Z11 = X + Y$$

If X + Y does not have a multiplier-n

$$Z11 = Z1^n$$

$$Z22 = Z2^n$$

where Z1 and Z2-inter simple, otherwise between X and

Z must be a common factor that is not valid.

Consider the case where X is divided into n;

Let X contains n. then let Z = Y + A :

$$X + Y = (Y + A)^n$$

After the conversion, we have:

$$\begin{aligned}
X & = n * Y^{n-1} * A + n * (n-1)/2 * Y^{n-2} * A^2 + \dots + \\
& + n * Y * A^{n-1} + A^n \quad [2.1]
\end{aligned}$$

First cut the prime number n would lead to his presence in A

member and since he is the only one in [2.1] without

multiplier of n and repeated reduction of n its presence and

will need to divide by n and Y.

That is not permissible; X and Y coprime numbers.

Similarly, these thoughts in the p2 we have:

$$Z - Y = X^n \quad [2.2]$$

$$Z - X = Y^n \quad [2.3]$$

3. take $Z = X + Y - B$, then:

$$X^n + Y^n = (X + Y - B)^n,$$

over the bracket and get:

$$\begin{aligned} X^n + Y^n &= (X + Y)^n - n * (X + Y)^{n-1} * B + \dots + \\ &+ n * (X + Y)^{n-1} * B - B^n \quad [3.1] \end{aligned}$$

Unveiling the $(X + Y)^n$ and leave out both parts of the X and Y

and then the only member of the B^n -factor n .

B has a factor of n .

Then, if $X + Y$ is not included n :

$$B = X^n - X^n = Y^n - Y^n = Z^n - Z^n \quad [3.2]$$

$$X = X^n + n * X^{n-1} * Y * Z^{n-2} * K \quad [3.3]$$

$$Y = Y^n + n * X * Y^{n-1} * Z^{n-2} * K \quad [3.4]$$

$$Z^n = X^n + n * X^{n-1} * Y * Z^{n-2} * K + Y^n \quad [3.5]$$

where is k - any integer

$$Z^n = X^n + Y^n$$

The other view will cause the numerical $K, X^n,$

Y^n , change that does not affect the [3.5]

$$2 * K = [(X^n + Y^n) - X^n - Y^n] / n * X^{n-1} * Y * Z^{n-2} \quad [3.6]$$

4. Let $X^n + Y^n = Z^n$

Right side [3.6]-cannot be even number.

According to Newton's binomial Z^n has the form:

$$\begin{aligned} X^{n-3} + E_2 * X^{n-4} * Y + \dots + E_S * X^{(n-3)/2} * Y^{(n-3)/2} + \dots + \\ + E_2 * X^{n-4} * Y + Y^{n-3} \quad [4.1] \end{aligned}$$

where E is the sum of the corresponding combinations

replication $X_1^{\{ \}} * Y_1^{\{ \}} .$

When X_1, Y_1 different parity obviously [4.1]-odd.

Cut on the Z_1 the right piece [3.5]

$$[(X_1 + Y_1)^{n-1} - (X_1^{n-1} - X_1^{n-2} * Y_1 + \dots - X_1 * Y_1^{n-2} + Y_1^{n-1})] /$$

$/n * X_1 * Y_1$ [4.2]

If X_1 and Y_1 -odd, the second brace contained n -odd members, and expression is odd.

We pass to the general case

$$Z_1 = X_1 + Y_1 + a$$

where a -any integer with a sign $+$ or $-$.

According to [3.4] we have:

$$(X_1 + Y_1 + a)^n = X_1^n + n * X_1^{n-1} * Y_1 + n * X_1^{n-2} * Y_1^2 + \dots + n * X_1 * Y_1^{n-1} + Y_1^n$$

and further we will present:

$$2K = K_1 + K_2$$

where K_1 -odd number, at which $Z_1 = X_1 + Y_1$

After transformation we have:

$$(X_1 + Y_1)^n + n * (X_1 + Y_1)^{n-1} * a + \dots + n * (X_1 + Y_1) * a^{n-1} + a^n = X_1^n + n * X_1^{n-1} * Y_1 * K_1 + n * X_1^{n-1} * Y_1 * K_2 + Y_1^n$$

From what follows:

$$n * X_1^{n-1} * Y_1 * (X_1 + Y_1) * K_2 = n * (X_1 + Y_1)^{n-1} * a + \dots + n * (X_1 + Y_1) * a^{n-1} + a^n [4.3]$$

a^n and $X_1 + Y_1$ has the general multiplier, we will call it (c) .

Thus

(c) belonging to $\in (a) - c^i$;

(c) belonging to $\in (X_1 + Y_1) - c^j$.

If $c^j > c^i$ that

$$K = [(X1 + Y1 + a)^n - (X1+Y1)(X1^{n-1} - \dots + Y1^{n-1})] / 2^n X1 Y1 (X1 + Y1 + a)$$

that K contains (c) that is impossible as between Z2 and Z1 there will

be the general multiplier that follows from [3.2]

$$Z1 - Z2 = n X1 Y1 K \text{ and it isn't admissible.}$$

We will note also that $(X1^{n-1} - \dots + Y1^{n-1})$ has no multiplier (c) of p2.

It agrees contains (a) all multipliers belonging above $X1+Y1$.

We will designate $a/(X1 + Y1)^j = L$.

$$Z1 = X1 + Y1 + L(X1 + Y1)^j$$

$$K2 = 2 * K - K1$$

$$K2 = [(X1 + Y1 + L(X1 + Y1)^j)^n - X1^n - Y1^n] / n X1 Y1 [(X1 + Y1 + L(X1 + Y1)^j)^j - (X1 + Y1)^n - X1^n - Y1^n] / n X1 Y1 (X1 + Y1)$$

after transformation we have:

$$K2 = [(X1 + Y1 + L(X1 + Y1)^j)^n * (X1 + Y1)^n - (X1 + Y1)^n * [(X1 + Y1 + L(X1 + Y1)^j)^j + L(X1 + Y1)^j * (X1 + Y1)^n] / n X1 Y1 (X1 + Y1) * [(X1 + Y1 + L(X1 + Y1)^j)^j]$$

$$K2 = L * (X1 + Y1)^{n-1+j-1} / n X1 Y1 + L * (X1 + Y1)^{j-1} * (X1 + Y1)^n /$$

$$n X1 Y1 [X1 + Y1 + L(X1 + Y1)^j]$$

$$K2 = [X1 + Y1 + L(X1 + Y1)^j]^n * L * (X1 + Y1)^{n+j-2} + L * (X1 + Y1)^{j-1} * (X1 + Y1)^n / n X1 Y1 [X1 + Y1 + L(X1 + Y1)^j]$$

$$K2 = (X1 + Y1)^{j-1} * \{ [X1 + Y1 + L(X1 + Y1)^j]^j * L * (X1 + Y1)^{n-1} + L * (X1 + Y1)^n \}$$

$$/ n X1 Y1 [X1 + Y1 + L(X1 + Y1)^j] \quad [4.4]$$

Brace {} contents after reduction on $(X1 + Y1)$ in a denominator doesn't may contain the specified multiplier as the first composed

him supports, and the second isn't present $L * (X1 + Y1)^n / (X1 + Y1)$

p.2. So the possible multiplier in $K2$ is $(X1 + Y1)^{j-1}$ [4.4]

That the decision in integers for [4.3] needed to be had in $K2$

multiplier $(X_1 + Y_1)$ in that minimum degree which contains member on the right side

$$n * (X_1 + Y_1)^{n-1} * a = n * (X_1 + Y_1)^{n-1} * (X_1 + Y_1)^j =$$

$$= n * (X_1 + Y_1)^{n-1+j}$$

There is no coincidence of possible and necessary K_2 .

Thus $a = 0$.

In a consequence of that in case of $X + Y$ doesn't contain n multiplier, in integers there is no decision.

5. If $X + Y$ is a factor of n , that the only possible according to p4.

The only available option

$$Z_{11} = n^{(m*n - 1)} * Z_1^n \quad [5.1]$$

$$Z_{22} = n * Z_2^n \quad [5.2]$$

as set out in paragraph2 Z_{22} is divided into general n and more.

The option where in the X or Y is a factor of n has no solution in integers as Z already contains n .

Which means:

$$2 * Z = X_1^n + Y_1^n + Z_{11}$$

$X_1^n + Y_1^n$ contains a factor of n , which follows

from the

respective grouped expression based on Newton's

binomial:

$$(X_1 + Y_1)^n - n * X_1^{n-1} * Y_1 - \dots - n * X_1 * Y_1^{n-1} + Y_1^n =$$

$$- \dots - C_{n-1}^1 * X_1^{n-1} * Y_1 + C_{n-1}^2 * X_1^{n-2} * Y_1^2 - \dots - C_{n-1}^{n-1} * X_1 * Y_1^{n-1} + Y_1^n =$$

where odd degree have the multiplier $(X1 + Y1)$

together with the n .

So [5.1] minimum $m = 2$.

Suppose there exist integers $X = X1, Y = Y1, Z$ [1.1]

implementation is carried out. then as shown

above there is

$$X1 + Y1 = n^{m*n-1} * Z1^n$$

$$Z22 = n * Z2^n$$

$$Z = n^{m*n} * Z1^n * Z2^n$$

And with that Z [1.1] has more solutions to $X2 = n^m * X3;$

$$Y2 = n^m * Y3; Z = n^m * Z1^n * Z2^n$$

$$X3 + Y3 = Z1^n$$

$$Z11 = n^m * Z1^n$$

$$Z22 = n^{nm-m} * Z2^n$$

$$X1 + Y1 = X2 + Y2 = n^{m*n} * Z1^n * Z2^n = Z$$

However second decision

$$n^{m*n} * X3^n + n^{m*n} * Y3^n = n^{m*n} * (Z1 * Z2)^n$$

not solvable in whole numbers

Since the reduction of the n we get the equivalent:

$$X3^n + Y3^n = (Z1 * Z2)^n$$

where the $Z11$ is not n which means that there

is no integer solutions p.4 and p.5.

QED!