The temperature of charged black holes revisited

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Abstract

The radiation of particles by charged black holes in (1+1) dimensions is revisited. We consider the process of quantum tunnelling of particles through two (event and Cauchy) horizons. It is shown that the emission temperature for the ReissnerNordström background geometry is the same as the Hawking temperature for the Schwarzschild black hole and does not depend on the charge of a black hole.

The black hole radiation of scalar particles for the Schwarzschild background was investigated by Hawking [1]. Black holes can be considered as a quantum systems. The entropy and temperature of black holes may be evaluated within quantum mechanics. It was shown with the help of quantum mechanical calculations that the thermal radiation is due to a horizon of the Schwarzschild spacetime. This evaluation uses the classical curved spacetime without affecting the background. The radiation of scalar particles as a quantum tunnelling effect near the horizon was considered also in [2], [3], [4], [5]. That approach is based on the WKB approximation for the calculation of the tunnelling probability via the horizon. The trajectory is classically forbidden. Such method was also applied for studying the radiation of black hole in different spacetime backgrounds. As a result, the quantum tunnelling method allows us to learn thermodynamic properties of black holes and to investigate the black hole radiation.

The popular point of view is that the temperature is associated with each horizon of the black hole in the backgrounds possessing several horizons. We consider the evaporation of the charged black hole with the ReissnerNordström background to have the unique temperature. This fact is based on the equilibrium thermodynamics. Thus, we imply the thermodynamic stability of the charged black holes. As a result, the radiation temperature for two horizons is the same and the spacetime is stable.

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We apply the tunnelling method for the calculation of the imaginary part of the action for the emission processes through the horizons that are classically forbidden. The imaginary part of the action for the particle emission at some temperature is similar to the Boltzmann factor. Thus, the tunnelling probability within the WKB approximation for the classically forbidden path from inside to outside the horizons is

$$P = \exp\left(-2\mathrm{Im}\frac{S}{\hbar}\right), \tag{1}$$

where S is the classical action of the trajectory at the leading order in $\hbar$. With the aid of Eq.(1) one may evaluate the quantum tunnelling probability. We also note that a black hole emission spectrum is similar to a black body radiation [6]. In this letter we explore the tunnelling method for the case of the emission of particles from non-rotating charged black holes. The system of units $c = k_B = 1$ is used.

We consider the black hole in (1+1) dimension with the Reissner-Nordström metric

$$ds^2 = -A(r)dt^2 + \frac{1}{A(r)}dr^2, \qquad A(r) = 1 - \frac{2GM}{r} + \frac{G^2Q^2}{r^2}. \tag{2}$$

where G is gravitational (Newton's) constant, and M, Q are the mass and charge of a black hole respectively. We study only a motion of particles in the radial direction and, therefore, the spherically symmetric two dimensional part is neglected here. The event horizon and an internal Cauchy horizon [7] are in the points $r = r_+$ and $r = r_-$, correspondingly, where $1/A(r)$ diverges, $A(r_\pm) = 0$:

$$r_\pm = GM \pm G\sqrt{M^2 - Q^2}. \tag{3}$$

Applying the WKB approximation, solutions to wave equations for particles with different spins can be obtained in the form

$$\psi = \psi_0 \exp\left(\frac{i}{\hbar}S(t,r)\right), \tag{4}$$

where ψ_0 is constant spinor for fermions or a vector for spin-1 bosons or a scalar for spinless particles. The action is given by

$$S(t,r) = S_0(t,r) + \hbar S_1(t,r) + \hbar^2 S_2(t,r) + ... \tag{5}$$

The solution to wave equations in the leading order to $\hbar$ may be found in the form

$$S_0 = -Et + W(r) + C, \tag{6}$$

where E is an energy, C is a complex constant and $W(r)$ is as follows [8], [9], [10]:

$$W_{\pm}(r) = \pm \int \frac{\sqrt{E^2 - m^2 A(r)}}{A(r)} dr. \tag{7}$$

For massless particles one has to put $m = 0$ in Eq.(7). For the outgoing ($p_r = \partial_r S_0 > 0$) motion of particles we use W_+, and for ingoing ($p_r = \partial_r S_0 < 0$) - W_-. We study a trajectory of particles in the direction r from the inside to the outside of the horizons, and therefore, the W_+ will be used. From Eq.(2) one finds

$$\frac{1}{A(r)} = \frac{r^2}{2G\sqrt{M^2 - Q^2}} \left(\frac{1}{r - r_+} - \frac{1}{r - r_-} \right). \tag{8}$$

Thus, the expression (7) possesses a simple poles at the horizons. If $r \neq r_{\pm}$ the integral (7) is well defined and real, but for a path going through the points $r_{\pm}$ the integral is not defined because $A^{-1}(r_{\pm}) = \infty$. We use a replacement $r_{\pm} \to r_{\pm} - i\varepsilon$ for outgoing particles [11] to calculate the integral for crossing the horizons $r_{\pm}$. As a result, we specify the complex contour that may be used for the calculating the integral around $r = r_{\pm}$. To normalize the probability, one should use the relation ImC=-ImW_-=ImW_+ [12], so that from Eq.(6) we have $\text{Im}S_0 = 2\text{Im}W_+$. In this case there is not a reflection. We need the imaginary part of C for singular coordinates (that can be considered as boundary conditions) which are not well-defined across the horizon. One may evaluate the imaginary part of the integral (7) with the help of [13]

$$\frac{1}{r - i\varepsilon} = i\pi\delta(r) + \mathcal{P}\left(\frac{1}{r}\right), \tag{9}$$

and $\mathcal{P}\left(\frac{1}{r}\right)$ means the principal value of $1/r$. Calculating the integral in Eq.(7) using Eqs.(8),(9), we obtain

$$\text{Im}W_+ = 2\pi GEM. \tag{10}$$

From Eqs.(1), in leading order of $\hbar$, one finds the tunnelling probability

$$P = \exp\left(-\frac{4}{\hbar}\text{Im}W_+\right) = \exp\left(-\frac{8\pi GME}{\hbar}\right). \tag{11}$$

From (11) and the Boltzmann expression one finds the emission temperature of the charged black hole

$$T = \frac{\hbar}{8\pi GM} \tag{12}$$

which coincides with the Hawking's temperature for the Schwarzschild black hole and does not depend on the charge. This conclusion is in contrast with commonly accepted expression $T = k_+\hbar/(2\pi) = \hbar(r_+ - r_-)/(4\pi r_+^2)$ for the temperature of charged black holes. Our claim was based on the quantum tunnelling method applied for two horizons and implying the equilibrium temperature associated with the event and Cauchy horizons.

Thus, we have demonstrated with the aid of the quantum tunnelling method that the Hawking temperature takes place for charged black holes emission of particles because of two horizons. Here only eternal black holes were considered, which forms gravitational background geometry and a thermal spectrum obtained is inconsistent with the energy conservation as the background is fixed. Taking into account energy conservation one may obtain E^2 corrections to the black hole radiation spectrum [5].

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