Slowly Rotating Black Holes in Brans-Dicke-Maxwell Theory

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In this paper, we construct a class of (n+1)-dimensional ($n \geq 4$) slowly rotating black hole solutions in Brans-Dicke-Maxwell theory with a quadratic potential. These solutions can represent black holes with inner and outer event horizons, an extreme black hole and a naked singularity and they are neither asymptotically flat nor (anti)-de Sitter. We compute the Euclidean action and use it to obtain the conserved and thermodynamics quantities such as entropy, which does not obey the area law. We also compute the angular momentum and the gyromagnetic ratio for these type of black holes where the gyromagnetic ratio is modified in Brans-Dicke theory compared to the Einstein theory.

Keywords Black Holes, Thermodynamics, Brans-Dicke Theory.

I. INTRODUCTION

Brans-Dicke(BD) theory [1] is perhaps the most common alternative theory to the Einstein's general relativity. This theory contains both Mach's principle and Dirac's large number hypothesis. The theory has recently received interest as it arises naturally as the low energy limit of many theories of quantum gravity such as the supersymmetric string theory or Kaluza-Klein theory, and is also found to be consistent with present cosmological observations [2]. The theory contains an adjustable parameter ,ω, that represents the strength of the coupling between matter and the scalar field. For certain values of ω, the BD theory agrees with GR in the post-Newtonian limit up to any desired accuracy and hence weak-field observations cannot rule out the BD theory in favor of general relativity [3], although the singularity problem remains.

Shortly after the appearance of this theory one of its authors, C. Brans, obtained the statically spherically symmetric solutions [4]. Since then many authors have investigated black holes in Brans-Dicke theory [5]. Hawking proved in four dimensions the stationary and vacuum Brans-Dicke solution is just the Kerr solution with a constant scalar field everywhere [6]. Cai and Myung have proved that in four dimensions, the charged black hole solution in the Brans-Dicke-Maxwell theory is just the Reissner-Nordstrom solution with a constant scalar field [7]. The Kerr-Newman type black hole solutions, which are different from the solutions in general relativity, have been

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constructed for $-5/2 < \omega < -3/2$ in [8]. Thermodynamics of black holes in Brans-Dicke theory is investigated [9].

On the other hand, the rotating black hole solutions in higher dimensional Einstein gravity was found by Myers and Perry [10]. The solutions were uncharged and can be considered as a generalization of the four dimensional Kerr solutions. Moreover it has recently been shown that the gravity in higher dimensions contains much richer dynamics than in four dimensions. As an example, there exists a black ring solution in five dimensions with the horizon topology of $S^2 \times S^2$ [11], which has the same mass and angular momentum as the Myers-Perry solution and therefore contradicts the uniqueness theorem in five dimensions. Although the non-rotating black hole solutions in higher dimensional Einstein-Maxwell gravity was found many years ago [12], the analytic solution of a generalization of the charged Myers-Perry solution in $(n+1)$-dimensional Einstein Maxwell gravity has not been found yet. Solutions of different kinds of charged rotating black holes in higher dimensions have been discussed in the framework of supergravity and string theory [13–15]. In [16], the solutions of charged rotating black hole in $(n+1)$-dimensional Einstein-Maxwell theory with a single rotation parameter in the limit of slow rotation have been constructed. In addition, [17] contains a class of charged slowly rotating black hole solutions in Gauss-Bonnet gravity. Rotating black holes in Einstein-Maxwell-Dilaton gravity is discussed in [26].

In this paper we investigate charged slowly rotating black holes in Brans-Dicke theory by using the conformal transformation between dilaton fields and Brans-Dicke theory. The structure of this paper is as follows: In section II we obtain the solution of charged rotating Brans-Dicke black holes and discuss their causal structure, then in section III we obtain the conserved quantities of the finite action by using the Euclidean action and section IV contains our results.

II. SLOWLY ROTATING BLACK HOLES IN BRANS-DICKE-MAXWELL THEORY

The action of the Brans-Dicke-Maxwell theory in $(n+1)$-dimensions with a scalar field Φ and a self-interacting potential $V(\Phi)$ is

$$I_G = -\frac{1}{16\pi}\int_{\mathcal{M}} d^{n+1}x\sqrt{-g}(\Phi\mathcal{R} - \frac{\omega}{\Phi}(\nabla\Phi)^2 - V(\Phi) - F_{\mu\nu}F^{\mu\nu}) \tag{1}$$

where $\mathcal{R}$ is the Ricci scalar, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the electromagnetic tensor field, A_μ the vector potential, ω the coupling constant and Φ is the BD scalar field. By varying the the action (1) with respect to the metric $g_{\mu\nu}$, the scalar field Φ and vector field A_μ, one can obtain the following field equations

$$\begin{aligned} G_{\mu\nu} &= \frac{\omega}{\Phi^2}(\nabla_\mu\Phi\nabla_\nu\Phi - \frac{1}{2}g_{\mu\nu}(\nabla\Phi)^2) - \frac{V(\Phi)}{2\Phi}g_{\mu\nu} + \frac{1}{\Phi}(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^2\Phi) \\ &+ \frac{2}{\Phi}(F_{\mu\lambda}F_\nu^\Lambda - \frac{1}{2}F_{\rho\sigma}F^{\rho\sigma}g_{\mu\nu}) \end{aligned} \tag{2}$$

$$\nabla^2\Phi = -\frac{n-3}{2[(n-1)\omega+n]}F^2 + \frac{1}{2[(n-1)\omega+n]}\left[(n-1)\Phi\frac{dV(\Phi)}{d\Phi} - (n+1)V(\phi)\right] \tag{3}$$

$$\nabla_\mu F^{\mu\nu} = 0 \tag{4}$$

where $G_{\mu\nu}$ is the Einstein tensor. It is not easy to solve the field equations (2)-(4) directly because the right hand side of eq. (2) includes the second derivatives of the scalar field. Fortunately, we can transform these field equations to the dilaton field equations by conformal transformations. If one uses the following conformal transformations

$$\begin{aligned} \bar{g}_{\mu\nu} &= \Phi^{2/(n-1)}g_{\mu\nu} \\ \bar{\Phi} &= \frac{n-3}{4\alpha}ln\Phi \end{aligned} \tag{5}$$

where

$$\alpha = (n-3)/\sqrt{4(n-1)\omega+4n} \tag{6}$$

then the action (1) takes the form

$$\bar{I}_G = -\frac{1}{16\pi}\int_{\mathcal{M}} d^{n+1}x\sqrt{-\bar{g}}\{\bar{\mathcal{R}} - \frac{4}{n-1}(\bar{\nabla}\ \bar{\Phi})^2 - \bar{V}(\bar{\Phi}) - e^{-\frac{4\alpha\bar{\Phi}}{n-1}}\bar{F}_{\mu\nu}\bar{F}_{\mu\nu}\} \tag{7}$$

where $\bar{\mathcal{R}}$ and $\bar{\nabla}$ are the Ricci scalar and covariant derivative corresponding to the metric $\bar{g}_{\mu\nu}$ and $\bar{V}(\bar{\Phi})$ is:

$$\bar{V}(\bar{\Phi}) = \Phi^{-(n+1)/(n-1)}V(\Phi) \tag{8}$$

Eq. (7) is simply the action of (n+1)-dimensional Einstein-Maxwell-dilaton gravity, where $\bar{\Phi}$ is the dilaton field, $\bar{V}(\bar{\Phi})$ a potential for $\bar{\Phi}$ and α is a constant that determines the strength of coupling of the scalar and electromagnetic field $\bar{F}_{\mu\nu}$. By varying the action (7) with respect to $\bar{g}_{\mu\nu}$, $\bar{\Phi}$ and $\bar{F}_{\mu\nu}$, we obtain

$$\bar{\mathcal{R}}_{\mu\nu} = \frac{4}{n-1}(\bar{\nabla}_\mu\bar{\Phi}\bar{\nabla}_\nu\bar{\Phi} + \frac{1}{4}\bar{V}(\bar{\Phi})\bar{g}_{\mu\nu}) + 2e^{-4\alpha\bar{\Phi}/(n-1)}(\bar{F}_{\mu\lambda}\bar{F}_\nu^\lambda - \frac{1}{2(n-1)}\bar{F}_{\rho\sigma}\bar{F}^{\rho\sigma}\bar{g}_{\mu\nu}) \tag{9}$$

$$\bar{\nabla}^2\bar{\Phi} = \frac{n-1}{8}\frac{\partial \bar{V}}{\partial \bar{\Phi}} - \frac{\alpha}{2}e^{-4\alpha\bar{\Phi}/(n-1)}\bar{F}_{\rho\sigma}\bar{F}^{\rho\sigma} \tag{10}$$

$$\bar{\nabla}_\mu[\sqrt{-\bar{g}}e^{-4\alpha\bar{\Phi}/(n-1)}\bar{F}^{\mu\nu}] = 0 \tag{11}$$

Many authors have obtained the solutions of above field equations [18–26]. It is now simple to obtain the solutions of field equations (2)-(4) by applying the conformal transformations (5) to the solution of the field equations (9)-(11). In [26], the solution of field equations (9)-(11) has been obtained in the slowly rotating case where $a \ll 1$. In this case the only term in the metric which changes to $O(a)$ is $g_{t\phi}$, and A_ϕ is the only component of the vector potential that changes, where the dilaton field does not change to $O(a)$. Therefore, for infinitesimal angular momentum up to $O(a)$, we can take the following form for the metric in (n+1)-dimension for Einstein-Maxwell-dilaton theory [26]

$$\begin{aligned} ds^2 = & - U(r)dt^2 + \frac{dr^2}{U(r)} - 2af(r)sin^2\theta dtd\phi \\ & + r^2R^2(r)(d\theta^2 + sin^2\theta d\phi^2 + cos^2\theta d\Omega_{n-3}^2) \end{aligned} \tag{12}$$

where U(r), R(r) and f(r) are functions of r, and a is a parameter associated with its angular momentum and $d\Omega_{n-3}^2$ denotes the metric of an unit $(n-3)$-sphere. For small values of a, U(r) is a function only of r. From equation (11) we can obtain the t component of the Maxwell equations

$$\bar{F}_{tr} = \frac{qe^{4\alpha\bar{\Phi}/(n-1)}}{(rR)^{n-1}} \tag{13}$$

where q is an integration constant related to the electric charge of the black hole. By using the definition $Q = \frac{1}{4\pi}\int exp[-4\alpha\bar{\Phi}/(n-1)]Fd\Omega$ we obtain the electric charge as

$$Q = \frac{q\omega_{n-1}}{4\pi} \tag{14}$$

where ω_{n-1} represents the volume of a hypersurface with constant curvature. In general, when we have a rotational parameter, there is also a vector potential of the form

$$A_\phi = aqh(r)sin^2\theta \tag{15}$$

In [26], for a Liouville-type dilaton potential, is introduced for $\bar{V}(\bar{\Phi})$

$$\bar{V}(\bar{\Phi}) = 2\Lambda^{2\zeta\bar{\Phi}} \tag{16}$$

where Λ and ζ are constants. The unknown functions $U(r), f(r)$ and $R(r)$ are obtained using the ansatz in [26]

$$R(r) = e^{2\alpha\bar{\Phi}/(n-1)} \tag{17}$$

By substituting eq. (17), the Maxwell fields (13) and (15) and the metric (13) into the field equations (9)-(11), we can obtain

$$U(r) = -\frac{(n-2)(\alpha^2+1)^2 b^{-2\gamma} r^{2\gamma}}{(\alpha^2-1)(\alpha^2+n-2)} - \frac{m}{r^{(n-1)(1-\gamma)-1}} + \frac{2q^2(\alpha^2+1)^2 b^{-2(n-2)\gamma}}{(n-1)(\alpha^2+n-2)} r^{2(n-2)(\gamma-1)} \tag{18}$$

$$f(r) = \frac{m(\alpha^2+n-2)b^{(n-3)\gamma}}{\alpha^2+1} r^{(n-1)(n-\gamma)+1} - \frac{2q^2(\alpha^2+1)b^{(1-n)\gamma}}{n-1} r^{2(n-2)(\gamma-1)} \tag{19}$$

$$\bar{\Phi} = \frac{(n-1)\alpha}{2(1+\alpha^2)} ln(\frac{b}{r}) \tag{20}$$

$$h(r) = r^{(n-3)(\gamma-1)-1} \tag{21}$$

where $\gamma = \alpha^2/(1+\alpha^2)$ and b is an arbitrary constant. In addition we should have

$$\zeta = \frac{2}{\alpha(n-1)}, \qquad \Lambda = \frac{(n-1)(n-2)\alpha^2}{2b^2(\alpha^2-1)} \tag{22}$$

in order to fully satisfy the field equations.

To obtain the solutions of the field equations (2)-(4) in the Brans-Dicke-Maxwell theory, we take a metric of the form

$$\begin{aligned} ds^2 = & - A(r)dt^2 + \frac{dr^2}{B(r)} - 2ag(r)sin^2\theta dt d\phi \\ & + r^2 H^2(r)(d\theta^2 + sin^2\theta d\phi^2 + cos^2\theta d\Omega^2_{n-3}) \end{aligned} \tag{23}$$

To determine the unknown functions $A(r), B(r), g(r)$ and $H(r)$ we apply the conformal transformations (5), (6) and (8) to eqs. (17),(18) and (20). Leading to

$$\begin{aligned} A(r) = & -\frac{(n-2)(\alpha^2+1)^2 b^{-2\gamma(\frac{n-5}{n-3})} r^{2\gamma(\frac{n-5}{n-3})}}{(\alpha^2-1)(\alpha^2+n-2)} \\ & + \frac{2q^2(\alpha^2+1)^2 b^{2\gamma(2-n+\frac{2}{n-3})}}{(n-1)(\alpha^2+n-2)} r^{2(n-2)(\gamma-1)-\frac{4\gamma}{n-3}} - \frac{m b^{\frac{4\gamma}{n-3}} r^{\gamma(n-1-\frac{4}{n-3})}}{r^{(n-2)}} \end{aligned} \tag{24}$$

$$
\begin{aligned}
B(r) \ &= \ -\frac{(n-2)(\alpha^2+1)^2 b^{-2\gamma(\frac{n-1}{n-3})} r^{2\gamma(\frac{n-1}{n-3})}}{(\alpha^2-1)(\alpha^2+n-2)} \\
&+ \ \frac{2q^2(\alpha^2+1)^2 b^{-2\gamma(n-2+\frac{2}{n-3})}}{(n-1)(\alpha^2+n-2)} r^{2(n-2)(\gamma-1)+\frac{4\gamma}{n-3}} - \frac{m b^{-\frac{4\gamma}{n-3}} r^{\gamma(n-1+\frac{4}{n-3})}}{r^{(n-2)}}
\end{aligned}
\tag{25}
$$

and

$$
g(r) = \frac{m(\alpha^2+n-2) b^{(n-3+\frac{4}{n-3})\gamma}}{\alpha^2+1} r^{(n-1)(n-\gamma)+1-\frac{4\gamma}{n-3}} - \frac{2q^2(\alpha^2+1) b^{(1-n+\frac{4}{n-3})\gamma}}{n-1} r^{2[(n-2)(\gamma-1)-\frac{2\gamma}{n-3}]}
\tag{26}
$$

$$
H(r) = (\frac{b}{r})^{\gamma\frac{n-5}{n-3}}
\tag{27}
$$

$$
\Phi(r) = (\frac{b}{r})^{2\gamma\frac{n-1}{n-3}}
\tag{28}
$$

$$
F_{tr} = \frac{q b^{\gamma(3-n)}}{r^{(n-3)(1-\gamma)+2}}
\tag{29}
$$

$$
V(\Phi) = 2\Lambda\Phi^{\frac{1}{\alpha(n-2)}[(\alpha+1)-4]}
\tag{30}
$$

At this point, it is worthwhile to investigate the physical properties of these solutions. We can show that the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $r=0$, and it is finite for $r \neq 0$ and

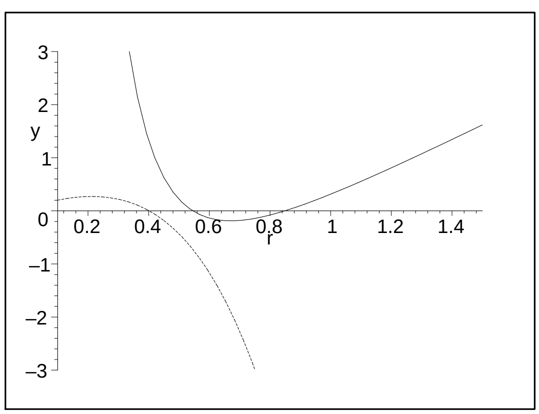

FIG. 1: The function B(r) versus r for $n=4$, $m=2$, $b=1$ and $q=1$. $\alpha=0.5$ (bold line) and $\alpha=1.2$ (dashed line)

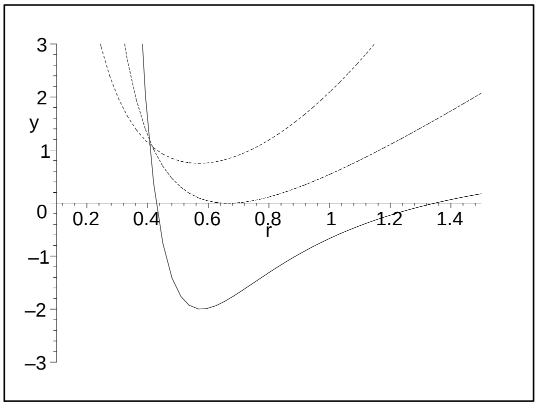

FIG. 2: The function B(r) versus r for $n = 4$, $m = 2$, $b = 1$ and $q = 1$, $\alpha = 0$ (bold line), $\alpha = 0.54$ (dashed line) and $\alpha = 0.7$ (dotted line)

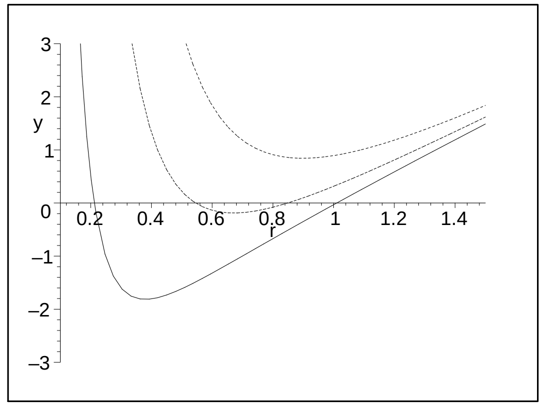

FIG. 3: The function V(r) versus r for $n = 4$, $m = 2$ and $b = 1$, $\alpha = 0.5$ $q = 0.5$(bold line), $q = 1$ (dashed line) and $q = 1.5$ (dotted line)

approaches to zero as $r \to 0$. We find that there is an essential singularity at r=0. In addition, the solution is ill-defined for $\alpha = 1$ and the cases $\alpha > 1$ and $\alpha < 1$ should be considered separately. For $\alpha > 1$, there exists a cosmological horizon (fig. 1) whereas there is no cosmological horizon for $\alpha < 1$. In this case eq. (24) contains a wide range of causal structure which depends on the values of the metric parameters α, m, q and k (fig. 2-3).

Moreover we can obtain some information about causal structure by considering the temperature of the horizons. By using the definition of Hawking temperature on the outer horizon r_+, which may be obtained through the definition of surface gravity

$$T_+ = \frac{1}{2\pi}\sqrt{-\frac{1}{2}(\nabla_\mu \chi_\nu)(\nabla^\mu \chi^\nu)} \tag{31}$$

where χ is the Killing vector ∂_t, we can write

$$T_+ \;=\; -\frac{(\alpha^2+n-2)m}{4\pi(\alpha^2+1)} r_+^{(n-1)(\gamma-1)} + \frac{(n-2)(\alpha^2+1)b^{-2\gamma}}{2\pi(1-\alpha^2)} r_+^{2\gamma-1} \tag{32}$$

We see from the above equation that the temperature is invariant under conformal transformations [26], which is a result of the regularity of the conformal parameter at the horizon. For $\alpha > 1$ we find that the temperature is negative from eq. (32). Numerical calculations show that the temperature of the event horizon goes to zero as the black hole approaches an extreme one. In addition we can show that for ($\alpha < 1$)

$$m_{ext} = \frac{2(n-2)(\alpha^2+1)^2 b^{-2\gamma}}{(n-\alpha^2)(\alpha^2+n-2)} r_+^{(2-n)(\gamma-1)+\gamma} \tag{33}$$

In summary, the metric (24) can represent a rotating black hole with two inner and outer horizons located at r_+ and r_- provided that the mass parameter m is greater than m_{ext}, an extreme black hole when $m = m_{ext}$, and a naked singularity when $m < m_{ext}$.

The electric potential U, measured at infinity with respect to the horizon, is defined by [27]

$$U = A_\mu \chi^\mu \mid_{\to\infty} - A_\mu \chi^\mu \mid_{\to 0} \tag{34}$$

where χ is the null generator of the event horizon. Therefore we obtain:

$$U = \frac{q b^{\gamma(3-n)}}{\Xi \Gamma r_+^\Gamma} \tag{35}$$

where $\Gamma = \gamma(3-n) + n - 2$.

III. EUCLIDEAN ACTION AND CONSERVED QUANTITIES

The ADM (Arnowitt-Deser-Misner) mass M, entropy S and electric potential U of the topological black hole can be calculated through the use of the Euclidean action method [28]. In this approach, the electrical potential and the temperature are initially fixed on a boundary with a fixed radius r_+. The Euclidean action has two parts; bulk and surface. The first step to make the Euclidean action is to substitute t with $i\tau$ (this change does not affect our physical parameters, especially the angular momentum, see eq. (44)). This makes the metric positive definite:

$$\begin{aligned} ds^2 = &+ A(r)dt^2 + \frac{dr^2}{B(r)} - 2ag(r)sin^2\theta dt d\phi \\ &+ r^2 H^2(r)(d\theta^2 + sin^2\theta d\phi^2 + cos^2\theta d\Omega^2_{n-3}) \end{aligned} \tag{36}$$

There is a conical singularity at the horizon $r = r_+$ in the Euclidean metric. To eliminate it, the Euclidean time τ is made periodic with period β, where β is the inverse of Hawking temperature. We can now calculate the Euclidean action of $(n+1)$-dimensional Brans-Dicke-Maxwell theory. It can be obtained analytically and by continuously changing of action (1) to Euclidean time τ, i.e.,

$$I_{GE} = -\frac{1}{16\pi} \int_{\mathcal{M}} d^{n+1}x\sqrt{g} \left(\Phi R - \frac{\omega}{\Phi}(\nabla\Phi)^2 - V(\Phi) - F_{\mu\nu}F^{\mu\nu} \right) + \frac{1}{8\pi} \int d^n x \sqrt{h} \Phi (K - K_0), \tag{37}$$

where K represents the extrinsic curvature on the induced metric h, and K_0 is the extrinsic curvature on the metric h for flat space-time, which must be added so that the Euclidean action is normalized to zero in flat space-time. Using the metric (37), we find to $O(1)$

$$R = -g^{-1/2}(g^{1/2}U'V/U)' - 2G_0^0 + \mathrm{O}(a^2) + \mathrm{O}(a^4)$$
$$K = -\frac{\sqrt{B(r)}}{2}\frac{A'(r)rH(r) + 2(n-2)A(r)H(r) + (2n-7)A(r)rH'(r)}{A(r)rH(r)}$$
$$K_0 = \frac{2(n-3)-\gamma}{2r}\sqrt{\frac{(n-2)(\alpha^2+1)^2}{(\alpha^2-1)(\alpha^2+n-2)}}(\frac{r}{b})^{-\frac{n-1}{n-3}} \tag{38}$$

where G_0^0 is the 0-0 component of the Einstein tensor. By substituting eq. (38) in action (37) and using eqs. (24)-(30), we obtain

$$\begin{aligned} I_{GE} &= \beta\frac{\omega_{n-1}}{16\pi}\left(\frac{b^{(n-1)\gamma}(n-1)}{(\alpha^2+1)}\right) - \frac{\omega_{(n-1)}}{4l^{n-2}}\left(b^{(n-1)\gamma}r_+^{(n-1)(1-\gamma)}\right) \\ &- \beta\frac{\omega_{n-1}}{8\pi}\frac{(n-\alpha^2)(\alpha^2+n-2)b^{2(n-2)\gamma}\omega_{n-1}}{n(n-2)(\alpha^2+1)^2}ma^2 - \beta\frac{q^2}{8\pi\Gamma r_+^{\Gamma}} \end{aligned} \tag{39}$$

where $\Gamma = (n-3)(1-\gamma)+1$. According to Ref. [29], the thermodynamical potential can be given by I_{GE}

$$I_{GE} = \beta M - S - \beta U q - \beta\Omega J \tag{40}$$

where M is the ADM mass, S the entropy and, U the electric potential corresponding to the conservation of charge q and $\Omega = a$ in this case. Comparing eqs. (39) and (40), we find

$$M = \frac{b^{(n-1)\gamma}}{16\pi}\frac{n-1}{1+\alpha^2}\omega_{(n-1)}m, \tag{41}$$

$$S = \frac{b^{(n-1)\gamma}r_+^{(n-1)(1-\gamma)}}{4l^{(n-2)}} \tag{42}$$

and

$$Q = \frac{q\omega_{(n-1)}}{4\pi} \tag{43}$$

$$J = \frac{(n-\alpha^2)(\alpha^2+n-2)b^{2(n-2)\gamma}\omega_{n-1}}{8\pi n(n-2)(\alpha^2+1)^2}ma \tag{44}$$

We can see from the above equations that the ADM mass, entropy and electric potential are invariant under the conformal transformation [18]. In addition, in the context of BD gravity, where we have the additional gravitational scalar degree of freedom, the entropy of the black hole does not follow the area law. This is due to the fact that the black hole entropy comes from the boundary term in the Euclidean action formalism.

In addition, the charge which is calculated in eq. (43) is the same as the one which was calculated in eq. (14). By combining eqs. (41) and (44), we can write:

$$J = \frac{2Ma}{n-1} \tag{45}$$

In order to calculate the gyromagnetic ratio of this type of black hole, we first need the magnetic dipole moment for slowly rotating black holes, i.e., $\mu = Qa$, then the gyromagnetic ratio is given by

$$g = \frac{2\mu M}{QJ} = \frac{n(n-1)(n-2)(\alpha^2+1)}{(n-\alpha^2)(\alpha^2+n-2)b^{(n-3)\gamma}} \tag{46}$$

As our solutions are neither asymptotically flat nor (A)dS, we get $g \geq 2$ in four dimension, in contrast to asymptotically flat or (A)dS which have $g \leq 2$ in four dimensions [30]. In the absence of a non-trivial dilaton ($\alpha = \gamma = 0$), the gyromagnetic ratio reduces to:

$$g = n - 1 \tag{47}$$

IV. CONCLUSIONS

In this paper we construct the solutions of slowly rotating black holes in $(n+1)$-dimensional Brans-Dicke-Maxwell theory with a liouville-type potential in the limit of a slow rotation parameter, with an arbitrary value of the coupling constant ω. Our solutions are neither asymptotically flat nor (A)dS, in contrast to the rotating black holes in the Einstein-Maxwell theory. The solutions are ill-defined for $\alpha = 1$ and for $\alpha > 1$ we have cosmological horizons and there are no cosmological horizons for $\alpha < 1$. In the latter case ($\alpha < 1$), we can have a black hole with inner and outer event horizons if $m > m_{ext}$, an extreme black hole if $m = m_{ext}$ and a naked singularity for $m < m_{ext}$. The cosmological horizons have a negative temperature for $\alpha > 1$. We computed the Euclidean action and obtained the thermodynamics and conserved quantities. The temperature and entropy for this type of black hole were found to equal those in the static case to $O(a)$. In addition entropy does not follow the area law. Moreover we obtained angular momentum and gyromagnetic ratio for this rotating Brans-Dicke black hole. The gyromagnetic ratio is modified in this theory.

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