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Gravitational "Doppler Boosting / De-boosting" Effect within the Framework of General Relativity

M. Zilberman[1]

Shiny World Corp., Canada

Abstract

The "Doppler boosting / de-boosting" relativistic effect increases / decreases the apparent luminosity of approaching / receding sources of radiation. This effect was analyzed in detail within the Special Relativity framework and was confirmed in many astronomical observations. It is however not clear if "Doppler boosting / de-boosting" exists in the framework of General Relativity as well, and if it exists, which equations describe it.

The "Einstein's elevator" and Einstein's "Equivalence principle" allow to obtain the formula for "Doppler boosting / de-boosting" for a uniform gravitational field within the vicinity of the emitter/receiver. Under these simplified conditions, the ratio $\mathcal{M}$ between apparent (L) and intrinsic (L_o) luminosity can be conveniently represented using source's spectral index α and gravitational redshift z as $\mathcal{M}(z, \alpha) \equiv L/L_o=(z+1)^{\alpha-3}$. This is the first step towards the complete set of equations that describe the gravitational "Doppler boosting / de-boosting" effect within the General Relativity framework including radial gravitational field and arbitrary values of distance *h* between emitter and receiver.

Keywords: General Relativity, Doppler boosting, Equivalence principle, de-boosting, Einstein elevator

Introduction. "Doppler boosting / de-boosting" in Special Relativity

The "Doppler boosting / de-boosting" relativistic effect increases / decreases the apparent luminosity of approaching / receding sources of radiation.

The illustration fig.1 (re-drawn from the figure 11.2 of [1]) illustrates the distribution of energy in "boosting" and "de-boosting" zones of relativistic radiation sources. The arrows show the directions of photons in the observer's rest frame for a source that emits isotropically in its own rest frame and moves to the right at various velocities. In accordance to [1] "for a relativistic source moving with a Lorentz factor $\Gamma = (1-\beta^2)^{-1/2} \gg 1$ (in the lab frame), half of the photons and most (3/4) of the emitted energy are within an angle of $1/\Gamma$ around its direction of motion".

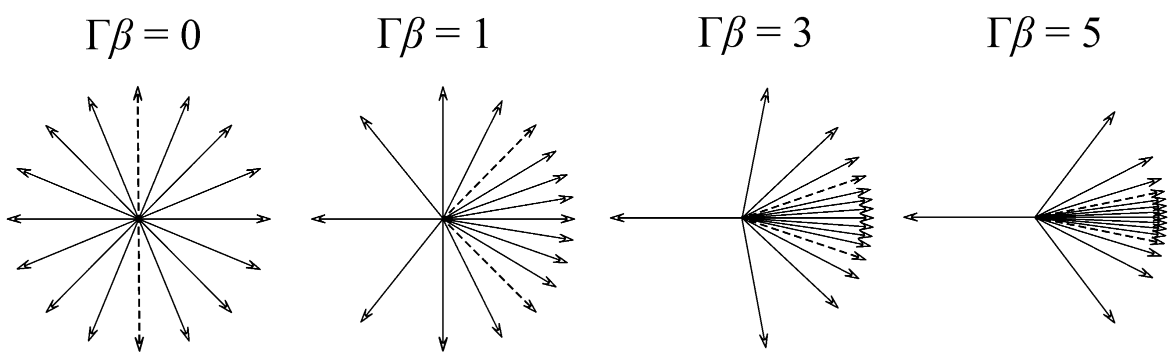

Figure 1. Relativistic boosting and de-boosting. Half of the photons (and 3/4 of the radiated energy) are within an angle of $1/\Gamma$ around the direction of motion (between the dashed arrows, which correspond to θ=90° in rest frame.) Re-drawn from the figure 11.2 of [1].

[1] email address: MZilberman137@gmail.com

This effect was analyzed in detail within the Special Relativity framework and was confirmed in many astronomical observations. “Doppler boosting” and “de-boosting” were successfully taken into account in the analysis of relativistic jets of active galactic nuclei (AGN), in the observation of double white dwarfs, in research of pulsars, in search of exoplanets and stars in binary systems, and in the analysis of gamma-ray bursts (GRBs) [2, 3, 4, 5, 6, 7, 8, 9].

The relationship between the apparent luminosities (L), intrinsic luminosities (L_o) and spectral index given by formula (1) from Lister 2003, [3].

$L = L_o \delta^p$ (1).

Here the Doppler factor δ is

$\delta=\gamma^{-1}(1-\beta \cos \theta)^{-1}$ (2),

the Lorentz factor γ is

$\gamma = (1-\beta^2)^{-1/2}$ (3),

the velocity β is the velocity v of a relativistic light source normalized to the speed of light c

$\beta = v/c$ (4),

θ is the angle between line of sight and the velocity direction, α is the spectral index ($S_\nu \propto \nu^{\alpha}$) of emitter, and p=3-$\alpha$ for a discrete emitting region.

Analysis. “Doppler boosting / de-boosting” in General Relativity

It is however not clear if “Doppler boosting / de-boosting” also exists in the framework of General Relativity, and if it exists, which equations describe it. The following section will analyze this in further detail.

“Doppler boosting” is a combination of 3 individual relativistic effects, namely relativistic aberration, time dilation and Doppler shifts. All 3 of these effects also exist within General Relativity. Gravitational aberration has been known since the famous experiment that detected gravitational deflection in 1919 [10] to the modern observation of gravitational lensing and black hole shadows [11, 12]. Gravitational time dilation was confirmed since the Pound and Rebka experiment [13] and has continued with recent research of gravitational redshift tracked by “Radioastron” [14]. Doppler shifts are the basis of cosmology.

As per the thought experiment, described by Misner, Thorne and Wheeler in “Einstein’s elevator” model [15], we can expect that the observer in the elevator that moves with a constant acceleration g>0 will observe “Doppler de-boosting” from the radiation emitter, which is located in the same elevator, below the receiver. (Authors in [15] use word “rocket” instead of “elevator”, but we will continue to use “elevator” here as per Einstein’s term.)

Indeed, in the thought experiment [15] the receiver, which is located within the elevator above the emitter, observes Doppler’s redshift as the velocity of the receiver is greater than the velocity of the emitter (located within the same elevator) as soon as the acceleration g>0. This difference in velocities between the emitter and receiver occurs due to the fact that during the time interval

ΔT required for photons radiated by the emitter to reach the receiver, the receiver's velocity increases due to the elevator's acceleration.

For the same reason, luminosity of the emitter measured by the receiver will appear reduced, since the velocity of the receiver is greater than the velocity of the emitter (due to acceleration) and the luminosity of the receding emitter is de-boosted.

Since the observer within the elevator does not know if the acceleration *g* is produced by gravity or by the elevator's acceleration, in accordance with the Einstein's "Equivalence principle", we can expect that "Doppler de-boosting" will be observed for gravity as well.

In the first approximation we may try to calculate the gravitational boosting/de-boosting effect using the relationship between the apparent luminosities (L), intrinsic luminosities (L_o) and spectral index given by formula (1) above.

In the model with "Einstein's elevator" the velocity v of the receiver is $v = g\Delta T$, where ΔT is the time required by photons radiated by the emitter to reach the receiver and g is the acceleration.

If h is the distance between the emitter and receiver, then for a short h the time interval ΔT is $\Delta T \approx h/c$. (The distance h is "short" if photon's additional travel due to distancing of receiver is negligible compared to h.)

Therefore, velocity v of the receiver is $v = g\Delta T = gh/c$ (5),

and normalized velocity $\beta = v/c = gh/c^2$ (6).

Correspondingly $\gamma = (1-\beta^2)^{-1/2} = [1-(gh/c^2)^2]^{-1/2}$ (7).

Since $\delta = \gamma^{-1}(1-\beta \cos \theta)^{-1}$, we have $\delta = [1-(gh/c^2)^2]^{1/2} (1-gh/c^2 \cos \theta)^{-1}$ (8).

Assuming that the receiver is located exactly above the emitter, in the direction of acceleration g (i.e. that $\theta = 180^0$), we have

$$\delta = [1-(gh/c^2)^2]^{1/2} (1+gh/c^2)^{-1} \quad (9),$$

and

$$L = L_o \{[1-(gh/c^2)^2]^{1/2} (1+gh/c^2)^{-1}\}^p \quad (10).$$

Since p=3-α for a discrete emitting region we have

$$L = L_o \{[1-(gh/c^2)^2]^{1/2} (1+gh/c^2)^{-1}\}^{(3-\alpha)} \quad (11),$$

where α is the spectral index ($S_\nu \propto \nu^{\alpha}$) of the emitter.

Formula (11) can be conveniently represented using the Doppler shift parameter z. This parameter depends only on the relative velocities of the transmitter and receiver, which allows us to get rid of the distance h. In addition, it can be measured directly by the receiver at any time.

Since velocity v of the receiver is $v = g\Delta T = gh/c$ we can replace part gh/c in formula (11) with the velocity v and rewrite formula (11) as

$$L = L_o \{[1- (v/c)^2]^{1/2} (1+v/c)^{-1} \}^{(3-\alpha)} \qquad (12),$$

$$\text{or } L = L_o \{(1+v/c) [1- (v/c)^2]^{-1/2}\}^{(\alpha-3)} \qquad (13).$$

Since the redshift parameter z and velocity v are linked as

$$z+1 = (1+v/c) (1-v^2/c^2)^{-1/2} \qquad (14),$$

we can replace in (13) entire part $\{(1+v/c) [1- (v/c)^2]^{-1/2}\}$ with (z+1) and finally have the formula (11) rewritten using redshift parameter z as

$$L = L_o (z+1)^{\alpha-3} \qquad (15)$$

$$\text{or } \mathcal{M}(z, \alpha) \equiv L/L_o = (z+1)^{\alpha-3} \qquad (16)$$

where $\mathcal{M}(z, \alpha)$ is the ratio between the apparent and intrinsic luminosities as a function of gravitational redshift z and spectral index α.

Formulas (11) and (16) were obtained in simplified conditions of a uniform gravitational field (since we used the model with "Einstein's elevator" and the Equivalence principle) and for short distances h between the emitter and receiver. This is the first step towards the complete set of equations that describe the "Doppler boosting / de-boosting" effect within the General Relativity framework.

Conclusions

The "Einstein's elevator" and Einstein's "Equivalence principle" allow to obtain the formula for "Doppler boosting / de-boosting" for a uniform gravitational field within the vicinity of the emitter/receiver. This is the first step towards the complete set of equations that describe the "Doppler boosting / de-boosting" effect within the General Relativity framework including radial gravitational field and arbitrary values of distance h.

References

1. Granot J. & Ramirez-Ruiz E. Jets and gamma-ray burst unification schemes, 2012, in Gamma-ray Bursts, eds. Chryssa Kouveliotou, Ralph A. M. J. Wijers and Stan Woosley. Published by Cambridge University Press, 2012

2. Kellermann, K.I., Kovalev, Y.Y., Lister, M.L. et al. Doppler boosting, superluminal motion, and the kinematics of AGN jets. 2007, Astrophys Space Sci 311, 231–239

3. Lister M. L. Altered luminosity functions of relativistically beamed jet populations, 2003, ApJ, 599,105–115

4. Shporer A., Kaplan D. L., Steinfadt J. D. R., et al. A Ground-based Measurement of the Relativistic Beaming Effect in a Detached Double White Dwarf Binary. The Astrophysical Journal Letters, Volume 725, Issue 2, pp. L200-L204 (2010).

5. Zhou, J., & Su, Y. Doppler boosting and de-boosting effects in relativistic jets of AGNs and GRBs, 2006, Proceedings of the IAU, 2(S238), 477-478

6. Yang J., Brocksopp C., Corbel S., Paragi Z., Tzioumis T., Fender R. P.. A decelerating jet observed by the EVN and VLBA in the X-ray transient XTE J1752−223, 2010, Monthly Notices of the Royal Astronomical Society: Letters, 409, 1, L64–L68

7. Li K. L. et al The X-Ray Modulation of PSR J2032+4127/MT91 213 during the Periastron Passage in 2017, 2018, ApJ 857, 123

8. Placek B. Relativistic Beaming as a Probe of Stellar and Planetary Masses. IOP Conf. Series: Journal of Physics: Conf. Series 1239 (2019) 012008, doi:10.1088/1742-6596/1239/1/012008

9. Massi M., Torricelli-Ciamponi G. Intrinsic physical properties and Doppler boosting effects in LS I +61^{0}303, 2014, A&A 564, A23

10. Dyson F., Eddington A., Davidson C. 1920 IX. A determination of the deflection of light by the sun's gravitational field, from observations made at the total eclipse of May 29, 1919. Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character 220: 291–333

11. Chang Z., Zhu Q. Black hole shadow in the view of freely falling observers. Journal of Cosmology and Astroparticle Physics, Volume 2020, June 2020

12. Arakida, H. General Relativistic Aberration Equation and Measurable Angle of Light Ray in Kerr–de Sitter Spacetime. Universe 2021, 7, 173.

13. Pound R. V., Rebka G. A., Jr., Phys. Rev. Lett. 3, (1959)

14. Nunes, N.V., Bartel, N, Bietenholz, M.F., et al. The gravitational redshift monitored with RadioAstron from near Earth up to 350,000 km, 2020, Adv. Space Res. Vol. 65, Issue 2, 790-797

15. Misner, Charles W.; Thorne, Kip S.; Wheeler, John Archibald (1973), Gravitation, San Francisco: W. H. Freeman, ISBN 978-0-7167-0344-0, pp.189-190