The radiation by space-time periodic dielectric medium

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Abstract

We derive the power spectrum of photons generated by the plane wave in dielectric medium. The experimental observation of such radiation in such medium can be considered as the integral part of the aurora borealis and australis. The consequence of the dielectric plane wave form of vacuum generated by the gravitational waves is mentioned.

Key words: Space-time variable dielectric medium, Vavilov-Čerenkov radiation, energy loss, gravitational waves.

1 Introduction

The plane wave in the dielectric medium is the specific case of the non-stationary dielectric medium which is defined as a medium where some parameters such as index of refraction, magnetic permeability, electric permitivity are changed by external influences, in the form of electric field, magnetic field, acoustical and ultrasound field, thermal fluctuations, thermal waves, mechanical pressure, phase transitions of medium and so on (Bass et al. 1965).

The radiation of the non-stationary dielectric medium follows from the Maxwell equations and it is produced also in the case where the velocities of charges are subluminal in contradiction to the case of the Vavilov-Čerenkov radiation, where the fast moving charged particle radiates only if its speed is faster than the speed of light in this medium.

While the discovery idea of the Vavilov-Čerenkov radiation was originated by Heaviside (1889), who calculated radiation of the charged object moving in a medium faster than electromagnetic waves in the same medium, the non-stationary dielectric origin of radiation was established long after the discovery of the transition radiation by Ginzburg in 1940 (Ginzburg, 1986).

Kelvin (1901), presented also an idea that the light emission of particles is possible at the speed of them greater than that of the velocity of light. However, he never considered the no-stationary medium. Sommerfeld (1904), proposed the hypothetical radiation with a sharp angular distribution, but not the radiation by the non-stationary medium.

While the electromagnetic Vavilov-Čerenkov radiation was first observed in the early 1900's by experiments developed by Marie and Pierre Curie when studying radioactivity emission via observation of the emission of a bluish-white light from transparent substances in the neighborhood of strong radioactive source, there is no information on so called non-stationary medium radiation. The first attempt to understand the origin of the so called Vavilov-Čerenkov effect was made by Mallet (1926, 1929a, 1929b), who observed that the light emitted by a variety of transparent bodies placed close to a radioactive source always had the same bluish-white quality, and that the spectrum was continuous, with no line or band structure characteristic of fluorescence. However, the transition radiation and the radiation by the non-stationary medium was observed after long time when investigations were forgotten for many years and when Čerenkov experiments (Čerenkov, 1934) was performed at the suggestion of Vavilov who opened a door to the true physical nature of the this effect (Bolotovskii, 2009).

The Vavilov-Čerenkov radiation was first theoretically interpreted by Tamm and Frank (1937) in the framework of the classical electrodynamics. The source theoretical description of this effect was given by Schwinger et al. (1976) at the zero temperature regime and the classical spectral formula was generalized to the finite temperature situation and for the massive photons by author (Pardy, 1989; 2002). The Vavilov-Čerenkov effect was also used by author (Pardy, 1997) to possible measurement of the Lorentz length contraction. It was supposed in all cases that the dielectric medium was stationary. The novelty of this article is to consider effect with the non-stationary medium.

2 The Schwinger formulation of the Vavilov-Čerenkov radiation in the stationary dielectric medium

Let us first remember the Vailov-Čerenkov radiation in the stationary medium calculated by means of the Schwinger source theory methods. The Schwinger source theory (Schwinger et al., 1976) is the theoretical construction which uses quantum mechanical particle language. Initially it was constructed for description of the particle physics situations occurring in the high-energy physics experiments. However, it was found that the original formulation simplifies the calculations in the electrodynamics and gravity where the interactions are mediated by photon or graviton respectively. In case that the index of refraction is some constant, then it is possible to use the Schwinger source theory formulation of the problem.

The basic formula in the source theory is the vacuum to vacuum amplitude (Schwinger et al., 1976):

$$< 0_+|0_- >= e^{\frac{i}{\hbar}W(S)}, \tag{1}$$

where the minus and plus tags on the vacuum symbol are causal labels, referring to any time before and after space-time region where sources are manipulated. The exponential form is introduced with regard to the existence of the physically independent experimental arrangements which has a simple consequence that the associated probability amplitudes multiply and corresponding W expressions add (Schwinger, 1970; Schwinger et al., 1976).

The electromagnetic field is described by the amplitude (1) with the action

$$W(J) = \frac{1}{2c^2}\int (dx)(dx')J^\mu(x)D_{+\mu\nu}(x-x')J^\nu(x'), \tag{2}$$

where the dimensionality of $W(J)$ is the same as the dimensionality of the Planck constant $\hbar$. J_μ is the charge and current densities. The symbol $D_{+\mu\nu}(x-x')$ is the photon propagator and its explicit form will be determined later.

It may be easy to show that the probability of the persistence of vacuum is given by the following formula (Schwinger et al., 1976):

$$|< 0_+|0_- >|^2 = \exp\{-\frac{2}{\hbar}\mathrm{Im}\, W\} \stackrel{d}{=} \exp\left\{-\int dtd\omega\frac{P(\omega,t)}{\hbar\omega}\right\}, \tag{3}$$

where we have introduced the so called power spectral function (Schwinger et al., 1976) $P(\omega,t)$. In order to extract this spectral function from $\mathrm{Im}\, W$, it is necessary to know the explicit form of the photon propagator $D_{+\mu\nu}(x-x')$.

The electromagnetic field is described by the four-potentials $A^\mu(\phi,\mathbf{A})$ and it is generated by the four-current $J^\mu(c\varrho,\mathbf{J})$ according to the differential equation (Schwinger et al., 1976):

$$\left(\Delta - \frac{\mu\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}\right)A^\mu = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right)J_\nu \tag{4}$$

with the corresponding Green function $D_{+\mu\nu}$:

$$D_+^{\mu\nu} = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right)D_+(x-x'), \tag{5}$$

where $\eta^\mu \equiv (1,\mathbf{0})$, μ is the magnetic permeability of the dielectric medium with the dielectric constant ε, c is the velocity of light in vacuum, n is the index of refraction of this medium, and $D_+(x-x')$ was derived by Schwinger et al. (1976) in the following form:

$$D_+(x-x') = \frac{i}{4\pi^2 c}\int_0^\infty d\omega\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}e^{-i\omega|t-t'|}. \tag{6}$$

Using formulas (2), (3), (5) and (6), we get for the power spectral formula the following expression (Schwinger et al., 1976):

$$P(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt'\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\cos[\omega(t-t')]$$

$$\times\left\{\varrho(\mathbf{x},t)\varrho(\mathbf{x}',t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x},t)\cdot\mathbf{J}(\mathbf{x}',t')\right\}. \tag{7}$$

However, in case that the index of refraction is some function of space and time, the above method cannot be used it means that there is no such formula as the formula (7) and it is necessary to use appropriate methods for he calculating of radiation of charges in the non-stationary dielectric medium.

3 The radiation of charges by the plane wave index of refraction

Let us consider the non-stationary dielectric medium with the plane wave index of refraction as the analogue of the Volkov plane wave potential potential, (Volkov 1935), where the Volkov potential is a plane wave with $A_\mu = A_\mu(kx)$, where $kx = \mathbf{k}\cdot\mathbf{x} - \omega t = \chi$ in the Dirac equation. In this case, the so called the Volkov solution of the Dirac equation for an electron moving in a field of a such plane wave was derived and used also by author in the form (Berestetzkii et al., 1989; Pardy, 2003; Pardy, 2004; Pardy, 2007):

$$\psi_p = \frac{u(p)}{\sqrt{2p_0}}\left[1 + e\frac{(\gamma k)(\gamma A(\chi))}{2kp}\right]\exp\left[(i/\hbar)S\right] \tag{8}$$

and S is an classical action of an electron moving in the potential $A(\chi)$:

$$S = -px - \int_0^{kx}\frac{e}{(kp)}\left[(pA) - \frac{e}{2}(A)^2\right]d\chi. \tag{9}$$

So, in our case we define the specification of non-stationary dielectric medium by relations

$$\varepsilon(\mathbf{x},t) = \varepsilon(kx); \quad \mu(\mathbf{x},t) = \mu(kx); \quad n(\mathbf{x},t) = n(kx); \quad kx = \mathbf{k}\cdot\mathbf{x} - \omega t = \chi. \tag{10}$$

The charge and current density of electron moving with the velocity $\mathbf{v}$ and charge e is as it is well known:

$$\varrho = e\delta(\mathbf{x} - \mathbf{v}t) \tag{11}$$

$$\mathbf{j} = e\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t). \tag{12}$$

The equations for the four potential A_μ are given by equations:

$$\Delta\mathbf{A} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{\mu}{c}\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t) \tag{13}$$

and

$$\Delta\varphi - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{1}{\varepsilon}\delta(\mathbf{x} - \mathbf{v}t) \tag{14}$$

with the additional Lorentz calibration condition: $\partial_\mu A^\mu - (\mu\varepsilon - 1)(\eta\partial)(\eta A) = 0$, or,

$$\mathrm{div}\mathbf{A} + \frac{\mu\varepsilon}{c}\frac{\partial\varphi}{\partial t} = 0, \tag{15}$$

where $\eta^\mu = (1, \mathbf{0})$ is the unit time-like vector in the rest frame of medium and

$$J^\mu = (c\varrho, \mathbf{J}), \quad x^\mu = (ct, \mathbf{r}), \quad k^\mu = (k^0, \mathbf{k}). \tag{16}$$

Our goal is to solve modified equation (4), where parameters of dielectric medium are given by eq. (10). Or,

$$\left(\Delta - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\right) A^\mu = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right) J_\nu; \quad n = n(\chi); \quad \mu = \mu(\chi). \tag{17}$$

With regard to the fact that it is not possible to use the mentioned Schwinger method in order to get the solution in the Volkov form, we suppose

$$A^\mu = A^\mu(\chi). \tag{18}$$

Then it is suitable to suppose that current is dependent on the wave symbol χ, or, $J^\mu = J^\mu(\chi)$, which means that he left side of eq. (17) is compatible with its right side. Then, we can look for the solution in the form $A^\mu = A^\mu(\chi)$.

After insertion of $A^\mu(\chi)$ into eq. (17), we get the differential equation for $A^\mu(\chi)$, with $J_\nu = J_\nu(\chi)$.

$$\frac{d^2}{d\chi^2}A^\mu(\chi) = \frac{\frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right)}{k^2 - \frac{n^2}{c^2}\omega^2} J_\nu(\chi); \quad n = n(\chi); \quad \mu = \mu(\chi). \tag{19}$$

After insertion of $J_\nu = const = a_\nu$ into eq. (19), we get:

$$A^\mu(\chi) = \int d\chi \int d\chi \frac{\frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right)}{k^2 - \frac{n^2}{c^2}\omega^2} a_\nu; \quad \chi = \mathbf{k}\cdot\mathbf{x} - \omega t. \tag{20}$$

In case, we take $J_\nu = a_\nu\delta(\chi)$, we get solution :

$$A^\mu(\chi) = \frac{\frac{\mu(0)}{c}\left(g^{\mu\nu} + \frac{n(0)^2-1}{n(0)^2}\eta^\mu\eta^\nu\right)}{k^2 - \frac{n(0)^2}{c^2}\omega^2} a_\nu\chi + const = C^{\mu\nu}a_\nu + const; \quad \chi = \mathbf{k}\cdot\mathbf{x} - \omega t. \tag{21}$$

Let us remark that $\delta(\chi)$ in the current is not zero at point $\mathbf{k}\cdot\mathbf{x} - \omega t = 0$, which can be transcribed as $ax + by + cz + d = 0$, which is the equation of a plane in the coordinate system xyz, where parameter d depends on time and it means that the plane, or the sheet moves in space. So, we solve the radiation problem of the charged sheet moving in the plane wave in dielectric medium, supposing that such charged sheet can be realized in experiment. The moving charged thread, or string is an analogue of our situation.

4 The power spectral formula for plane wave in dielectric medium

Let us define the permitivity, magnetic permeability and consequently dielectric constant and current in the form

$$\varepsilon = \varepsilon(kx); \quad \mu = \mu(kx); \quad n = n(kx); \quad J_\nu = a_\nu\delta(p\chi); \quad a_\nu = const; \quad p = const. \tag{22}$$

Then, with regard to eq. (21) we write the solution in the form:

$$A^\mu = C^{\mu\nu}\frac{a_\nu}{p}\chi + const, \tag{23}$$

where $C^{\mu\nu}$ are the corresponding constants in eq. (21) and constant p defines the difference between potential A_μ phase and the source phase.

It follows from eqs.

$$\mathbf{B} = \mathrm{rot}\mathbf{A}; \quad \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \mathrm{grad}\varphi \tag{24}$$

and eq. (23) that

$$\varphi = A^0 = C^{0\nu}\frac{a_\nu}{p}\chi + const; \quad \mathbf{E}^l = \frac{\omega}{c}C^{l\nu}\frac{a_\nu}{p}\chi - C^{0\nu}\frac{a_\nu}{p}k^l; \quad l = 1, 2, 3. \tag{25}$$

The energy loss is defined by the formula (Sokolov et al., 1974):

$$W_{loss} = Q\int \mathbf{J}\cdot\mathbf{E}d\mathbf{x} = Q\int \frac{a_\nu}{p}\delta(\chi)\left\{\frac{\omega}{c}C^{l\nu}\frac{a_\nu}{p}\chi - C^{0\nu}\frac{a_\nu}{p}k^l\right\}d\mathbf{x}; \quad l = 1, 2, 3 \tag{26}$$

where Q corresponds to a moving charges.

The power spectrum $P(\omega)$ follows from the energy loss formula as its Fourier mapping

$$P(\omega') = const\int_{-\infty}^{\infty} dt W_{loss}(t)e^{i\omega' t} = const\frac{a^l}{p}\frac{1}{\omega'}\left\{\frac{\omega}{c}C^{l\nu}\frac{a_\nu}{p} - C^{0\nu}\frac{a_\nu}{p}k^l\right\}\delta(\mathbf{k}). \tag{27}$$

The corresponding electrical and magnetic intensities are given by the obligate formulas from eq. (24). The corresponding Poynting vector which expresses the amount of radiation energy is as follows:

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B}. \tag{28}$$

After performing the necessary operation for the electric intensity and magnetic induction and after inserting the results into the formula for the Poynting vector, we get

$$(\mathbf{S})_l = \frac{c}{4\pi}\varepsilon_{lmn}\varepsilon_{nuv}E_m\partial_u A_v, \tag{29}$$

where symbols $\varepsilon_{lmn}, \varepsilon_{nuv}$ are the Levi-Civita anti-symmetrical tensors with $\varepsilon_{123} = 1$. Using $\varepsilon_{lmn}\varepsilon_{nuv} = \delta_{lu}\delta_{mv} - \delta_{lv}\delta_{mu}$, we get final formula for the l-component of the Poynting vector:

$$(\mathbf{S})_l = \frac{c}{4\pi}\left(E_v\frac{\partial A_v}{\partial x^l} - E_v\frac{\partial A_l}{\partial x^v}\right) \tag{30}$$

The formula (27) is the original one and it is not excluded that in will play the similar role in physics as the older Vavilov-Čerenkov formula.

5 Discussion

It is interesting to compare the physical features of the stationary Vavilov-Čerenkov effect with the radiation of the non-stationary dielectric medium.

1) While the V-Č radiation arises only for particle velocity greater than the velocity of light in the dielectric medium, the radiation by the non-stationary medium is generated also by particles with subluminal velocities.

2) The V-Č radiation depends only on the charge and not on mass of the moving particles.

The same statement is valid for radiation generated in the non-stationary medium.
3) The V-Č radiation is produced in the visible interval of the light frequencies and partly in the ultraviolet part of the frequency spectrum. The radiation does not exist for very short waves because from the dispersion theory of the index of refraction n it follows that $n < 1$ in a such situation. The radiation of the non-stationary medium is generated over the all spectrum and for every index of refraction
4) The spectral frequency formula is linear function of the frequency for the 3D homogeneous medium. In case of the space-time variable medium, the corresponding spectrum depends on the specification of the variability of medium.
5) The radiation generated in the given point of the trajectory spreads on the surface of cone with the vertex in this point and with the axis identical with the direction of motion of the particle. The vertex angle of the cone is given by the relation $\cos\theta = c/nv$.
6) There is no Mach cone in the 2D dielectric medium. There is only the Mach angle. It follows from the fact that Vavilov-Čerenkov effect is the result of the collective motion of the 2D dielectric medium and it also follows from the quantum definition of the Vavilov-Čerenkov effect in the 2D structures (Pardy, 2012). The same statement is valid for 2D non-stationary dielectric medium.
7) The energy loss of a particle caused by the Vavilov-Čerenkov radiation are approximately equal to 1% of all energy losses in the condensed matter such as the bremsstrahlung and so on. The energy loss of particle depends on the specification of the non-stationary medium.
8) The fundamental importance of the Čerenkov radiation is in its use for the modern detectors of very speed charged particles in the high energy physics (Kleinknecht, 1986). It is not excluded that the radiation generated by the non-stationary medium is of the same importance as the V-Č radiation .
9) The detection of the Vavilov-Čerenkov radiation enables to detect not only the existence of the particle, however also the direction of motion and its velocity and according to eq. $\cos\theta = c/nv$ and also its charge. The same statement is not valid for the non-stationary medium.
10) While the V-Č effect is considered as the special representation of the laboratory condition, the radiation of the non-stationary dielectric medium with pane wave can be considered as the integral part of the astronomical and sky effect. We mean that the description of aurora borealis (nothern lights), or, aurora australis (southern lights), is incomplete without consideration the radiation of the non-stationary sky.

Let us remark in conclusion that the deflection of light by gravitational field can be described as the trajectory of light in vacuum presented as the optical medium with the index of refraction n_g. This index of refraction is $n_g(kx)$ in the presence of the gravitational waves. Then, charged cosmical rays in such space-time periodic index of refraction form the cosmical aurora and this is the easy way how to detect the gravitational waves in cosmical space.

References

Bass, F. G. and Yakovenko, V. M. (1965). Theory of radiation from a charge passing through an electrically inhomogeneous medium, *Physics-Uspekhi* **8** (2), 420-444.
Berestetzkii, V. B.; Lifshitz, E. M. and Pitaevskii, L. P. (1989). *Quantum electrodynamics*

(Moscow, Nauka). (in Russian).
Bolotovskii, B. M. (2009). Vavilov-Cherenkov radiation: its discovery and application, *Physics-Uspekhi* **52**(11), 1099-1110.
Čerenkov, P. A. (1934). The visible radiation of pure liquids caused by γ-rays, *Comptes Rendus Hebdomadaires des Scances de l' Academic des Sciences USSR* **2**, 451.
Ginzburg, V. L. (1986). Radiation by uniformly moving sources, in: *The lessons of Quantum Theory*, ed. by J. DeBoer, E. Dal and O. Ulfbeck (Elsevier Science Publishers) and references therein.
Ginzburg, V. N. and Tsytovič, V. L. (1984). *The transition radiation and the transition scattering*, Moscow. (in Russian).
Heaviside, O. (1889). On the electromagnetic effects due to the motion of electrification through a dielectric, *Philos. Mag.*, S. 5, **27**, 324-339.
Kelvin, L. (1901). Nineteenth century clouds over the dynamical theory of heat and light, *Philos. Mag.*, S. **6**, **2**, 10.
K. Kleinknecht, K. (1986). *Detectors for Particle Radiation*, (Cambridge Univ. Press,).
Mallet, L. (1926). Spectral research of luminescence of water and other media with gamma radiation, *Comptes Rendus*, **183**, 274.; ibid. (1929a).*Comptes Rendus*, **187**, 222.; ibid. (1929b). *Comptes Rendus*, **188**, 445.
Pardy, M. (1989). Finite-temperature Čerenkov radiation, *Physics Letters A* **134**(6), 357.
Pardy, M. (1997). Čerenkov effect and the Lorentz contraction, *Phys. Rev. A* **55**, 1647.
Pardy, M. (2002). Čerenkov effect with massive photons, *International Journal of Theoretical Physics*, **41**(5), 887.
Pardy, M. (2003). Electron in the ultrashort laser pulse: *Int. Journal of Theoretical Physics* 42, No. 1, 99-110.
Pardy, M. (2004). Massive photons and the Volkov solution of the Dirac equation: Int. Journal of Theoretical Physics 43, No. 1, January, 127-139.
Pardy, M. (2007).The synchrotron radiation from the Volkov solution of the Dirac equation: e-print hep-ph/0703102
Pardy, M. (2012). The two-dimensional Vavilov-Čerenkov radiation, arXiv:1205.4019v1 [quant-ph] 17 May 2012.; ibid. The Čererenkov effect in graphene-like structures: in: *Graphene - Research and Applications*, ISBN 980-953-307-516-2, Book edited by: Prof. Jian Ru Gong National Center for Nanoscience and Technology, China, February 19, 2012.
Schwinger, J. (1970). *Particles, Sources and Fields*, Vol. I (Addison-Wesley, Reading, Mass.).
Schwinger, J., Tsai, W. Y. and Erber, T. (1976). Classical and quantum theory of synergic synchrotron-Čerenkov radiation, *Annals of Physics (NY)* **96**, 303.
Sokolov, A. A. and Ternov, I. M. (1983). *The relativistic electron*, Second ed.,(Moscow, Nauka). (in Russian).
Sommerfeld, A. (1904). Zur Elektronentheorie: II. Grundlagen für eine allgemeine Dynamik des Elektrons, *Göttingen Nachr.*, **99**, 363-439.
Tamm, I. E. and Frank, I. M. (1937). The coherent radiation of a fast electron in a medium, *Dokl. Akad. Nauk SSSR* **14**, 109.