

# Non-BCS superconductivity for underdoped cuprates by spin-vortex attraction

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## Abstract

Within a gauge approach to the  $t$ - $J$  model, we propose a new, non-BCS mechanism of superconductivity for underdoped cuprates. The gluing force of the superconducting mechanism is an attraction between spin vortices on two different Néel sublattices, centered around the empty sites described in terms of fermionic holons. The spin fluctuations are described by bosonic spinons with a gap generated by the spin vortices. Due to the no-double occupation constraint, there is a gauge attraction between holon and spinon binding them into a physical hole. Through gauge interaction the spin vortex attraction induces the formation of spin-singlet (RVB) spin pairs with a lowering of the spinon gap. Lowering the temperature the approach exhibits two crossover temperatures: at the higher crossover a finite density of incoherent holon pairs are formed leading to a reduction of the hole spectral weight, at the lower crossover also a finite density of incoherent spinon RVB pairs are formed, giving rise to a gas of incoherent preformed hole pairs, and magnetic vortices appear in the plasma phase. Finally, at a even lower temperature the hole pairs become coherent, the magnetic vortices become dilute and superconductivity appears. The superconducting mechanism is not of BCS-type since it involves a gain in kinetic energy (for spinons) coming from the spin interactions.

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## 1. Introduction

We propose a new, non-BCS mechanism of superconductivity (SC) for hole-underdoped cuprates relying in essential way upon a “compositeness” [1] of the low-energy hole excitation appearing in the spin-charge gauge approach [2] to the 2D  $t$ - $J$  model, used to describe the CuO planes. This “composite” structure involves a gapful bosonic constituent carrying spin 1/2 (spinon  $z_\alpha$ ) and a gapless spinless fermionic constituent carrying charge (holon  $h$ ), supported on the empty sites. An attractive interaction mediated by an emergent slave-particle gauge field ( $A_\mu, \mu = 0, 1, 2$ ) binds them into a hole resonance. In terms of this “composite” structure we interpret two crossovers appearing in the normal state of cuprates that are view as “precursors” of superconductivity and the recovery of full coherence of the hole at the superconducting transition.

## 2. “Normal” state

To give the key ingredients of the proposed SC mechanism we start shortly reviewing some basic features of the spin-charge gauge approach to the normal state. In the underdoped region of the model the disturbance of hole doping on the anti-ferromagnetic (AF) background originates spin vortices dressing the holons, with opposite chirality in the two Néel sublattices (see Fig.1).

Propagating in this gas of slowly moving vortices the AF spinons, originally gapless in the undoped Heisenberg model, acquire a finite gap, leading to a short range AF order with inverse correlation length

$$m_s \approx (\delta |\log \delta|)^{1/2}. \quad (1)$$

In eq.(1)  $\delta$  is the doping concentration and the logarithmic correction is due to the long-range tail of the spin vortices. Eq. (1) agrees with experimental data in [3]. From the no-double occupation constraint of the  $t$ - $J$  model emerges the slave-particle gauge field  $A_\mu$ . It is minimally coupled to holon and spinon and it takes care of the redundant  $U(1)$  degrees of freedom coming from the spin-charge decomposition of the hole ( $c_\alpha$ ) of the  $t$ - $J$  model into spinon and holon. The dynamics of the transverse mode of the gauge field is dominated by the contribution of the gapless holons. Their Fermi surface produces an anomalous skin effect, with momentum scale

$$Q \approx (T k_F^2)^{1/3}, \quad (2)$$

the Reizer momentum, where  $k_F$  is the holon Fermi momentum. As a consequence of the  $T$ -dependence of the Reizer momentum, the hole (holon-spinon) and the magnon (spinon-antispinon) resonances formed by the gauge attraction have a strongly  $T$ -dependent life-time leading to a behaviour of these excitations less coherent than in a standard Fermi-liquid. For the appearance of Reizer skin effect the presence of a gap for spinons is crucial, because gapless spinons would condense

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at low  $T$  thus gapping the gauge field through the Anderson-Higgs mechanism and destroying the  $T$ -dependent skin effect that decreases the coherence of hole and magnon.

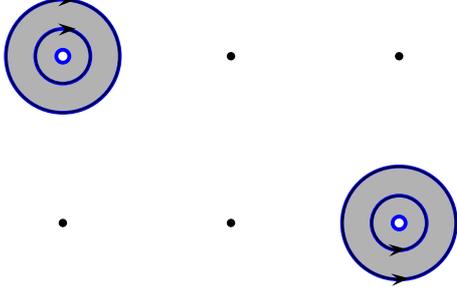


Figure 1: Pictorial representation of the spin vortices dressing the holons represented by white circles at their center.

### 3. Superconductivity mechanism

The gluing force of the proposed superconductivity mechanism is a long-range attraction between spin vortices centered on holons in two different Néel sublattices. Therefore its origin is magnetic, but it is not due to exchange of AF spin fluctuations as e.g. in the proposal of [4], [5]. Explicitly the relevant term in the effective Hamiltonian is:

$$J(1 - 2\delta)\langle z^* z \rangle \sum_{i,j} (-1)^{|i|+|j|} \Delta^{-1}(i-j) h_i^* h_j^* h_i h_j, \quad (3)$$

where  $\Delta$  is the 2D lattice laplacian and

$$\langle z^* z \rangle \sim \int d^2 q (\vec{q}^2 + m_s^2)^{-1} \sim (\Lambda^2 + m_s^2)^{1/2} - m_s, \quad (4)$$

with  $\Lambda \approx 1$  as a UV cutoff. We propose that, lowering the temperature, superconductivity is reached with a three-step process: at a higher crossover a finite density of incoherent holon pairs are formed, at a lower crossover a finite density of incoherent spinon RVB pairs are formed, giving rise to a gas of incoherent preformed hole pairs and a gas of magnetic vortices appears in the plasma phase, at a even lower temperature both the holon pairs and the RVB pairs, hence also the hole pairs, become coherent and the gas of magnetic vortices becomes dilute. This last temperature identifies the superconducting transition. Clearly this mechanism relies heavily on the "composite" structure of the hole appearing in the "normal" state. Let us analyze in a little more detail these three steps.

### 4. Holon pairing

At the highest crossover temperature, denoted as

$$T_{ph} \approx J(1 - 2\delta)\langle z^* z \rangle, \quad (5)$$

a finite density of incoherent holon pairs appears, as consequence of the attraction of spin vortices with opposite chirality. We propose to identify this temperature with the experimentally observed (upper) pseudogap (PG) temperature, where the

in-plane resistivity deviates downward from the linear behavior. The formation of holon pairs, in fact, induces a reduction of the spectral weight of the hole, starting from the antinodal region [6]. The mechanism of holon pair formation is BCS-like in the sense of gaining potential energy from attraction and losing kinetic energy, as shown by the reduction of the spectral weight. As natural due to its magnetic origin, its energy scale is however related to  $J$  and not  $t$ , since the attraction originates from the  $J$ -term of the  $t$ - $J$  model. We denote the BCS-like holon-pair field by  $\Delta^h$ .

### 5. Spinon pairing and incoherent hole pairs

The holon pairing alone is not enough for the appearance of superconductivity, since its occurrence needs the formation and condensation of *hole* pairs. In the previous step instead we have only the formation of holon-pairs. One then firstly needs the formation also of spinon-pairs. It is the gauge attraction between holon and spinon, that, roughly speaking, using the holon-pairs as sources of attraction induces in turn the formation of short-range spin-singlet (RVB) spinon pairs (see Fig.2).

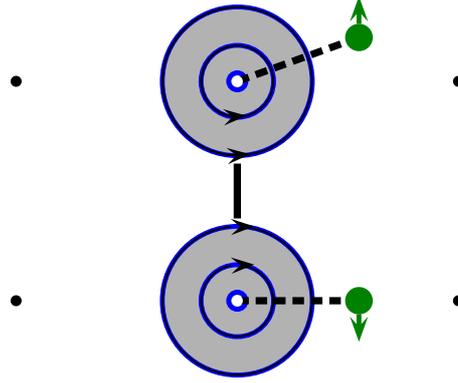


Figure 2: Pictorial representation of hole pairs, holons are represented by white circles surrounded by vortices, spinons by black circles with spin (arrow); the black line represents spin-vortex attraction, the dashed line the gauge attraction

This phenomenon occurs, however, only when the density of holon-pairs is sufficiently high, since this attraction has to overcome the original AF-repulsion of spinons caused by the Heisenberg  $J$ -term which is positive in our approach, in contrast with the more standard RVB [7] and slave-boson [8] approaches. Summarizing, at an intermediate crossover temperature, denoted as  $T_{ps}$ , lower than  $T_{ph}$  in agreement with previous remarks, a finite density of incoherent spinon RVB pairs are formed, which, combined with the holon pairs, gives rise to a gas of incoherent preformed hole pairs. We denote the RVB spinon-pair field by  $\Delta^s$ . It turns out that for a finite density of spinon pairs there are two (positive energy) excitations, with different energies, but the same spin and momenta. They are given, e.g., by creating a spinon up and destructing a spinon down in one of the RVB pairs. The corresponding dispersion relation, thus exhibits two (positive) branches (see Fig.3):

$$\omega(\vec{k}) = 2t \sqrt{(m_s^2 - |\Delta^s|^2) + (|\vec{k} \pm |\Delta^s||)^2}. \quad (6)$$

The lower branch exhibits a minimum with an energy lower than  $m_s$ , analogous to the one appearing in a plasma of relativistic fermions [9]; it implies a backflow of the gas of spinon-pairs dressing the “bare” spinon.

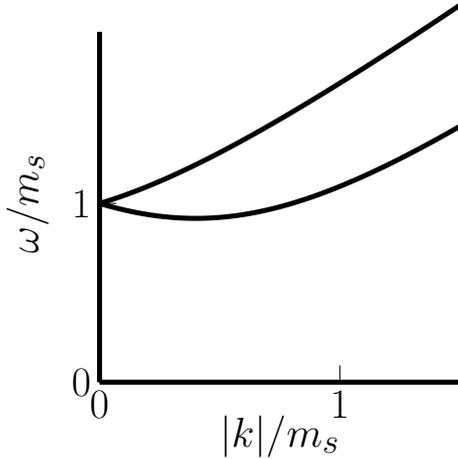


Figure 3: The positive branches of the spinon dispersion relation in presence of finite density of RVB spinon pairs

Hence RVB condensation lower the spinon kinetic energy, but, as explained above, its occurrence needs the gauge interaction to overcome the spinon Heisenberg repulsion. The two-branches dispersion of the spinon (6) is reminiscent of the hour-glass shape of the neutron resonance found in the superconducting region and slightly above in temperature for underdoped samples [10]. If a suitable attraction mechanism for spinon and antispinon works, one can show that a similar dispersion is induced for the magnon resonance [6], directly comparable with experimental data. As soon as we have a finite density of hole pairs the RVB-singlet hole-pair field  $\Delta^c \approx \Delta^s/\Delta^h$  is non-vanishing and the gradient of its phase describes magnetic vortices. Hence below  $T_{ps}$  a gas of magnetic vortice (vortex-loops in space-time) appears, in the plasma phase, because the incoherence of the hole pairs leads to a vanishing expectation value of the phase of  $\Delta^c$ . Therefore, we propose to identify  $T_{ps}$  with the experimental crossover corresponding to the appearance of the diamagnetic and (vortex) Nernst signal [11]. This interpretation is reinforced by the computation of the contour-plot of the spinon pair density in the  $\delta - T$  plane [12], ressembling the contour-plot of the diamagnetic signal. The presence of holon pairs is required in advance to have RVB pairs, and two factors contribute to the density of holon pairs: the density of holons and the strength of the attraction,  $\approx J(1 - m_s)$  from (3). These two effects act in opposite way increasing doping, this yields a finite range of doping for a non-vanishing expectation value of  $|\Delta^s|$ , starting from a non-zero doping concentration, producing a “dome” shape of  $T_{ps}$  and of the contour-plot. The RVB spinon pair formation is clearly not BCS-like, the energy gain coming from the kinetic energy of spinons as discussed above; its energy scale is again related to  $J$ .

## 6. Hole-pair coherence and superconductivity

Finally, at a even lower temperature, the superconducting transition temperature  $T_c$ , both holon pairs and RVB pairs, hence also the hole pairs, become coherent and a d-wave hole condensate

$$\langle \sum_{\alpha,\beta} \epsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \quad (7)$$

appears, corresponding to a non-vanishing expectation value of the hole-pair field  $\Delta^c$ . As soon as the holon and RVB pairs condense the slave-particle global gauge symmetry is broken from  $U(1)$  to  $\mathbf{Z}_2$ . The Anderson-Higgs mechanism then implies a gap for the gauge field  $A_\mu$  increasing with the density of RVB pairs. In this “Higgs”-phase the magnetic vortices become dilute. Therefore the SC transition appears as a 3D XY-type transition for magnetic vortices in presence of a dynamical gauge field, similar in this respect to the transition in the phase-fluctuation scenario proposed in [13]. One can prove that in our model a gapless gauge field is inconsistent with the coherence of holon pairs, i.e. coherent holon pairs cannot coexist with incoherent spinon pairs; hence the condensation of both occurs simultaneously. Since the gauge field binding spinon and holon into a hole resonance becomes massive in the superconducting state, one expects that the life-time of such resonance becomes  $T$ -independent, because the  $T$ -dependent anomalous skin effect appearing in the normal state is suppressed by the mass. Therefore the hole become coherent at the superconducting transition.

The appearance of two temperatures, one for pair formation and a lower one for pair condensation, is typical of a BEC-BCS crossover regime for a fermion system with attractive interaction [14]. In this sense the incoherent hole pairs discussed in previous section play a role analogous to that of the “preformed pairs” considered e.g. in [15]. In our approach, however, the gas of hole pairs appears only at finite doping, implying a fortiori a “dome” shape for the superconductivity region starting from a non-vanishing critical doping concentration at  $T = 0$ . This result is in agreement with experimental data, but at odds with standard fermionic BEC-BCS attractive systems, where the condensation usually persists in the extreme BEC limit [14], and with the original Mean Field version of the slave boson approach [8], where holon condensation occur for arbitrary small holon density at  $T = 0$ .

The non-vanishing critical doping for the “dome” exhibited in our approach appears also in [16], where, however, a nodal structure is argued to be present for the hole already in the region where the magnetic vortices are not dilute. On the contrary in that region our approach preserves a finite FS, as consequence of the incoherence of the holon pairs, and nodes for holes appear only below the superconducting transition. The superconducting transition being of XY-type is “kinetic energy” driven, but again related to the  $J$ - scale since the dynamics of vortices is triggered by the mass of the spinon and of the gauge field, both originated from the Heisenberg term. The “kinetic energy” driven character of the superconducting transition appears consistent with some experimental data for underdoped and optimally doped samples [17].

## 7. Final comments

Our approach exhibits another crossover [2],

$$T^* \approx t/8\pi|\log \delta|, \quad (8)$$

intersecting  $T_{ps}$ . Such crossover is not directly related to superconductivity. It corresponds to a change in the holon dispersion. It is characterized by the emergence of a "small" holon FS around the momenta  $\pm\pi/2, \pm\pi/2$ , with complete suppression of the spectral weight for holes in the antinodal region and partial suppression outside of the magnetic Brillouin zone. Induced physical effects are observable experimentally both in transport and thermodynamics [18]. This crossover appears only in two-dimensional bipartite lattices. Below  $T^*$  the effect of short-range AF fluctuations become stronger and the transport physics of the corresponding normal state region ("pseudogap") is dominated by the interplay between the short-range AF of spinons and the thermal diffusion induced by the gauge fluctuations triggered by the Reizer momentum, producing in turn the metal-insulator crossover [2]. We identify  $T^*$  in experimental data with the inflection point of in-plane resistivity and the broad peak in the specific heat coefficient [18].

The above presentation is just a sketch of the approach, many details have been already worked out explicitly, others remain admittedly conjectural, but the mechanism of SC proposed here is rather complete in its main structure and qualitatively consistent with many experimental features, as partially discussed above. A more complete presentation of our approach to superconductivity will appear in [12].

## References

- [1] P. A. Marchetti, A. Ambrosetti, Z. B. Su and L. Yu, *J. of Phys. and Chem. of Solids* **69**, 3277 (2008).
- [2] P. A. Marchetti, Z. B. Su and L. Yu, *J. Phys. Condens. Matter* **19**, 125209 (2007) and references therein.
- [3] B. Keimer *et al.*, *Phys. Rev. B* **46**, 14034 (1992).
- [4] D. Pines, in *Proc. Conf. on Gap Symmetry in High-Tc Superconductors*, Plenum, New York, 1998; D.J. Scalapino, *J. Low Temp. Phys.* **117** 179 (1999).
- [5] M. Y. Kuchiev and O.P. Sushkov, *Physica C* **218**, 197 (1993); V.V. Flambaum, M. Y. Kuchiev and O.P. Sushkov, *Physica C* **227**, 267 (1994).
- [6] P. A. Marchetti and M. Gambaccini, in preparation.
- [7] P.W. Anderson *et al.*, *J Phys. Condens. Matter* **16**, R755-R769 (2004).
- [8] P.A. Lee and N. Nagaosa, *Phys. Rev. B* **46** 5621 (1992).
- [9] H. A. Weldon, *Phys. Rev. D* **40**, 2410 (1989).
- [10] C. Stock *et al.* *Phys. Rev. B* **71**, 024522 (2005)
- [11] Y. Wang *et al.*, *Phys. Rev. Lett.* **88**, 257003 (2002); Y. Wang, L. Li and N. P. Ong, *Phys. Rev. B* **73**, 024510 (2006).
- [12] P. A. Marchetti, F. Ye, Z. B. Su and L. Yu, to be published.
- [13] V.J. Emery and S.A. Kivelson, *Nature* **374**, 434 (1995).
- [14] P. Nozieres, S. Schmitt-Rink, *J. Low Temp. Phys.* **59**, 195 (1985).
- [15] Y.J. Uemura, *Physica* **282**, 194 (1997).
- [16] Z. Tesanovic, *Nat. Phys.* **4**, 408 (2008).
- [17] G. Deutscher *et al.*, *Phys. Rev. B* **72**, 092504 (2005); E. van Heumen *et al.*, *Phys. Rev. B* **75**, 054522 (2007).
- [18] P. A. Marchetti and A. Ambrosetti, *Phys. Rev. B* **78**, 085119 (2008).