A Definition of Work

Alejandro A. Torassa

(2014) Buenos Aires, Argentina
atorassa@gmail.com

Abstract

In classical mechanics, this paper presents a definition of work, which can be used in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

Definition of Work

If we consider two particles A and B then the definition of the total work W_{ab} done by the forces $\mathbf{F}_a$ and $\mathbf{F}_b$ acting on particles A and B respectively is:

$$W_{ab} = \frac{1}{2}\, m_a m_b \left[2\int_1^2 \left(\frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_b}{m_b} \right) \cdot d(\mathbf{r}_a - \mathbf{r}_b) + \Delta \left(\frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_b}{m_b} \right) \cdot (\mathbf{r}_a - \mathbf{r}_b) \right]$$

where m_a and m_b are the masses of particles A and B, and $\mathbf{r}_a$ and $\mathbf{r}_b$ are the positions of particles A and B.

The total work W_{ab} is equal to the change in kinetic energy.

$$W_{ab} = \Delta\, \frac{1}{2}\, m_a m_b \left[(\mathbf{v}_a - \mathbf{v}_b)^2 + (\mathbf{a}_a - \mathbf{a}_b) \cdot (\mathbf{r}_a - \mathbf{r}_b) \right]$$

where $\mathbf{v}_a$ and $\mathbf{v}_b$ are the velocities of particles A and B, and $\mathbf{a}_a$ and $\mathbf{a}_b$ are the accelerations of particles A and B.

Therefore, the kinetic energy K_{ab} of particles A and B is:

$$K_{ab} = \frac{1}{2}\, m_a m_b \left[(\mathbf{v}_a - \mathbf{v}_b)^2 + (\mathbf{a}_a - \mathbf{a}_b) \cdot (\mathbf{r}_a - \mathbf{r}_b) \right]$$

And the potential energy U_{ab} of particles A and B is:

$$\Delta\, U_{ab} = -\frac{1}{2}\, m_a m_b \left[2\int_1^2 \left(\frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_b}{m_b} \right) \cdot d(\mathbf{r}_a - \mathbf{r}_b) + \Delta \left(\frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_b}{m_b} \right) \cdot (\mathbf{r}_a - \mathbf{r}_b) \right]$$