

# On the absorption of radiation by the negatively charged hydrogen ion

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## Abstract

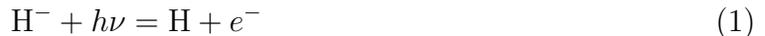
The absorption of infrared and visible radiation from stellar emission spectra by the negatively charged hydrogen ions  $\text{H}^-$  is considered. We derive the explicit formulas which can be used to determine the total absorption coefficient (per unit volume) for the negatively charged hydrogen ions  $^1\text{H}^-$  (protium) and  $^2\text{H}^-$  (deuterium or  $\text{D}^-$ ). The computed bound-free and free-free absorption coefficients  $a_\nu$  and  $k_\nu$  can be used to evaluate the actual absorption of infrared and visible radiation by the  $\text{H}^-$  ion in photospheres of many cold stars with surface temperatures  $T_s \leq 8,250 \text{ K}$ .

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## I. INTRODUCTION

As is well known (see, e.g., [1] - [3]) the negatively charged hydrogen ions  ${}^1\text{H}^-$  (protium) and  ${}^2\text{H}^-$  (deuterium or  $\text{D}^-$ ) are of great interest in Stellar Astrophysics. Indeed, the negatively charged hydrogen ions substantially determine the absorption of infrared and visible radiation in photospheres of all stars with temperatures bounded between  $T_{max} \approx 8,250$   $K$  (late A-stars) and  $T_{min} \approx 2,750$   $K$  (early M-stars). The main contribution to the light absorption by the negatively charged hydrogen ion  $\text{H}^-$  comes from its photodetachment



where the notation  $h\nu$  designates the incident light quantum, while  $e^-$  means the free electron, or photo-electron, for short. Here and below  $h$  is the Planck constant. Note that each of the negatively charged hydrogen ions, i.e. the  ${}^1\text{H}^-$ ,  ${}^2\text{H}^-$  (or  $\text{D}^-$ ),  ${}^3\text{H}^-$  (or  $\text{T}^-$ ) and  ${}^\infty\text{H}^-$  has only one bound (ground)  $1^1S$ -state with  $L = 0$ , where  $L$  is the angular momentum of this two-electron ion (see, e.g., [4]). The ground  $1^1S$ -state is a singlet state, i.e., the total electron spin  $S$  in this state equals zero, and therefore,  $2S + 1 = 1$ . Here and everywhere below the notation  ${}^\infty\text{H}/{}^\infty\text{H}^-$  stands for the model atom/ion with the infinitely heavy nucleus. The total energy of the ground  $1^1S$ -state in the  ${}^\infty\text{H}^-$  ion is  $-0.527\ 751\ 016\ 544\ 377\ 196\ 566\ 8$   $a.u.$  [4], while the total energy of the hydrogen atom with the infinitely heavy nucleus ( ${}^\infty\text{H}$ ) is  $-0.5$   $a.u.$  Therefore, the binding energy of the  ${}^\infty\text{H}^-$  ion is  $\chi_1 \approx 0.027\ 751$   $a.u. \approx 0.755\ 143\ 9$   $eV$  and the corresponding ionization frequency  $\nu_1 = \frac{\chi_1}{h}$  is located in the far infrared part of the spectrum. Note that the energies of ‘visible’ light quanta are  $\approx 2 - 3$   $eV$ , while the total energy of the ground state of hydrogen atom is  $13.65$   $eV$ . Light quanta with such energies ( $\approx 13.65$   $eV$ ) correspond to the vacuum ultraviolet region. The total energies of the ground  $1^1S$ -states of other  $\text{H}^-$ -like ions and  $\text{H}$ -like atoms can be found in Table I.

The final atomic state which arises after photodetachment of the negatively charged hydrogen ion, Eq.(1), includes the neutral hydrogen atom  $\text{H}$  and free electron  $e^-$ . The kinetic energy of this free electron can, in principle, be arbitrary. Therefore, it is easy to predict that all radiation quanta with the frequencies  $\nu \geq \nu_1 = \frac{\chi_1}{h}$  can produce photodetachment of the negatively charged hydrogen ion  $\text{H}^-$ . In other words, all light quanta with  $\nu \geq \nu_1$  can be absorbed during such a process. This also includes the quanta of visible light.

The absorption of radiation by the negatively charged hydrogen ions has been investigated in numerous earlier studies (see, e.g., [5] - [10] and references therein). In general, the light absorption by an arbitrary atomic system is determined by separate contributions from the corresponding bound-bound, bound-free and free-free transitions in this system. As mentioned above the negatively charged hydrogen ion has only one bound (ground)  $1^1S$ -state, i.e. there is no contribution from the bound-bound transitions for the  $H^-$  ion. The bound-free transitions in the  $H^-$  ion correspond to the photodetachment, Eq.(1), while the free-free (electron) transitions formally represent the radiation-stimulated electron scattering in the field of the neutral hydrogen atom (or inverse bremsstrahlung, for short)

$$e^- + h\nu + H = H + e^- \quad (2)$$

where the kinetic energy of the final electron differs from its incident energy due to the photon's absorption. If the kinetic energy of the both incident photon and electron in Eq.(2) are small, then the final state of the hydrogen atom coincides with its original (ground) state. Indeed, the lowest excitation energy of the ground state of the hydrogen atom is relatively high  $E \approx 0.375 \text{ a.u.} \approx 10.2043 \text{ eV}$ . Therefore, we can ignore all possible excitations of the central hydrogen atom during radiation-stimulated electron scattering in the field of the neutral hydrogen atom. Such an approximation has a good numerical accuracy, if the kinetic (or thermal) energies of electrons  $E_e \sim kT$  are significantly smaller than the lowest excitation energy of the ground state of the hydrogen atom ( $\approx 0.375 \text{ a.u.} \approx 10.2043 \text{ eV}$ ). It can be shown that such an assumption is always obeyed in all stars in which the absorption of radiation by the negatively charged hydrogen ions is important, i.e. it is true for the late A-stars and all F, G and K stars. Note that in the early F-stars and late A stars the absorption of radiation by the negatively charged hydrogen ions is consequently outweighed by the neutral hydrogen absorption. In the hot O- and B-stars, with surface temperatures  $T_s \geq 28,500 \text{ K}$  and  $T_s \geq 11,000 \text{ K}$ , respectively, the absorption of radiation quanta is mainly related with the transitions between different bound states in helium and hydrogen atoms. The negatively charged  $H^-$  ion does not exist at such temperatures as a stable, i.e. bound, system.

On the other hand, the  $H^-$  ions play practically no role for the cold M and N stars with  $T_s \leq 2,700 \text{ K}$  where the absorption of radiation by various molecular species and atoms of metals becomes important. However, in the late F, G and early K stars the absorption

of infrared and visible radiation by the negatively charged hydrogen ions is maximal. This includes our Sun which is a star of spectral type G2. The great role of the  $H^-$  ions for our Sun was suggested by R. Wild in 1939 (see discussions and references in [1], [2] and [3]).

In this study we analyze the bound-free and free-free transitions in the negatively charged hydrogen ions. Our main goal is to improve earlier evaluations of the photodetachment cross-section and the total cross-section of the free-free transitions in the  $H^-$  ion. In particular, we want to re-consider the validity of some approximations made in earlier works where the absorption of radiation by the negatively charged hydrogen ions has been evaluated [5] - [10]. Below, we pay a significant attention to the formulas which are used in numerical calculations of the absorption coefficient of the  $H^-$  ion. Results of different numerical calculations are discussed here very briefly, since we want to include such a discussion in our next paper.

## II. ABSORPTION COEFFICIENT OF THE NEGATIVELY CHARGED HYDROGEN ION

In general, the radiation absorption coefficient  $\alpha_\nu$  of the negatively charged hydrogen ion defined per unit volume is written in the form (see, e.g., [1])

$$\alpha_\nu = (n^- k_\nu + n_H p_e a_\nu) \left[ 1 - \exp\left(-\frac{h\nu}{kT}\right) \right] \quad (3)$$

where  $n^-$  and  $n_H$  are the spatial densities of the  $H^-$  ions and neutral hydrogen atoms  $H$ , respectively. Also, in this formula  $p_e = n_e kT$  is the electron pressure which corresponds to the temperature  $T = T_e$  and electron density  $n_e$ . The factor  $1 - \exp\left(-\frac{h\nu}{kT}\right)$  in Eq.(3) represents the ‘negative absorption’ of radiation. The spatial densities  $n_H, n^-$  and  $n_e$  are related to each other by the following equation

$$\frac{n_H n_e}{n^-} = \frac{g_1}{g_-} \frac{2(2\pi m_e kT)^{\frac{3}{2}}}{h^3} \exp\left(-\frac{\chi_1}{kT}\right) \quad (4)$$

where  $g_1 = 2$  and  $g_- = 1$  are the statistical weights of the ground states of the neutral hydrogen atom  $H$  and hydrogen negatively charged ion  $H^-$ , respectively. The notation  $\chi_1$  stands for the ionization potential of the  $H^-$  ion. In this equation and in all formulas below the notations  $m_e, h$  and  $e$  mean the electron mass, Planck constant and electron’s electric charge, respectively.

By using Eq.(4) we can express the spatial density of the  $H^-$  ions (i.e. the  $n^-$  factor in Eq.(3)) and reduce Eq.(3) to the form

$$\alpha_\nu = n_H p_e \left[ k_\nu \frac{h^3}{4(2\pi m_e)^{\frac{3}{2}} (kT)^{\frac{5}{2}}} \exp\left(\frac{\chi_1}{kT}\right) + a_\nu \right] \left[ 1 - \exp\left(-\frac{h\nu}{kT}\right) \right] \quad (5)$$

This formula is used for all current calculations of the absorption coefficient  $\alpha_\nu$  of the negatively charged ion  $H^-$ . In real calculations of  $\alpha_\nu$  it is always assumed that  $p_e = 1$  and  $n_H = 1$ , i.e. such calculations are performed for the unit electron pressure and per one hydrogen atom in unit volume. In this study the formula, Eq.(5), is also applied for numerical calculations of the absorption coefficient  $\alpha_\nu$  of the  $H^-$  ion. As it follows from Eq.(5) in order to determine the coefficient  $\alpha_\nu$  one needs to evaluate the  $k_\nu$  and  $a_\nu$  coefficients. The third unknown value, which can be found in Eq.(5), is the ionization potential  $\chi_1$  of the  $H^-$  ion(s). In reality, the ionization potentials  $\chi_1$  of all  $H^-$  ions are now known to very good numerical accuracy (see, e.g., [4] and Table I below). Therefore, below we restrict ourselves to the analysis of computational formulas used for the  $k_\nu$  and  $a_\nu$  coefficients. Note that the first numerical calculations of the absorption coefficient  $\alpha_\nu$  with the use of Eq.(5) were performed in [6]. It is also clear that if the total absorption coefficient  $\alpha_\nu$  as the function of  $r$  in stellar photosphere is known, then one finds the corresponding optical depth at the frequency  $\nu$

$$\tau_\nu(r) = \int_r^{+\infty} \alpha_\nu(y) dy \quad (6)$$

where  $r$  is the actual geometrical distance measured from the top of the Solar/stellar photosphere ( $\infty$ ).

Now, let us discuss the relations between the absorption coefficients  $a_\nu$  and  $k_\nu$  from Eqs.(3) and (5) and the corresponding cross-sections used below. Without loss of generality consider such a relation for the bound-free coefficient  $a_\nu$  and photodetachment cross-section  $\sigma_\nu$  of the  $H^-$  ion. The general relation produces the following formula  $a_\nu = \sigma_\nu \cdot 1 \cdot N = \sigma_\nu \frac{\rho}{M_H} N_A$ , where  $N_A$  is the Avogadro number,  $M_H$  is the mass of one mole of hydrogen atoms and  $\rho$  is the density of the stellar (hydrogenic) photosphere.

### III. PHOTODETACHMENT OF THE NEGATIVELY CHARGED HYDROGEN ION

In this Section we derive and discuss the explicit formulas which are used for numerical computation of the photodetachment cross-section of the  $H^-$  ion. The photodetachment

corresponds to the bound-free (optical) transitions, since the final state of one of the two electrons is an unbound state, or, in other words, the state from the unbound spectra of the  $H^-$  ion. For the two-electron  $H^-$  ion it is possible to obtain the closed analytical formula for the photodetachment cross-section, if the non-relativistic (or dipole) approximation is used. The derivation of this formula is the main goal of this Section.

Let us assume that the incident (i.e. second) electron was bound to the neutral hydrogen atom with the binding energy  $\epsilon_i = -I$ , where  $I = \chi_1$  is the ionization potential of the  $H^-$  ion. The incident photon has the momentum  $\mathbf{k}$  and the frequency  $\omega$  (or energy  $\hbar\omega = \omega$ ). In the final state the photo-electron moves as a free particle with the momentum  $\mathbf{p}$  and energy  $\epsilon_f$ . Since  $\mathbf{p}$  is a continuous variable, the photodetachment cross-section is written in the form

$$d\sigma = 2\pi |M_{i \rightarrow f}|^2 \delta(-I + \omega - \epsilon_f) \frac{d^3p}{(2\pi)^3} \quad (7)$$

where the wave function of the final state is normalized per one particle in unit volume  $V = 1$ . All wave functions and expressions used in computations of the transition amplitude  $M_{i \rightarrow f}$  are assumed to have a proper permutation symmetry upon spin-spatial coordinates of the two electrons 1 and 2. This allows one to write the formula, Eq.(7), and all formulas below in the one-electron form.

The delta-function in Eq.(7) is excluded by integrating over the momenta  $\mathbf{p}$  of the photo-electron. Indeed, by using the formula  $d^3\mathbf{p} = p^2 d|\mathbf{p}| d\epsilon$  and performing the integration over  $d\epsilon$  we can remove this delta-function and obtain the following general expression for the cross-section

$$d\sigma = e^2 \frac{|\mathbf{p}|}{2\pi\omega} |M_{i \rightarrow f}|^2 d\epsilon \quad (8)$$

The derivation of the analytical formula for the transition amplitude  $M_{i \rightarrow f}$  is drastically simplified (see, e.g., [12]) with the use of transverse gauge, i.e.

$$M_{i \rightarrow f} = -e\mathbf{A} \cdot \mathbf{j}_{i \rightarrow f} = -e\sqrt{\frac{2\pi}{\omega}} \int \int \psi_f^*(1, 2) [(\mathbf{e} \cdot \alpha_1) \exp(i\mathbf{k}_1 \cdot \mathbf{r}_1) + (\mathbf{e} \cdot \alpha_2) \exp(i\mathbf{k}_2 \cdot \mathbf{r}_2)] \psi_i(1, 2) d^3\mathbf{r}_1 d^3\mathbf{r}_2 \quad (9)$$

In this equation and everywhere below we shall use the so-called relativistic units where  $\hbar = 1$  and  $c = 1$ .

In this study all electrons are considered as non-relativistic particles, while the energy of the incident photon  $\hbar\omega (= \omega)$  is assumed to be larger than  $I$ , but it is substantially less

than the energy of electron at rest, i.e.  $\hbar\omega \ll m_e c^2$ . This means that the final velocity of the photoelectron is small, i.e. we are dealing with the non-relativistic problem. Therefore, the Dirac's  $\alpha$  matrixes can be replaced by the corresponding velocities  $\mathbf{v}_j = -\frac{\hbar}{m_e} \nabla_j$ , where  $j$  is the electron index. Moreover, in the non-relativistic approximation we can replace the factors  $\exp(i\mathbf{k}_j \cdot \mathbf{r}_j)$  ( $j = 1, 2$ ) by unity. Now, it is easy to obtain the following formula for the photodetachment cross-section

$$d\sigma_\nu = \frac{e^2 m_e |\mathbf{p}|}{2\pi\omega} |\mathbf{e} \cdot \left\{ \int \int \Psi_{fi}^*(\mathbf{r}_1, \mathbf{r}_2) \left( \frac{\partial}{\partial \mathbf{r}_1} + \frac{\partial}{\partial \mathbf{r}_2} \right) \Psi_{H^-}(\mathbf{r}_1, \mathbf{r}_2) d^3\mathbf{r}_1 d^3\mathbf{r}_2 \right\}|^2 d\omega \quad (10)$$

where  $\mathbf{e}$  is the vector which represents the polarization of the incident photon,  $\Psi_{fi}(\mathbf{r}_1, \mathbf{r}_2)$  is the wave function of the final state (after photodetachment), while  $\Psi_{H^-}(\mathbf{r}_1, \mathbf{r}_2)$  is the wave function of the ground  $1^1S$ -state of the hydrogen ion. In this study the ground state of the  $H^-$  ion is approximated by the following (exponential) variational expansion:

$$\Psi_{H^-} = \Psi_{H^-}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{i=1}^N C_i f_i = \sum_{i=1}^N C_i \exp(-\alpha_i r_1 - \beta_i r_2 - \gamma_i r_{12}) \quad (11)$$

where  $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$  is the electron-electron (or correlation) coordinate and  $C_i$  ( $i = 1, \dots, N$ ) are the linear variational coefficients of the approximate (or trial) wave function. The  $3N$ -parameters  $\alpha_i, \beta_i, \gamma_i$  ( $i = 1, \dots, N$ ) are the non-linear parameters (or varied parameters) of the trial wave function, Eq.(11). As follows from the results of our earlier studies (see, e.g., [4], [11]) the exponential variational expansion, Eq.(11), provides very high numerical accuracy for any bound state in arbitrary Coulomb three-body systems, i.e. for systems with arbitrary particle masses and electric charges. For the  $H^-$  ions such an accuracy is much higher than accuracy which any other variational expansion may provide, if the same number of basis functions  $N$  is used. The explicit form of the  $\Psi_{fi}^*(\mathbf{r}_1, \mathbf{r}_2)$  wave function is discussed below. Note that from Eq.(11) one finds

$$\begin{aligned} \mathbf{e} \cdot \left( \frac{\partial}{\partial \mathbf{r}_1} + \frac{\partial}{\partial \mathbf{r}_2} \right) \Psi_{H^-} &= \left( \mathbf{e} \cdot \frac{\mathbf{r}_1}{r_1} \right) \sum_{i=1}^N C_i \alpha_i f_i + \left( \mathbf{e} \cdot \frac{\mathbf{r}_2}{r_2} \right) \sum_{i=1}^N C_i \beta_i f_i \\ &= \left( \mathbf{e} \cdot \frac{\mathbf{r}_1}{r_1} \right) \left[ \frac{\partial}{\partial r_1} \Psi_{H^-} \right] + \left( \mathbf{e} \cdot \frac{\mathbf{r}_2}{r_2} \right) \left[ \frac{\partial}{\partial r_2} \Psi_{H^-} \right] \end{aligned} \quad (12)$$

After photodetachment of the  $H^-$  ion, Eq.(1), in the final state we have the neutral hydrogen atom in one of its bound state and free electron which moves as a free particle with the kinetic energy  $E_e = \frac{\hbar^2 k^2}{2m_e}$ , where  $\hbar = \frac{h}{2\pi}$  is the reduced Planck constant and  $m_e$  is the electron mass. The corresponding wave function of the final state is represented as a

product of the bound state wave function of the hydrogen atom and the wave function of the free electron. The wave function of the bound  $(n\ell m)$ -state of the hydrogen atom is written in the form (see, e.g., [13])  $\Phi_{n\ell m}(r, \Theta, \phi) = R_{n\ell}(Q, r)Y_{\ell m}(\Theta, \phi)$ , where  $Y_{\ell m}(\Theta, \phi) = Y_{\ell m}(\mathbf{n})$  is a spherical harmonic and  $R_{n\ell}(Q, r)$  is the one-electron radial function. The radial function  $R_{n\ell}(Q, r)$  takes the form (see, e.g., [13])

$$R_{n\ell}(Q, r) = \frac{1}{rn} \sqrt{\frac{Q(n-\ell-1)!}{(n+\ell)!}} \left[ \frac{2Qr}{n} \right]^{\ell+1} \left\{ \sum_{k=0}^{n-\ell-1} \frac{(-1)^k}{k!} \binom{n+\ell}{2\ell+k+1} \left[ \frac{2Qr}{n} \right]^k \right\} \exp\left(-\frac{Qr}{n}\right) \quad (13)$$

where  $Q$  ( $= 1$ ) is the nuclear charge, while  $n$  and  $\ell$  are the quantum numbers of this bound state. Note that this radial function has a unit norm.

The wave function of the final (i.e. free) electron is represented as the sum of the plane wave  $\phi(\mathbf{r}) = \exp(i\mathbf{k} \cdot \mathbf{r})$  and converging spherical wave which is usually designated as  $\phi_{\mathbf{p}}^{(-)}(\mathbf{r})$ . In the non-relativistic dipole approximation used here one finds that the following selection rule is applied: transition from the incident  $S(L = 0)$ -state of the  $\text{H}^-$  ion can only be into the final  $P(L = 1)$ -state. This means that in the final state we can detect either the H-atom in one of its  $s(\ell = 0)$ -states and free electron which moves away in the  $p$ -wave (the  $sp$ -channel of the  $\text{H}^-$  ion photodetachment), or the H-atom in one of its three  $p(\ell = 1)$ -states and free electron which moves away in the  $s$ -wave (the  $ps$ -channel of the  $\text{H}^-$  ion photodetachment). The wave function of the final state for the  $sp$ -channel takes the form

$$\psi_f(1, 2) = R_{n0}(Q = 1, r_1) \frac{3\iota}{2p} (\mathbf{n}_e \cdot \mathbf{n}_2) \Phi_{p1}(r_2) = \frac{3\iota}{2p} (\mathbf{n}_e \cdot \mathbf{n}_2) R_{n0}(r_1) \Phi_{p1}(r_2) \quad (14)$$

where  $\mathbf{n}_2 = \frac{\mathbf{r}_2}{r_2}$  is the unit vector of the second electron and  $\mathbf{n}_e = \frac{\mathbf{p}_e}{p_e}$  is the direction of the momentum of the outgoing photo-electron. The notation  $R_{n\ell=0}(Q = 1, r_1) = R_{n0}(r_1)$  stands for the radial function of the bound state of the hydrogen atom with quantum numbers  $n \geq 1$  and  $\ell = 0$ .

The wave function of the final state for the  $ps$ -channel can be written in the following form

$$\psi_f(1, 2) = R_{n\ell=1}(Q = 1, r_1) (\mathbf{n}_e \cdot \mathbf{n}_1) \frac{1}{2p} \Phi_{p\ell=0}(r_2) = \frac{1}{2p} (\mathbf{n}_e \cdot \mathbf{n}_1) R_{n1}(r_1) \Phi_{p0}(r_2) \quad (15)$$

where  $R_{n\ell=1}(Q = 1, r_1) = R_{n1}(r_1)$  is the radial function of the bound state of the hydrogen atom with quantum numbers  $n \geq 2$  and  $\ell = 1$ .

As follows from Eqs.(10) - (15) the formula, Eq.(10), is reduced to the form

$$| \left( \mathbf{e} \cdot \frac{\mathbf{r}_1}{r_1} \right) \Phi_1 + \left( \mathbf{e} \cdot \frac{\mathbf{r}_2}{r_2} \right) \Phi_2 |^2 \quad (16)$$

where  $\Phi_1$  and  $\Phi_2$  are the two scalar expressions. In stellar photospheres all hydrogen ions are systems free-oriented in space. This means that the expression from Eq.(16) must be averaged over possible directions of the unit vectors  $\mathbf{e}_1$  and  $\mathbf{e}_2$  which describe the polarization of the incident photon. In the incident light wave we always have  $\mathbf{e}_1 \perp \mathbf{e}_2 \perp \mathbf{n}$ , where the unit vector  $\mathbf{n}$  corresponds to the direction of propagation of this light wave, or the corresponding light quantum (photon). An arbitrary scalar expression can be averaged over all possible directions of the unit vectors  $\mathbf{e}_1$  and  $\mathbf{e}_2$  with the use of the following formulas

$$\overline{e_i e_k} = \frac{1}{2}(\delta_{ik} - n_i n_k) \quad (17)$$

and

$$\overline{(\mathbf{a} \cdot \mathbf{e})(\mathbf{b} \cdot \mathbf{e})} = \frac{1}{2}[(\mathbf{a} \cdot \mathbf{b}) - (\mathbf{a} \cdot \mathbf{n})(\mathbf{b} \cdot \mathbf{n})] = \frac{1}{2}(\mathbf{a} \times \mathbf{n})(\mathbf{b} \times \mathbf{n}) \quad (18)$$

where  $\mathbf{x} \times \mathbf{y}$  is the vector product of the two vectors  $\mathbf{x}$  and  $\mathbf{y}$ . Note that the final formula includes only the unit vector  $\mathbf{n}$  which describes the direction of photon propagation. If the direction of the propagation of the final photon is not known, then the last formula must also be averaged over all possible directions of the unit vector  $\mathbf{n}$ . This leads to the formula

$$\overline{(\mathbf{a} \cdot \mathbf{e})(\mathbf{b} \cdot \mathbf{e})} = \frac{1}{2} \overline{(\mathbf{a} \times \mathbf{n})(\mathbf{b} \times \mathbf{n})} = \frac{8\pi}{3} \cdot \frac{1}{2}(\mathbf{a} \cdot \mathbf{b}) = \frac{4\pi}{3}(\mathbf{a} \cdot \mathbf{b}) \quad (19)$$

For the *sp*- and *ps*-channels mentioned above the radial functions of the final states of the hydrogen atom can be designated as the  $|n0\rangle$  and  $|n1\rangle$ , respectively. Here the notations  $n$  and  $\ell$  designate the principal quantum number  $n$  and angular number  $\ell$  of the hydrogen atom. To designate the radial hydrogen functions we do not need to mention the magnetic number(s)  $m$ , where  $m = -\ell, -\ell + 1, \dots, \ell - 1, \ell$ . In reality, the largest contribution to the photodetachment of the  $\text{H}^-$  ion comes from that final state in which the final hydrogen atom forms in its ground state. This corresponds to the *sp*-channel. The final hydrogen atoms can also be formed in the excited  $2s(\ell = 0)-, 3s(\ell = 0)-, 4s(\ell = 0)-$ states. Such channels also contribute to the photodetachment of the  $\text{H}^-$  ion, but their contributions are much smaller than the contribution of the photodetachment channel in which the final hydrogen atom is formed the ground state. Analogous situation can be found for the *ps*-channel where the contribution from the  $2p(\ell = 1)-$ state substantially exceeds contributions from

other  $np(\ell = 1)$ -states in hydrogen atoms (here  $n \geq 3$ ). In general, the formation of the final hydrogen atoms in the highly excited  $s(\ell = 0)$ -states and  $p(\ell = 1)$ -states with  $n \geq 4$  is very unlikely, since the corresponding probabilities are very small.

It is also clear *a priori* that the contribution of the  $sp$ -channel in the photodetachment of the  $H^-$  ion is substantially larger than the contribution from the  $ps$ -channel. It directly follows from very large excitation energy of the  $2p(\ell = 1)$ -states in the hydrogen atom  $\Delta E \approx 10.2043 \text{ eV}$ . Another reason for this is almost exact orthogonality of the wave functions of the  $2p$ -state of the hydrogen atom  $H$  and ground state of the  $H^-$  ion. In reality, in the cold stars the  $ps$ -channel does not contribute into the photodetachment of the  $H^-$  ion. However, the contribution of this  $ps$ -channel rapidly increases with the temperature due to Maxwellian (thermal) distribution of the  $H^-$  ions upon temperatures in stellar photospheres. It follows from here that for stars with higher surface temperatures the  $ps$ -channel must play a larger role in the photodetachment of the  $H^-$  ions. In other words, there is a difference between the photodetachment of the  $H^-$  ions in photospheres of the late A-stars and in photospheres of earlier K-stars. Note also that in our non-relativistic approximation the rotationally excited channels with  $\ell \geq 2$  do not contribute in the total photodetachment cross-section.

#### IV. COMPUTATION OF THREE-BODY INTEGRALS IN PERIMETRIC COORDINATES

As we have shown above the problem of analytical and numerical computation of matrix elements arising in Eq.(10) with the trial wave function defined in Eq.(11) is reduced to the computation of the following three-body integrals with the Bessel functions  $j_0(kr_2)$  and  $j_1(kr_2)$  (see, e.g., [14] and [15]). In fact, each of these integrals takes the form

$$\int \int \int \phi_{n\ell}(r_1) j_m(kr_2) \exp(-\alpha r_1 - \beta r_2 - \gamma r_{12}) r_1 r_2 r_{12} dr_1 dr_2 dr_{12} \quad (20)$$

where  $m = 0, 1$  and  $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$  is the electron-electron coordinate. Note that the scalar  $r_{12}$  variable varies between the following limits  $|r_1 - r_2| \leq r_{12} \leq r_1 + r_2$ . In Eq.(20) the notation  $\phi_{n\ell}(r_1)$  stands for the unit norm wave functions of the bound states of the hydrogen atom. The integrals in Eq.(20) are computed analytically in perimetric coordinates  $u_1, u_2, u_3$ , where  $u_1 = \frac{1}{2}(r_2 + r_{12} - r_1)$ ,  $u_2 = \frac{1}{2}(r_1 + r_{12} - r_2)$  and  $u_3 = \frac{1}{2}(r_1 + r_2 - r_{12})$ . The three perimetric

coordinates  $u_1, u_2, u_3$  are independent of each other and each of them varies between 0 and  $+\infty$ .

Let us derive the closed analytical expression for the integrals, Eq.(20) with  $m = 0$  and  $m = 1$ . First, we need to obtain the general formula for the closely related auxiliary three-body integral  $\Gamma_{n,k,l}(\alpha, \beta, \gamma)$  which is written in the form

$$\Gamma_{n,k,l}(\alpha, \beta, \gamma) = \int \int \int r_1^k r_2^l r_{12}^m \exp(-\alpha r_1 - \beta r_2 - \gamma r_{12}) dr_1 dr_2 dr_{12} \quad (21)$$

where  $k \geq 0, l \geq 0, n \geq 0$  and  $\alpha + \beta > 0, \alpha + \gamma > 0$  and  $\beta + \gamma > 0$  (see below). Analytical computation of this integral has extensively been explained in a number of our earlier works. Correspondingly, below we restrict ourselves only to a few following remarks. In perimetric coordinates this integral, Eq.(21), takes the form

$$\Gamma_{n,k,l}(\alpha, \beta, \gamma) = 2 \int_0^\infty \int_0^\infty \int_0^\infty \exp[-(\alpha + \beta)u_1 - (\alpha + \gamma)u_2 - (\beta + \gamma)u_3] (u_1 + u_2)^n (u_1 + u_3)^m (u_2 + u_3)^l du_1 du_2 du_3 \quad (22)$$

where we took into account the fact that the Jacobian of transformation from the relative to perimetric coordinates equals 2. The integration over three independent perimetric coordinates  $u_i$  ( $0 \leq u_i < \infty$ ) in Eq.(22) is trivial and explicit formula for the  $\Gamma_{n,k,l}(\alpha, \beta, \gamma)$  integral is written in the form

$$\Gamma_{n,k,l}(\alpha, \beta, \gamma) = 2 \cdot l!m!n! \sum_{l_1=0}^l \sum_{m_1=0}^m \sum_{n_1=0}^n \frac{C_{m-m_1+l_1}^{l_1} C_{l-l_1+n_1}^{m_1} C_{n-n_1+m_1}^{m_1}}{(\alpha + \beta)^{m-m_1+l_1+1} (\alpha + \gamma)^{l-l_1+n_1+1} (\beta + \gamma)^{n-n_1+m_1+1}} \quad (23)$$

where  $C_k^m$  are the binomial coefficients (= number of combinations from  $k$  by  $m$ ). In many application all three integer numbers  $k, l, m$  are relatively small ( $\leq 3$ ) and this simplifies numerical applications of the formula, Eq.(23).

The parameters  $\alpha, \beta$  and  $\gamma$  in Eqs.(20) - (23) can be arbitrary real and/or complex numbers. Furthermore, one of these three parameters/numbers can be equal zero, but the three principal conditions for these parameters  $\alpha + \beta > 0, \alpha + \gamma > 0$  and  $\beta + \gamma > 0$  must always be obeyed. If these three parameters are complex, then these three conditions are written in the form  $Re(\alpha + \beta) > 0, Re(\alpha + \gamma) > 0$  and  $Re(\beta + \gamma) > 0$ . These conditions are needed to guarantee the convergence of the corresponding integrals in relative coordinates.

Now, the explicit formulas for the analogous three-body integrals which contain the Bessel functions  $j_0(kr_2)$  and/or  $j_1(kr_2)$  is derived in the following way. First, consider the integral,

Eq.(20), with the  $j_0(x)$  function which can also be written in the form  $j_0(x) = \frac{\sin x}{x}$ . In this case the integral Eq.(20) is written as the sum of the following two integrals

$$I_0(n, \ell; k) = \frac{1}{2ik} \int \int \int \phi_{n\ell}(r_1) \exp[-\alpha r_1 - (\beta - ik)r_2 - \gamma r_{12}] r_1 r_{12} dr_1 dr_2 dr_{12} \quad (24)$$

$$- \frac{1}{2ik} \int \int \int \phi_{n\ell}(r_1) \exp[-\alpha r_1 - (\beta + ik)r_2 - \gamma r_{12}] r_1 r_{12} dr_1 dr_2 dr_{12}$$

where  $i$  is the imaginary unit. Consider the case when the final hydrogen atom is formed in its ground state (or (10)-state). The integral  $I_0(n, \ell; k) = I_0(1, 0; k)$  can easily be determined in perimetric coordinates. Indeed, in this case  $\phi_{n\ell}(r_1) = \phi_{10}(r_1) = 2 \exp(-Ar)$ , where  $A = 1$ , and the last integral is reduced to the form

$$I_0(1, 0; k) = \frac{1}{ik} \int \int \int \exp[-(\alpha + A)r_1 - (\beta - ik)r_2 - \gamma r_{12}] r_1 r_{12} dr_1 dr_2 dr_{12} \quad (25)$$

$$- \frac{1}{ik} \int \int \int \exp[-(\alpha + A)r_1 - (\beta + ik)r_2 - \gamma r_{12}] r_1 r_{12} dr_1 dr_2 dr_{12}$$

$$= -\frac{2i}{k} \int_0^\infty \int_0^\infty \int_0^\infty \exp[-X_1 u_1 - X_2 u_2 - X_3 u_3] (u_1 + u_3)(u_1 + u_2) du_1 du_2 du_3$$

$$+ \frac{2i}{k} \int_0^\infty \int_0^\infty \int_0^\infty \exp[-X_1 u_1 - X_2^* u_2 - X_3^* u_3] (u_1 + u_3)(u_1 + u_2) du_1 du_2 du_3$$

where  $X_1 = \alpha + \gamma + A$ ,  $X_2 = \beta + \gamma - ik$ ,  $X_3 = \alpha + \beta + A - ik$  and  $X_2^* = \beta + \gamma + ik$ ,  $X_3^* = \alpha + \beta + A + ik$ . Note that each of the last two integrals is written in the form

$$J(a, b, c) = \int_0^\infty \int_0^\infty \int_0^\infty \exp[-ax - by - cz] (x + z)(x + y) dz dy dx = \frac{1}{abc} \left[ \frac{2}{a^2} \right. \\ \left. + \frac{1}{ab} + \frac{1}{ac} + \frac{1}{bc} \right] \quad (26)$$

Due to some historic reasons this integral is often called the Coulomb three-body integral.

The closed analytical expression for this integral allows one to obtain analytical formulas for all integrals, Eq.(20), which include the  $|n\ell\rangle$  radial functions of the final hydrogen atom. In reality, such integrals are computed as the partial derivatives from Eq.(26) in respect with the parameter  $\alpha$ . This solves the problem of analytical/numerical calculations of the three-body integrals from the exponential functions of the three relative coordinates, Eq.(20), which include the Bessel function  $j_0(kr)$ .

Analytical formulas for the analogous integrals with the Bessel function  $j_1(kr)$  are easily derived from Eq.(24) - (25) with the use of the relation

$$j_1(kr) = -\frac{1}{r} \frac{dj_0(kr)}{dk} \quad (27)$$

known for these two Bessel functions. Analytical and/or numerical computation of the corresponding derivatives and integrals is straightforward. In general, all analytical transformations of formulas are easily performed with the use of different platforms for analytical computations such as Mathematica and/or Maple (see, e.g., [16] and [17]). Note also that all integrals arising during this procedure are regular, since each derivative in respect to  $k$  gives an extra power of  $r_2$  which compensate the presence of this variable in the denominator of the right hand side of Eq.(27).

## V. A SIMPLE METHOD TO EVALUATE THE PHOTODETACHMENT CROSS-SECTION

The negatively charged hydrogen ion  $H^-$  is the bound two-electron atomic ion, i.e.  $N_e = 2$ , while the nuclear charge  $Q$  equals 1. As is well known (see, e.g., [9] - [10] and references therein) the long range one-electron asymptotic of the atomic wave function is written in the form

$$|\Psi(r)| = Cr^{\frac{Z}{t}-1} \exp(-tr) \quad (28)$$

where  $C$  is some numerical constant,  $Z = Q - N_e + 1$ , while  $t = \sqrt{2I_1}$  and  $I_1 = \chi_1$  is the first ionization potential which corresponds to the dissociation process  $H^- = H + e^-$ . For the  $H^-$  ion one finds in the last equation  $Q = 1, N_e = 2$ , and therefore,  $Z = 0$  and the long range asymptotic of the wave function is represented in the Yukawa-type form  $|\Psi(r)| = \frac{C}{r} \exp(-tr)$ , where  $C$  is a constant which must provide the best correspondence with the highly accurate wave function of the  $H^-$  ion at large  $r$ . The highly accurate wave function of the  $H^-$  ion is assumed to be known. The explicit formula which is used to determine the constant  $C$  is  $C = |\Psi_{H^-}(r)| r \exp(tr)$ , where  $\Psi_{H^-}(r) = \Psi_{H^-}(r_{32} = r, r_{31} = 0, r_{21} = r)$ . As follows from the results of actual computations the factor  $C$  does not change for relatively large interval of variations of  $r$ , e.g., for  $r$  bounded between 7.5 and 35 atomic units. The determined numerical value of this constant  $C$  for the negatively charged hydrogen ions varies between  $\approx 0.12595$  ( $^\infty H^-$ ) and  $\approx 0.12605$  ( $^1 H^-$ ).

The photodetachment cross-section  $\sigma(H^-)$  of the  $H^-$  ion is written in the form (in atomic units) (see, e.g., [8])

$$\sigma(H^-) = \frac{64\pi^2 C^2 \alpha a_0^2}{3} \cdot \frac{p^3}{(p^2 + \gamma^2)^3} = \frac{64\pi^2 C^2 \alpha}{3} \cdot \frac{p^3}{(p^2 + \gamma^2)^3} \quad (29)$$

where  $a_0$  is the Bohr radius and  $\gamma = \sqrt{2I_1}$ . In the usual units ( $cm^2$ ) the last formula takes the form

$$\sigma(H^-) = 2.7032847 \cdot 10^{-16} \cdot C^2 \cdot \frac{p^3}{(p^2 + \gamma^2)^3} \text{ cm}^2 \quad . \quad (30)$$

This formula with the known value of  $C$  can directly be used in calculations of the photodetachment cross-section  $\sigma(H^-)$ . This formula essentially coincides with Eq.(8) from [7]. In our case,  $C = 0.12595$ , and therefore, from the last formula one finds

$$\sigma(H^-) = 4.28840 \cdot 10^{-18} \cdot \frac{p^3}{(p^2 + \gamma^2)^3} \text{ cm}^2 \quad , \quad (31)$$

where  $p = |\mathbf{p}|$  is the momentum of the photo-electron (in atomic units) and  $\gamma = 0.23558869473885$  (see Table I).

The overall accuracy of this simple formula, Eq.(31), is outstanding. This remarkable fact can be explained by using the following inequality  $a \ll \lambda$ , where  $a \approx a_0$  is the effective radius of the atomic  $H^-$  ion ( $a_0$  is the Bohr radius), while  $\lambda$  is the wavelength of the photon which produces photodetachment of the  $H^-$  ion. For the  $H^-$  ion we have  $a \approx a_0 \approx 0.529 \text{ \AA}$  and  $\lambda \approx 1 \cdot 10^5 \text{ \AA}$ . Briefly, we can say that the incident photon can see only an outer asymptotics of the three-body wave function. Briefly, we can say that all details of the wave function of the  $H^-$  ion at small inter-particle distances are not important to describe the photodetachment of this ion. Recently, this simple method was often used in various applications related to the  $H^-$  ion. However, some important details of the photodetachment of the  $H^-$  ion cannot be investigated with the use of only this simple method.

## VI. INVERSE BREMSSTRAHLUNG IN THE FIELD OF THE HYDROGEN ATOM

In this Section we discuss the electron scattering stimulated by the electromagnetic radiation in the field of the heavy hydrogen atom, Eq.(2). In classical physics this process is called the inverse bremsstrahlung. In stellar photospheres of hydrogen stars inverse bremsstrahlung is responsible for absorption of infrared radiation with  $\lambda \geq 12,000 \text{ \AA}$ . Furthermore, it is very important to describe the total absorption of infrared radiation in photospheres of many cold stars.

As mentioned above at small energies of the incident photon  $h\nu \leq 5eV$  the original (ground) state of the hydrogen atom does not change. This means that for photons of small

energies we can ignore all possible excitations of the hydrogen atom, i.e. atomic transitions from the ground state of the H atom into other excited states. The incident photon is absorbed exclusively by the electron which moves in the field of the neutral hydrogen atom. In this case the electron scattering at the hydrogen atom ( $H + e^- + h\nu = H + e^-$ ) can be considered as a regular (but non-elastic!) scattering of the electron in the field of the heavy hydrogen atom. In the lowest order approximation we can replace the actual hydrogen atom by its polarization potential, which is a central and local potential  $V(r) = -\frac{a}{r^4}$ , where  $a$  is a positive constant. In particular, for the  ${}^\infty\text{H}$  atom in atomic units one finds  $a = \frac{9}{4}$ . Below, we shall consider the electron scattering in the field of the following central potential

$$V(r; a, b) = -\frac{a}{(r+b)^4} \quad (32)$$

which analytically depends upon the two real parameters  $a$  and  $b$ . We need to determine the cross-section of inverse bremsstrahlung for an electron which moves in the field of the central potential  $V(r; a, b)$ , Eq.(32). By using different numerical values for the  $a$  and  $b$  parameters we can obtain a variety of such cross-sections. The main interest, however, will be related to the cases when  $a \approx 4.5$  and  $b \approx 0$ , respectively.

First, let us obtain the incident and final wave functions which are the stationary one-electron wave functions of the continuous spectra of the radial Schrödinger equation (see, e.g., [13]) of the electron moving in the field of the central potential, Eq.(32). The solution of one-electron Schrödinger equation can be found by representing the unknown wave function  $\Psi(r)$  in the form  $\Psi_\ell(r) = A_\ell(r) \exp(i\delta_\ell(r))$ . Note that in the field of any central potential the angular momentum  $\ell$  is a good (i.e. conserving) quantum number. It reduces the original problem to the solution of the following differential equation

$$\frac{d\delta_\ell(r)}{dr} = -\frac{2V(r; a, b)}{k} \left[ j_\ell(kr) \cos \delta_\ell(r) - n_\ell(kr) \sin \delta_\ell(r) \right]^2 \quad (33)$$

where the phase  $\delta_\ell(r)$  of the wave function is assumed to be explicitly  $r$ -dependent and  $\delta_\ell(r=0) = 0$  (initial condition). With this initial condition the last equation is reduced to the form of the following integral equation

$$\delta_\ell(r) = -\frac{2}{k} \int_0^{+\infty} V(r; a, b) \left[ j_\ell(kr) \cos \delta_\ell(r) - n_\ell(kr) \sin \delta_\ell(r) \right]^2 dr \quad (34)$$

The equations Eq.(33) and Eq.(34) are the main equations of the variable phase method. The method of variable phase has been developed by V.V. Babikov and others in the middle

of 1960's [18] (see also references therein). Each of these two equations Eqs.(33) - (34) can be solved to high accuracy by using various numerical methods. In general, if we know the phase function  $\delta_\ell(r)$  for each  $r$ , then it is possible to determine the amplitude  $A_\ell(r)$  of the wave function and the wave function  $\Psi_k(r) = A_\ell(r) \exp(i\delta_\ell(r))$  itself. The corresponding equation for the amplitude  $A_\ell(r)$  takes the form [18]

$$\frac{dA_\ell(r)}{dr} = -\frac{A_\ell(r)V(r; a, b)}{k} \left[ j_\ell(kr) \cos \delta_\ell(r) - n_\ell(kr) \sin \delta_\ell(r) \right] \left[ j_\ell(kr) \sin \delta_\ell(r) + n_\ell(kr) \cos \delta_\ell(r) \right] \quad (35)$$

Note that the variable phase method works very well for central potentials which rapidly vanish at  $r \rightarrow +\infty$ , e.g., for  $|V(r)| \sim r^{-n}$ , where  $n \geq 3$  at  $r \rightarrow +\infty$ . This means that we can apply this method to determine the accurate wave functions of the continuous spectra in the field of the polarization potential, Eq.(32), created by the neutral H atom which is infinitely heavy. It appears that the variable phase method provides a large number of advantages in applications to the problems related with the absorption/emission of radiation from the states of continuous spectra.

Note that in the case of inverse bremsstrahlung the energy conservation law is written in the form

$$\frac{\hbar k_i^2}{2m_e} + \omega = \frac{\hbar k_f^2}{2m_e}, \quad \text{or} \quad \hbar\omega = \frac{p_f^2}{2m_e} - \frac{p_i^2}{2m_e} \quad (36)$$

where  $k_i$  and  $k_f$  are the wave numbers defined as  $k_a = |\mathbf{k}_a| = \frac{|\mathbf{p}_a|}{\hbar}$  and  $p_a$  is the momentum of the particle  $a$  and  $\mathbf{k}_a$  is the corresponding wave vector. In Eq.(36) the notation  $\hbar$  is the reduced Planck constant, while  $m_e$  is the electrons mass. From Eq.(36) one finds  $k_f = \sqrt{k_i^2 + \frac{2m_e}{\hbar}\omega}$ . By using the variable phase method described above we can obtain the initial and final state wave functions, i.e.  $\Psi_{k_i}(r)$  and  $\Psi_{k_f}(r)$ , respectively. In the variable phase method these wave functions are represented as the products of their amplitudes and phase parts, i.e.  $\Psi_{\ell; k_i}(r) = A_{\ell; k_i}(r) \exp(i\delta_{\ell; k_i}(r))$  and  $\Psi_{\ell; k_f}(r) = A_{\ell; k_f}(r) \exp(i\delta_{\ell; k_f}(r))$ . These two wave functions are used in the following computations of the cross-section of inverse bremsstrahlung. The explicit formula for the cross-section of inverse bremsstrahlung is (in relativistic units)

$$d\sigma_{\mathbf{k}\mathbf{p}_f} = \frac{\omega e^2}{(2\pi)^4 m_e p_i} |\mathbf{e}^* \mathbf{p}_{i \rightarrow f}|^2 d\Omega_{\mathbf{k}} d^3\mathbf{p}_f \quad (37)$$

where  $p_i = |\mathbf{p}_i|$  is the 'absolute' momentum of the incident electron,  $\mathbf{p}_f$  is the momentum

of the final electron and  $\mathbf{p}_{i \rightarrow f}$  is the following matrix element

$$\mathbf{p}_{i \rightarrow f} = -\imath \int \Psi_{k_f}^* \nabla \Psi_{k_i} d^3 \mathbf{r} \quad (38)$$

where  $\Psi_{k_i}$  and  $\Psi_{k_f}$  are the incident and final one-electron wave functions (see above).

Note that the wavelength  $\lambda$  of the light quantum which produces photodetachment of the  $\text{H}^-$  ion is  $\geq 12,000 \text{ \AA}$ , while the effective ‘scattering zone’  $R_s$  for the process, Eq.(2) is less than  $50 \text{ \AA}$ . Therefore, we can apply the approximation based on the fact that the dimensionless parameter  $\frac{R_s}{\lambda} \ll 1$ . This approximation allows one to replace the  $r$ -dependent phase factor  $\delta_{\ell; k_i}(r)$  in the wave function by its asymptotic value  $\delta_{\ell; k_i}(+\infty)$  determined at the infinity. In other words, the phase of the wave function does not depend upon  $r$ , i.e. it is a constant. The wave function  $\Psi_{\ell; k_i}(r)$  is represented in the form  $\Psi_{\ell; k_i}(r) = A_\ell(r) \exp(\imath \delta_\ell)$ , where  $\delta_\ell = \delta_\ell(r = +\infty)$  is a constant (or almost a constant). This drastically simplifies numerical computations of the integrals in Eq.(37) and Eq.(38).

## VII. CONCLUSION

We have discussed the absorption of infrared and visible radiation by the negatively charged hydrogen ions  $\text{H}^-$ . Such an absorption of radiation plays an extremely important role in Solar/Stellar Astrophysics. In general, the absorption of radiation by the  $\text{H}^-$  ions is mainly related with the photodetachment of these ions. The absorption of radiation by electrons scattered in the field of the neutral hydrogen atom(s) also contributes to the total cross-section at very large wavelength  $\lambda \geq 12,000 \text{ \AA}$  (infrared radiation). The photodetachment cross-section of the  $\text{H}^-$  ion has been determined with the use of highly accurate variational wave functions constructed recently for this ion. We explicitly discuss all details important for calculations of the photodetachment cross-section with the use of highly accurate (or truly correlated) wave functions of the three-body  $\text{H}^-$  ion. To evaluate the total cross-section of electron scattering in the field of the polarization potential of the neutral H atom we apply the variable phase method. The agreement between our cross-sections and results obtained in earlier works for different wavelength can be considered as very good.

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TABLE I: The non-relativistic total energies  $E$  and ionization potential  $\chi_1$  of the hydrogen negatively charged ions (isotopes) in atomic units.

ion	$E$	$\chi_1$
$^{\infty}\text{H}^-$	-0.527 751 016 544 377 196 567 0(2)	-0.027 751 016 544 377 196 567 0
$\text{T}^-$	-0.527 649 048 202 999 950 354 2(2)	-0.027 739 991 660 945 743 753 5
$\text{D}^-$	-0.527 598 324 686 477 854 280 7(2)	-0.027 734 509 437 460 880 345 7
$^1\text{H}^-$	-0.527 445 881 114 178 893 391 3(2)	-0.027 718 041 397 713 212 403 8