Linear Magnitudes

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Abstract

In classical mechanics, this paper presents definitions of linear magnitudes from vector magnitudes.

Linear Magnitudes

The linear magnitudes for a particle A of mass m_a are defined with respect to a position vector $\mathbf{r}$ which is constant in magnitude and direction.

Linear Mass	$Y_a = m_a\,(\mathbf{r} \cdot \mathbf{r}_a)$
Linear Momentum	$P_a = m_a\,(\mathbf{r} \cdot \mathbf{v}_a)$
Linear Force	$F_a = m_a\,(\mathbf{r} \cdot \mathbf{a}_a)$
Linear Work	$W_a = \int F_a \; d(\mathbf{r} \cdot \mathbf{r}_a)$
Theorem	$W_a = \Delta\, {}^1\!/_2\, m_a\,(\mathbf{r} \cdot \mathbf{v}_a)^2$

Where $\mathbf{r}_a$, $\mathbf{v}_a$, and $\mathbf{a}_a$ are the position, the velocity, and the acceleration of particle A.

The linear magnitudes for a system of particles are also defined with respect to a position vector $\mathbf{r}$ which is constant in magnitude and direction.

Linear Potential Energy

The linear potential energy U_a of a particle A on which a resultant force $\mathbf{F}_a$ acts, is given by:

$$U_a = -\int (\mathbf{r} \cdot \mathbf{F}_a)\, d(\mathbf{r} \cdot \mathbf{r}_a)$$

where $\mathbf{r}$ is a position vector which is constant in magnitude and direction, and $\mathbf{r}_a$ is the position of particle A.

If $\mathbf{F}_a$ is constant and since $\mathbf{F}_a = m_a\, \mathbf{a}_a$, it follows that:

$$U_a = -\, m_a (\mathbf{r} \cdot \mathbf{a}_a)(\mathbf{r} \cdot \mathbf{r}_a)$$

where m_a is the mass of particle A, and $\mathbf{a}_a$ is the constant acceleration of particle A.

Linear Mechanical Energy

The linear mechanical energy E_a of a particle A of mass m_a which moves in a uniform force field, is given by:

$$E_a = {}^1\!/_2\, m_a (\mathbf{r} \cdot \mathbf{v}_a)^2 - m_a (\mathbf{r} \cdot \mathbf{a}_a)(\mathbf{r} \cdot \mathbf{r}_a)$$

where $\mathbf{r}$ is a position vector which is constant in magnitude and direction, and $\mathbf{v}_a$, $\mathbf{a}_a$ and $\mathbf{r}_a$ are the velocity, the constant acceleration and the position of particle A.

The principle of conservation of the linear mechanical energy establishes that if a particle A moves in a uniform force field then the linear mechanical energy of particle A remains constant.

Principle of Least Linear Action

If we consider a single particle A of mass m_a then the principle of least linear action, is given by:

$$\delta \int_{t_1}^{t_2} \tfrac{1}{2}\, m_a (\mathbf{r} \cdot \mathbf{v}_a)^2 \, dt + \int_{t_1}^{t_2} (\mathbf{r} \cdot \mathbf{F}_a)\, \delta (\mathbf{r} \cdot \mathbf{r}_a) \, dt = 0$$

where $\mathbf{r}$ is a position vector which is constant in magnitude and direction, $\mathbf{v}_a$ is the velocity of particle A, $\mathbf{F}_a$ is the net force acting on particle A, and $\mathbf{r}_a$ is the position of particle A.

If $-\delta V_a = (\mathbf{r} \cdot \mathbf{F}_a)\, \delta (\mathbf{r} \cdot \mathbf{r}_a)$ and since $T_a = \tfrac{1}{2}\, m_a (\mathbf{r} \cdot \mathbf{v}_a)^2$, then:

$$\delta \int_{t_1}^{t_2} (T_a - V_a) \, dt = 0$$

And since $L_a = T_a - V_a$, then we obtain:

$$\delta \int_{t_1}^{t_2} L_a \, dt = 0$$

Bibliography

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