

# Lorentz Violation of the Photon Sector in Field Theory Models

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We compare the Lorentz violation terms of the pure photon sector between two field theory models, namely the minimal standard model extension (SME) and the standard model supplement (SMS). From the requirement of the identity of the intersection for the two models, we find that the free photon sector of the SMS can be a subset of the photon sector of the minimal SME. We not only obtain some relations between the SME parameters, but also get some constraints on the SMS parameters from the SME parameters. The CPT-odd coefficients  $(k_{AF})^\alpha$  of the SME are predicted to be zero. There are 15 degrees of freedom in the Lorentz violation matrix  $\Delta^{\alpha\beta}$  of free photons of the SMS related with the same number of degrees of freedom in the tensor coefficients  $(k_F)^{\alpha\beta\mu\nu}$ , which are independent from each other in the minimal SME, but are inter-related in the intersection of the SMS and the minimal SME. With the related degrees of freedom, we obtain the conservative constraints ( $2\sigma$ ) on the elements of the photon Lorentz violation matrix. The detailed structure of the photon Lorentz violation matrix suggests some applications to the Lorentz violation experiments for photons.

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## I. INTRODUCTION

Lorentz symmetry is one of the basic principles of modern physics, and it stands as one of the basic foundations of the standard model of particle physics. The minimal standard model has achieved a great success in predictions and explanations of various experiments. Nevertheless, some fundamental questions remain to be answered. One of the most essential questions is whether the Lorentz invariance holds exactly or to what extent it holds. Through theoretical researches and recent available experiments on the Lorentz invariance violation (LIV or LV), we can obtain a deeper insight into the nature of Lorentz symmetry and clarify these fundamental questions.

The possible Lorentz symmetry violation (LV) effects have been investigated for decades from various theories, motivated by the unknown underlying theory of quantum gravity together with various phenomenological applications [1, 2]. The existence of an “æther” or “vacuum” can bring the breaking down of Lorentz invariance [3, 4]. From basic consideration, there are investigations on the concepts of space-time such as whether the space-time is discrete or continuous [5–7], or whether a fundamental length scale should be introduced to replace the Newtonian constant  $G$  [8]. The Lorentz violation can happen in many alternative theories, e.g., the doubly spe-

cial relativity (DSR) [9–11], torsion in general relativity [12, 13], non-covariant field theories [14], and large extra-dimensions [18, 19]. Among these theoretical investigations of Lorentz violation, it is a powerful framework to discuss various LV effects based on traditional techniques of effective field theory in particle physics. It starts from the Lagrangian of the standard model, and then includes all possible terms containing the Lorentz violation effects. The magnitudes of these LV terms can be constrained by various experiments. The standard model extension (SME) [20] is an example within such field theory frameworks, in which the LV terms are measured with several tensor fields as coupling constants, and modern experiments have built severe constraints on the relevant Lorentz violation parameters [21].

The standard model extension is an effective framework for phenomenological analysis. We still need a fundamental theory to derive the Lorentz violation terms from basic principles. An attempt for such a purpose has been offered in Refs. [22, 23], in which a more basic principle, denoted as physical invariance or physics independence, is proposed to extend the basic principle of relativity. Instead of the requirement that the equations describing the laws of physics have the same form in all admissible frames of reference, it requires that the equations describing the laws of physics have the same form in all admissible mathematical manifolds. The principle leads to the following replacement of the ordinary partial  $\partial_\alpha$  and the covariant derivative  $D_\alpha$

$$\partial^\alpha \rightarrow M^{\alpha\beta} \partial_\beta, \quad D^\alpha \rightarrow M^{\alpha\beta} D_\beta, \quad (1)$$

where  $M^{\alpha\beta}$  is a local matrix. We separate it to two matri-

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ces like  $M^{\alpha\beta} = g^{\alpha\beta} + \Delta^{\alpha\beta}$ , where  $g^{\alpha\beta}$  is the metric tensor of space-time and  $\Delta^{\alpha\beta}$  is a new matrix which is particle-type dependent generally. Since  $g^{\alpha\beta}$  is Lorentz invariant,  $\Delta^{\alpha\beta}$  contains all the Lorentz violating degrees of freedom from  $M^{\alpha\beta}$ . Then  $\Delta^{\alpha\beta}$  brings new terms violating Lorentz invariance in the standard model and is called Lorentz invariance violation matrix. This new framework is referred as the Standard Model Supplement (SMS) [22, 23], and it has been applied to discuss the Lorentz violation effects for protons [22] and photons [24]. It is also extended to neutrinos recently [25] for the discussion of the recent neutrino speed anomaly reported by the OPERA collaboration [26, 27].

Before accepting the SMS as a fundamental theory, one can take the SMS as an effective framework for phenomenological applications by confronting with various experiments to determine and/or constrain the Lorentz violation matrix  $\Delta^{\alpha\beta}$  for various particles. From a more general sense, the SMS should be a subset of a general version of the SME. However, in the case of the minimal version of the SME, the relationship between the SMS and the SME is unclear yet. The purpose of this paper is to compare the Lorentz violation effects of the photon sector between the two models. As have been well known, light has always played a significant role in the developments of physics, and the Lorentz violation in the photon sector is also under active investigations both theoretically and experimentally [2]. The Lorentz violation parameters of photons in the SME have been well constrained by various experiments, which have been summarized in Ref. [21]. By confronting with the collected data in Ref. [21], we can obtain bounds and detailed structure of the Lorentz violation parameters in the model SMS [22–24].

This article is organized as follows. Sec. II provides the relation between the photon Lorentz violation matrix in the SMS and various tensor fields in the minimal SME, in the case that the two models give the same results. These relations define the boundary of the intersection of these two theories too. In Sec. III, we discuss the general structure of the photon Lorentz violation matrix in our model, and its implications for the potential property of the space-time structure for photons. At the same time, we obtain the constraints on the elements of the photon Lorentz violation matrix. Then conclusion is given in the last section.

## II. RELATIONS BETWEEN TWO MODELS

In the standard model extension (SME), the terms that violate Lorentz invariance are added by hands from some considerations such as gauge invariance, Hermitean, power-counting renormalizability and etc. We consider just the minimal SME [20] here. The Lorentz violation terms in the minimal SME contain many tensor fields as coupling constants, with their magnitudes to be determined/constrained by experiments. Though these

Lorentz violation terms are allowed by some general considerations, the reason for their existence still needs to be provided from theoretical aspects. In the SMS [22, 23], the Lorentz violation terms arise from the replacement of Eq. (1), which is considered as a necessary requirement from the basic principle of physical invariance. As both of the two models are built within the framework of effective field theory in particle physics, they can be considered as two special cases of a general standard model extension within the effective field theory. It is therefore necessary to study the relationship between the two models.

The Lagrangians of the pure photon sector in the SMS and the minimal SME are

$$\mathcal{L}_{\text{SMS}} = -\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} + \mathcal{L}_{\text{GV}}, \quad (2)$$

where

$$\begin{aligned} \mathcal{L}_{\text{GV}} = & -\frac{1}{2}\Delta^{\alpha\beta}\Delta^{\mu\nu}(g_{\alpha\mu}\partial_\beta A^\rho\partial_\nu A_\rho - \partial_\beta A_\mu\partial_\nu A_\alpha) \\ & -F_{\mu\nu}\Delta^{\mu\alpha}\partial_\alpha A^\nu, \end{aligned} \quad (3)$$

and

$$\mathcal{L}_{\text{SME}} = -\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} + \mathcal{L}_{\text{photon}}^{\text{CPT-even}} + \mathcal{L}_{\text{photon}}^{\text{CPT-odd}}, \quad (4)$$

where

$$\mathcal{L}_{\text{photon}}^{\text{CPT-even}} = -\frac{1}{4}(k_F)_{\alpha\beta\mu\nu}F^{\alpha\beta}F^{\mu\nu}, \quad (5)$$

$$\mathcal{L}_{\text{photon}}^{\text{CPT-odd}} = \frac{1}{2}(k_{AF})^\alpha\epsilon_{\alpha\beta\mu\nu}A^\beta F^{\mu\nu}. \quad (6)$$

We denote the matrix  $\Delta^{\alpha\beta}$  above as Lorentz invariance violation matrix, whose dimension is massless. When we ignore the field redefinition, there are 16 independent dimensionless degrees of freedom in  $\Delta^{\alpha\beta}$  generally [23]. As coupling constants, the vacuum expectation value of  $\Delta^{\alpha\beta}$  is CPT-even, and the vacuum expectation value of its derivative  $\partial_\mu\Delta^{\alpha\beta}$  is CPT-odd. Coefficients  $(k_F)_{\alpha\beta\mu\nu}$  and  $(k_{AF})^\alpha$  are CPT-even and CPT-odd respectively. The CPT-even terms in SME might be understood as originated from some general relativity consideration as proposed in Ref. [12].  $(k_F)_{\alpha\beta\mu\nu}$  is antisymmetric for the first pair indices  $\alpha$  and  $\beta$ , antisymmetric for the second pair  $\mu$  and  $\nu$ , and symmetric for the interchange of the two pairs of indices. Hence there are 21 degrees of freedom in  $(k_F)_{\alpha\beta\mu\nu}$ . With the redefinition of the gauge field, there are 2 degrees of freedom to be reduced in  $(k_F)_{\alpha\beta\mu\nu}$ . So there are 19 independent degrees of freedom under the redefinition of the fields and 21 degrees of freedom in general without considering this redefinition. Another 4 degrees of freedom are in  $(k_{AF})^\alpha$ . After all consideration, there are  $19 + 4 = 23$  independent degrees of freedom for  $(k_{AF})^\alpha$  and  $(k_F)_{\alpha\beta\mu\nu}$  to consider the field redefinition, and  $21 + 4 = 25$  ones without considering this redefinition in general in the pure photon sector of the minimal

SME [21]. Given the situation that the Lorentz violation matrix  $\Delta^{\alpha\beta}$  here is coupled with other types of fermions and bosons, there are no universal redefinitions for all the fields of different particles yet. So we discuss mainly the general form of  $\Delta^{\alpha\beta}$  with all the 16 degrees of freedom, fit data from various experiments, and obtain the magnitudes or constraints by the experiments, avoiding any a priori assumption on  $\Delta^{\alpha\beta}$ .

We can make a direct correspondence between the two Lagrangians in Eqs. (2) and (4) (cf. the table in Ref. [22]), when considering the vacuum expectation values of both  $\Delta^{\alpha\beta}$  (CPT-even) and its derivative  $\partial_\mu \Delta^{\alpha\beta}$  (CPT-odd) as coupling constants and Lorentz violation parameters. In the case that just the Lorentz violation matrix  $\Delta^{\alpha\beta}$  is adopted as the violation parameters, there are terms left in Eq. (2) which can not be covered by the Lagrangian in Eqs. (5) and (6). Comparing directly Eq. (3) with Eqs. (5) and (6), we can not find a direct term-to-term equivalence between the Lagrangians of the free photon sector in the SMS and the minimal SME.

Here, we treat  $\Delta^{\alpha\beta}$  as its vacuum expectation value, i.e. as coupling constants in the field theory framework. Then any derivatives  $\partial_\mu \Delta^{\alpha\beta}$  vanish in the following partial integrations during the derivations.

In the standard model supplement, the motion equation for free photons is

$$\Pi_{\text{SMS}}^{\gamma\rho} A_\rho = 0,$$

and the Lagrangian in Eq. (2) reads also

$$\mathcal{L}_{\text{SMS}} = -\frac{1}{2} A_\gamma \Pi_{\text{SMS}}^{\gamma\rho} A_\rho, \quad (7)$$

where

$$\begin{aligned} \Pi_{\text{SMS}}^{\gamma\rho} = & -g^{\gamma\rho} \partial^2 + \partial^\gamma \partial^\rho \\ & + \Delta^{\gamma\alpha} \partial^\rho \partial_\alpha + \Delta^{\rho\alpha} \partial^\gamma \partial_\alpha + \Delta^{\gamma\beta} \Delta^{\rho\nu} \partial_\beta \partial_\nu \\ & - g^{\gamma\rho} (2\Delta^{\mu\alpha} \partial_\mu \partial_\alpha + g_{\alpha\mu} \Delta^{\alpha\beta} \Delta^{\mu\nu} \partial_\beta \partial_\nu). \end{aligned} \quad (8)$$

From Eq. (2) to Eq. (7), partial integrations are used. We use the Fourier transformation  $A_\rho(x) = A_\rho(p) \exp(-ip \cdot x)$  to get

$$\begin{aligned} \Pi_{\text{SMS}}^{\gamma\rho}(p) = & g^{\gamma\rho} p^2 - p^\gamma p^\rho \\ & - \Delta^{\gamma\alpha} p^\rho p_\alpha - \Delta^{\rho\alpha} p^\gamma p_\alpha - \Delta^{\gamma\beta} \Delta^{\rho\nu} p_\beta p_\nu \\ & + g^{\gamma\rho} (2\Delta^{\mu\alpha} p_\mu p_\alpha + g_{\alpha\mu} \Delta^{\alpha\beta} \Delta^{\mu\nu} p_\beta p_\nu). \end{aligned}$$

For the free photon in the minimal SME, the Lagrangian is similar

$$\mathcal{L}_{\text{SME}} = -\frac{1}{2} A_\gamma \Pi_{\text{SME}}^{\gamma\rho} A_\rho, \quad (9)$$

where

$$\begin{aligned} \Pi_{\text{SME}}^{\gamma\rho} = & -g^{\gamma\rho} \partial^2 + \partial^\gamma \partial^\rho + 2(k_F)^{\gamma\alpha\beta\rho} \partial_\alpha \partial_\beta \\ & + 2(k_{AF})_\alpha \epsilon^{\gamma\alpha\beta\rho} \partial_\beta, \end{aligned} \quad (10)$$

and the representation in momentum space is

$$\begin{aligned} \Pi_{\text{SME}}^{\gamma\rho}(p) = & g^{\gamma\rho} p^2 - p^\gamma p^\rho - 2(k_F)^{\gamma\alpha\beta\rho} p_\alpha p_\beta \\ & - 2i(k_{AF})_\alpha \epsilon^{\gamma\alpha\beta\rho} p_\beta. \end{aligned}$$

We see that  $\Pi_{\text{SMS}}^{\gamma\rho}(p)$  and  $\Pi_{\text{SME}}^{\gamma\rho}(p)$  are the inverse of the photon propagator in the momentum space. The propagator determines the propagating properties of photons.

When the two Lagrangians Eq. (2) and Eq. (4) are equivalent to each other for free photons, i.e. we consider the common part (intersection) between the two models, some enlightenments are expected to come. We can get  $\Pi_{\text{SMS}}^{\gamma\rho} = \Pi_{\text{SME}}^{\gamma\rho}$ . Then the matrix equation is satisfied

$$\begin{aligned} & g^{\gamma\rho} (2\Delta^{\mu\alpha} p_\mu p_\alpha + g_{\alpha\mu} \Delta^{\alpha\beta} \Delta^{\mu\nu} p_\beta p_\nu) \\ & - \Delta^{\gamma\alpha} p^\rho p_\alpha - \Delta^{\rho\alpha} p^\gamma p_\alpha - \Delta^{\gamma\beta} \Delta^{\rho\nu} p_\beta p_\nu \\ = & -2(k_F)^{\gamma\alpha\beta\rho} p_\alpha p_\beta - 2i(k_{AF})_\alpha \epsilon^{\gamma\alpha\beta\rho} p_\beta. \end{aligned}$$

Making derivative with respect to momentum  $p^\alpha$  for two times, we obtain

$$\begin{aligned} & 2g_{\gamma\rho} (\Delta_{\alpha\beta} + \Delta_{\beta\alpha} + g^{\mu\nu} \Delta_{\mu\alpha} \Delta_{\nu\beta}) \\ & - \Delta_{\gamma\alpha} g_{\rho\beta} - \Delta_{\gamma\beta} g_{\rho\alpha} - \Delta_{\rho\alpha} g_{\gamma\beta} - \Delta_{\rho\beta} g_{\gamma\alpha} \\ & - \Delta_{\gamma\alpha} \Delta_{\rho\beta} - \Delta_{\gamma\beta} \Delta_{\rho\alpha} \\ = & -4(k_F)_{\gamma(\alpha\beta)\rho} - 4i(k_{AF})^\mu \epsilon_{\mu\gamma\rho(\alpha} l_{\beta)}^s. \end{aligned} \quad (11)$$

We accept conventions of general relativity for the notation of indices here and in the following derivations. The coefficient  $l_\beta^s$  is introduced here, and its dimension is [length] or [mass]<sup>-1</sup>.  $l_\beta^s$  represents the characteristic length of the physical process. Based on the symmetric/antisymmetric properties of indices  $\gamma$  and  $\rho$ , we get two matrix equations further

$$-i(k_{AF})^\mu \epsilon_{\mu\gamma\rho(\alpha} l_{\beta)}^s = (k_F)_{[\gamma|(\alpha\beta)|\rho]} = 0, \quad (12)$$

and

$$\begin{aligned} & 2g_{\gamma\rho} (\Delta_{\alpha\beta} + \Delta_{\beta\alpha} + g^{\mu\nu} \Delta_{\mu\alpha} \Delta_{\nu\beta}) \\ & - \Delta_{\gamma\alpha} g_{\rho\beta} - \Delta_{\gamma\beta} g_{\rho\alpha} - \Delta_{\rho\alpha} g_{\gamma\beta} - \Delta_{\rho\beta} g_{\gamma\alpha} \\ & - \Delta_{\gamma\alpha} \Delta_{\rho\beta} - \Delta_{\gamma\beta} \Delta_{\rho\alpha} \\ = & -4(k_F)_{(\gamma|(\alpha\beta)|\rho)} = -4(k_F)_{\gamma(\alpha\beta)\rho}. \end{aligned} \quad (13)$$

The general formula Eq. (13) here demonstrates the relations of the Lorentz violation matrix  $\Delta_{\alpha\beta}$  and the coefficient  $(k_F)_{\gamma\alpha\beta\rho}$ , for the intersection of the SMS and the minimal SME.

When we take  $k_{AF}$  and  $k_F$  of Eqs. (12) and (13) into the Lagrangians of the minimal SME of Eqs. (5) and (6), we find that the Lagrangian of the minimal SME of Eq. (4) can be converted to that of the SMS of Eq. (2). This tells us that the free photon sector of the SMS can be considered as a subset of the minimal SME, provided that the coefficients  $(k_{AF})^\mu$  and  $(k_F)_{\gamma\alpha\beta\rho}$  are constrained by Eqs. (12) and (13).

The identity Eq. (12) tells us the relations between  $(k_F)_{\gamma\alpha\beta\rho}$  and  $(k_{AF})^\mu$ . The constraints mean that the

violation coefficient  $(k_{AF})^\mu$  (CPT-odd) vanishes in the photon sector of the minimal SME, i.e.

$$(k_{AF})^\mu = 0.$$

There is a maximal sensitivity  $10^{-42} \sim 10^{-43}$  GeV for the coefficients  $k_{(V)00}^{(3)}$ ,  $k_{(V)10}^{(3)}$ ,  $\text{Re}k_{(V)11}^{(3)}$  and  $\text{Im}k_{(V)11}^{(3)}$  in Tab. II. These four parameters are defined in terms of coefficient  $(k_{AF})^\alpha$  in Tab. III. In Eq. (13), tensor  $(k_F)_{\gamma\alpha\beta\rho}$  is antisymmetric for indices  $\gamma, \alpha$ , and antisymmetric for indices  $\beta, \rho$ . At the same time,  $(k_F)_{\gamma\alpha\beta\rho} = (k_F)_{\beta\rho\gamma\alpha}$ . So there are 21 degrees of freedom in  $(k_F)_{\gamma\alpha\beta\rho}$  generally, and  $21 - 6 = 15$  independent elements in tensor  $(k_F)_{\gamma(\alpha\beta)\rho}$ . We have already known that there are 16 degrees of freedom for  $\Delta^{\alpha\beta}$ , without the field redefinitions being considered. Therefore, any one of  $\Delta^{\alpha\beta}$  and  $(k_F)_{\gamma\alpha\beta\rho}$  can not completely determines the other one. In Eq. (13), there are 15 degrees of freedom of the Lorentz violation matrix  $\Delta_{\alpha\beta}$  related with 15 ones of the tensor  $(k_F)_{\gamma\alpha\beta\rho}$ . A definite  $\Delta^{\alpha\beta}$  can determine at most 15 degrees of freedom of  $(k_F)_{\gamma\alpha\beta\rho}$ , and vice versa.

We consider an example that  $\Delta_{\alpha\beta}$  is a symmetric matrix. Then we can get the explicit form for it. Multiplying  $g^{\gamma\rho}$  on both sides of Eq. (13), we obtain

$$3g^{\mu\nu}\Delta_{\mu\alpha}\Delta_{\nu\beta} + 6\Delta_{(\alpha\beta)} = -2(k_F)_{\alpha\beta},$$

where we define  $(k_F)_{\alpha\beta} = g^{\gamma\rho}(k_F)_{\gamma\alpha\beta\rho}$ . The tensor  $(k_F)_{\gamma\alpha\beta\rho}$  has most of the properties of the Riemann curvature tensor. So  $(k_F)_{\alpha\beta}$  is a ‘‘Ricci tensor’’ and satisfies that  $(k_F)_{\alpha\beta} = (k_F)_{\beta\alpha}$ . As we know that  $(k_F)_{\alpha\beta} \ll 1$  and  $\Delta_{\alpha\beta} \ll 1$ , the solution of  $\Delta_{\alpha\beta}$  in terms of  $(k_F)_{\alpha\beta}$ , to the second order, is

$$\Delta_{\alpha\beta} = -\frac{1}{3}(k_F)_{\alpha\beta} - \frac{1}{18}g^{\mu\nu}(k_F)_{\mu\alpha}(k_F)_{\nu\beta}. \quad (14)$$

With the assumption of  $\Delta_{\alpha\beta}$  being a symmetric matrix, there are 10 independent elements in it. Then the

Lorentz invariance violation matrix for the free photon can be obtained from the tensor  $(k_F)_{\gamma\alpha\beta\rho}$  completely, and can be considered as somewhat a kind of Ricci tensor. In the following part, the general case of  $\Delta_{\alpha\beta}$  with all the 16 degrees of freedom is considered.

### III. LORENTZ VIOLATION MATRIX OF PHOTONS

The Lorentz violation parameters of the minimal SME have been constrained from various recent experiments. The data can also provide bounds on the magnitudes of the elements of the Lorentz violation matrix of photons in the SMS, through Eqs. (12) and (13) above. The Lorentz violation parameters of the SME commonly used in experiments are four matrices  $(\tilde{\kappa}_{e+})^{jk}$ ,  $(\tilde{\kappa}_{e-})^{jk}$ ,  $(\tilde{\kappa}_{o+})^{jk}$  and  $(\tilde{\kappa}_{o-})^{jk}$  in Ref. [21]:

$$\begin{aligned} (\tilde{\kappa}_{e+})^{jk} &= -(k_F)^{0j0k} + \frac{1}{4}\epsilon^{j pq}\epsilon^{k rs}(k_F)^{pqrs}, \\ (\tilde{\kappa}_{e-})^{jk} &= -(k_F)^{0j0k} - \frac{1}{4}\epsilon^{j pq}\epsilon^{k rs}(k_F)^{pqrs} + \frac{2}{3}(k_F)^{0l0l}\delta^{jk}, \end{aligned}$$

$$(\tilde{\kappa}_{o+})^{jk} = -\frac{1}{2}\epsilon^{j pq}(k_F)^{0k pq} + \frac{1}{2}\epsilon^{k pq}(k_F)^{0j pq},$$

and

$$(\tilde{\kappa}_{o-})^{jk} = \frac{1}{2}\epsilon^{j pq}(k_F)^{0k pq} + \frac{1}{2}\epsilon^{k pq}(k_F)^{0j pq}.$$

The same indices mean summation. More parameters related with the Lorentz violation matrix are listed in Tab. III.

In terms of  $(k_F)^{\alpha\beta\mu\nu}$ , the four matrices can be rewritten as follows,

$$(\tilde{\kappa}_{e+})^{jk} = \begin{pmatrix} -(k_F)^{0101} + (k_F)^{2323} & -(k_F)^{0102} + (k_F)^{2331} & -(k_F)^{0103} + (k_F)^{2312} \\ \text{''} & -(k_F)^{0202} + (k_F)^{3131} & -(k_F)^{0203} + (k_F)^{3112} \\ \text{''} & \text{''} & -(k_F)^{0303} + (k_F)^{1212} \end{pmatrix},$$

which is symmetric for the indices  $j$  and  $k$ ;

$$(\tilde{\kappa}_{e-})^{jk} = \begin{pmatrix} -(k_F)^{0101} - (k_F)^{2323} + \alpha & -(k_F)^{0102} - (k_F)^{2331} & -(k_F)^{0103} - (k_F)^{2312} \\ \text{''} & -(k_F)^{0202} - (k_F)^{3131} + \alpha & -(k_F)^{0203} - (k_F)^{3112} \\ \text{''} & \text{''} & -(k_F)^{0303} - (k_F)^{1212} + \alpha \end{pmatrix},$$

which is symmetric for the indices  $j$  and  $k$ , and  $\alpha \equiv \frac{2}{3}(k_F)^{0l0l}$ ;

$$(\tilde{\kappa}_{o+})^{jk} = \begin{pmatrix} 0 & (k_F)^{0131} - (k_F)^{0223} & (k_F)^{0112} - (k_F)^{0323} \\ \text{''} & 0 & (k_F)^{0212} - (k_F)^{0331} \\ \text{''} & \text{''} & 0 \end{pmatrix},$$

which is antisymmetric for the indices  $j$  and  $k$ ; and

$$(\tilde{\kappa}_{o-})^{jk} = \begin{pmatrix} 2(k_F)^{0123} & (k_F)^{0131} + (k_F)^{0223} & (k_F)^{0112} + (k_F)^{0323} \\ & 2(k_F)^{0231} & (k_F)^{0212} + (k_F)^{0331} \\ & & 2(k_F)^{0312} \end{pmatrix},$$

which is symmetric for the indices  $j$  and  $k$ . The symbol " " above means that the matrix element is not written explicitly for brevity and it can be obtained from the property of symmetry/antisymmetry of the correspond-

ing matrix.

With Eq. (13), we can replace the tensor  $(k_F)^{\alpha\beta\mu\nu}$  with the Lorentz violation matrix  $\Delta^{\alpha\beta}$ . So the four matrices above are rewritten like:

$$(\tilde{\kappa}_{e+})^{jk} = \begin{pmatrix} \frac{1}{2}\Delta & \frac{1}{2}(\Delta^{10}\Delta^{20} + \Delta^{23}\Delta^{13}) & \frac{1}{2}(\Delta^{10}\Delta^{30} + \Delta^{32}\Delta^{12}) \\ & \frac{1}{2}\Delta & \frac{1}{2}(\Delta^{20}\Delta^{30} + \Delta^{31}\Delta^{21}) \\ & & \frac{1}{2}\Delta \end{pmatrix}, \quad (15)$$

which is symmetric for the indices  $j$  and  $k$ , with  $\Delta = \Delta^{\alpha\beta}g_{\alpha\beta}$ ;

$$(\tilde{\kappa}_{e-})^{jk} = \begin{pmatrix} \frac{1}{2}\Delta^{00} + \frac{1}{6}\Delta^{11} - \frac{5}{6}\Delta^{22} - \frac{5}{6}\Delta^{33} & \frac{1}{2}(\Delta^{10}\Delta^{20} - \Delta^{23}\Delta^{13}) & \frac{1}{2}(\Delta^{10}\Delta^{30} - \Delta^{32}\Delta^{12}) \\ & \frac{1}{2}\Delta^{00} + \frac{1}{6}\Delta^{22} - \frac{5}{6}\Delta^{11} - \frac{5}{6}\Delta^{33} & \frac{1}{2}(\Delta^{20}\Delta^{30} - \Delta^{31}\Delta^{21}) \\ & & \frac{1}{2}\Delta^{00} + \frac{1}{6}\Delta^{33} - \frac{5}{6}\Delta^{11} - \frac{5}{6}\Delta^{22} \end{pmatrix}, \quad (16)$$

which is symmetric for the indices  $j$  and  $k$ , with  $\alpha \equiv \frac{2}{3}(k_F)^{0i0l}$ ;

$$(\tilde{\kappa}_{o+})^{jk} = \begin{pmatrix} 0 & -\frac{1}{2}(\Delta^{02}\Delta^{32} + \Delta^{01}\Delta^{31}) & \frac{1}{2}(\Delta^{03}\Delta^{23} + \Delta^{01}\Delta^{21}) \\ & 0 & -\frac{1}{2}(\Delta^{03}\Delta^{13} + \Delta^{02}\Delta^{12}) \\ & & 0 \end{pmatrix}, \quad (17)$$

which is antisymmetric for the indices  $j$  and  $k$ ; and

$$(\tilde{\kappa}_{o-})^{jk} = \begin{pmatrix} ? & \frac{1}{2}(\Delta^{02}\Delta^{32} - \Delta^{01}\Delta^{31}) & \frac{1}{2}(\Delta^{01}\Delta^{21} - \Delta^{03}\Delta^{23}) \\ & ? & \frac{1}{2}(\Delta^{03}\Delta^{13} - \Delta^{02}\Delta^{12}) \\ & & ? \end{pmatrix}, \quad (18)$$

which is symmetric for the indices  $j$  and  $k$ . The metric tensor  $g_{\alpha\beta} = \text{diag}(-1, 1, 1, 1)$  is used here. The symbol "?" in matrix  $(\tilde{\kappa}_{o-})^{jk}$  denotes that the involved elements  $(k_F)^{0123}$ ,  $(k_F)^{0231}$  and  $(k_F)^{0312}$  can not be written in terms of the Lorentz violation matrix  $\Delta^{\alpha\beta}$  through Eq. (13). We have known from the preceding section that only 15 degrees of freedom in matrix  $\Delta^{\alpha\beta}$  are inter-related with the same number of independent degrees of freedom in tensor  $(k_F)_{\gamma\alpha\beta\rho}$ , and vice versa. Besides the representation of  $(\tilde{\kappa}_{e+})^{jk}$ ,  $(\tilde{\kappa}_{e-})^{jk}$ ,  $(\tilde{\kappa}_{o+})^{jk}$  and  $(\tilde{\kappa}_{o-})^{jk}$  in terms of  $\Delta^{\alpha\beta}$ , the relations between other Lorentz violation parameters commonly used in the minimal SME and the Lorentz violation matrix here are summarized in Tab. III. There are 15 independent expressions in terms of  $\Delta^{\alpha\beta}$  appearing in the four above matrices  $(\tilde{\kappa}_{e+})^{jk}$ ,  $(\tilde{\kappa}_{e-})^{jk}$ ,  $(\tilde{\kappa}_{o+})^{jk}$  and  $(\tilde{\kappa}_{o-})^{jk}$ . These 15 independent expressions help to determine 15 degrees of freedom of the Lorentz violation matrix  $\Delta^{\alpha\beta}$ .

With the recent maximal sensitivities attained from current experiments for Lorentz violation parameters  $(\tilde{\kappa}_{e+})^{jk}$ ,  $(\tilde{\kappa}_{e-})^{jk}$ ,  $(\tilde{\kappa}_{o+})^{jk}$  and  $(\tilde{\kappa}_{o-})^{jk}$  of the free photon sector in the minimal SME (see Tab. II), we get the maximal sensitivities or the conservative bounds from experiments for Lorentz invariance matrix  $\Delta_{\text{photon}}^{\alpha\beta}$

$$\begin{pmatrix} 3\Delta^{33} + 10^{-17} & 10^{-5} & 10^{-5} & 10^{-6} \\ 10^{-9} & \Delta^{33} + 10^{-17} & 10^{-9} & 10^{-9} \\ 10^{-9} & 10^{-9} & \Delta^{33} + 10^{-17} & 10^{-9} \\ 10^{-9} & 10^{-8} & 10^{-8} & \Delta^{33} \end{pmatrix}, \quad (19)$$

in the Sun-centered inertial reference frame [21, 28]. The publication [21] claimed a  $2\sigma$  limit on Lorentz violation coefficients  $(\tilde{\kappa}_{e+})^{jk}$ ,  $(\tilde{\kappa}_{e-})^{jk}$ ,  $(\tilde{\kappa}_{o+})^{jk}$ ,  $(\tilde{\kappa}_{o-})^{jk}$  and etc. in Tab. II. The 15 independent degrees of freedom of  $\Delta^{\alpha\beta}$  are determined, and there is still one freedom  $\Delta^{33}$  remaining unclear. The maximal sensitivity for the elements of  $\Delta^{\alpha\beta}$  is listed in Tab. I.

Eq. (19) demonstrates that the Lorentz violation matrix  $\Delta^{\alpha\beta}$  needs not to be a symmetric matrix in general. The non-symmetric structure of the photon Lorentz violation matrix suggests preferred directions and potential anisotropy of space-time [24, 29] for propagating of the free photon, even in the case of no gravitation. More experiments will give more details for  $\Delta^{\alpha\beta}$ .

There are different representations for the Lorentz violation matrix  $\Delta^{\alpha\beta}$  in different coordinate systems. These representations are related with each other by a coordinate transformation matrix  $T^{\alpha\beta}$  in group  $SO(1,3)$ . When the relative velocity between these two coordinate systems is much smaller than the light speed, the element of  $T^{\alpha\beta}$  is either order  $O(1)$  or close to zero. An element of the Lorentz violation matrix in a coordinate system is the linear combinations of the elements of  $\Delta^{\alpha\beta}$  in the other coordinate system. Then the magnitudes of the Lorentz violation matrix in these two coordinates are not different too much with each other. So we expect that the upper bound on the violation parameters appearing in Ref. [24] is compatible with the maximal sensitivity shown in Eq. (19). The limit of order  $10^{-14}$  in Ref. [24] for the photon Lorentz violation matrix is indeed compatible with the bound  $10^{-8}$  here.

From Eq. (19), we find that the trace  $\Delta \equiv \text{tr}(\Delta^{\alpha\beta}) = g_{\alpha\beta}\Delta^{\alpha\beta} \simeq 10^{-17}$ . A competitive upper bound  $1.6 \times 10^{-14}$  on the photon Lorentz matrix  $\Delta^{\alpha\beta}$  was obtained in Ref. [24]. In that article [24], we made an assumption about the form of the matrix  $\Delta^{\alpha\beta}$  theoretically for the analysis on the data there. There is no a priori assumptions here about the general structure of  $\Delta^{\alpha\beta}$ . We see that the maximal attained sensitivity  $10^{-17}$  for the trace of the Lorentz violation matrix is stronger than the upper limit  $10^{-14}$  gotten in Ref. [24]. Compared with the stringent bound on the trace  $\Delta$ , the maximal attained sensitivities put looser limits on the non-diagonal elements of  $\Delta^{\alpha\beta}$ , shown in Eq. (19).

Through this work, we have seen that the two theories of the SMS and the minimal SME can give same results for free photons. Eq. (13) shows the correlations between the Lorentz invariance violation matrix  $\Delta^{\alpha\beta}$  of our model and the coupling tensor  $(k_F)_{\alpha\beta\mu\nu}$  appearing in the photon sector of the minimal SME. The relations of the violation parameters  $\Delta^{\alpha\beta}$  with the parameters  $(k_F)_{\alpha\beta\mu\nu}$  uncover the detailed structure of the Lorentz violation matrix of free photons in Eq. (19). Up to now, there have been no compelling experimental evidences for the existence of Lorentz violation for photons. All that we have gotten so far are the theoretical analysis and the maximal sensitivities attained from the recent experiments.

#### IV. CONCLUSION

Two Lorentz violation models, the minimal standard model extension (SME) and the standard model supplement (SMS), are compared here for the photon sector. For all the terms in the Lagrangians of the pure pho-

ton sector, there is no direct one-to-one correspondence between the two models in general. However, some interesting results can be obtained by the requirement that the two models are identical with each other in the intersection. We find that the free photon sector of the SMS can be a subset of the minimal SME provided with some connections in the SME parameters. (i) We consider the photon sector of the two models, and two main equations are obtained between  $\Delta^{\alpha\beta}$ ,  $(k_F)^{\alpha\beta\mu\nu}$  and  $(k_{AF})^\alpha$ , through the propagator of photons in the momentum space. (ii) These equations suggest that the CPT-odd coefficients  $(k_{AF})^\alpha$  are zero. Such a suggestion is supported by available experimental bounds, for example, there is the maximal sensitivity  $10^{-42} \sim 10^{-43}$  GeV from experiments for the coefficients  $k_{(V)00}^{(3)}$ ,  $k_{(V)10}^{(3)}$ ,  $\text{Re}k_{(V)11}^{(3)}$  and  $\text{Im}k_{(V)11}^{(3)}$ . (iii) There are 15 degrees of freedom in the Lorentz violation matrix  $\Delta^{\alpha\beta}$  and the same number of degrees of freedom in tensor  $(k_F)^{\alpha\beta\mu\nu}$  to be inter-related. We got the conservative bound Eq. (19) on the detailed structure of the photon Lorentz violation matrix in our model. The bounds on  $\Delta^{\alpha\beta}$  are gotten from the limits on the Lorentz parameters of the minimal SME. The detailed structure of the photon Lorentz violation matrix can play an important role for applications to Lorentz violation experiments.

For  $\Delta^{\alpha\beta}$  of free photons, due to the factor that a universal constant can be absorbed into the gauge field  $A^\mu$ , there are 15 independent degrees of freedom in  $\Delta^{\alpha\beta}$  of free photons to describe Lorentz violation. In the paper, we do not use these 15 independent degrees of freedom to derive the magnitudes of all the 16 elements of  $\Delta^{\alpha\beta}$ , but use the relations of it with the parameters in the minimal SME to get the constraints on  $\Delta^{\alpha\beta}$  of free photons from the constraints of various experiments on the minimal SME.

The strong constraints on the matrix elements of  $\Delta^{\alpha\beta}$  mean that Lorentz violation is small for photons if it exists. The matrix  $\Delta^{\alpha\beta}$  is not symmetric generally. The non-symmetry property of  $\Delta^{\alpha\beta}$  implies that the space-time for free photons can be not isotropic very well, even if in the case of no gravitation. To date, there has been theoretical analysis on Lorentz violation and there is no strong experimental evidence supporting Lorentz violation for photons. Generally, we should study the minimal standard model extension and the standard model supplement separately, and then determine whether these two models are equivalent to each other by directly confronting with relevant experiments.

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### Appendix A: Tables

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- [1] W. Bietenholz, Phys. Rept. **505**, 145 (2011).
- [2] For a brief review on Lorentz violation effects through very high energy photons of astrophysical sources, see, e.g., L. Shao, B.-Q. Ma, Mod. Phys. Lett. A **25**, 3251 (2010) [arXiv:1007.2269], and references therein.
- [3] P.A.M. Dirac, Nature **168**, 906 (1951).
- [4] J.D. Bjorken, Ann. Phys. **24**, 174 (1963).
- [5] M. Planck, Sitzber. K. Preuss. Akad. Berlin **5**, 440 (1899).
- [6] H.S. Snyder, Phys. Rev. **71** (1947) 38; **72** (1947) 68.
- [7] Y. Xu and B.-Q. Ma, Mod. Phys. Lett. A **26**, 2101 (2011) [arXiv:1106.1778 [hep-th]].
- [8] L. Shao, B.-Q. Ma, Sci. China Phys. Mech. Astro. **54**, 1771 (2011) [arXiv:1006.3031 [hep-th]].
- [9] G. Amelino-Camelia, Int. J. Mod. Phys. D **11**, 35 (2002) [arXiv:gr-qc/0012051].
- [10] J. Magueijo and L. Smolin, Phys. Rev. Lett. **88**, 190403 (2002) [arXiv:hep-th/0112090].
- [11] X. Zhang, L. Shao, B.-Q. Ma, Astropart. Phys. **34**, 840 (2011).
- [12] W.-T. Ni, Phys. Rev. Lett. **35**, 319 (1975); W.-T. Ni, Rept. Prog. Phys. **73**, 056901 (2010) [arXiv:0912.5057 [gr-qc]].
- [13] M.L. Yan, Commun. Theor. Phys. **2**, 1281 (1983).
- [14] H.B. Nielsen and M. Ninomiya, Nucl. Phys. B **141**, 153 (1978);
- [15] S. Chadha and H.B. Nielsen, Nucl. Phys. B **217**, 125 (1983);
- [16] H.B. Nielsen and I. Picek, Phys. Lett. **114B**, 141 (1982);
- [17] H.B. Nielsen and I. Picek, Nucl. Phys. **B211**, 269 (1983).
- [18] V. Ammosov and G. Volkov, hep-ph/0008032.
- [19] H. Pas, S. Pakvasa, T.J. Weiler, Phys.Rev. D **72**, 095017 (2005).
- [20] D. Colladay, V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998).
- [21] V. A. Kostelecky, N. Russell, Rev. Mod. Phys. **83**, 11 (2011) [arXiv:0801.0287].
- [22] Zhou L., B.-Q. Ma, Mod. Phys. Lett. A **25**, 2489 (2010) [arXiv:1009.1331].
- [23] Zhou L., B.-Q. Ma, Chin. Phys. C (HEP & NP) **35**, 987 (2011) [arXiv:1109.6387].
- [24] Zhou L., B.-Q. Ma, arXiv:1009.1675.
- [25] Zhou L., B.-Q. Ma, arXiv:1109.6097.
- [26] T. Adam et al., arXiv:1109.4897.
- [27] For a brief review on experiments for superluminal neutrinos and some theoretical investigations, see, e.g., B. Q. Ma, Mod. Phys. Lett. A **27**, in press (2012), arXiv:1111.7050 [hep-ph].
- [28] R. Bluhm et al., Phys. Rev. D **68**, 125008 (2003).
- [29] J.-P. Bocquet et al., Phys. Rev. Lett. **104**, 241601 (2010).

TABLE I: Maximal sensitivities ( $2\sigma$ ) for the Lorentz violation matrix of photons.

Coefficient	Sensitivity
$\Delta^{00} - 3\Delta^{33}$	$10^{-17}$
$\Delta^{11} - \Delta^{33}$	$10^{-17}$
$\Delta^{22} - \Delta^{33}$	$10^{-17}$
$\Delta^{01}$	$10^{-5}$
$\Delta^{02}$	$10^{-5}$
$\Delta^{03}$	$10^{-6}$
$\Delta^{10}$	$10^{-9}$
$\Delta^{20}$	$10^{-9}$
$\Delta^{30}$	$10^{-9}$
$\Delta^{12}$	$10^{-9}$
$\Delta^{13}$	$10^{-9}$
$\Delta^{21}$	$10^{-9}$
$\Delta^{23}$	$10^{-9}$
$\Delta^{31}$	$10^{-8}$
$\Delta^{32}$	$10^{-8}$



TABLE II: Maximal sensitivities ( $2\sigma$ ) for the photon sector (from Ref. [21]). The superscripts  $X, Y, Z$  there are converted to 1, 2, 3 here respectively for consistence with the notation of the Lorentz violation matrix.

Coefficient	Sensitivity
$(\tilde{\kappa}_{e+})^{12}$	$10^{-32}$
$(\tilde{\kappa}_{e+})^{13}$	$10^{-32}$
$(\tilde{\kappa}_{e+})^{23}$	$10^{-32}$
$(\tilde{\kappa}_{e+})^{11} - (\tilde{\kappa}_{e+})^{22}$	$10^{-32}$
$(\tilde{\kappa}_{e+})^{33}$	$10^{-32}$
$(\tilde{\kappa}_{o-})^{12}$	$10^{-32}$
$(\tilde{\kappa}_{o-})^{13}$	$10^{-32}$
$(\tilde{\kappa}_{o-})^{23}$	$10^{-32}$
$(\tilde{\kappa}_{o-})^{11} - (\tilde{\kappa}_{o-})^{22}$	$10^{-32}$
$(\tilde{\kappa}_{o-})^{33}$	$10^{-32}$
$(\tilde{\kappa}_{e-})^{12}$	$10^{-17}$
$(\tilde{\kappa}_{e-})^{13}$	$10^{-17}$
$(\tilde{\kappa}_{e-})^{23}$	$10^{-17}$
$(\tilde{\kappa}_{e-})^{11} - (\tilde{\kappa}_{e-})^{22}$	$10^{-17}$
$(\tilde{\kappa}_{e-})^{33}$	$10^{-16}$
$(\tilde{\kappa}_{o+})^{12}$	$10^{-13}$
$(\tilde{\kappa}_{o+})^{13}$	$10^{-14}$
$(\tilde{\kappa}_{o+})^{23}$	$10^{-14}$
$\tilde{\kappa}_{\text{tr}}$	$10^{-14}$
$k_{(V)00}^{(3)}$	$10^{-43}$ GeV
$k_{(V)10}^{(3)}$	$10^{-42}$ GeV
$\text{Re}k_{(V)11}^{(3)}$	$10^{-42}$ GeV
$\text{Im}k_{(V)11}^{(3)}$	$10^{-42}$ GeV

TABLE III: Definitions for the photon sector in the minimal SME, together with relations with the Lorentz violation matrix of the SMS. (The two left columns are from Ref. [21].)

Symbol	Definition	Relation
$(\tilde{\kappa}_{e+})^{jk}$	$-(k_F)^{0j0k} + \frac{1}{4}\epsilon^{j pq}\epsilon^{krs}(k_F)^{pqrs}$	Eq. (15)
$(\tilde{\kappa}_{e-})^{jk}$	$-(k_F)^{0j0k} - \frac{1}{4}\epsilon^{j pq}\epsilon^{krs}(k_F)^{pqrs} + \frac{2}{3}(k_F)^{0l0l}\delta^{jk}$	Eq. (16)
$(\tilde{\kappa}_{o+})^{jk}$	$-\frac{1}{2}\epsilon^{j pq}(k_F)^{0k pq} + \frac{1}{2}\epsilon^{k pq}(k_F)^{0j pq}$	Eq. (17)
$(\tilde{\kappa}_{o-})^{jk}$	$\frac{1}{2}\epsilon^{j pq}(k_F)^{0k pq} + \frac{1}{2}\epsilon^{k pq}(k_F)^{0j pq}$	Eq. (18)
$\tilde{\kappa}_{\text{tr}}$	$-\frac{2}{3}(k_F)^{0l0l}$	$-\frac{2}{3}\Delta^{11} + \frac{1}{3}\Delta$
$k^1$	$(k_F)^{0213}$	?
$k^2$	$(k_F)^{0123}$	?
$k^3$	$(k_F)^{0202} - (k_F)^{1313}$	$-\frac{1}{2}\Delta$
$k^4$	$(k_F)^{0303} - (k_F)^{1212}$	$-\frac{1}{2}\Delta$
$k^5$	$(k_F)^{0102} + (k_F)^{1323}$	$-\frac{1}{2}(\Delta^{10}\Delta^{20} + \Delta^{23}\Delta^{13})$
$k^6$	$(k_F)^{0103} - (k_F)^{1223}$	$-\frac{1}{2}(\Delta^{10}\Delta^{30} + \Delta^{32}\Delta^{12})$
$k^7$	$(k_F)^{0203} + (k_F)^{1213}$	$-\frac{1}{2}(\Delta^{20}\Delta^{30} + \Delta^{31}\Delta^{21})$
$k^8$	$(k_F)^{0112} + (k_F)^{0323}$	$-\frac{1}{2}(\Delta^{03}\Delta^{23} - \Delta^{01}\Delta^{21})$
$k^9$	$(k_F)^{0113} - (k_F)^{0223}$	$\frac{1}{2}(\Delta^{01}\Delta^{31} - \Delta^{02}\Delta^{32})$
$k^{10}$	$(k_F)^{0212} - (k_F)^{0313}$	$\frac{1}{2}(\Delta^{03}\Delta^{13} - \Delta^{02}\Delta^{12})$
$k_{(V)00}^{(3)}$	$-\sqrt{4\pi}(k_{AF})^0$	0
$k_{(V)10}^{(3)}$	$-\sqrt{4\pi/3}(k_{AF})^3$	0
$\text{Re}k_{(V)11}^{(3)}$	$\sqrt{2\pi/3}(k_{AF})^1$	0
$\text{Im}k_{(V)11}^{(3)}$	$-\sqrt{2\pi/3}(k_{AF})^2$	0