

Stochastic Physics & Bell's correlation

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Preliminaries

Starting from the set theoretic result in the ASTP2010 paper of the author, the question was raised whether or not a local realistic model is actually possible. In the present draft this possibility will be demonstrated using stochastic argumentation. For completeness the author would also like to refer to a draft paper on arXiv. In the latter paper the author showed that from a local hidden variable model the quantum correlation may be obtained. The proposed basic flaw in Bell's reasoning is also indicated in that paper.

The final goal of the present draft is to show that a 2LHV based correlation of Bell can be construed that follows the structure laid down in ASTP2010. This is to show that A and B functions under a strict locality condition produce correlations that have a non-zero probability to violate the CHSH. With the stochastic of the example the LHV partitioning of the Omega sets that, according to critics can only be realized with non-local hidden variables, is then established.

Stochastic reasoning

In the present draft the A and B functions are stochastic in certain intervals of the respective hidden variables while in other intervals those functions are not stochastic sign functions. We believe that two aspects of the ASTP2010 paper need to be recapitulated. In the first place we have the integral

$$\frac{1}{2} P(x, y) = \int_{\lambda \in \Omega_{+P(a,b)=0}(x,y)} \rho_{\lambda} - \int_{\lambda \in \Omega_{-P(a,b)=0}(x,y)} \rho_{\lambda} \quad (1.1)$$

The Omega sets are defined by

$$\Omega_{0|P(a,b)=0}(x, y) = \{ \lambda \in \Lambda \mid A_{\lambda}(a)B_{\lambda}(b) = -A_{\lambda}(x)B_{\lambda}(y) = \pm 1 \} \quad (1.2)$$

Together with

$$\Omega_{\pm|P(a,b)=0}(x, y) = \{ \lambda \in \Lambda \mid A_{\lambda}(a)B_{\lambda}(b) = A_{\lambda}(x)B_{\lambda}(y) = \pm 1 \} \quad (1.3)$$

The plus in the Omega index indicates a +1 for the product while the minus in the Omega index indicates a -1 result for the product.

The previous mentioned integral (1.1) will be treated 'stochastically'. This means we will argue for:

$$\Pr \{P_{LHV}(x, y) = -(x \cdot y)\} > 0 \quad (1.4)$$

Meaning the probability that LHV based Bell correlations exist that violate the CHSH is non-zero.

In the second place the expression of the LHV correlation in (1.1) must match the one below for consistency. This is so because we argued for a partitioning of the Λ set into three Ω sets, $\Omega_{0P(a,b)=0}(x, y)$, $\Omega_{+P(a,b)=0}(x, y)$ and $\Omega_{-P(a,b)=0}(x, y)$. Hence,

$$\frac{1}{2} P(x, y) = \int_{\lambda \in \Omega_{0P(a,b)=0}(x, y)} \rho_{\lambda} A_{\lambda}(x) B_{\lambda}(y) \quad (1.5)$$

Functions A and B

In this draft the A and B function represent the measurement functions and a strict locality rule must be obeyed. The rule is simply that in no way whatsoever the behavior of the A function can be related to the setting of B and vice versa. Because in our model of ASTP2010 we assumed that partitioning of the universal set Λ of the hidden variables we also reject indirect communication through sets related to settings. In effect a measurement function may hence only be determined by its settings and its particular hidden variable(s) plus set(s) associated to it.

Let us inspect a two-hidden variable model and associate λ_1 to A and λ_2 to B. This implies that the universal set Λ is a Cartesian product of Λ_1 for the A side LHV and Λ_2 for the B side LHV. Hence, $\Lambda = \Lambda_1 \times \Lambda_2$. If we subsequently introduce

$$A_{\lambda_1}(x) = \begin{cases} \alpha_{\lambda_1}(x), & \text{for } \lambda_1 \in I. \\ \alpha'_{\lambda_1}(x), & \text{for } \lambda_1 \in \Lambda_1 \setminus I. \end{cases} \quad (1.6)$$

with, $I \subset \Lambda_1$ partitioning Λ_1 into two sets. Similarly for the B function we see:

$$B_{\lambda_2}(y) = \begin{cases} \beta_{\lambda_2}(y), & \text{for } \lambda_2 \in J. \\ \beta'_{\lambda_2}(y), & \text{for } \lambda_2 \in \Lambda_2 \setminus J. \end{cases} \quad (1.7)$$

Similarly, $J \subset \Lambda_2$ creates a partitioning of Λ_2 into two sets. The α and β functions also project in the set $\{-1, 1\}$. The dot denotes the place where an index value can be filled in (see the selection rule below).

It is supposed that proper $A_{\lambda_1}(a)$ and $B_{\lambda_2}(b)$ always can be obtained for convenient orthogonal A and B settings such that Omega sets can be construed with the definitions in (1.6) and (1.7). Later on we will have more to say about this matter.

Probability density of the hidden variables

In our example let us postulate a density for $(\lambda_1, \lambda_2) \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \times \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \equiv \Lambda$. Here, $[p, q] = \{x \in \mathbb{R} \mid p \leq x \leq q\}$. The density is (for $n = 1, 2$)

$$\rho_{\lambda_n} = \begin{cases} \frac{1}{\sqrt{2}}, & \lambda_n \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \\ 0, & \text{elsewhere} \end{cases} \quad (1.8)$$

Form of the integrals in the example 2LHV density

Using the density we now may write for this two LHV model

$$P(x, y) = \iint_{(\lambda_1, \lambda_2) \in \Omega_{+P(a,b)=0}(x,y)} d\lambda_1 d\lambda_2 - \iint_{(\lambda_1, \lambda_2) \in \Omega_{-P(a,b)=0}(x,y)} d\lambda_1 d\lambda_2 \quad (1.9)$$

Moreover, the consistency condition in (1.5) now becomes

$$P(x, y) = \iint_{(\lambda_1, \lambda_2) \in \Omega_{0P(a,b)=0}(x,y)} A_{\lambda_1}(x) B_{\lambda_2}(y) d\lambda_1 d\lambda_2 \quad (1.10)$$

Parameter selection rules

In the example for the ease of presentation a two dimensional parameter vector system is supposed. Let us select, $1_A = (1, 0)$ and $2_A = (0, 1)$ for the A wing of the experiment. For the B wing let us take

$1_B = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ and $2_B = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$. The indices on the numbers indicate the wing. The number

notation is a short hand for indicating the parameter vectors.

In ASTP2010 it was described how settings of a measurement instrument restrict the λ_1 and λ_2 intervals. In the two tables below the selection rule is recapitulated. For the A instrument:

Setting	Interval
1_A	$I_1 = \left[\frac{-1}{\sqrt{2}}, 1 - \frac{1}{\sqrt{2}} \right] \subset \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right]$
2_A	$I_2 = \left[-1 + \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right] \subset \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right]$

For the B instrument:

Setting	Interval
1_B	$J_1 = \left[\frac{-1}{\sqrt{2}}, 0 \right] \subset \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right]$
2_B	$J_2 = \left[0, \frac{1}{\sqrt{2}} \right] \subset \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right]$

In terms of measurement one can imagine a first ‘stage gate’ where the selection rule in the two tables is implemented. In a subsequent ‘stage gate’ the proper A and B response function, given in (1.6) and (1.7), are implemented. If the entrance of the to-be-measured particle is the second ‘stage gate’ the selection of the A and B function in the measurement instrument is also affected by the event of the entrance of the particle. Hence, third stage gate A and B function selection will warrant proper stochastic behavior for the functions.

In order to violate the CHSH, four pairs, $(1_A, 1_B)$, $(1_A, 2_B)$, $(2_A, 1_B)$ and $(2_A, 2_B)$ need to be inspected on their LHV correlation and consistency with the idea of partitioning. If

$$\Pr \{ | P_{LHV}(1_A, 1_B) - P_{LHV}(1_A, 2_B) - P_{LHV}(2_A, 1_B) - P_{LHV}(2_A, 2_B) | > 2 \} > 0 \quad (1.11)$$

then Bell’s expression for correlation is unfit for discriminating between all LHV models and quantum mechanical result.

Setting pair $(1_A, 1_B)$

1. The correlation

As can be seen from the definitions of 1_A and 1_B the quantum correlation in this case is equal to

$$P_{QM}(1_A, 1_B) = -\frac{1}{\sqrt{2}}. \text{ The LHV expression is expected to behave like presented in (1.4).}$$

From the selection rule in the previous two tables we may obtain, in the first stage gate, the partitioning into the set $I_1 \times J_1$ and the set $\Lambda \setminus (I_1 \times J_1)$. Note that $\Lambda = \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right] \times \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right]$.

Let us suppose that the $\alpha_{\lambda_1}(x)$ from (1.6) and the $\beta_{\lambda_2}(y)$ from (1.7) are ± 1 signs obtained from tossing fair (but separated) coins. Let us denote this with $\alpha_{\lambda_1}(x) = F_{cr}(1)$ and $\beta_{\lambda_2}(y) = F_{cr}(2)$. Under the convenient assumption that $A_{\lambda_1}(a)$ and $B_{\lambda_2}(b)$ always can be obtained for suitable orthogonal A and B settings we find in probabilistic terms that

$$\Pr \left\{ \Omega_{+|P(a,b)=0}(1_A, 1_B) = \emptyset \ \& \ \Omega_{-|P(a,b)=0}(1_A, 1_B) = I_1 \times J_1 \right\} > 0 \quad (1.12)$$

This is so because with $\alpha_{\lambda_1}(x) = F_{cr}(1)$ and $\beta_{\lambda_2}(y) = F_{cr}(2)$ there is a non-zero probability that, under convenient $A_{\lambda_1}(a)$ and $B_{\lambda_2}(b)$, $A_{\lambda_1}(1_A)B_{\lambda_2}(1_B) = -1$.

Note that if we drop the convenient $A_{\lambda_1}(a)$ and $B_{\lambda_2}(b)$ assumption and make $A_{\lambda_1}(a)$ and $B_{\lambda_2}(b)$ stochastic too the probability in (1.12) will be smaller but still nonzero. The latter fact is what counts so it is allowed to argue under convenient $A_{\lambda_1}(a)$ and $B_{\lambda_2}(b)$.

If it is subsequently noted that (1.12) implies that

$$\Pr \left\{ \iint_{(\lambda_1, \lambda_2) \in \Omega_{+|P(a,b)=0}(1_A, 1_B)} d\lambda_1 d\lambda_2 - \iint_{(\lambda_1, \lambda_2) \in \Omega_{-|P(a,b)=0}(1_A, 1_B)} d\lambda_1 d\lambda_2 = -\frac{1}{\sqrt{2}} \right\} > 0 \quad (1.13)$$

then we see $\Pr \left\{ P(1_A, 1_B) = -\frac{1}{\sqrt{2}} \right\} > 0$.

2. The consistency condition

For the sake of easy argumentation let us introduce the notation $P(1_A, 1_B)|_{\Omega_{\pm}}$ for the LHV correlation based on the form in (1.9) and $P(1_A, 1_B)|_{\Omega_0}$ for the LHV correlation based on the form (1.10). In our case the consistency condition is: $P(1_A, 1_B)|_{\Omega_0}$ is equal to $P(1_A, 1_B)|_{\Omega_{\pm}}$.

When either the stochastic assignment leads to $\Omega_{+|P(a,b)=0}(1_A, 1_B) = \emptyset \ \& \ \Omega_{-|P(a,b)=0}(1_A, 1_B) = I_1 \times J_1$ or $\Omega_{-|P(a,b)=0}(1_A, 1_B) = \emptyset \ \& \ \Omega_{+|P(a,b)=0}(1_A, 1_B) = I_1 \times J_1$ we see that, under convenient $A_{\lambda_1}(a)$ and $B_{\lambda_2}(b)$, Ω_0 is equal to

$$\Omega_{0|P(a,b)=0}(1_A, 1_B) = \Lambda \setminus (I_1 \times J_1) \quad (1.14)$$

Reformulating Ω_0 into a more handy form this leads to

$$\Omega_{0|P(a,b)=0}(1_A, 1_B) = (I_1 \times J_2) \cup ((\Lambda_1 \setminus I_1) \times J_1) \cup ((\Lambda_1 \setminus I_1) \times J_2) \quad (1.15)$$

Recall that $\Lambda \equiv \Lambda_1 \times \Lambda_2$ with $\Lambda_1 = \Lambda_2 = \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right]$ and the sets I_1, J_1 and J_2 as defined in the selection rule tables.

From the definition of the A and B functions in (1.6) and (1.7) and $\alpha_{\lambda_1}(x) = F_{ct}(1) = \alpha$ together with $\beta_{\lambda_2}(y) = F_{ct}(2) = \beta$ brings us to the following expression for $P(1_A, 1_B)|_{\Omega_0}$

$$\begin{aligned} P(1_A, 1_B)|_{\Omega_0} = & \iint_{(\lambda_1, \lambda_2) \in I_1 \times J_2} \alpha \beta'_{\lambda_2}(1_B) d\lambda_1 d\lambda_2 + \iint_{(\lambda_1, \lambda_2) \in (\Lambda_1 \setminus I_1) \times J_1} \alpha'_{\lambda_1}(1_A) \beta d\lambda_1 d\lambda_2 + \\ & + \iint_{(\lambda_1, \lambda_2) \in (\Lambda_1 \setminus I_1) \times J_2} \alpha'_{\lambda_1}(1_A) \beta'_{\lambda_2}(1_B) d\lambda_1 d\lambda_2 \end{aligned} \quad (1.16)$$

This is true because for e.g. $\lambda_1 \in I_1$ we have $A_{\lambda_1}(1_A) = \alpha_{\lambda_1}(1_A) = \alpha \in \{-1, 1\}$ based on a fair coin toss F_{ct} . Similarly for $\lambda_2 \in J_2$ we have $B_{\lambda_2}(1_B) = \beta'_{\lambda_2}(1_B) \in \{-1, 1\}$ while for $\lambda_2 \in J_1$ we see that $B_{\lambda_2}(1_B) = \beta_{\lambda_2}(1_B) = \beta \in \{-1, 1\}$ etc.

Because of the consistency condition we need to show that: $P(1_A, 1_B)|_{\Omega_0} = P(1_A, 1_B)|_{\Omega_{\pm}} = \frac{-1}{\sqrt{2}}$. Let us introduce the following notation. We have

$$U = \int_{\lambda_2 \in J_2} \beta'_{\lambda_2}(1_B) d\lambda_2 \quad (1.17)$$

together with

$$V = \int_{\lambda_1 \in \Lambda_1 \setminus I_1} \alpha'_{\lambda_1}(1_A) d\lambda_1 \quad (1.18)$$

Because, $\int_{\lambda_1 \in I_1} d\lambda_1 = \left(1 - \frac{1}{\sqrt{2}}\right) - \left(\frac{-1}{\sqrt{2}}\right) = 1$ and $\int_{\lambda_2 \in J_1} d\lambda_2 = 0 - \left(\frac{-1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$ the following algebraic equation occurs after multiplying (1.16) on both sides with α

$$U - \frac{1}{\sqrt{2}}V + \alpha UV = \frac{-\alpha}{\sqrt{2}} \quad (1.19)$$

Here, $\alpha\beta = -1$, is employed because of $\Omega_{+|P(a,b)=0}(1_A, 1_B) = \emptyset$ & $\Omega_{-|P(a,b)=0}(1_A, 1_B) = I_1 \times J_1$ while the definitions in (1.17) and (1.18) are used that lead us to (1.19).

3. Numerics of the consistency condition on partitioning

The equation in (1.19) appears to be difficult to solve exactly. In this section the existence of $\alpha'_{\lambda_1}(1_A)$ and $\beta'_{\lambda_2}(1_B)$ will be shown by approximately solve the U,V equation. To this end let us select mathematical forms for $\alpha'_{\lambda_1}(1_A)$ and $\beta'_{\lambda_2}(1_B)$.

Suppose, for proper, $\eta(1_B)$ and $\zeta(1_A)$ that

$$\exists_{\eta(1_B) \in J_2} \beta'_{\lambda_2}(1_B) = \text{sign}(\eta(1_B) - \lambda_2) \quad (1.20)$$

For completeness: $J_2 = \left[0, \frac{1}{\sqrt{2}}\right]$. As can be verified we may write

$$U = \int_0^{\eta(1_B)} d\lambda_2 - \int_{\eta(1_B)}^{\frac{1}{\sqrt{2}}} d\lambda_2 = 2\eta(1_B) - \frac{1}{\sqrt{2}} \quad (1.21)$$

Hence, $U \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. In addition, suppose

$$\exists_{\zeta(1_A) \in \Lambda_1 \setminus I_1} \alpha'_{\lambda_1}(1_A) = \text{sign}(\zeta(1_A) - \lambda_1) \quad (1.22)$$

For completeness: $\Lambda_1 \setminus I_1 = \left[1 - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. As can be verified we also have

$$V = \int_{1-\frac{1}{\sqrt{2}}}^{\zeta(1_A)} d\lambda_1 - \int_{\zeta(1_A)}^{\frac{1}{\sqrt{2}}} d\lambda_1 = 2\zeta(1_A) - 1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 2\zeta(1_A) - 1 \quad (1.23)$$

This implies that in numerical exploration of possible zeroes of (1.19) we see $V \in \left[1 - \sqrt{2}, \sqrt{2} - 1\right]$.

With the interval conditions $U \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ and $V \in \left[1 - \sqrt{2}, \sqrt{2} - 1\right]$ and exploring $\alpha = 1$ the

following numerical results were found for $\delta(U, V) = U - \frac{1}{\sqrt{2}}V + \alpha UV - \left(\frac{-\alpha}{\sqrt{2}}\right)$

U	V	Step size h	$\delta(U, V)$
-0.60711	0.075786	0.01	0.0004
-0.45511	0.215786	0.001	9.1×10^{-6}
-0.45371	0.218186	0.0001	9.9×10^{-7}

Similarly for $\alpha = -1$ we found

U	V	Step size h	$\delta(U, V)$
0.31000	-0.40421	0.01	2.1×10^{-5}
0.32300	-0.37421	0.001	4.7×10^{-6}
0.32760	-0.36691	0.0001	8.0×10^{-7}

The numerical exactness of the approximation can of course always be better. For the first row in the $\alpha = -1$ case the following plot of the error $\delta(U, V)$ can be shown.

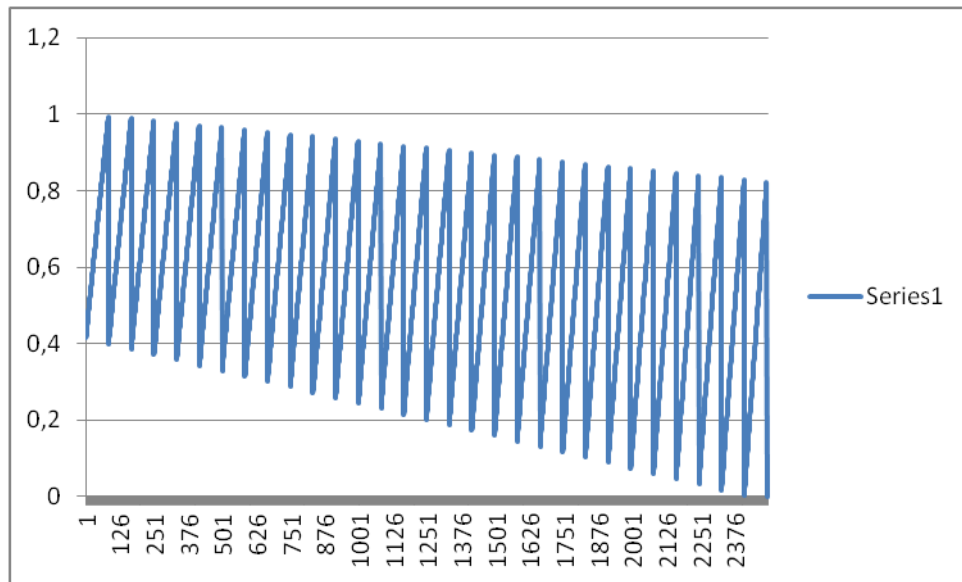


Figure 1 Plot of error term in iteration step.

Hence, there exists a solution to (1.19) 'in principle' within the boundaries of the approximate open intervals $U \in (-0,70711, 0,70711)$ and $V \in (-0,414214, 0,414214)$. This implies that both $\zeta(1_A)$ and $\eta(1_B)$ are possible 'in principle'. Hence, $\alpha'_{\lambda_1}(1_A)$ and $\beta'_{\lambda_2}(1_B)$ can be obtained when $\Omega_{+|P(a,b)=0}(1_A, 1_B) = \emptyset$ & $\Omega_{-|P(a,b)=0}(1_A, 1_B) = I_1 \times J_1$.

4. Concluding remarks on correlation and consistency for $(1_A, 1_B)$

In the previous paragraphs of this section it was demonstrated that $\Pr \left\{ P(1_A, 1_B) |_{\Omega_{\pm}} = -\frac{1}{\sqrt{2}} \right\} > 0$ in a manner that is consistent with partitioning of the Λ set of LHV's, or, because of numerical

demonstration only: $\Pr\left\{P(1_A, 1_B) |_{\Omega_0} \approx -\frac{1}{\sqrt{2}}\right\} > 0$. This allows the conclusion that (misusing slightly the Bayesian condition notation)

$$\Pr\left\{P_{LHV}(1_A, 1_B) = -\frac{1}{\sqrt{2}} | Consistency\right\} > 0 \quad (1.24)$$

The consistency refers to the partitioning of the Λ set of LHVs without the need to renormalize the probability density function defined in equation (1.8).

Concluding remarks

Although much more numerical work and demonstration needs to be done that fits into one weekend of the author, we may draw the conclusion that not only for the pair $(1_A, 1_B)$,

$\Pr\left\{P_{LHV}(1_A, 1_B) = -\frac{1}{\sqrt{2}} | Consistency\right\} > 0$ can be obtained but also for $(1_A, 2_B)$, $(2_A, 2_B)$ and $(2_A, 1_B)$ pairs we may see,

$$\begin{aligned} \Pr\left\{P_{LHV}(1_A, 2_B) = \frac{1}{\sqrt{2}} | Consistency\right\} &> 0, \\ \Pr\left\{P_{LHV}(2_A, 2_B) = \frac{1}{\sqrt{2}} | Consistency\right\} &> 0, \\ \Pr\left\{P_{LHV}(2_A, 1_B) = \frac{1}{\sqrt{2}} | Consistency\right\} &> 0 \end{aligned} \quad (1.25)$$

Please do observe that the ‘stage gating’ of the measurement instrument such as described in the previous sections implies that the event of a particle hitting the ‘measurement area’ of the instrument ensures proper randomization of function response. Meaning that $\alpha_{\lambda_1}(x)$ and $\beta_{\lambda_2}(y)$ in their proper I and J sets are stochastic random projections in $\{-1, 1\}$, with $\alpha\beta = 1$ such that $\Pr\left\{\Omega_{-|P(a,b)=0}(x, y) = \emptyset \ \& \ \Omega_{+|P(a,b)=0}(x, y) = I(x) \times J(y)\right\} > 0$ for (x, y) equal to $(1_A, 2_B)$, $(2_A, 2_B)$ or $(2_A, 1_B)$. The functions $\alpha'_{\lambda_1}(1_A)$ and $\beta'_{\lambda_2}(1_B)$ are adapted to them. The, $I(x) \times J(y)$, is determined according to the selection rule in the selection rule tables.

In (1.25) the consistency conditions reflect the partitioning of the set Λ without the necessity to renormalize the probability density in (1.8). This finally may lead to the already anticipated conclusion in ASTP2010 that there is a non-zero probability, small as it may be, such that LHVs violate the CHSH metric. Please observe that strict locality is obeyed in all steps of the computations and in all stage gates of the separate measurement instruments.

Hence, the conclusion of the author in ASTP2010 that Bell’s correlation is an unfit measure to differentiate between LHV and quantum non-locality in experiment can still be upheld. This

conclusion is related to the one expressed by Joy Christian in an arXiv prepublication (CHRIST2011). The present author believes that this paper of Christian still holds because previous attacks on it were rightfully rejected. Hopefully, we can read Christian's rebuttal on the attacks on his views in arXiv¹ soon. Note that the claim of the author also is supported by earlier publications such as IJTP2007.

References

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¹ arXiv's admittance policy aims to make a level field.