Standard Model Parameters from Large Deviation Theory

(Rev. 1)

Part 1: Motivation and Mathematical Background

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Abstract

The Standard Model of Particle Physics (SM) contains a large number of empirical parameters whose origin remains unexplained. Among these, fermion masses, mixing angles, CP-violating phases, and the existence of three fermion generations are independent inputs rather than consequences of a deeper organizing principle. This work explores the hypothesis that these parameters emerge from large deviation properties of the multifractal attractor describing the endpoint of the entropy flow. In this first part, we motivate the relevance of Large Deviation Theory (LDT) to fundamental physics and introduce the necessary mathematical background. We emphasize the role of convex rate functions, their minima, and their overlap structure, illustrating how discrete, stable saddles emerge dynamically. Representative examples are provided to guide physical intuition and prepare the ground for subsequent application to SM parameters.

Key words: Large Deviation Theory, complex dynamics, nonequilibrium systems, Standard Model parameters, Rényi entropy.

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1. Introduction

Despite its outstanding phenomenological success, SM leaves unanswered the question of why its "free" parameters take the observed values. The Yukawa couplings, in particular, span many orders of magnitude without an apparent organizing principle. The existence of three replicated generations of fermions (quarks and leptons) continues to resist explanation. This situation suggests that the SM may be an effective description emerging from a deeper statistical or dynamical structure.

For reference, SM contains the following free parameters:

Table 1: Free parameters of the Standard Model

Category	**Parameters**
Gauge couplings	3 (g_1, g_2, g_3 for $U(1)$, $SU(2)$, $SU(3)$)
Fermion masses	9 (6 quarks, 3 charged leptons)
CKM mixing	4 (3 angles, 1 CP phase)
Higgs sector	2 (VEV, self-coupling)

Note that, with massive neutrinos, additional parameters enter the picture (masses, PMNS mixing angles, and CP phases), bringing the total number of SM parameters to 26.

Viewed in the context of *complex dynamics* of systems outside equilibrium, Large Deviation Theory (LDT) provides a natural framework for understanding how sharply peaked distributions, discrete hierarchies, and exponential suppression arise from underlying probability measures. Originally developed in probability theory and statistical mechanics [1, 2, 3], LDT has found applications ranging from turbulence to nonequilibrium thermodynamics [4, 5]. Its core object, the *rate function*, encodes the exponential scaling of rare fluctuations and has a geometric structure that is particularly suggestive for hierarchy formation.

In this paper, we argue that convex rate functions and their saddle structure provide a natural mathematical language for discussing the emergence of discrete physical parameters. This first part is a brief account of motivation and mathematical background; applications to fermion masses, mixings and the triplication of fermion generations will be developed in the second part.

2. Basics of Large Deviation Theory

2.1 The rate function

Consider a family of random variables X_N depending on a large parameter N. One says that X_N satisfies a large deviation principle if

$$P(X_N \approx \alpha) \;\sim\; \exp\bigl[-N\, I(\alpha)\bigr], \tag{1}$$

where $I(\alpha)$ is the *rate function*. The symbol "$\sim$" indicates logarithmic equivalence in the limit $N \to \infty$.

The rate function plays a role analogous to an *effective potential* or *free-energy density* in Statistical Physics.

2.2 Convexity and stability

Under very general conditions, $I(\alpha)$ is a convex function:

$$I(\alpha) \geq 0, \qquad I(\alpha_0) = 0 \quad \text{at its minimum}, \tag{2}$$

where

$$\frac{d^2 I}{d\alpha^2} > 0. \tag{3}$$

Condition (3) ensures the stability of the minimum α_0, which corresponds to the most probable or dominant configuration. In physical terms:

- minima of $I(\alpha)$ correspond to *stable macroscopic states*,
- fluctuations away from the minimum are exponentially suppressed.

3. Representative Convex Rate Functions

To build intuition, we now consider a simple example of convex rate functions of quadratic form [6],

$$I_n(\alpha) = k\,(\alpha - \alpha_n)^2, \tag{4}$$

where:

- $k > 0$ controls the curvature,
- α_n labels the position of the minimum.

This form captures the local structure of a generic convex rate function near its stable saddle point. Additional representative examples of rate functions in physical applications are given in [6].

3.1 Isolated minima

Figure 1 shows several such rate functions with distinct and isolated minima α_n. The curves represent convex, large-deviation rate functions $I_n(\alpha)$ with unique minima at $\alpha = \alpha_n$. The minima correspond to *stable entropy saddles.* Convexity guarantees stability and exponential suppression of fluctuations away from these points.

An isolated minimum suggests the emergence of *discrete preferred configurations*, a feature directly relevant to quantized or hierarchical parameters.

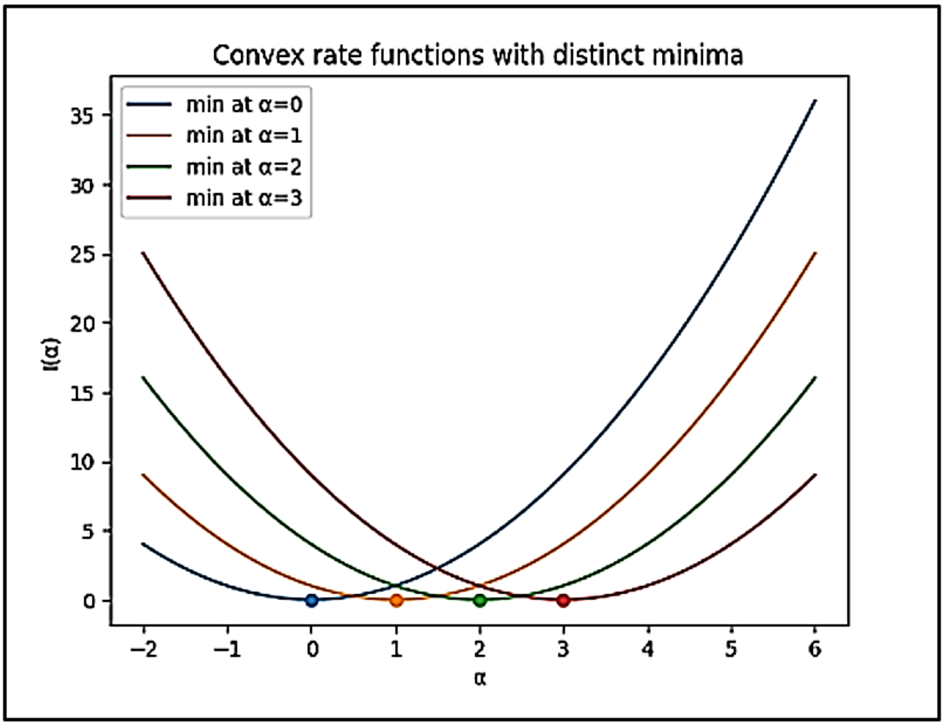

Figure 1: Convex rate functions $I_n(\alpha)$ with isolated minima at $\alpha = \alpha_n$. Each minimum represents a stable entropy saddle. Convexity guarantees exponential suppression of fluctuations away from these points.

3.2 Overlap of adjacent saddles

While isolated minima correspond to stable states, *overlap* between nearby rate functions plays an equally important role. Figure 2 illustrates the partial overlap between adjacent convex rate functions when their minima are sufficiently close and/or the curvature is moderate. Adjacent rate functions exhibit regions where their exponential weights are comparable. Such overlap regions correspond to *mixing between neighboring saddles*, while more distant saddles remain effectively decoupled.

This overlap structure naturally leads to:

- nearest-neighbor interactions,
- hierarchical mixing patterns,
- exponential suppression of long-range couplings.

These features mirror qualitative properties observed in fermion mixing matrices.

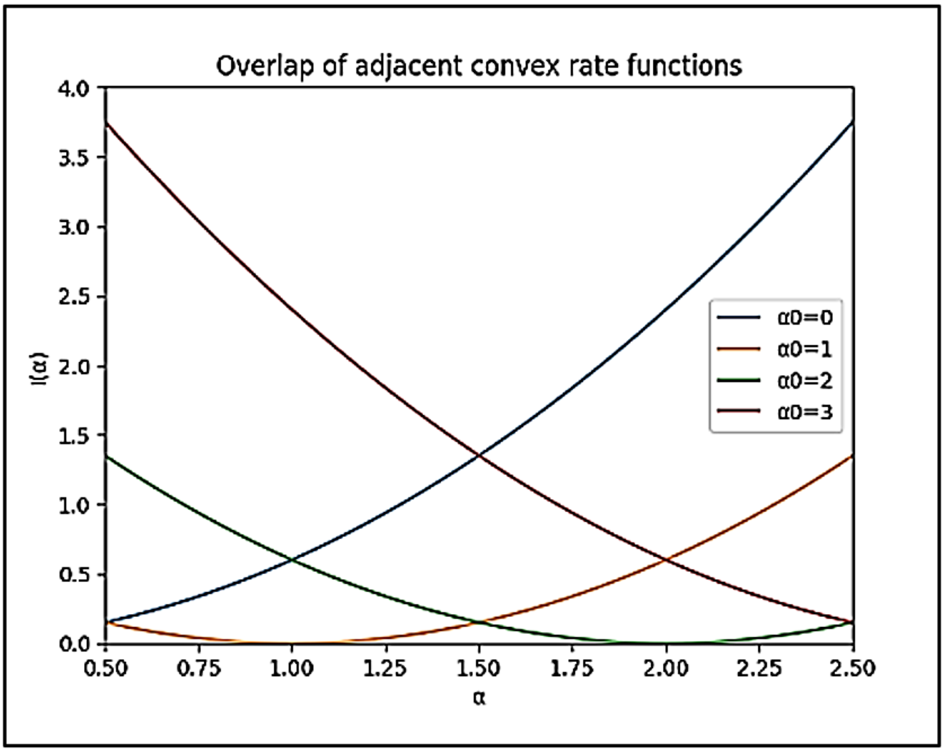

Figure 2: Overlap between adjacent convex rate functions. Regions where exponential weights are comparable correspond to mixing between neighboring saddles; distant saddles remain decoupled.

4. Physical Interpretation

The relevance of LDT to Particle Physics can be summarized as follows:

Discrete minima

Stable minima of rate functions provide a natural origin for discrete physical parameters without imposing them by hand.

Hierarchies

Exponential suppression away from minima explains large numerical hierarchies as geometric effects in configuration space.

Mixing and correlations

Overlap between neighboring saddles generates controlled mixing, while distant saddles remain decoupled.

These properties of rate functions suggest that parameters traditionally viewed as independent inputs may instead be emergent consequences of an underlying statistical structure. The underlying justification is that phenomena unfolding far above the SM scale likely evolve in a highly unstable environment characterized by large amplitude fluctuations and non-Gaussian probability distributions.

5. Outlook

This brief introduction surveys the conceptual foundations required to apply LDT to the origin of SM parameters. In follow-up submissions, attempts will be made to apply this framework to fermion mass hierarchies, mixing angles, and the remaining SM parameters. The next sequel will show that the hierarchy of fermion masses follows the exponential scaling

$$m = v\, e^{-I(\alpha)}, \tag{5}$$

where v is the vacuum expectation value of the Higgs scalar and α denotes the set of scaling exponents defining the multifractal attractor.

A. Rényi Entropy, Multifractals, and Large Deviations

A.1 Motivation

The emergence of convex rate functions in LDT suggests that the underlying probability measure is not mono-fractal (single scaling exponent), but rather multifractal, characterized by a spectrum of local scaling exponents α. In this context, Rényi entropy provides the natural functional encoding scale-dependent fluctuations and non-Gaussian statistics.

Unlike Shannon entropy, which probes only the typical set of events, Rényi entropy systematically weights rare and atypical fluctuations—precisely the regime governing SM parameters when interpreted as emergent large-deviation minima above the electroweak scale.

A.2 Definition of Rényi Entropy

Let $p_i(\ell)$ be a probability distribution defined at resolution scale ℓ. The Rényi entropy of order $q \in \mathbb{R}$, $q \neq 1$, is defined as [7, 8, 9, 10]

$$S_q = \frac{1}{1-q} \ln\left(\sum_i p_i^q\right). \tag{6}$$

The Shannon entropy is recovered smoothly in the limit $q \to 1$:

$$\lim_{q\to 1} S_q = -\sum_i p_i \ln p_i. \tag{7}$$

Physical interpretation:

- $q > 1$ enhances regions of large probability (stable configurations),
- $q < 1$ enhances rare fluctuations (large deviations),
- $q = \frac{1}{2}$ plays a special role in diffusion-dominated multifractal flows and in the applications of Rényi entropy in Foundational Physics [7, 8, 9, 10].

A.3 Multifractal Measures and Generalized Dimensions

A probability measure is multifractal if it follows the local scaling behavior

$$p_i(\ell) \sim \ell^{\alpha_i}, \tag{8}$$

where α_i is the Hölder exponent. The Rényi entropy defines the *generalized dimensions*

$$D_q = \lim_{\ell\to 0} \frac{S_q(\ell)}{\ln(1/\ell)}. \tag{9}$$

For a mono-fractal measure, D_q is constant. In contrast, multifractality corresponds to a nontrivial q-dependence of D_q.

A.4 Legendre Structure and the Rate Function

The multifractal spectrum $f(\alpha)$ corresponds to a set of different local Hölder exponents and is obtained via a Legendre transform of the scaling exponent

$$\tau(q) = (q-1)\, D_q. \tag{10}$$

Defining

$$\alpha_q = \frac{d\tau}{dq}, \qquad f(\alpha) = q\alpha - \tau(q), \tag{11}$$

one finds that the large deviation rate function is identical (up to normalization) to the multifractal spectrum:

$$I(\alpha) = \alpha - f(\alpha). \tag{12}$$

Thus, the convexity of $I(\alpha)$ follows directly from the concavity of $f(\alpha)$, which itself arises from the maximization of entropy.

A.5 Connection to Large Deviation Theory

In LDT, the probability of observing a fluctuation characterized by α obeys

$$P_\ell(\alpha) \asymp \exp\!\big[-\ln(\ell^{-1})\, I(\alpha)\big]. \tag{13}$$

Comparing (13) and (9) reveals that Rényi entropy governs the exponential suppression rate of atypical (rare) events. The minima of $I(\alpha)$ correspond to:

- maximal entropy configurations,
- dynamically stable fixed points,
- phenomenologically observed SM parameters.

This establishes Rényi entropy as the **entropic generator of rate functions**.

A.6 Overlapping Minima and Parameter Universality

When families of convex rate functions $I_n(\alpha)$ arise from nearby scales or sectors,

$$I_n(\alpha) = I(\alpha - \delta_n), \tag{14}$$

their minima overlap provided

$$|\delta_n| \lesssim \sigma_\alpha, \tag{15}$$

where σ_α is the curvature width near the minimum:

$$\sigma_\alpha^{-2} = I''(\alpha_{\min}). \tag{16}$$

Physical meaning of (14)–(16):

- overlapping minima imply robust parameter values,
- separation beyond σ_α leads to symmetry breaking or flavor hierarchies,
- SM masses and mixing angles arise as outcomes of entropy-saturated regions.

A.7 Why Rényi Entropy is Essential and Not Optional

Shannon entropy cannot generate:

- non-quadratic rate functions,
- scale-dependent convexity,
- overlapping minima across sectors.

In contrast, Rényi entropy uniquely provides:

- a tunable sensitivity to fluctuations (q),
- a natural multifractal interpretation,
- a direct bridge to large deviations.

This is why Rényi entropy is the best tool for uncovering the emergent SM parameters. Summarizing the logical chain of this Appendix:

$$\text{Rényi entropy} \implies D_q \implies f(\alpha) \implies I(\alpha) \implies \text{SM parameter selection}$$

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