A Brief Note on Charge Quantization from Fractal Distributions

Ervin Goldfain

Photonics CoE, Welch Allyn Inc., Skaneateles Falls, NY 13153, USA

Abstract

This brief note points out that fractal spacetimes having minimal deviations from integer dimensionality naturally lead to the emergence of *fractional magnetic charges*. Although these are un-observable at energy scales significantly lower than the electroweak scale (M_{EW}), their cumulative contribution may become relevant for charge quantization according to Dirac's theory of magnetic monopoles.

Key words: fractal spacetime, minimal fractal manifold, fractional magnetic charges, charge quantization, Dirac monopole.

In contrast with the standard formulation of classical electrodynamics, Maxwell equations on fractal distribution of charged particles generate *fractional magnetic charges* or *fractional monopoles* (q_m) [1-2]. The non-vanishing divergence of an external magnetic field $\mathbf{B}$ applied to a fractal distribution of charges is given by

$$\nabla \cdot \mathbf{B} = -\mathbf{B} \cdot \nabla c_2(d, \mathbf{r}) \tag{1}$$

in which the correction coefficient assumes the form

$$c_2(d, \mathbf{r}) = \frac{2^{2-d}}{\Gamma(d/2)} |\mathbf{r}|^{d-2} \tag{2}$$

Fractional monopoles depend on the gradient of (2) according to

$$q_m \sim \mathbf{B}\cdot\nabla c_2(d,\mathbf{r}) \tag{3}$$

We assume herein that the magnitude of the radial vector $\mathbf{r}$ is normalized to a reference length r_0 or, equivalently, to a reference mass scale $\mu_0 = r_0^{-1}$. Hence,

$$\mathbf{r} = (\frac{r}{r_0})\mathbf{u_r} = (\frac{\mu_0}{\mu})\mathbf{u_r} \tag{4}$$

in which $\mathbf{u_r}$ stands for the unit vector in the radial direction. Since the deviation from two dimensionality on a minimal fractal manifold is quantified as $d = 2 \pm \varepsilon$, with ε << 1, (2) is well approximated by

$$c_2(d,\mathbf{r}) \sim (\frac{\mu_0}{\mu})^{\pm\varepsilon}\mathbf{u_r} \tag{5}$$

Combined use of (2) and (5) yields

$$\nabla c_2(\varepsilon,\mathbf{r}) \sim \pm\varepsilon(\frac{\mu_0}{\mu})^{-1}\mathbf{u_r} = \pm\varepsilon(\frac{\mu}{\mu_0})\mathbf{u_r} \tag{6}$$

Because our analysis is carried out in a classical framework, we choose $\mu_0 = M_{EW}$ and the regime of mesoscopic scales $\mu << M_{EW}$, with $\mu/M_{EW} = O(\varepsilon)$. Relation (6) turns into

$$\nabla c_2(\varepsilon,\mathbf{r}) \sim \pm\varepsilon^2\mathbf{u_r} \tag{7}$$

The quadratic dependence on ε suggests that fractional magnetic charges are likely to be unobservable on mesoscopic scales. Substituting (7) into the Dirac charge quantization condition [3] gives

$$e q_m \sim \frac{n}{2} \Rightarrow e(\pm \varepsilon^2 \mathbf{B} \cdot \mathbf{u_r}) \sim \frac{n}{2} \tag{8}$$

where natural units are assumed and $n = \pm 1, \pm 2, \ldots$. It is readily seen that, in contrast with fractional magnetic charges, the quantization of free electric charges scales as ε^{-2} and is likely to be observable at mesoscopic distances on the order of $O(\mu^{-1})$.

Needless to say, this short analysis is far from being either rigorous or complete. Our sole intent is opening an unexplored research avenue which, to the best of our knowledge, has not received any prior consideration.

References

1. Tarasov V. E., "Electromagnetic field of fractal distribution of charged particles" http://arxiv.org/pdf/physics/0610010.pdf

2. Tarasov V. E., *"Fractional Dynamics: Application of Fractional Calculus to Dynamics of Particles, Fields and Media"*, (2011), Springer-Verlag.

3. see e. g., Shnir, Yakov M., "*Magnetic Monopoles"*, (2005), Springer-Verlag.