

Antigravity in AFT

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Abstract

We show how antigravity effects emerge from arrangement field theory. AFT is a proposal for an unifying theory which joins gravity with gauge fields by using the Lie group E_6 or $Sp(6)$. Details of theory have been exposed in the papers 1206.3663 and 1206.5665 (2012).

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1 Introduction

Arrangement field theory is a quantum theory defined by means of probabilistic spin-networks. These are spin-networks where the existence of an edges is regulated by a quantum amplitude. AFT is a proposal for an unifying theory which joins gravity with gauge fields. See [1] and [2] for details. The unifying group is $U(6, \mathbf{H})$ for the euclidean theory ($Sp(6)$ in other notation), while it is $E6$ for the lorentzian theory. The unifying group contains three indistinguishable copies of gauge fields, mixed by gravitational field. Moreover, commutators between gravitational and gauge fields are non null and give new terms for the Einstein equations. In what follows we focus on the term which mixes gravity with electromagnetism, showing that its contribution to Einstein equations could generate antigravity. In the end we verify that new interactions don't affect the making of nucleus and nucleons.

2 Antigravity

The term which mixes gravity with electromagnetism is given by space-time integration of the following expression:

$$-\frac{1}{4}f^{(G)(EM1)(EM2)}A_{\mu}^{(G)}A_{\nu}^{(EM1)}\left(F^{(EM2)\mu\nu} + \alpha f^{(EM3)(EM1)(EM2)}A^{(EM3)\mu}A^{(EM1)\nu}\right) \quad (1)$$

Remember that AFT includes three indistinguishable electro-magnetic fields, with non-trivial commutators. In this way $A^{(G)}$ is the gravitational gauge field, $A^{(EMn)}$ is the n-th electromagnetic field and α is the fine structure constant. In the realistic case of null torsion, the gravitational gauge field can be rewritten in function of the tetrad field:

$$A_{\mu}^{(G)bc} = \frac{1}{2}e^{\nu[b}\partial_{[\mu}e_{\nu]}^{c]} + \frac{1}{4}e_{\mu d}e^{\nu b}e^{\sigma c}\partial_{[\sigma}e_{\nu]}^d$$

From now we take a low energy limit so defined: $e_{ii} = 1$ with $i = 1, 2, 3$, $e_{00} = \theta(x)$ and $\partial_0\theta(x) = 0$. Varying with respect to e we obtain:

$$\frac{\delta A_\mu^{(G)bc}}{\partial e_\tau^s} = \frac{1}{2}e^{\nu[b}\delta_s^{c]}\delta_{[\nu}^\tau\partial_{\mu]} + \frac{1}{4}e_{\mu s}e^{\nu b}e^{\sigma c}\delta_{[\nu}^\tau\partial_{\sigma]}$$

$$\frac{\delta A_\mu^{(G)bc}}{\partial g_{\omega\tau}} = 2e^{\omega s}\frac{\delta A_\mu^{(G)bc}}{\partial e_\tau^s} = e^{\omega[c}e^{b]\nu}\delta_{[\nu}^\tau\partial_{\mu]} + \frac{1}{2}\delta_\mu^\omega e^{\nu b}e^{\sigma c}\delta_{[\nu}^\tau\partial_{\sigma]}$$

The component with $c = \omega = \tau = 0$ and $b \neq 0$ results:

$$\frac{\delta A_\mu^{(G)b0}}{\partial g_{00}} = -\theta^{-1}\delta_\mu^0\partial_b - \frac{1}{2}\theta^{-1}\delta_\mu^0\partial_b = -\frac{3}{2\theta}\delta_\mu^0\partial_b$$

$$A^{(EM)\rho}A_\rho^{(EM)}A^{(EM)\mu}\frac{\delta A_\mu^{(G)b0}}{\partial g_{00}} = \frac{3}{2\theta}\partial_b A^{(EM)0}A^{(EM)\rho}A_\rho^{(EM)}$$

The minus sign has disappeared because we have reversed the derivative. The quartic term in (1) becomes:

$$-\frac{\alpha}{4}f^b\frac{3}{2\theta}\partial_b A^{(EM)0}A^{(EM)\rho}A_\rho^{(EM)} = -\partial_b f^b\frac{3\alpha}{8\theta}V(\theta^2V^2 - A^2)$$

$$f^b = \sum_{cade} f^{(bo)ca}f^{dea} \approx 4\frac{x^b}{r}.$$

Here we have indicated with V the electric potential and with A the magnetic vector potential. The sum inside f is over the three electromagnetic fields.

It's so clear that varying the complete action with respect to $g_{\mu\nu}$ we obtain a new term for Einstein equations. In the Newtonian limit we can substitute $g_{00} = -(1 - 2\phi)$ and $R_{00} - (1/2)Rg_{00} = \nabla^2\phi$ where ϕ is the newtonian potential. Hence:

$$2\nabla^2\phi \approx 8\pi T^{00} = 8\pi\frac{-2}{\sqrt{-g}}\frac{\delta\sqrt{-g}L_{matt}}{\delta g_{00}}$$

$$\approx \partial_b\frac{x^b}{r}24\pi\alpha V(\theta V^2 - \theta^{-1}A^2) \quad (2)$$

For radial potential we have

$$\partial_b \phi = \frac{x^b}{r} \partial_r \phi.$$

In such case

$$C_G = \partial_r \phi \approx 12\pi\alpha V(\theta V^2 - \theta^{-1} A^2)$$

Now we insert the appropriate universal constants and approximate θ with 1:

$$C_G \approx 12\pi\alpha \frac{(G\varepsilon_0)^{3/2}}{c^4 L_p} V(V^2 - c^2 A^2) = kV(V^2 - c^2 A^2) \quad (3)$$

Here L_p is the Planck length, equal to $\sqrt{\hbar G/c^3}$. The multiplicative constant is

$$k = \frac{12\pi}{137} \cdot \frac{(6,67 \cdot 10^{-11} \cdot 8,85 \cdot 10^{-12})^{3/2}}{(3 \cdot 10^8)^4 \cdot (1,62 \cdot 10^{-35})} = 30,27 \cdot 10^{-33} \left(\frac{C^3 s^4}{Kg^3 m^5} \right).$$

This means that for having a weight variation (on Earth) of about 10% ($\Delta C_G = 1$) we need an electrical potential of 10^{11} Volts. These are 100 billions of Volts. For $V = Q/r$ and $A = 0$ we have:

$$C_G = \frac{k}{(4\pi\varepsilon_0)^3} \cdot \frac{Q^3}{r^3} = 2,198 \cdot 10^{-2} \left(\frac{m^4}{s^2 C^3} \right) \frac{Q^3}{r^3}$$

Note that the sign of C_G is the sign of Q and then we obtain antigravity for negative Q . We associate to this interaction an equivalent mass m , substituting $C_G = Gm/r^2$. We have

$$m = \frac{k}{G} V^3 r^2 = \frac{k}{G(4\pi\varepsilon_0)^3} \frac{Q^3}{r} = 3,293 \cdot 10^8 \left(\frac{Kg m}{C^3} \right) \frac{Q^3}{r}$$

which is a negative mass for negative Q . Negative mass implies negative energy via the relation $E = mc^2$. Intuitively, if we search a similar relation for gravi-magnetic field (which is $\nabla \times (g^{0i})$, $i = 1, 2, 3$), we should find the same formula (3) with an exchange between V and cA .

We calculate now at what distance the gravitational attraction between two protons is equal to their electromagnetic repulsion.

$$G \frac{m^2}{r^2} = \frac{k^2}{G^2(4\pi\epsilon_0)^6} \frac{Q_p^6}{r^4} = \frac{1}{4\pi\epsilon_0} \frac{Q_p^2}{r^2}$$

$$\frac{k^2 Q_p^4}{G^2(4\pi\epsilon_0)^5} = r^2$$

$$\implies r^2 = 79,49 \cdot 10^{-70} m^2 \implies r = 8,916 \cdot 10^{-35} m = 5,516 L_p$$

Note that we are 20 orders of magnitude under the range of strong force and 23 orders of magnitude under the range of weak force. In this way the gravitational force doesn't affect the making of nucleus and nucleons.

3 Conclusion

We have seen that a potential of 10^{11} Volts can induce relevant gravitational effects. They are too many for notice variations in the experiments with particles accelerators. However they sit at the border of our technological capabilities. The possibility to rule gravitation is very attractive and constitutes a good reason for try experiments with high electric potentials. Such experiments can be connected to the work of Nikola Tesla and can also be a good test for the arrangement field theory.

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