ON LAMINAR CONVECTION IN SOLAR TYPE STARS

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We present a new model of large-scale multilayer convection in solar type stars. This model allows us to understand such self-similar structures observed at solar surface as granulation, supergranulation and giant cells. We study the slow-rotated hydrogen star without magnetic field with the spherically-symmetric convective zone. The photon's flux comes to the convective zone from the central thermonuclear zone of the star. The interaction of these photons with the fully ionized hydrogen plasma with $T > 10^5 K$ is carried out by the Tomson scattering of photon flux on protons and electrons. Under these conditions plasma is optically thick relative to the Tomson scattering. This fact is the fundamental one for the multilayer convection formation. We find the stationary solution of the convective zone structure. This solution describes the convective layers responsible to the formation of the structures on the star's surface.

KEY WORDS Large-scale convection, Tomson scattering, solar atmosphere structures

INTRODUCTION

The systematic extreme ultraviolet and X-ray emission observations from Skylab station, Yohkoh, SoHO and Trace satellites give us the very interesting images of solar corona. After the previous images modifications (the partial gain of some interesting details) we can see large-scale corona structures around the solar disk and these structures are not associated with active regions (Priest et al., 1990; Chertoc, 2002). The structures are similar to standard coronal loops that connected separate active regions together (Beck, 1998), but their "foots" lean on the photosphere out of active regions. These regular structures cover the hole solar disk as the more large-scale chromocpheric network. The lifetime of such loops is about a week for relatively small loops with length approximately equal to $2 \cdot 10^4$ km and with plasma concentration approximately equal to 10^{15} m^{-3} but for the most great loops with length approximately equal to $3 \cdot 10^5$ km and with plasma concentration approximately $2 \cdot 10^{14}$ m^{-3} the lifetime is about some months.

These large-scale structures (chains, loops) are observed for some years. We see that all the observed loop systems associated with quiet Sun permanent exist as a regular part of solar corona. It's necessary note that photosphere and chromosphere have regular structures such as grains, supergrains and giant grains. The giant grains are discovered by the helioseismology's methods (Bec, 1998). These giant grains have the regular structure, their sizes are about $3 \cdot 10^5 km$ with regular plasma speeds of $100m/s$. It's well known that granulation, chromospheric network, supergranulation and giant loops is the consequence of under-photosphere convective zone existence. The solar-like stars photospheres have the similar structures as Sun has: grains and supergrains.

In this paper we present the simple model of the hydrogen star convection zone.

The necessary condition of free convection (rises in plasma layers with thickness of only some times smaller then solar radius) is the Schwarzschild criterion – the specific entropy of plasma decreases with moving away from the star center. Such convection will develop when the temperature inside of small convective volume (convective cell) decreases slower than the temperature decreases in neighboring plasma (Rudiger, 1989).

We use the Schwarzschild criterion later in our paper.

If solar convection is laminar so such processes as granulation, chromosphere network and supergranulation may exist in the convective layers of different thickness. Therefore solar convective zone consists of the three layers at least.

Under large-scale laminar convection conditions the small-scale turbulent convection appears owing to development of different plasma instabilities. The ejections of matter at granulation and supergranulation scales are connected more than likely with instabilities dynamics. The lifetime of these ejections is small and they can't change the regular structure of quiet Sun but these ejections outline this structure by effective way.

In this paper we propose the laminar convection model. In this model all the structures – granulation, chromosphere network, supergranulation and regular large-scale coronal structure – are examined as result of laminar convection action. We suppose that these structures are the different realizations of solution spectrum.

Let's consider the main assumptions of this model: The convective zone is the layer with the spherically symmetry distribution of plasma around the radiative transfer energy zone. In this layer the condition of real hydrostatic equilibrium is carried out.

In this paper we consider a case when the layer under study consists of the ionized hydrogen plasma only (protons and electrons). This consideration may not be applied for the Sun atmosphere conditions but it significantly simplifies the mathematical description of convection and allow us understand the mechanism of convection zone structures formation. In our model this layer is open system through which the energy flux moves upwards. So let's consider that the plasma conditions we can described as polytropic equation:

$$\frac{N}{N_0} = \left(\frac{T}{T_0}\right)^n \tag{1}$$

where n is the polytropic index, N and T are the plasma concentration and temperature. These values are $N_0 \approx 5 \cdot 10^{27} m^{-3}$ and $T_0 \approx 2 \cdot 10^6 K$ at the bottom border of the layer. The layer thickness (the convective zone depth) is approximately $0.3R_\odot$, where $R_\odot$ is the solar radius. The energy emission come to convective zone, the temperature $T_r \approx T_0$ near the bottom border of the convective zone. This flux is the reason of the development of laminar convection.

The emission and ionized plasma interaction is carried out by photon scattering on electrons and protons in case where the photon energy don't exceed the value kT. The time of energy transmission from photons to plasma don't excess the value (Kaplan, Tsytovich, 1973):

$$t_0 = \frac{3m_p c}{8\sigma_T \epsilon_r}, \tag{2}$$

if $kT_r \ll m_p c^2$, where k is the Boltzmann's constant, σ_T is the Thomson probability section of scattering, m_p is the electron mass, $\epsilon_r = \frac{4\sigma_B}{c} {T_r}^4$, σ_B is the Stephan – Boltzmann constant. If $T_r = T_0$ then $t_0 \approx 0.1s$.

The distance of free run for photons is equal to $\Delta \approx (\sigma_T N)^{-1}$. If $N = N_0$ then $\Delta \approx 3$ m.

In Δ^3 volume plasma and emission are in thermodynamic equilibrium almost because of the radiation is connected with matter.

The thermal conductivity mechanism is made available by the next processes in our case. The plasma (heated by radiation in value Δ^3) loses the energy by bremsstrahlung. The speed of these losses is $\epsilon_{ep} = 1.6 \cdot 10^{-40} N^2 \sqrt{T}$ $J \cdot m^{-3} s^{-1}$. The characteristic time of this process is equal to (Kaplan, Tsytovich, 1973): $t_2 = 2.6 \cdot 10^{17} \sqrt{T} N^{-1} s$. If $N = N_0, T = T_0$ we derive $t_2 \approx 10^{-6} s$. At the other hand the bremsstrahlung heats up the electrons in the vicinity of the volume Δ^3. The time taken for this process

$$t_1 = \frac{3m_e c}{8\sigma_T E}, \tag{3}$$

where $E = 3/2 NkT m_e/m_p$ is the electron energy density. If $T_* \leq T \leq T_0$ *and* $N_* \leq N \leq N_0, N_* = 2.5 \cdot 10^{26} m^{-3}$ we find $20s < t_1 < 160s$.

The characteristic rate of thermal conductivity is equal to $v_\chi = \frac{\Delta}{t_1}$. So the thermal conductivity coefficient for the process described (for the same order of magnitude) is equal to:

$$\chi = v_\chi \lambda = \frac{\lambda\Delta}{t_1} \qquad (4),$$

where λ is the thickness of the shell warmed up.

The convectional energy transfer is carried out thanks to macroscopic transports of the value Δ^3. The temperature inside the volume Δ^3 is higher then the plasma temperature in the layers which are situated higher then the bottom border of the convective zone, see Section 1. Thus the Archimedean raising force acts on this value and gives him the acceleration $\frac{g \cdot \Delta T}{T}$, where g is the free fall acceleration on the bottom border of the convective zone, $\Delta T = T_0 - T, T < T_0$.

The flotation process is retarded by viscosity. In our case the viscosity is the consequence of the Tomson scattering. The value Δ^3 is full of plasma and radiation. When this value moves the radiation is scattered by electrons of neighboring plasma. Thanks to the scattering the equalization of electron momentum takes place inside the volume Δ^3 and outside of one. This viscosity they called radiation viscosity. It characterized by the viscosity coefficient

$$\nu = \frac{1}{3} \frac{c}{\sigma_T N} \qquad (5).$$

If $N = N_0$ then $\nu \approx 6 \cdot 10^9 \ m^2/s$. This value is similar to the value estimation taken from the analysis of observations. The floating is ended when the raising force is in equilibrium with viscosity forces. The characteristic time of convective floating is equal to

$$t_2 = \frac{\nu}{g\lambda\frac{\Delta T}{T}},$$

where λ is correspond to characteristic scale of the convective layer (the mixing length). If $\lambda \approx 2 \cdot 10^8 m$, $T = T_0$ and $\frac{\Delta T}{T} \approx 1$, $g = 2g_\odot$, where $g_\odot \approx 274 m/s^2$ is the gravity force acceleration on the solar surface then we have $t_2 \approx 0.05s$. So at the bottom border of the convective zone the relation $t_1 > t_2$ is taken place. In this case the convective transfer is more effective then the heat conduction.

Near the top border of the convective zone $N = 4 \cdot 10^{22} \ m^{-3}$ and $\Delta_* = (\sigma_T N)^{-1} \approx 10^3 \ km$. In this case we can ignore the Tomson effect. The radiation of plasma propagates free up to solar photosphere.

In the section 1. we give the solutions of stationary convective zone structures in the hydrodynamics approximation with the heat conduction (4) and viscous (5) coefficients.

These solutions have the solitary wave structure and describe the model of multi-layer convection. All the convective cells have the torus contour.

1 THE EQUATIONS OF THE STATIONARY CONVECTIVE ZONE STRUCTURE

The set of simultaneous equations for the spherically symmetric stationary convective zone which rotates about z axis (because of the hydrodynamics approximation is correct) have the form:

$$(\vec{v},\nabla)\vec{v} = \frac{\nabla(p+p_r)}{\rho} + \vec{g} + \nu\cdot\nabla\vec{v} - 2[\vec{v},\vec{\omega}] \tag{6}$$

is the motion equation, where $\vec{\omega}$ is the angular velocity of convective zone rotation, ρ is the plasma density, p is the plasma pressure, p_r is the pressure of radiation.

$$(\vec{v},\nabla)T = \chi\cdot\Delta T \tag{7}$$

is the heat conduction equation,

$$\nabla(p+p_r) + \rho\vec{g} = 0 \tag{8}$$

is the hydrostatics equilibrium equation,

$$\frac{dp_r}{dr} = -\frac{\sigma_T N}{c}\frac{1}{4\pi r^2}L \tag{9}$$

is the radiation transfer equation outside of the volume Δ^3,

$$\frac{dL}{dr} = 4\pi r^2\epsilon_{ep} \tag{10}$$

is the bremsstrahlung of plasma equation inside the volume Δ^3,

$$dM = 4\pi r^2\rho\cdot dr \tag{11}$$

is the mass conservation equation.

In the equation (8) we don't take the density of radiation ρ_r because of $\rho_r \ll \rho$ in solar-like stars.

The set of simultaneous equations (8-10) have used by A.S.Eddington in 1926 (Eddington, 1926).

Let $p = NkT, N = N(r), T = T(r)$ and state of plasma is described by the polytropic equation (1). Let's take the variable $x = \frac{r}{\zeta}$, where

$$\zeta = \Big(\frac{kT_0}{4\pi G m_p \rho_0}\Big)^{1/2} \approx 13 \cdot 10^5 km > R_\odot.$$

Then function $\tau = \frac{T}{T_0}$ with equations (8) - (11) can be transformed to the following equation

$$(n+1)\frac{1}{x^2}(x^2\tau_x)_x = -\tau^n + \alpha\tau^{2n+\frac{1}{2}}, \tag{12}$$

where index x means the differentiation with respect to x and

$$\alpha = \frac{\sigma_T \epsilon_{ep}(N_0, T_0)}{4\pi G m_p \rho_0} \approx 10^{17}.$$

Let velocity vector $\vec{v}$ has the $\{V, W, Z\}$ components in spherical coordinate system. The vector of angular velocity $\vec{\omega}$ has the following components: $\{\omega cos\theta, -\omega sin\theta, 0, 0\}$.

Let $\theta \ll 1$. From the equations of the structure

$$(\vec{v}, \nabla)\vec{v} = \nu \cdot \Delta\vec{v} - 2[\vec{v}, \vec{\omega}]$$

$$(\vec{v}, \nabla)T = \chi \cdot \Delta T, \tag{13}$$

one can find the equation for $V(x)$ and $\tau(x)$:

$$VV_x = \frac{\nu}{\zeta} \cdot \frac{1}{x^2} \cdot (x^2 V_x)_x$$

$$V\tau_x = \frac{\chi}{\zeta} \cdot \frac{1}{x^2} \cdot (x^2 \tau_x)_x \tag{14}$$

The equations (12) and (14) are simplified when we assume that the component of velocity V is decreased with the depth. This condition is in agreement with solar observations: the plasma spread out velocity in the photosphere decreases with the scale increasing from grains to giant grains.

We choose the solution in the next form:

$$V = \frac{\sigma}{\zeta}\nu \tag{15}$$

where σ is the free parameter.

This permits us to simplify the equation (14) and transform it to the following:

$$(n+1)\beta\tau_x = -\tau^{2n+1}(1-\alpha\tau^{n+1/2}), \tag{16}$$

where $\beta = 7.5, \sigma = \frac{\nu_0}{\chi_0}\sigma$. The equation (16) has different solutions for the different values of n. Let's choose the value n (use the Schwarzschild criterion). According to this criterion the temperature inside the small element Δ^3 has to decrease with increasing of the distance from the star center slower then decreasing of plasma temperature occurs. The plasma is in the hydrostatic equilibrium and the radiation is absence.

Substitute $p_r = 0, p = NkT$ and $\rho = m_p \cdot N$ to (8).

Use the polytropic equation (1), we find the relative change of the plasma temperature (radiation doesn't take into account)

$$|\frac{T_x}{T}|_0 = \frac{m_p g\zeta}{(n+1)kT_0}\tau^{-1}.$$

In the volume Δ^3 the temperature changes according to (16). Also let's take into account $\alpha \gg 1$ and $\tau \leq 1$. Then $\alpha\tau^{n+1/2} \gg 1$ and relative change of the temperature inside the volume Δ^3 is approximately equal to:

$$|\frac{T_x}{T}| \approx \frac{\alpha}{(n+1)\beta}\tau^{3n+1/2}$$

In our case of the evolution of the convective instability $|\frac{T_x}{T}|_0 > |\frac{T_x}{T}|$ the number β is evaluate as: $\beta > \frac{\alpha}{223}\tau^{3n+3/2}$. In this case we have $\tau^n \gg (\alpha\sqrt{\tau})^{-1}$. Therefore $\beta > (223\alpha^2)^{-1}$.

At the other hand one can integrate the equation (16) because of ignoring the first member of the right part of the equation.

Thus we obtain the next algebraic equation:

$$\tau^{-3n-1/2} - 1 = \frac{\alpha}{\beta}\frac{3n+1/2}{n+1}(x-x_0).$$

Using this equation and the consideration that $\tau^n \gg (\alpha\sqrt{\tau})^{-1}$ we find that $\beta < \frac{3n+1/2}{n+1}\frac{x-x_0}{\alpha^2\tau}$. The value of polytropic index n (1) is necessary to choose as to make up the next unequality:

$$\frac{1}{223\alpha^2} < \beta < \frac{3n+1/2}{n+1}\frac{x-x_0}{\alpha^2\tau}. \tag{17}$$

For the solar-like star we have $\frac{1}{303} \leq \tau \leq 1$ and $10^{-4} \leq x - x_0 \leq \frac{2}{57}$. In this case the unequality (17) is realized for all n having the positive values. Let's choose $n = 3/2$. Then the accurate solution of the equation (16) for $\tau(x)$ one can find from the next algebraic equation:

$$\frac{2}{5\beta}(x-x_0) = \frac{1}{3}(1-\frac{1}{\tau^3}) + \alpha(1-\frac{1}{\tau}) - \frac{\alpha^{3/2}}{2}\Big(ln\frac{\alpha^{1/2}+1}{\alpha^{1/2}-1} - ln\frac{\alpha^{1/2}\tau+1}{\alpha^{1/2}\tau-1}\Big) \tag{18}$$

We have taken into account that $\tau(x_0) = 1$ here.

The solution for $\tau(x)$ has the solitary wave form. It's clear from the form of the equation (16).

If $\alpha^{1/2}\tau >> 1$ we have the asymptotic solution

$$\left(\frac{T_0}{T}\right)^5 \approx \frac{2\alpha}{\beta\zeta}(r-r_0) \tag{19}$$

For $x \to x_0$ one can find that

$$\tau \approx e^{-\frac{2}{3\alpha\beta}(x-x_0)} \to 1.$$

At last we can find the expression for the speed components W and Z. Then we examine the most simple case of the symmetric spreading out on the sphere surface when $W = Z$. Let's consider also that the angular velocity w we can take from the equation

$$V\frac{\partial W}{\partial x} + 2W \cdot \omega \cdot cos\theta = \frac{\nu}{\zeta^2} \cdot \frac{1}{x^2} \cdot \frac{\partial}{\partial x}\Big(x^2\frac{\partial W}{\partial x}\Big). \tag{20}$$

As follows from the equation (20) the convective zone rotates differently. Thanks to the convection the redistribution of the rotatory moment inside the star takes place. This effect is accurately studied in (Rudiger, 1989).

Under conditions selected in our paper we can find the equation for W from the first equation of the set of simultaneous equations (14). His form becomes simple enough

$$\frac{dW^2}{dl} = \frac{\nu}{\zeta^2} \cdot \frac{1}{x} \cdot \frac{dW^2}{dl^2}, \tag{21}$$

if we change the θ and ϕ angular variables to l variable and $dl = \sqrt{d\theta^2 + sin^2\theta d\varphi^2}$. Among the multiple numbers of solutions of the equation (21) there is periodic solution. This periodic solution has the next form:

$$W = W_0 \cdot tg(W_0\frac{\zeta^2}{\nu} \cdot x \cdot (l-l_0)), \tag{22}$$

where W_0 is the speed peak value W, the point l_0 is situated at the radius x and is the start reading for l coordinate. On the surface of sphere with radius x_* plasma spreads out from l_0 point. So our model is symmetric there are many points $l_{0,i}$ on the surface of the sphere of radius x_*. The distance between the neighboring points is equal to $2\xi = x_0(l_{0,i} - l_{0,i-1}) = x_* \cdot \Delta l = \frac{\pi\nu}{W_0\zeta^2}$. Between these points there are two opposing plasma streams with velocities of opposite direction. These streams compensate each other at the distance ξ from the each points.

So all the surface of the radius x_* breaks-down to the cells with diameters which are equal to ξ. All the number of these cells L we can calculate when we the surface square πx_*^2 divide by the cell square $\pi(\xi/2)^2$: $L = 4 \cdot (x_*/\xi)^2$. Then the velocity amplitude is equal to $W_0 = \pi\nu/2\xi^2 x_* \sqrt{L}$. The kinetic energy density ϵ is proportional to W_0^2. So the convective streams have the spectral energy distribution $\epsilon \sim L \sim \epsilon^{-2}$. The solutions of the convective zone structures (21) and (22) describes the stationary convection when all zone of the convective energy transfer consists of the layers with the different thickness. Every convective cell have the torus form. These solutions of this important problem are made for the first time. From the equation (19) follows the next conclusion: the convective zone differently rotates. Thanks to the convection the rotation moment redistribution inside the star is taken place. This effect is studied in detail in (Rudiger, 1989).

2 SUMMARUY AND CONCLUSIONS

This model qualitatively describes the deep convective layers of the star under the supergrains layer. In case of the star's convective transfer it's important that plasma at these layers is the fully ionized. We don't study star's plasma at the highest convective under-photospheric layer where the turbulent processes are possible. In this turbulent layer there are necessary conditions for the generation of the long-scale magnetic field of the star. At the layers under this turbulent under-photospheric layer the convection is the stationary convection.

Let's use the asymptotic solution (19) for the convective zone analysis. We have the convective zone consists of some layers with thickness of $\lambda_i, i = 0, 1, 2, ...$ The temperature on the lower part of the layer's border is equal to T_i, on the top part is T_{i^*}. If we take the dependence of parameters from (15) and (19) on T_0 into account so we can find the relation between the velocity and temperature at the bottom and top borders of the neighboring layers:

$$V_{i-1}/V_i = (\lambda_{i-1}/\lambda_i)^2 (T_i/T_{i-1})^{3/2} (T_{i-1^*}/T_{i^*})^5 \tag{23}$$

For the qualitative estimation let's substitude the characteristics of the convective layers associated with giant cells and supergrains into (23):

$$V_0 = 10 m/s, T_0 \sim 2 \cdot 10^6 K, \lambda_0 \sim 3 \cdot 10^5 km$$

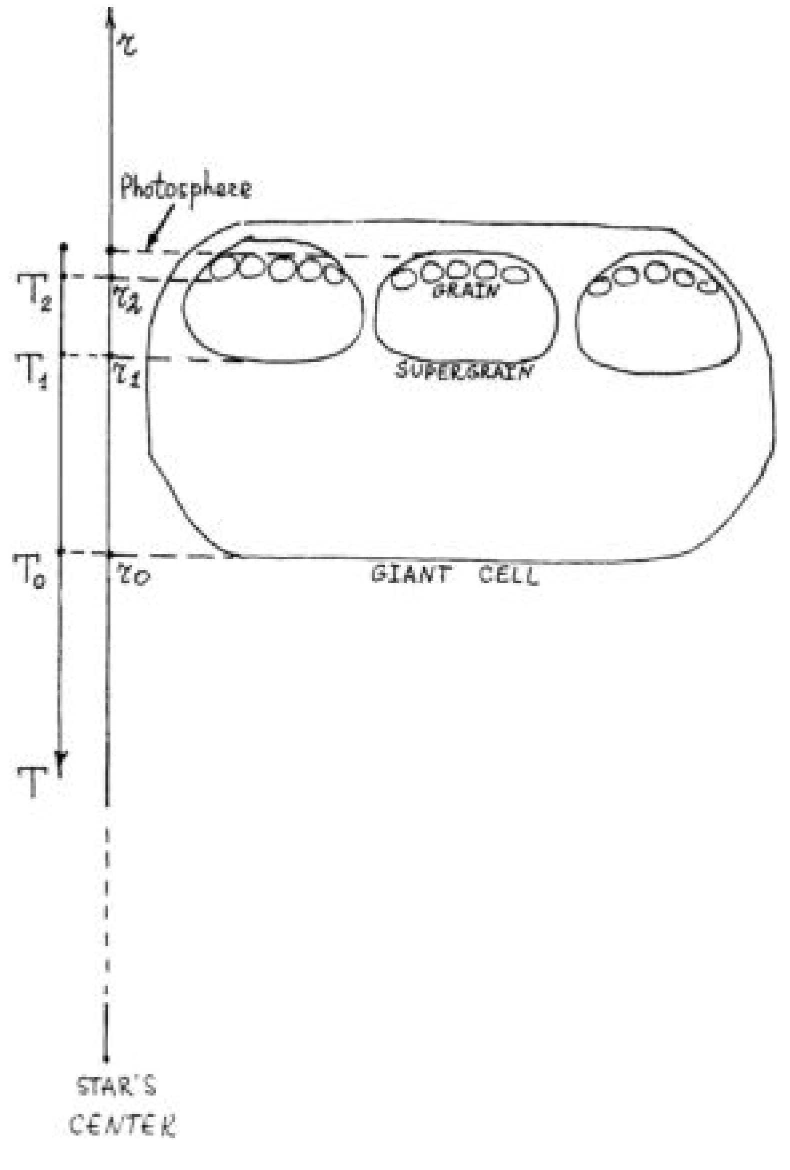

Figure 1: The convective zone structure

$$V_1 = 100m/s, T_1 \sim 10^6 K, \lambda_1 \sim 3 \cdot 10^4 km.$$

In this case we obtain that $T_{0*} \approx 0.4T_{1*}$ and the temperature on the top border of the layer λ_0 is smaller than the temperature on the top border of the layer $\lambda_1 < \lambda_0$. So we can see that λ_1 torus are situated into λ_0 torus.

This qualitative analysis of the formulae (23) allows us to make the conclusion about relatively disposed convective layers in the hydrogen star. The layers are put one into another as we can see at the Figure 1.

In (Rozgacheva et al., 2003; Rozgacheva et al., 2004) was shown, that the torus typical scales may form the geometric progression. This fact is one of our model tests. Such geometric progression that described stationary structures at Solar surface is probably form the fractal set.

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