Observational constrains on two cosmological models

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Abstract

Recently the model-independent approach to calculating a plot of scale factor a versus lookback time t_L has been developed by Ringermacher & Mead [1]. In the present paper we compare the dependence obtained in their work with predicted ones given by cosmological model with scalar meson field and by two-component (warm massive fermions + cosmological constant) cosmological model. For these models we fit cosmological parameters and estimate corresponding confidence intervals. It was found that there exist a broad region in the parameter space of two-component model satisfying the observed data. We also estimate the transition redshift of the universe in the noisy $a(\tau_L)$-data using anisotropic Gaussian filter on $a - t_L$ plane. The estimated value of $z_t = 0.72 \pm 0.05$ imposes stronger restrictions on the models' parameters.

KEY WORDS: cosmological models, scale factor, transition redshift.

1. Introduction

One of the most important task of the observable cosmology is a choice of our world model and its support by our observations. Here we use one cosmological test for two cosmological models. It is the cosmological scale factor as a function of cosmological time.

The outline of the paper is as follows. In Section 2, we give a brief review of cosmological model with scalar meson field and obtain the hyperbolic solution of the equation for the scale factor. In Section 3 we do the same for the two-component cosmological model with warm massive fermions and cosmological constant. Section 4 is dedicated to chi-square analysis of the $a(t_L)$-data obtained in [1] in view of estimation of cosmological models' parameters and corresponding confidence intervals. In Section 5 we estimate the transition redshift of the universe with the purpose to impose stronger restrictions on the parameters of two-component model. In Section 6 we make final conclusions and remarks.

2. The cosmological models with complex field

Let us consider a cosmological model with conformally flat space. The metric can be written

$$ds^2 = g_{mn} dx^m dx^n = c^2 dt^2 - a^2 \left[\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 \right], \tag{1}$$

where $a(t)$ is the scale factor, and t is the cosmological time of the model.

We consider the space-time (1) which is made up of the charged scalar meson field (complex field)

$$\psi = \varphi_1 + i\varphi_2$$
$$\psi\psi^* = \Psi^2 = \text{const}, \tag{2}$$

where the asterisk denotes complex conjugation and Ψ is the field amplitude relating to the field charge $Q - \Psi^2$. We choose the complex field Lagrangian

$$L = \frac{1}{hc}\left(g^{mn}\frac{\partial\psi}{\partial x^m}\frac{\partial\psi^*}{\partial x^n} - U\left(\psi\psi^*\right)\right) + \frac{dF}{dt},$$

where h is Planck's constant, c is the light velocity, U_0 is an amplitude of the field potential, and dF/dt is a total derivative of some differentiable function.

This cosmological model was considered in papers [2, 3, 4].

The Einstein equations with the energy-momentum tensor

$$T_{mn} = \frac{2}{hc}\Psi^2\frac{\partial\varphi}{\partial x^m}\frac{\partial\varphi}{\partial x^n} - g_{mn}\frac{dF}{dt}$$

lead to two equations:

$$3\left(\frac{a_t}{a}\right)^2 = \frac{2\kappa}{hc}\Psi^2\left(\frac{\partial\varphi}{\partial t}\right)^2 - \kappa c^2\frac{dF}{dt},$$
$$\frac{1}{c^2}\left[2\left(\frac{a_t}{a}\right)_t + 3\left(\frac{a_t}{a}\right)^2\right] = -\frac{2\kappa}{hc}\Psi^2\frac{1}{a^2}\left(\frac{\partial\varphi}{\partial x}\right)^2 - \kappa\frac{dF}{dt}. \tag{3}$$

Here the subscript t denotes differentiation in t, $\kappa = \frac{8\pi G}{c^4}$ is the Einstein gravity constant, and we have taken into account that the solution of the equation (2) is

$$\psi = \Psi \mathrm{e}^{i\varphi}, \quad \psi^* = \Psi \mathrm{e}^{-i\varphi},$$

$\varphi\left(x^m\right)$ is a field phase, and in isotropic case

$$\frac{\partial\varphi}{\partial x^1} = \frac{\partial\varphi}{\partial x^2} = \frac{\partial\varphi}{\partial x^3} = \frac{\partial\varphi}{\partial x}.$$

The Lagrange equation of the field ψ lead to two equations:

$$\left(\frac{1}{c}\frac{\partial\varphi}{\partial t}\right)^2 - \frac{3}{a^2}\left(\frac{\partial\varphi}{\partial x}\right)^2 = U_0,$$

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{3}{a^2}\frac{\partial^2\varphi}{\partial x^2} + \frac{3}{c^2}\frac{a_t}{a}\frac{\partial\varphi}{\partial t} = 0. \tag{4}$$

Four equations (3) and (4) lead to the equation for the scale factor:

$$\frac{a_{tt}}{a} + \left(\frac{a_t}{a}\right)^2 = \frac{\kappa c}{3h}U_0\Psi^2 - \frac{2\kappa c^2}{3}\frac{dF}{dt}. \tag{5}$$

Using the notations of Hubble constant $H = a_t/a$ and the deceleration parameter $q = -a_{tt}a/a_t^2$ one can rewrite last equation in a form

$$H^2(1-q) = \frac{\kappa c}{3h}U_0\Psi^2 - \frac{2\kappa c^2}{3}\frac{dF}{dt}. \tag{5'}$$

In the hyperbolic case (corresponding to $L = 0$, $dF/dt = 0$) the right-hand side of the equation (5) is a constant, so the left-hand side remains constant, too. One can determine the value of it through the current values of Hubble constant and the deceleration parameter

$$\frac{\kappa c}{3h}U_0\Psi^2 = H_0^2(1-q_0) > 0.$$

The hyperbolic solution of the equation (5) takes the form

$$a = a_*\sqrt{\sinh\left(\frac{t}{t_*} + \phi\right)}, \tag{6}$$

where

$$a_* = a_0\left(\frac{1+q_0}{1-q_0}\right)^{1/4},$$

$$t_* = \frac{H_0^{-1}}{\sqrt{2(1-q_0)}},$$

$$\phi = \frac{1}{2}\ln\left(\frac{\sqrt{2}+\sqrt{1-q_0}}{\sqrt{2}-\sqrt{1-q_0}}\right) - \frac{t_0}{t_*}.$$

The solution (6) is singular at time $t = -t_*\phi$. If we put it equal to zero, the current time will be equal to the age of the universe in our model

$$t_0 = \frac{1}{2}\ln\left(\frac{\sqrt{2}+\sqrt{1-q_0}}{\sqrt{2}-\sqrt{1-q_0}}\right)t_*.$$

The solution (6) also contains the inflexion point, which corresponds to transition to an accelerating regime of the model's expansion. It localized at

$$t_t = \left(\ln(1+\sqrt{2}) - \phi\right)t_*. \tag{7}$$

Thus we can see that the hyperbolic solution of equation (5) qualitatively reflects our state of knowledge on the history of cosmological expansion. In sections 4 and 5 we will make qualitative comparison of it with the empirical $a(t)$ dependency obtained in [1].

3. The two-component cosmological model with warm massive fermions and cosmological constant

We now consider a two-component model in a flat space-time with metrics (1) filled with the gas of free massive particles and dark energy in the form of cosmological constant. The energy-density tensor for free particles has the form [5, 6]

$$(T^i_k)_\nu = c\int \frac{p^i p_k}{p_0} f(x^i, p^\alpha)\sqrt{-g}d^3p,$$

where $g = |g_{ik}|$, $f(x^i, p^\alpha)$ is the single-particle distribution function depending on 4-coordinates and 3-momenta ($p^i p_i = (m_\nu c)^2$). The function f is normalized by the condition $(T^0_0)_\nu = \varepsilon_\nu$ for the energy-density of the gas. As the model expands its dependence on momenta remains unchangeable. For massive neutrinos that become free before the derelativization epoch the distribution function is

$$f(q) = \frac{f^*}{e^{cq/\theta_0 a_0} + 1}$$

where θ_0 is a temperature parameter, $q^2 = q^\alpha q_\alpha$, q^α are components of conformal momentum ($p^\alpha = q^\alpha/a^2$) and

$$f^* = \frac{\varepsilon_{0\nu}}{4\pi c}\left(\frac{c}{\theta_0}\right)^2\left(\int_0^\infty \frac{\sqrt{u^2+\gamma_\nu^2}}{e^u+1}u^2du\right)^{-1},$$

$\gamma_\nu = m_\nu c^2/\theta_0$. The expansion of the model is governed by the equation for the scale factor

$$H^2 = \left(\frac{a_t}{a}\right)^2 = H_0^2\left(\Omega_M\left(\frac{a_0}{a}\right)^4\frac{I_1(\gamma_\nu, a/a_0)}{I_1(\gamma_\nu, 1)} + \Omega_\Lambda\right) \tag{8}$$

where

$$I_1(\gamma_\nu, x) = \int_0^\infty \frac{\sqrt{u^2 + (\gamma_\nu x)^2}}{e^u+1}u^2du, \tag{9}$$

the quantities $\Omega_M = (8\pi G/3H_0^2c^2)\varepsilon_{0\nu}$, $\Omega_\Lambda = c^2\Lambda/3H_0^2$ are the cosmological densities of the neutrino gas and dark energy, $\varepsilon_{0\nu}$ and $\Lambda c^4/8\pi G$ are the energy densities of neutrinos and dark energy, respectively. For flat universe $\Omega_M + \Omega_\Lambda = 1$. From (8) one can obtain

$$t = \frac{1}{H_0}\int_0^{a/a_0}\frac{dx}{x\sqrt{\Omega_M\frac{I_1(\gamma_\nu,x)}{I_1(\gamma_\nu,1)}x^{-4} + \Omega_\Lambda}}. \tag{10}$$

It easily can be seen that if $\gamma_\nu \to \infty$ ($\theta_0 \to 0$) the ratio $I_1(\gamma_\nu, a/a_0)/I_1(\gamma_\nu, 1) \to a/a_0$ and the model tend to standard ΛCDM case

$$t_{\Lambda CDM} = \frac{1}{H_0}\int_0^{a/a_0}\frac{dx}{x\sqrt{\Omega_M x^{-3} + \Omega_\Lambda}}. \tag{11}$$

4. Fitting cosmological parameters

In this section we compare theoretical predictions on $a(t)$ dependency given by two above mentioned models with the model-independent results obtained by Ringermacher & Mead [1]. They utilized the Hubble diagram data for more then 500 standard candles (SNe + radio-galaxies) up to $z = 1.8$ to perform a robust numerical integration leading to dimensionless lookback time[1]

$$\tau_L \equiv H_0(t_0 - t) = -\int_y^0 \frac{a}{a_0} dy, \tag{12}$$

where $t_0 = t(a = a_0)$ is the cosmic time, $y = aD_L/D_H$ is a dimensionless coordinate distance, $D_H = c/H_0$, $a/a_0 = (1+z)^{-1}$, z is a standard candle's redshift. The luminosity distance D_L is related to standard candle's modulus $\mu = m - M_\odot = 5 \lg D_L(Mpc) + 25$.

To perform the estimation of models' parameters and goodness-of-fit we calculate the log-likelihood function

$$\log L(\omega) \propto -\frac{1}{2}\sum_{i=1}^{N} \left(\frac{a_i - a(\tau_{Li}|\omega)}{\sigma_i}\right)^2,$$

where ω denotes the parameter space of a model. For model with complex field $\omega = \{q_0\}$, and for two-component model $\omega = \{\Omega_M, \gamma_\nu\}$. Unfortunately, individual errors σ_i are not known in advance because observers often do not provide a classical error estimation due to spurious problems such as SN misclassification, uncertain extragalactic extinction laws, poorly constrained colors, etc (see e.g. [7]). To reflect the differences in quality of the observational material the objects are often only divided into "gold" and "silver" subsets. Moreover, the error is accumulating during the process of numerical integration in (12): the error grows towards earlier times with some rough law. As such a law we choose a second order polynomial with respect to τ_L, approximating the absolute deviations of empirical points from the best-fitting ΛCDM model with parameters $\Omega_\Lambda = 0.735$ and $\Omega_M = 0.265$ [1]. Obviously, this approach prohibits an independent assessment of goodness-of-fit [8].

In the table 1 we give the best-fit parameters for our models.

Table 1: Best-fit models' parameters

Meson scalar field	$q_0 = 0.72 \pm 0.015$
Neutrino + cosmological constant	$\gamma_\nu = 15$, $\Omega_M = 0.26$

5. Estimating transition redshift

[1]The authors of the original paper [1] actually calculate different quantity with the same notation $\tau_L = 1 - \int_y^0 \frac{a}{a_0} dy$, but it is not equal to $H_0 t$ in our notation, because the dimensionless cosmic time $H_0 t_0$ is not necessarily equal to 1.

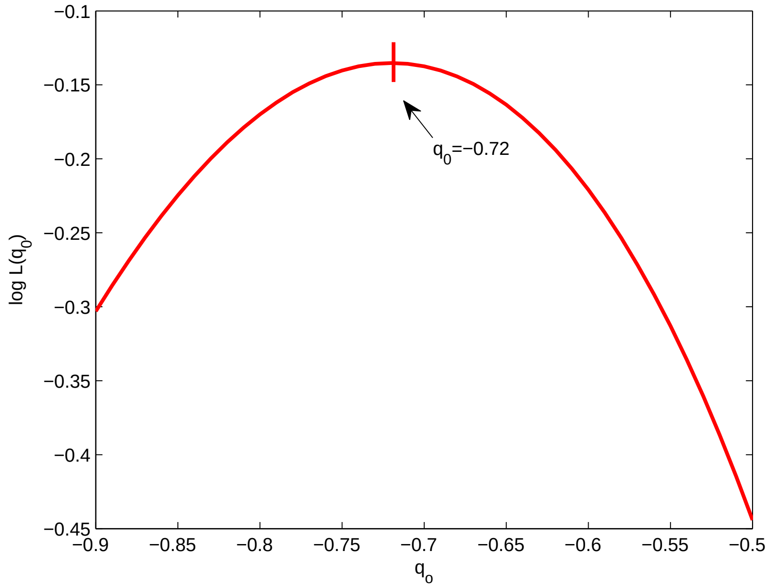

Figure 1: The log-likelihood function for the model with scalar meson field

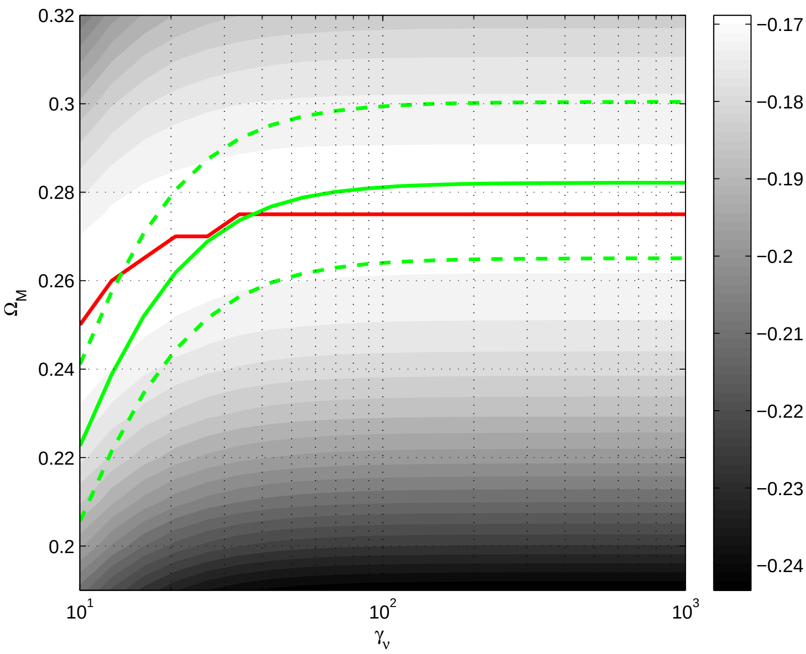

Figure 2: The log-likelihood function for the two-component model

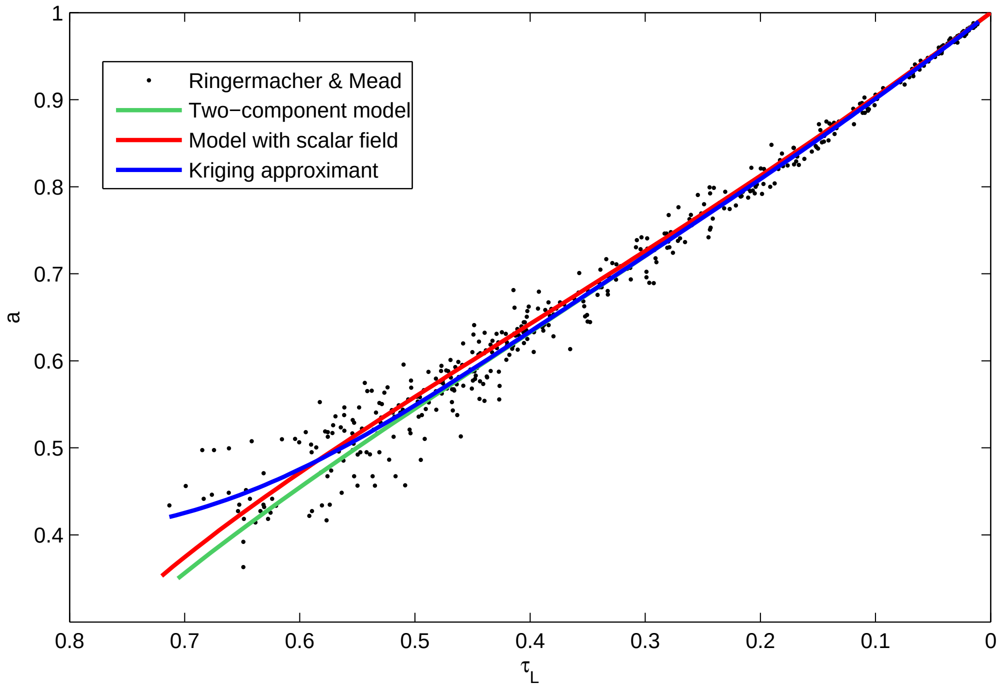

Figure 3: The best-fit $a(t_L)$ dependencies for different models

As we can see there is a broad region in the two-component model's parameter space satisfying the observed data in a least-squares sense. In this Section we are trying to impose stronger restrictions on the parameters of our models localizing the transition redshift z_t which corresponds to the position of inflexion point on the $a(t)$ plot.

For the model with scalar field the location of z_t uniquely defines the parameter q_0

$$q_0(z_t) = \frac{1-(z_t+1)^4}{1+(z_t+1)^4}.$$

For the two-component model it defines a subset of possible solutions, lying on a curve

$$\Omega_M(\gamma_\nu|z_t) = \left((1+z_t)^4 \frac{I_1\left(\gamma_\nu,(1+z_t)^{-1}\right)}{I_1\left(\gamma_\nu,1\right)} - \frac{1}{2}(1+z_t)\frac{I_2\left(\gamma_\nu,(1+z_t)^{-1}\right)}{I_1\left(\gamma_\nu,1\right)} + 1\right)^{-1},$$

where

$$I_2(\gamma_\nu, x) \equiv \frac{\partial I_1}{\partial \gamma_\nu} = x\gamma_\nu^2 \int_0^\infty \frac{u^2 du}{(e^u+1)\sqrt{u^2+(\gamma_\nu x)^2}}.$$

The problem of searching of inflexion points in noisy data is widely known. Nevertheless there is no method, that can guarantee an adequate result. Among sophisticated methods there are applications of wavelet analysis, Gaussian process regression (kriging) [8], radial basis function networks, anisotropic Gaussian filtering, etc. After some experiments with the data we have chosen the last one.

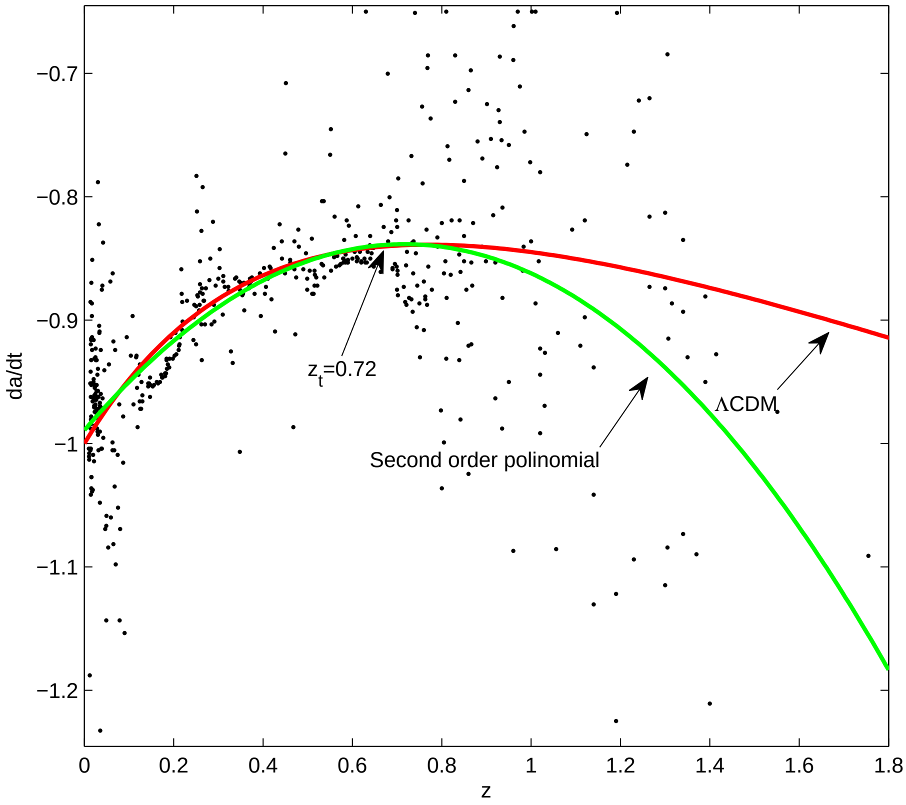

Figure 4: The derivative $\frac{da}{dt}$

6. Conclusions

In the present paper we have compared the empirical $a(t)$ dependency obtained in the work [1] with predicted dependencies given by cosmological model with scalar meson field and by two-component cosmological model with massive fermions and dark energy in a form of cosmological constant. Both of the models have a unique inflexion point. So, this models qualitatively agree with modern views on expansion history. For these models we have chosen the best-fit parameters. The model with scalar meson field has shown a significant correspondence to the observational data. In the case with two-component model we have found that there exist a broad region in the parameter space of this model satisfying the observational data in a least-squares sense. We also have estimated the transition redshift of the universe in the noisy $a(\tau_L)$-data using anisotropic Gaussian filter on $a - t_L$ plane. The estimated value of $z_t = 0.72 \pm 0.05$ imposes stronger restrictions on the cosmological parameters.

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