Alternative Classical Mechanics

Alejandro A. Torassa

(2013) Buenos Aires, Argentina
atorassa@gmail.com

Abstract

This paper presents an alternative classical mechanics, which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

Universal Reference Frame

The universal reference frame is a reference frame fixed to the center of mass of the universe.

The universal position $\mathring{\mathbf{r}}_a$, the universal velocity $\mathring{\mathbf{v}}_a$, and the universal acceleration $\mathring{\mathbf{a}}_a$ of a particle A relative to the universal reference frame $\mathring{\text{S}}$, are given by:

$$\mathring{\mathbf{r}}_a = (\mathbf{r}_a)$$

$$\mathring{\mathbf{v}}_a = d(\mathbf{r}_a)/dt$$

$$\mathring{\mathbf{a}}_a = d^2(\mathbf{r}_a)/dt^2$$

where $\mathbf{r}_a$ is the position of particle A relative to the universal reference frame $\mathring{\text{S}}$.

The dynamic position $\breve{\mathbf{r}}_a$, the dynamic velocity $\breve{\mathbf{v}}_a$, and the dynamic acceleration $\breve{\mathbf{a}}_a$ of a particle A of mass m_a, are given by:

$$\breve{\mathbf{r}}_a = \int\int (\mathbf{F}_a/m_a)\, dt\, dt$$

$$\breve{\mathbf{v}}_a = \int (\mathbf{F}_a/m_a)\, dt$$

$$\breve{\mathbf{a}}_a = (\mathbf{F}_a/m_a)$$

where $\mathbf{F}_a$ is the net force acting on particle A.

General Principle

The total position $\tilde{\mathbf{R}}_{ij}$ of a system of biparticles of mass M_{ij} $(M_{ij} = \sum_i \sum_{j>i} m_i m_j)$, is given by:

$$\tilde{\mathbf{R}}_{ij} = \sum_i \sum_{j>i} \frac{m_i m_j}{M_{ij}} \left[(\mathring{\mathbf{r}}_i - \mathring{\mathbf{r}}_j) - (\breve{\mathbf{r}}_i - \breve{\mathbf{r}}_j) \right] = 0$$

The total position $\tilde{\mathbf{R}}_i$ of a system of particles of mass M_i $(M_i = \sum_i m_i)$, is given by:

$$\tilde{\mathbf{R}}_i = \sum_i \frac{m_i}{M_i} (\mathring{\mathbf{r}}_i - \breve{\mathbf{r}}_i) = 0$$

Therefore, the total position $\tilde{\mathbf{R}}_{ij}$ of a system of biparticles and the total position $\tilde{\mathbf{R}}_i$ of a system of particles are always in equilibrium.

Observations

From the general principle the following equations are obtained:

12 equations for a biparticle AB:

$$\frac{1}{x} \left[(\mathbf{r}_a - \mathbf{r}_b)^y \times \left[\frac{d^z(\mathring{\mathbf{r}}_a - \mathring{\mathbf{r}}_b)}{dt^z} \right]_{\mathring{\mathrm{S}}} \right]^x - \frac{1}{x} \left[(\mathbf{r}_a - \mathbf{r}_b)^y \times \left[\frac{d^z(\breve{\mathbf{r}}_a - \breve{\mathbf{r}}_b)}{dt^z} \right]_{\mathring{\mathrm{S}}} \right]^x = 0$$

12 equations for a particle A:

$$\frac{1}{x} \left[(\mathbf{r}_a)^y \times \left[\frac{d^z(\mathring{\mathbf{r}}_a)}{dt^z} \right]_{\mathring{\mathrm{S}}} \right]^x - \frac{1}{x} \left[(\mathbf{r}_a)^y \times \left[\frac{d^z(\breve{\mathbf{r}}_a)}{dt^z} \right]_{\mathring{\mathrm{S}}} \right]^x = 0$$

Where:

x takes the value 1 or 2 (1 vector equation, and 2 scalar equation)

y takes the value 0 or 1 (0 linear equation, and 1 angular equation)

z takes the value 0 or 1 or 2 (0 position equation, 1 velocity equation, and 2 acceleration equation)

Observations:

If y takes the value 0 then the symbol $\times$ should be removed from the equation.

$\mathbf{r}_a$ and $\mathbf{r}_b$ are the positions of particles A and B relative to a reference frame S.

$\left[d^z(\ldots)/dt^z \right]_{\mathring{\mathrm{S}}}$ means the z-th time derivative relative to the universal reference frame $\mathring{\mathrm{S}}$.

Reference Frame

The universal position $\mathring{\mathbf{r}}_a$, the universal velocity $\mathring{\mathbf{v}}_a$, and the universal acceleration $\mathring{\mathbf{a}}_a$ of a particle A relative to a reference frame S, are given by:

$$\mathring{\mathbf{r}}_a = \mathbf{r}_a + \breve{\mathbf{r}}_s$$

$$\mathring{\mathbf{v}}_a = \mathbf{v}_a + \breve{\omega}_s \times \mathbf{r}_a + \breve{\mathbf{v}}_s$$

$$\mathring{\mathbf{a}}_a = \mathbf{a}_a + 2\,\breve{\omega}_s \times \mathbf{v}_a + \breve{\omega}_s \times (\breve{\omega}_s \times \mathbf{r}_a) + \breve{\alpha}_s \times \mathbf{r}_a + \breve{\mathbf{a}}_s$$

where $\mathbf{r}_a$, $\mathbf{v}_a$, and $\mathbf{a}_a$ are the position, the velocity, and the acceleration of particle A relative to the reference frame S; $\breve{\mathbf{r}}_s$, $\breve{\mathbf{v}}_s$, $\breve{\mathbf{a}}_s$, $\breve{\omega}_s$, and $\breve{\alpha}_s$ are the dynamic position, the dynamic velocity, the dynamic acceleration, the dynamic angular velocity and the dynamic angular acceleration of the reference frame S.

The dynamic position $\breve{\mathbf{r}}_s$, the dynamic velocity $\breve{\mathbf{v}}_s$, the dynamic acceleration $\breve{\mathbf{a}}_s$, the dynamic angular velocity $\breve{\omega}_s$, and the dynamic angular acceleration $\breve{\alpha}_s$ of a reference frame S fixed to a particle S, are given by:

$$\breve{\mathbf{r}}_s = \int\int (\mathbf{F}_0/m_s)\; dt\; dt$$

$$\breve{\mathbf{v}}_s = \int (\mathbf{F}_0/m_s)\; dt$$

$$\breve{\mathbf{a}}_s = (\mathbf{F}_0/m_s)$$

$$\breve{\omega}_s = \left|(\mathbf{F}_1/m_s - \mathbf{F}_0/m_s)/(\mathbf{r}_1 - \mathbf{r}_0)\right|^{1/2}$$

$$\breve{\alpha}_s = d(\breve{\omega}_s)/dt$$

where $\mathbf{F}_0$ is the net force acting on the reference frame S in a point 0, $\mathbf{F}_1$ is the net force acting on the reference frame S in a point 1, $\mathbf{r}_0$ is the position of the point 0 relative to the reference frame S (the point 0 is the center of mass of particle S and the origin of the reference frame S) $\mathbf{r}_1$ is the position of the point 1 relative to the reference frame S (the point 1 does not belong to the axis of rotation) and m_s is the mass of particle S (the vector $\breve{\omega}_s$ is along the axis of rotation)

A reference frame S is rotating if $\breve{\omega}_s \neq 0$, it is non-rotating if $\breve{\omega}_s = 0$, and it is inertial if $\breve{\omega}_s = 0$ and $\breve{\mathbf{a}}_s = 0$.

The magnitudes $\breve{\mathbf{r}}$, $\breve{\mathbf{v}}$, $\breve{\mathbf{a}}$, $\breve{\omega}$, and $\breve{\alpha}$ are invariant under transformations between reference frames.

In this paper it is assumed that the dynamic position $\breve{\mathbf{r}}_{cm}$, the dynamic velocity $\breve{\mathbf{v}}_{cm}$, the dynamic acceleration $\breve{\mathbf{a}}_{cm}$, the dynamic angular velocity $\breve{\omega}_{cm}$, and the dynamic angular acceleration $\breve{\alpha}_{cm}$ of the universal reference frame $\mathring{\mathrm{S}}$ fixed to the center of mass of the universe are always zero.

In addition, the universal position $\mathring{\mathbf{r}}_{cm}$, the universal velocity $\mathring{\mathbf{v}}_{cm}$, and the universal acceleration $\mathring{\mathbf{a}}_{cm}$ of the center of mass of the universe relative to the universal reference frame $\mathring{\mathrm{S}}$ are always zero.

Unified Force

The unified force $\mathbf{U}$ exerted on a particle A of mass m_a by another particle B of mass m_b, caused by the interaction between particle A and particle B, is given by:

$$\mathbf{U} = \frac{m_a m_b}{m_{cm}} \left[(\mathring{\mathbf{a}}_a - \mathring{\mathbf{a}}_b) - (\breve{\mathbf{a}}_a - \breve{\mathbf{a}}_b) \right]$$

where m_{cm} is the mass of the center of mass of the universe, $\mathring{\mathbf{a}}_a$ and $\mathring{\mathbf{a}}_b$ are the universal accelerations of particles A and B, $\breve{\mathbf{a}}_a$ and $\breve{\mathbf{a}}_b$ are the dynamic accelerations of particles A and B.

From the above equation it follows that the net unified force $\mathbf{U}_{ab}$ acting on a biparticle AB of mass $m_a m_b$, is given by:

$$\mathbf{U}_{ab} = m_a m_b \left[(\mathring{\mathbf{a}}_a - \mathring{\mathbf{a}}_b) - (\breve{\mathbf{a}}_a - \breve{\mathbf{a}}_b) \right]$$

where $\mathring{\mathbf{a}}_a$ and $\mathring{\mathbf{a}}_b$ are the universal accelerations of particles A and B, $\breve{\mathbf{a}}_a$ and $\breve{\mathbf{a}}_b$ are the dynamic accelerations of particles A and B.

From the second above equation it follows that the net unified force $\mathbf{U}_a$ acting on a particle A of mass m_a, is given by:

$$\mathbf{U}_a = m_a (\mathring{\mathbf{a}}_a - \breve{\mathbf{a}}_a)$$

where $\mathring{\mathbf{a}}_a$ and $\breve{\mathbf{a}}_a$ are the universal acceleration and the dynamic acceleration of particle A.

Finally, from the general principle it follows that the net unified force $\mathbf{U}_{ab}$ acting on a biparticle AB and the net unified force $\mathbf{U}_a$ acting on a particle A are always in equilibrium.

Bibliography

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