The excited mesons due to the laser pulse

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Abstract

We consider the meson as the string of a large gluon tube with mass M, the left end of which is fixed to a quark mass $m << M$ and the second quark mass m is fixed to the right end of the string. The force of the delta-function form is applied to the left side of the string. We find the propagation of the pulse in the system and the excitation states of a meson. The problem of excitated states of mesons plays the crucial role in the meson physics.

Key words: Meson, string, quarks, gluon tube, delta-function pulse.

1 Introduction

The determination of the hadron and meson mass spectrum in the framework of quantum chromodynamics (QCD), or dynamical states of mesons still remains a unsolved problem. So, the potential methods and the string methods are used to solve these problems.

In the string model of hadrons the quarks are treated to be tied together by a gluon tube which can be approximated by the tube of vanishing width, or by string (Nambu, 1974). Then, dynamics of hadrons can be approximated by the Nambu-Goto action for the relativistic string. The merit of the string model is the natural explanation of the quark confinement and the dynamics of the system. On the other hand, there are some mathematical problems when the quark masses are different from zero. So, there are many trials to give the final words to the problem of hadron dynamics and hadron masses. It is well known that some models of string theory involves also the so called extra-dimension. However, it is well known that this theory is at present time not predictable. The goal of theoretical physics is predictions based on the mathematical knowledge. On the other hand, the goal of the experimental physic is the confirmation of theory with appropriate experimental simplicity. We use here approach to the string dynamics which consists in experimental situation in laser physics that the pulse is applied to the left side of a string and then we determine the string dynamics. We do not consider in this simplification the rotating string.

So, let us consider the elastic string of large mass M, the left end of which is joined with mass $m << M$ and body of mass m is fixed to the right end of the string (Pardy, 2005). The force of the delta-function form is applied to the left side of the string. The delta-function is chosen for simplicity. This function can be replaced by the different functions. We show that the internal motion of the elastic string medium is controlled by the wave equation. We show that the momentum of the massive ends is not conserved in time. Our problem represents the missing problem in textbooks on mechanics.

The pedagogical benefit of this article is in the rigorous definition of the problem in the university mechanics and in the proof that the Dirac delta-function elegantly works in mechanics.

The experimental demonstration of the delta-function tension can be easily performed as it is described in the text. The mathematical formulation of the problem and the solution gives deep insight in the dynamics of strings with the massive ends. We give new possibilities of the university mechanics. We hope that our approach can form the serious motivation for study the string theory on LHC.

In particle physics, mesons are hadronic subatomic particles composed of one quark and one antiquark, bound together by the strong interaction. Because mesons are composed of sub-particles, they have a physical size, with a diameter of roughly one fermi, which is about 2/3 the size of a proton or neutron. All mesons are unstable, with the life-time a few hundredths of a microsecond. Charged mesons decay (sometimes through intermediate particles) to form electrons and neutrinos. Uncharged mesons may decay to photons.

Because quarks have a spin of 1/2, the difference in quark number between mesons and baryons results in conventional two-quark mesons being bosons, whereas baryons are fermions.

2 Classical particle interaction with an impulsive force

Let us first show that use of the impulsive force of the delta-function form is physically meaningful in a classical mechanics of a point particle. We idealize the impulsive force by the Dirac δ-function. Then we apply the δ-function to the string model of mesons.

Newton's second law in the one-dimensional form for the interaction of a massive particle with mass m with force F is $ma = F$. Then, with F being an impulsive force $P\delta(\alpha t)$ it is as follows:

$$m\frac{d^2x}{dt^2} = P\delta(\alpha t), \tag{2}$$

where P and α are some constants, with MKSA dimensionality $[P] = \text{kg.m.s}^{-2}$, $[\alpha] = \text{s}^{-1}$. We put $|\alpha| = 1$.

Using the Laplace transform (Arfken, 1967) in the last equation, with

$$\int_0^\infty e^{-st}x(t)dt \stackrel{d}{=} X(s), \tag{3}$$

$$\int_0^\infty e^{-st}\ddot{x}(t)dt = s^2X(s) - sx(0) - \dot{x}(0), \tag{4}$$

$$\int_0^\infty e^{-st}\delta(\alpha t)dt = \frac{1}{\alpha}, \tag{5}$$

we obtain:

$$ms^2X(s) - msx(0) - m\dot{x}(0) = P/\alpha. \tag{6}$$

For a particle starting from the rest with $\dot{x}(0) = 0, x(0) = 0$, we get

$$X(s) = \frac{P}{ms^2\alpha}. \tag{7}$$

Using the inverse Laplace transform, we obtain

$$x(t) = \frac{P}{m\alpha}t \tag{8}$$

and

$$\dot{x}(t) = \frac{P}{m\alpha}. \tag{9}$$

Let us remark, that the Laplace transform of the delta-function in eq. (5) is absolutely rigorous if we use first the Laplace transform of $\delta(\alpha(t - \varepsilon))$ and then we take $\varepsilon = 0$.

In case of the harmonic oscillator with the damping force and under influence of the general force $F(t)$, the Newton law is as follows:

$$m\frac{d^2x(t)}{dt^2} + b\dot{x}(t) + kx(t) = F(t). \tag{10}$$

After application of the Laplace transform (3) and with regard to the same initial conditions as in the preceding situation, $\dot{x}(0) = 0, x(0) = 0$, we get the following algebraic equation:

$$ms^2X(s) + bsX(s) + kX(s) = F(s), \tag{11}$$

or,

$$X(s) = \frac{F(s)}{m\omega_1}\frac{\omega_1}{(s + b/2m)^2 + \omega_1^2} \tag{12}$$

with $\omega_1^2 = k/m - b^2/4m^2$.

Using inverse Laplace transform denoted by symbol $\mathcal{L}^{-1}$ applied to multiplication of functions $f_1(s)f_2(s)$,

$$\mathcal{L}^{-1}(f_1(s)f_2(s)) = \int_0^t d\tau F_1(t-\tau)F_2(\tau), \tag{13}$$

we obtain with $f_1(s) = F(s)/m\omega_1$, $f_2(s) = \omega_1/((s + b/2m)^2 + \omega_1^2)$, $F_1(t) = F(t)/m\omega_1, F_2(t) = \exp(-bt/2m)\sin\omega_1 t$.

$$x(t) = \frac{1}{m\omega_1}\int_0^t F(t-\tau)e^{-\frac{b}{2m}\tau}\sin(\omega_1\tau)d\tau. \tag{14}$$

For impulsive force $F(t) = P\delta(\alpha t)$, we have from the last formula

$$x(t) = \frac{(P/\alpha)}{m\omega_1}e^{-\frac{b}{2m}t}\sin\omega_1 t. \tag{15}$$

3 The pulse propagating in a string

In this section we will solve the motion of a string with the massive ends (the body with mass m is fixed to the every end of the string) on the assumption that the tension in the string is linear and the applied force is of the Dirac delta-function. First, we will derive the Euler wave equation from the Hook law of tension and then we will give the rigorous mathematical formulation of the problem. Linearity of the wave equation enables to solve this problem by the Laplace transform method. We follow monograph by Tikhonov et al. (1977) and the author preprint (Pardy, 1996) where this method was used to solve the Gassendi model of gravity. Although Gassendi (Fraser et al., 1998) is known in physics as the founder of the modern atomic theory of matter, his string model of gravity was not accepted. The Newton reaction to this model was empirical. He said: "Hypotheses non fingo". It seems that Gassendi ideas was applied later by Faraday in his theory of electromagnetism.

Let us remark, that Gassendi solved the philosophical question: where is the force between body A and B, when the intermediate distance is L? The answer is, that it is hidden in the straight-line string connecting A and B. The tension is given by the Newton law and it is constant along the string. This is a consequence of the nonlinear Hook law. In case of the planetary situation, the motion of a planet around Sun is performed along an ellipse which is slightly undulated, or, wavy. The undulation is so small, that it cannot be observed by any the most modern laser technique.

The present problem can be also defined as a central collision of two bodies (balls). While in the basic mechanics the central collision is considered as a contact collision of the two balls, here, the collision is mediated by the string.

To our knowledge, the present problem is not involved in the textbooks of mathematical physics or in the mathematical journals. This problem was not possible to define and solve in the Newton period, because the method of solution is based on the Euler partial wave equation, the Laplace transform, The Riemann-Mellin transform, the Bromwich integral and Bromwich contour and other ingredients of the operator calculus which was elaborated after the Newton period.

Now, let us consider the string of the length L, the left end of which is joined with mass m and the right end is joined with mass m. The force of the delta-function form is applied to the left end and the initial state of the string is the state of equilibrium. The deflection of the string element dx at point x and time t let be $u(x,t)$ where $x \in (0, L)$.

The differential equation of motion of string elements can be derived by the following way (Tikhonov et al., 1977). We suppose that the force acting on the element dx of the string is given by the law:

$$T(x,t) = ES\left(\frac{\partial u}{\partial x}\right), \qquad (16)$$

where E is the modulus of elasticity, S is the cross section of the string. We easily derive that

$$T(x + dx) - T(x) = ESu_{xx}dx. \qquad (17)$$

The mass dm of the element dx is $\varrho ESdx$, where $\varrho = const$ is the mass density of the string matter and the dynamical equilibrium gives

$$\varrho Sdxu_{tt} = ESu_{xx}dx. \qquad (18)$$

So, we get

$$\frac{1}{c^2}u_{tt} - u_{xx} = 0; \quad c = \left(\frac{E}{\varrho}\right)^{1/2}. \qquad (19)$$

Now, we get the problem of the mathematical physics in the form of the differential wave equation:

$$u_{tt} = c^2 u_{xx} \qquad (20)$$

with the initial conditions

$$u(x,0) = 0; \quad u_t(x,0) = 0 \qquad (21)$$

and with the boundary conditions

$$mu_{tt}(0,t) = au_x(0,t) + P\delta(\alpha t); \quad mu_{tt}(L,t) = au_x(L,t), \tag{22}$$

where we have put

$$a = -ES; \quad P = \text{some constant}. \tag{23}$$

The delta-function can be approximatively realized by the strike of the hammer to the left end of the string.

The equation (20) with the initial and boundary conditions (21) and (22) represents one of the standard problems of the mathematical physics and can be easily solved using the Laplace transform (Arfken, 1967):

$$\hat{L}u(x,t) \stackrel{d}{=} \int_0^\infty e^{-pt}u(x,t)dt \stackrel{d}{=} \varphi(x,p). \tag{24}$$

Using (24) and (20) we get:

$$\hat{L}u_{tt}(x,t) = p^2\varphi(x,p) - pu(x,0) - u_t(x,0) = p^2\varphi(x,p), \tag{25}$$

$$\hat{L}u_{xx}(x,t) = \varphi_{xx}(x,p); \quad \hat{L}\delta = 1/\alpha; \quad \hat{L}u(0,t) = \varphi(0,p) = 0. \tag{26}$$

After elementary mathematical operations we get the differential equation for φ in the form

$$\varphi_{xx}(x,p) - k^2\varphi(x,p) = 0; \quad k = p/c. \tag{27}$$

with the boundary condition in eq. (22).

We are looking for the the solution of eq. (27) in the form

$$\varphi(x,p) = c_1 \cosh kx + c_2 \sinh kx. \tag{28}$$

We get from the boundary conditions in eq. (22)

$$c_1 = \frac{1}{p}\,\frac{ac(P/\alpha)\cosh(pL/c) - (P/\alpha)mpc^2\sinh(pL/c)}{\sinh(pL/c)(a^2 - m^2p^2c^2)}, \tag{29}$$

$$c_2 = -\frac{(P/\alpha)c}{ap} + \frac{(P/\alpha)mac^2\cosh(pL/c) - (P/\alpha)pm^2c^3\sinh(pL/c)}{a\sinh(pL/c)(a^2 - m^2c^2p^2)}. \tag{30}$$

The corresponding $\varphi(x,p)$ is of the form:

$$\varphi(x,p) = \frac{1}{p}\,\frac{ac(P/\alpha)\cosh(pL/c) - (P/\alpha)mpc^2\sinh(pL/c)}{\sinh(pL/c)(a^2 - m^2p^2c^2)}\cosh(px/c) \quad +$$

$$\left[-\frac{(P/\alpha)c}{ap} + \frac{a(P/\alpha)mc^2\cosh(pL/c) - bpm^2c^3\sinh(pL/c)}{a\sinh(pL/c)(a^2 - m^2c^2p^2)}\right]\sinh(px/c). \tag{31}$$

The corresponding function $u(x,t)$ follows from the theory of the Laplace transform as the mathematical formula (res is residuum)(Arfken, 1967):

$$u(x,t) = \frac{1}{2\pi i}\oint e^{pt}\varphi(x,p)dp = \sum_{p=p_n} \text{res } e^{pt}\varphi(x,p) =$$

$$\sum_{p=p_n} \text{res } e^{pt}\frac{1}{p}\frac{ac(P/\alpha)\cosh(pL/c)}{\sinh(pL/c)(a^2-m^2p^2c^2)}\cosh(px/c) \quad -$$

$$\sum_{p=p_n} \text{res } e^{pt}\frac{(P/\alpha)mc^2}{(a^2-m^2p^2c^2)}\cosh(px/c) \quad -$$

$$\sum_{p=p_n} \text{res } e^{pt}\left[\frac{(P/\alpha)c}{ap}\right]\sinh(px/c) \quad +$$

$$\sum_{p=p_n} \text{res } e^{pt}\left[\frac{m(P/\alpha)c^2\cosh(pL/c)}{\sinh(pL/c)(a^2-m^2p^2c^2)}\right]\sinh(px/c) \quad -$$

$$\sum_{p=p_n} \text{res } e^{pt}\left[\frac{(P/\alpha)pm^2c^3}{a}\frac{1}{(a^2-m^2c^2p^2)}\right]\sinh(px/c) \quad =$$

$$u_1 - u_2 - u_3 + u_4 - u_5, \tag{32}$$

where

$$u_j = \sum \text{res } e^{pt}\frac{A_j}{B_j}; \quad j = 1,2,3,4,5 \tag{33}$$

and

$$A_1 = ac(P/\alpha)\cosh(pL/c)\cosh(px/c); \quad B_1 = p\sinh(pL/c)(a^2-m^2p^2c^2) \tag{34}$$

$$A_2 = (P/\alpha)mc^2\cosh(px/c); \quad B_2 = (a^2-m^2p^2c^2) \tag{35}$$

$$A_3 = (P/\alpha)c\sinh(px/c); \quad B_3 = ap \tag{36}$$

$$A_4 = (P/\alpha)mc^2\cosh(pL/c)\sinh(px/c); \quad B_4 = \sinh(pL/c)(a^2-m^2p^2c^2) \tag{37}$$

$$A_5 = (P/\alpha)pm^2c^3\sinh(px/c); \quad B_5 = a(a^2-m^2p^2c^2). \tag{38}$$

We know from the theory of the complex functions that if the pole of some function $f(z)/g(z)$ is simple and it is at point a, then the residuum is as follows (Arfken, 1967):

$$\text{residuum} = \frac{f(a)}{g'(a)}. \tag{39}$$

If the pole at point a of the function $f(z)$ is multiple of the order m, then the residuum is defined as follows:

$$\text{residuum} = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left[(z-a)^m f(z)\right]. \tag{40}$$

Let us first determine the function

$$u_1 = \sum \text{res}\; e^{pt}\frac{A_1}{B_1}. \tag{41}$$

Poles of B_1 are at points $p = 0$, (this is a pole of the order 2), $p = +a/mc, p = -a/mc$ and $p_n = +i\pi nc/L, p_n = -i\pi nc/L, n = 1, 2, 3, \ldots$. So, the function u_1 is as follows:

$$u_1 = \frac{(P/\alpha)c^2}{La}t - \frac{(P/\alpha)c}{a}\cosh\left(\frac{aL}{mc^2}\right)\cosh\left(\frac{ax}{mc^2}\right)\sinh\left(\frac{at}{mc}\right) \quad +$$

$$\sum_{n=1}^{n=\infty}\frac{2a(P/\alpha)c}{\pi n}\frac{L^2}{a^2L^2+m^2\pi^2n^2c^4}\cos\left(\frac{\pi nx}{L}\right)\sin\left(\frac{\pi nct}{L}\right). \tag{42}$$

.

For the function u_2 we get:

$$u_2 = \left(-\frac{(P/\alpha)c}{a}\right)\sinh\left(\frac{at}{mc}\right)\cosh\left(\frac{ax}{mc^2}\right). \tag{43}$$

$$u_3 = 0. \tag{44}$$

For u_4 and u_5 we get:

$$u_4 = \left(-\frac{(P/\alpha)c}{a}\right)\coth\left(\frac{aL}{mc^2}\right)\sinh\left(\frac{ax}{mc^2}\right)\sinh\left(\frac{at}{mc}\right) \tag{45}$$

$$u_5 = \left(-\frac{(P/\alpha)c}{a}\right)\sinh\left(\frac{ax}{mc^2}\right)\sinh\left(\frac{at}{mc}\right) \tag{46}$$

The dimensionality of u is $[u] = \text{m}$ and $u(x, 0) = 0$. The momentum of a left particle $p = m\dot{u}(0, t)$, or right particle $p = m\dot{u}(L, t)$ is not conserved. Only the total momentum of a system is conserved.

The electronical verification of the propagation of a pulse in the string and by the computer elaboration is not problem of the electronic and computer laboratory. It can be performed using so called tensiometer applying to the rod. At the same time it is possible to realize the optical verification of our model using the infrared rays reflected by the element of a string, because the phonons forming the pulse can be absorbed by the infrared rays. So, we see that our mathematical model of a string can be considered as an integral part of electronics and of quantum solid state physics.

Now, let us give some general ideas following from the wave equation. It is well known that the solution of this equation is in general in the form (Landau et al., 2000):

$$f\left(t-\frac{x}{c}\right); \quad g\left(t+\frac{x}{c}\right), \tag{47}$$

where functions f, g are general. It means it involves also the function of the delta-form. For the wave propagating from the left side to the right side, we take function f. The corresponding tension in the string is

$$T = ESu_x(x,t) = ESf'\left(t-\frac{x}{c}\right)\left(\frac{-1}{c}\right). \tag{48}$$

We easily see that $T(x=0, t=0) = T(x=L, t=L/c)$, and it means that when the pulse force is created at the left end of the string then it propagates in the string and after time L/c it is localized in the right end of the string.

4 Discussion

Our article is the modification of the author article (Pardy, 2005), which is reformulation and elaboration of some problems involved in the textbooks on mathematical physics. However, our approach is pedagogically original in the sense that we use the initial force of a delta-function form to show the internal motion of the string. The delta-function form of electromagnetic pulse was used also by author (Pardy, 2002; 2003) to discuss the quantum motion of an electron in the laser pulse. We have considered here the real strings in the real space and we do not use extra-dimensions and unrealistic strings (Pardy, 2004).

The linear string model can be generalized to the nonlinear strings, strings with the internal structure and variable cross-section, the magnetic strings, the dielectric strings, or the strings can be considered as the linear chains composed from the massive elements. Then, we can define the Born-Kárman chain, Heisenberg chain, Bethe chain, Ising chain, Thirring chain, and many others quantum chains.

It seems, there is no information in textbooks on mechanics on the central collision of two particle where the force is mediated by string (Pars,1964).

The vibration energy of the string states are involved in formula (42). The analysis of such problem was performed for instance by Lambiase and Nesterenko (1996) and Nesterenko and Pirozhenko (1997), and others. We hope that our approach is the starting point of the new way of the string theory of mesons.

REFERENCES

Arfken, G. (1967). *Mathematical Methods for Physicists*, Academic Press, New York and London.

Barbashov B. M. and Nesterenko, V. V. (1990). *Introduction to the relativistic string theory*, Singapore, World Scientific.

Fraser, G. E., Lillestøl, E. and Sellevåg, I. (1998). *The Search for Infinity*, George Philip

Limited.

Lambiase, G. and Nesterenko, V. V. (1996). Quark mass correction to the string potential. *Phys. Rev. D* **54**, 6387.

Landau, L. D. and Lifschitz, E. M. (2000). *The Classical Theory of Fields*, Fourth revised English edition, Butterworth-Heinemann, Linacre house, Jordan Hill, Oxford OX2 8DP.

Nambu Y. (1974). Strings, Monopoles and gauge fields. *Physical Review D* **10**, 4262.

Nesterenko, V. V. (1990). On the radial motion of quarks bound by a string. *Z. Phys. C - Particles and Fields* **47**, 111.

Nesterenko, V. V. and Pirozhenko, I. G. (1997). Calculation of the interquark potential generated by a string with massive ends. *Phys. Rev. D* **55**, 6603.

Pardy, M. (1996). The string model of gravity. e-print gr-qc/9602007.

Pardy, M. (2002). Electron in the ultrashort laser pulse. e-print hep-ph/0207274.

Pardy, M. (2003). Electron in the ultrashort laser pulse. *Int. Journal of Theoretical Physics*, **41**, No. 1, 99.

Pardy, M. (2004). Light in metric space-time and its deflection by the screw dislocation. e-print hep-ph/0402249; ibid., Spacetime Physics Research Trends. Horizons in World Physics, Volume **248**, ISBN 1-59454-322-4 (2005).

Pardy, M. (2005). The propagation of a pulse in the real strings and rods, e-print math-ph/0503003.

Pars, L. A. (1964). *A Treatise on Analytical Mechanics*, Heineman, London.

Tikhonov, A. N. and Samarskii, A. A. (1977). *The Equations of Mathematical Physics.* Nauka, Moscow. (in Russsian).