

$GL(2)$ -structures of the Langlands global program

Christian PIERRE

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Abstract

All kinds of global correspondences of Langlands are evaluated from the functional representation spaces of the algebraic bilinear semigroups $GL_2(L_{\bar{v}} \times L_v)$ with entries in products, right by left, of sets of archimedean increasing completions.

Degenerate singularities on these functional representation spaces can give rise, by versal deformations and blowups of these, to one or two new covering functional representation spaces of $GL_2(\cdot \times \cdot)$ according to the type of considered singularities.

The discovered correspondences of Langlands are associated with singular and nonsingular universal $GL(2)$ -structures.

1 Introduction

The generalization of the concept of Dirichlet character led Langlands [Lan1] to formulate the (global) reciprocity conjecture [Lan2] which asserts that:

For any irreducible representation σ of $\text{Gal}(\overline{F}/F)$ in $\text{GL}_n(\mathcal{C})$, there exists a cuspidal (unramified) automorphic representation Π of $\text{GL}_n(\mathbb{A}_F)$ in such a way that the Artin L -function of σ agrees with the Langlands L -function Π at almost every place of the adèle ring of a number field F of characteristic 0 [Kna], [Shi].

Recent advances were realized in the so-called Langlands program essentially by M. Harris and R. Taylor [H-T] in the case of p -adic number fields and by L. Lafforgue [Laf] in the case of function fields.

In the frame of the Langlands global program developed by the author [Pie1], an interesting question consists in **evaluating all kinds of correspondences of Langlands from the functional representation spaces of $\text{GL}_2(F)$ being affected or not by (degenerate) singularities.**

The discovered correspondences are then associated with singular and nonsingular universal $\text{GL}(2)$ -structures.

The basic mathematical frame (considered in chapter 2 of this work) is essentially bilinear in such a way that the **Langlands global correspondences consist in bijections between the equivalence classes of Galois representations given by bilinear algebraic semigroups $\text{GL}_2(L_{\overline{v}} \times L_v)$ and the corresponding conjugacy classes of their cuspidal representations** where L_v (resp. $L_{\overline{v}}$) denotes the set of left (resp. right) increasing archimedean pseudo-ramified completions. These completions, resulting from the corresponding compactified Galois extensions, are characterized by increasing extension degrees which are integers modulo N .

Let $GL_2(L_{\overline{v}} \times L_v)$ be the bilinear algebraic semigroup with entries in the products, right by left, of symmetric completions. It decomposes according to:

$$\text{GL}_2(L_{\overline{v}} \times L_v) = T_2^t(L_{\overline{v}}) \times T_2(L_v)$$

where $T_2^t(L_{\overline{v}})$ (resp. $T_2(L_v)$) is a right (resp. left) linear semigroup of lower (resp. upper) triangular matrices referring to the lower (resp. upper) half space.

The representation space of the algebraic bilinear semigroup $\text{GL}_2(L_{\overline{v}} \times L_v)$ is given by the $\text{GL}_2(L_{\overline{v}} \times L_v)$ -bisemimodule $M_R(L_{\overline{v}}) \times M_L(L_v)$ and corresponds to the representation space $\sigma(W_{\tilde{L}_{\overline{v}}} \times W_{\tilde{L}_v})$ of the product, right by left, of global Weil groups $W_{\tilde{L}_{\overline{v}}}$ and $W_{\tilde{L}_v}$, respectively over the sets of extensions $\tilde{L}_{\overline{v}}$ and \tilde{L}_v .

The set of differentiable bifunctions $\{\phi_R(M_{\tilde{v}_{\mu, m_{\mu}}}) \otimes \phi_L(M_{v_{\mu, m_{\mu}}})\}_{\mu, m_{\mu}}$, $1 \leq \mu \leq \infty$, is the set of bisections of **the bisemisheaf of rings** $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$ **on the algebraic bilinear semigroup** $M_R(L_{\bar{v}}) \otimes M_L(L_v)$.

This leads to **the Langlands global correspondence**

$$\text{LGC} : \quad \sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v}) \longrightarrow \Pi(\text{GL}_2(L_{\bar{v}} \times L_v))$$

between the set $\sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v})$ of two-dimensional representative subspaces of the Weil global subgroups, given by the algebraic bilinear semigroups $(M_R(L_{\bar{v}}) \otimes M_L(L_v))$, and their cuspidal representation given by $\Pi(\text{GL}_2(L_{\bar{v}} \times L_v))$ and obtained from a toroidal compactification of the bisemisheaf $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$.

As **the bisemisheaf** $\phi_R(M_R(L_{\bar{v}})) \otimes_{L_{\bar{v}} \times L_v} \phi_L(M_L(L_v))$ is a vector bisemispace giving rise to an inner product bisemispace, it **can generate an orthogonal complement bisemisheaf** $\phi_R^{\perp}(M_R^{\perp}(L_{\bar{v}})) \otimes \phi_L^{\perp}(M_L^{\perp}(L_v))$ **by means of endomorphisms based on Galois antiautomorphisms**.

By this way, the following **Langlands general global correspondence can be stated**:

$$\text{LGGC}_{\text{ST}} : \quad \sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v}) \longrightarrow \Pi(\text{GL}_2^{(r)}(L_{\bar{v}} \times L_v)) + \Pi(\text{GL}_2^{\perp}(L_{\bar{v}} \times L_v))$$

where:

- $\Pi(\text{GL}_2^{(r)}(L_{\bar{v}} \times L_v))$ is the cuspidal representation on the reduced algebraic bilinear semigroup $\text{GL}_2^{(r)}(L_{\bar{v}} \times L_v)$;
- $\Pi(\text{GL}_2^{\perp}(L_{\bar{v}} \times L_v))$ is the cuspidal representation on the orthogonal complement algebraic bilinear semigroup $\text{GL}_2^{\perp}(L_{\bar{v}} \times L_v)$.

Consequently, the functional representation bisemispace [F-H] of $\sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v})$, given by **the direct sum of the reduced bisemisheaf** $\phi_R(M_R^{(r)}(L_{\bar{v}})) \otimes \phi_L(M_L^{(r)}(L_v))$ over $\text{GL}_2^{(r)}(L_{\bar{v}} \times L_v)$ (and written in condensed notation $(\widetilde{M}_{\text{ST}_R}^T \otimes \widetilde{M}_{\text{ST}_L}^T)$) **and of the orthogonal bisemisheaf** $\phi_R^{\perp}(M_R^{\perp}(L_{\bar{v}})) \otimes \phi_L^{\perp}(M_L^{\perp}(L_v))$ over $\text{GL}_2^{\perp}(L_{\bar{v}} \times L_v)$ (and written in condensed notation $(\widetilde{M}_{\text{ST}_R}^S \otimes \widetilde{M}_{\text{ST}_L}^S)$), **constitutes a nonsingular universal mathematical structure**. It is also a nonsingular universal physical structure corresponding to the space-time string fields of an elementary particle.

Degenerate singularities on the time or space shifted bisemisheaf $(\widetilde{M}_{\text{ST}_R}^{T_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p})$ or $(\widetilde{M}_{\text{ST}_R}^{S_p} \otimes \widetilde{M}_{\text{ST}_L}^{S_p})$ can give rise, by versal deformations [G-R] and blowups of these, **to one or two new covering (shifted) bisemisheaves** according to the kind of considered degenerate singularities: this constitutes the content of chapter 3.

More concretely, **according to the existence or the absence of degenerate singularities on the basic space-time bisemisheaf** $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$, where $\widetilde{M}_{\text{ST}_R}^{T_p-S_p} = \widetilde{M}_{\text{ST}_R}^{T_p} + \widetilde{M}_{\text{ST}_R}^{S_p}$, **three sets of Langlands general global correspondences (LGGC) can be built**:

- a) If there are no degenerate singularities on $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$, we get the **one-level LGGC_{ST} correspondences** mentioned above.
- b) If there are degenerate singularities of corank 1 and codimension inferior to 3 or degenerate singularities of corank 2 and codimension inferior or equal to 3 on $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$, we get, after a process of desingularizations [Hir], [Hau], and toroidal compactification of $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$, the **two-level LGGC_{ST-MG} correspondences**:

$$\begin{aligned} \text{LGGC}_{\text{ST-MG}} : \quad & \sigma(W_{\widetilde{L}_{\overline{v}}} \times W_{\widetilde{L}_v}) + \sigma(W_{\widetilde{L}_{\overline{v}S \circ T(1)}} \times W_{\widetilde{L}_{vS \circ T(1)}}) \\ & \longrightarrow \Pi(\text{GL}_2^{(r)}(L_{\overline{v}} \times L_v)) + \Pi(\text{GL}_2^\perp(L_{\overline{v}} \times L_v)) \\ & \quad + \Pi(\text{GL}_2^{(r)}(L_{\overline{v}_{\text{cov}(1)}} \times L_{v_{\text{cov}(1)}})) + \Pi(\text{GL}_2^\perp(L_{\overline{v}_{\text{cov}(1)}} \times L_{v_{\text{cov}(1)}})) \end{aligned}$$

where:

- $\sigma(W_{\widetilde{L}_{\overline{v}S \circ T(1)}} \times W_{\widetilde{L}_{vS \circ T(1)}})$ corresponds to the first level bisemisheaf $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$ covering the basic bisemisheaf $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$;
 - $\Pi(\text{GL}_2^{(r)}(L_{\overline{v}_{\text{cov}(1)}} \times L_{v_{\text{cov}(1)}})) + \Pi(\text{GL}_2^\perp(L_{\overline{v}_{\text{cov}(1)}} \times L_{v_{\text{cov}(1)}}))$ is the cuspidal representation of the first level covering algebraic bilinear semigroups over $L_{\overline{v}_{\text{cov}(1)}} \times L_{v_{\text{cov}(1)}} \approx L_{\overline{v}S \circ T(1)} \times L_{vS \circ T(1)}$.
- c) If there are degenerate singularities of corank 1 and codimension 3 or $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$, we get, after a process of desingularizations and toroidal compactifications of the singular bisemisheaves $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$ and $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$, the **three-level LGGC_{ST-MG-M} correspondences**:

$$\begin{aligned} & \sigma(W_{\widetilde{L}_{\overline{v}}} \times W_{\widetilde{L}_v}) + \sigma(W_{\widetilde{L}_{\overline{v}S \circ T(1)}} \times W_{\widetilde{L}_{vS \circ T(1)}}) + \sigma(W_{\widetilde{L}_{\overline{v}S \circ T(2)}} \times W_{\widetilde{L}_{vS \circ T(2)}}) \\ & \longrightarrow \Pi(\text{GL}_2^{(r)}(L_{\overline{v}} \times L_v)) + \Pi(\text{GL}_2^\perp(L_{\overline{v}} \times L_v)) \\ & \quad + \Pi(\text{GL}_2^{(r)}(L_{\overline{v}_{\text{cov}(1)}} \times L_{v_{\text{cov}(1)}})) + \Pi(\text{GL}_2^\perp(L_{\overline{v}_{\text{cov}(1)}} \times L_{v_{\text{cov}(1)}})) \\ & \quad + \Pi(\text{GL}_2^{(r)}(L_{\overline{v}_{\text{cov}(2)}} \times L_{v_{\text{cov}(2)}})) + \Pi(\text{GL}_2^\perp(L_{\overline{v}_{\text{cov}(2)}} \times L_{v_{\text{cov}(2)}})) \end{aligned}$$

where:

- $\sigma(W_{\widetilde{L}_{\overline{v}S \circ T(2)}} \times W_{\widetilde{L}_{vS \circ T(2)}})$ corresponds to the second level bisemisheaf $(\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{M}_L}^{T_p-S_p})$ covering the first level bisemisheaf $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$;
- $\Pi(\text{GL}_2^{(r)}(L_{\overline{v}_{\text{cov}(2)}} \times L_{v_{\text{cov}(2)}})) + \Pi(\text{GL}_2^\perp(L_{\overline{v}_{\text{cov}(2)}} \times L_{v_{\text{cov}(2)}}))$ is the cuspidal representation of the second level covering algebraic bilinear semigroups.

It then results that:

- 1) the one-level correspondences LGGC_{ST} are related to nonsingular universal mathematical and physical $\text{GL}(2)$ -structures given by the bisemisheaves $(\widetilde{M}_{\text{STR}}^{T_p-S_p} \otimes \widetilde{M}_{\text{STL}}^{T_p-S_p})$;
- 2) the two-level correspondences $\text{LGGC}_{\text{ST-MG}}$ are related to singular universal mathematical and physical GL_2 -structures given by the bisemisheaves $(\widetilde{M}_{\text{STR}}^{T_p-S_p} \oplus \widetilde{M}_{\text{MGR}}^{T_p-S_p}) \otimes (\widetilde{M}_{\text{STL}}^{T_p-S_p} \oplus \widetilde{M}_{\text{MGL}}^{T_p-S_p})$;
- 3) the three-level correspondences $\text{LGGC}_{\text{ST-MG-M}}$ are related to singular universal mathematical and physical GL_2 -structures given by the bisemisheaves $(\widetilde{M}_{\text{STR}}^{T_p-S_p} \oplus \widetilde{M}_{\text{MGR}}^{T_p-S_p} \oplus \widetilde{M}_{\text{ML}}^{T_p-S_p}) \otimes (\widetilde{M}_{\text{STL}}^{T_p-S_p} \oplus \widetilde{M}_{\text{MGL}}^{T_p-S_p} \oplus \widetilde{M}_{\text{ML}}^{T_p-S_p})$.

2 Nonsingular universal $GL(2)$ -structures

2.1 Archimedean places

Let k be a global number field of characteristic 0.

Let $\tilde{L} = \tilde{L}_R \cup \tilde{L}_L$ - denote a finite symmetric complex splitting field of k composed of right and left algebraic extension semifields \tilde{L}_R and \tilde{L}_L in one-to-one correspondence in such a way that \tilde{L}_L (resp. \tilde{L}_R) is the set of complex (resp. conjugate complex) simple roots.

Similarly, **let $\tilde{L}^+ = \tilde{L}_R^+ \cup \tilde{L}_L^+$ be a finite symmetric real splitting field of k** where \tilde{L}_L^+ (resp. \tilde{L}_R^+) is the left (resp. right) algebraic extension semifield composed of the set of positive (resp. symmetric negative) simple real roots.

Let $\tilde{L}_{\omega_1} \subset \dots \subset \tilde{L}_{\omega_\mu} \subset \dots \subset \tilde{L}_{\omega_t}$ (resp. $\tilde{L}_{\bar{\omega}_1} \subset \dots \subset \tilde{L}_{\bar{\omega}_\mu} \subset \dots \subset \tilde{L}_{\bar{\omega}_t}$) denote **the set of increasing complex subsemifields of \tilde{L}_L (resp. \tilde{L}_R)** and let $\tilde{L}_{v_1} \subset \dots \subset \tilde{L}_{v_\mu} \subset \dots \subset \tilde{L}_{v_t}$ (resp. $\tilde{L}_{\bar{v}_1} \subset \dots \subset \tilde{L}_{\bar{v}_\mu} \subset \dots \subset \tilde{L}_{\bar{v}_t}$) be the **set of increasing real subsemifields of \tilde{L}_L^+ (resp. \tilde{L}_R^+)**.

To each complex extension \tilde{L}_{ω_μ} (resp. $\tilde{L}_{\bar{\omega}_\mu}$) is associated the complex completion L_{ω_μ} (resp. $L_{\bar{\omega}_\mu}$) obtained by **an isomorphism of compactification** of \tilde{L}_{ω_μ} (resp. $\tilde{L}_{\bar{\omega}_\mu}$) onto a closed compact subset of the complex numbers \mathcal{C} (resp. their conjugate complex \mathcal{C}^*).

Similarly, a real completion L_{v_μ} (resp. $L_{\bar{v}_\mu}$) can be obtained from the real extension \tilde{L}_{v_μ} (resp. $\tilde{L}_{\bar{v}_\mu}$) by **an isomorphism of compactification** of \tilde{L}_{v_μ} (resp. $\tilde{L}_{\bar{v}_\mu}$) onto a closed compact subset of \mathbb{R}^+ (resp. \mathbb{R}^-).

Let $L_\omega = \{L_{\omega_\mu}\}_\mu$ (resp. $L_{\bar{\omega}} = \{L_{\bar{\omega}_\mu}\}_\mu$) denote the set of left (resp. right) complex completions (with multiplicity equal to 1) associated with \tilde{L}_L (resp. \tilde{L}_R) and let $L_v = \{L_{v_\mu, m_\mu}\}_{\mu, m_\mu}$ (resp. $L_{\bar{v}} = \{L_{\bar{v}_\mu, m_\mu}\}_{\mu, m_\mu}$) be the set of left (resp. right) real equivalent completions (with multiplicity m_μ) associated with \tilde{L}_L^+ (resp. \tilde{L}_R^+).

Each real left (resp. right) pseudoramified completion (as well as the equivalent extension) is characterized by a degree

$$[L_{v_\mu, m_\mu} : k] = * + \mu \cdot N \quad (\text{resp. } [L_{\bar{v}_\mu, m_\mu} : k] = * + \mu \cdot N)$$

which is an integer modulo N where $*$ denotes an integer inferior to N , and where N is responsible for the pseudoramification [Pie1].

Each complex left (resp. right) pseudoramified completion (as well as the equivalent extension) is characterized by a degree:

$$[L_{\omega_\mu} : k] = * + \mu \cdot N \cdot m^{(\mu)} \quad (\text{resp. } [L_{\bar{\omega}_\mu} : k] = * + \mu \cdot N \cdot m^{(\mu)})$$

where $m^{(\mu)}$ denotes the multiplicity of the μ -th equivalent real completions L_{v_μ, m_μ} covering the complex corresponding completion L_{ω_μ} according to [Pie1].

The set $\{L_{v_\mu, m_\mu}\}_{m_\mu}$ (resp. $\{L_{\bar{v}_\mu, m_\mu}\}_{m_\mu}$) of real left (resp. right) pseudoramified equivalent completions define the **infinite pseudoramified archimedean left (resp. right) real place**

v_μ (resp. \bar{v}_μ) and the set $\{L_{\omega_\mu, m_\mu}\}_{m_\mu}$ (resp. $\{L_{\bar{\omega}_\mu, m_\mu}\}_{m_\mu}$) of complex left (resp. right) pseudoramified equivalent completions define **the infinite pseudoramified archimedean left (resp. right) complex space ω_μ (resp. $\bar{\omega}_\mu$)**.

2.2 Algebraic bilinear semigroups

Let

$$W_{\tilde{L}_\omega} = \{\text{Gal}(\tilde{L}_{\omega_\mu}/k)\}_{\mu=1}^t \quad (\text{resp. } W_{\tilde{L}_{\bar{\omega}}} = \{\text{Gal}(\tilde{L}_{\bar{\omega}_\mu}/k)\}_{\mu=1}^t)$$

be **the global Weil group** defined as the Galois subgroup of the complex pseudoramified extensions \tilde{L}_{ω_μ} (resp. $\tilde{L}_{\bar{\omega}_\mu}$) characterized by extension degrees $d = 0 \pmod N$ and let $W_{\tilde{L}_v} = \{\text{Gal}(\tilde{L}_{v_\mu}/k)\}_{\mu=1}^t$ (resp. $W_{\tilde{L}_{\bar{v}}} = \{\text{Gal}(\tilde{L}_{\bar{v}_\mu}/k)\}_{\mu=1}^t$) be the corresponding global Weil group of the real pseudoramified extensions \tilde{L}_{v_μ} (resp. $\tilde{L}_{\bar{v}_\mu}$).

In the context of bisemistruures [Pie4], let $G^{(2)}(L_{\bar{v}} \times L_v)$ be the 2-dimensional real compactified representation space of $(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v})$ covering in the sense of [Pie5] its complex equivalent $G^{(1)}(L_{\bar{\omega}} \times L_\omega)$ being the complex compactified representation space of $(W_{\tilde{L}_{\bar{\omega}}} \times W_{\tilde{L}_\omega})$, $G^{(2)}(L_{\bar{v}} \times L_v)$ being identified with $G^{(1)}(L_{\bar{\omega}} \times L_\omega)$ [Del] or with $G^{(2)}(L_{\bar{\omega}} \times L_\omega)$ (particular “restricted” case considered in [Pie5]).

So, **the algebraic representation of the algebraic bilinear semigroup of matrices** $\text{GL}_2(L_{\bar{v}} \times L_v)$ in the $\text{GL}_2(L_{\bar{v}} \times L_v)$ -bisemimodule $G^{(2)}(L_{\bar{v}} \times L_v)$, also noted $M_R(L_{\bar{v}}) \otimes M_L(L_v)$, results from the morphism of $\text{GL}_2(L_{\bar{v}} \times L_v)$ into the group of automorphisms $\text{GL}(M_R(L_{\bar{v}}) \otimes M_L(L_v))$ of $M_R(L_{\bar{v}}) \otimes M_L(L_v)$.

The algebraic bilinear semigroup $\text{GL}_2(L_{\bar{v}} \times L_v)$ over products of symmetric pairs of real completions **covers its classical linear equivalent** as proved in [Pie5] and has **the structure of a bisemigroup given by the triple $(G_L, G_R, G_{R \times L})$** [Pie4] where:

- a) G_L (resp. G_R) is a left (resp. right) semigroup under the addition of its left (resp. right) elements g_{L_i} ((resp. g_{R_i}) restricted to (or referring to) the upper (resp. lower) half space.
- b) $G_{R \times L}$ is a bilinear semigroup whose bielements $(g_{R_i} \times g_{L_i})$ are submitted to the cross binary operation $\underline{\times}$ according to:

$$\begin{aligned} G_{R \times L} \underline{\times} G_{R \times L} &\longrightarrow G_{R \times L} \\ (g_{R_i} \times g_{L_i}) \underline{\times} (g_{R_j} \times g_{L_j}) &\longrightarrow (g_{R_i} + g_{R_j}) \times (g_{L_i} + g_{L_j}) \end{aligned}$$

leading to the cross products $(g_{R_i} \times g_{L_j})$ and $(g_{R_j} \times g_{L_i})$.

The algebraic bilinear semigroup $\text{GL}_2(L_{\bar{v}} \times L_v)$ decomposes according to:

$$\text{GL}_2(L_{\bar{v}} \times L_v) = T_2^t(L_{\bar{v}}) \times T_2(L_v)$$

where $T_2^t(L_{\bar{v}})$ (resp. $T_2(L_v)$) is a right (resp. left) linear semigroup of lower (resp. upper) triangular matrices which entries in $L_{\bar{v}}$ (resp. L_v).

The algebraic representation space $M_R(L_{\bar{v}}) \times M_L(L_v)$ of $\mathrm{GL}_2(L_{\bar{v}} \times L_v)$ decomposes into a set of conjugacy class representatives on the real pseudoramified completions $L_{\bar{v}, m_\mu}$ and L_{v, m_μ} according to:

$$M_R(L_{\bar{v}}) \otimes M_L(L_v) = \{M_{\bar{v}, m_\mu} \otimes M_{v, m_\mu}\}_{v, m_\mu}$$

where m_μ labels the multiplicity of the μ -th conjugacy class representative $M_{\bar{v}, m_\mu} \otimes M_{v, m_\mu}$. Let $\phi_R(M_{\bar{v}, m_\mu})$ (resp. $\phi_L(M_{v, m_\mu})$) be a complex-valued one-dimensional differentiable function over the (μ, m_μ) -th conjugacy class representative of $T_2^t(L_{\bar{v}})$ (resp. $T_2(L_v)$) and let $\phi_R(M_{\bar{v}, m_\mu}) \otimes \phi_L(M_{v, m_\mu})$ denote the corresponding bifunction on $M_{\bar{v}, m_\mu} \otimes M_{v, m_\mu}$.

Then, the set $\{\phi_R(M_{\bar{v}, m_\mu})\}_{\mu, m_\mu}$ (resp. $\{\phi_L(M_{v, m_\mu})\}_{\mu, m_\mu}$) of \mathcal{C} -valued differentiable functions, localized in the lower (resp. upper) half space is the set $\Gamma(\phi_R(M_R(L_{\bar{v}})))$ (resp. $\Gamma(\phi_L(M_L(L_v)))$) of right (resp. left) sections of the semisheaf of rings $\phi_R(M_R(L_{\bar{v}}))$ (resp. $\phi_L(M_L(L_v))$) [Mum], [Ser].

And, the set $\{\phi_R(M_{\bar{v}, m_\mu}) \otimes \phi_L(M_{v, m_\mu})\}_{\mu, m_\mu}$ of differentiable bifunctions constitutes the set $\Gamma(\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v)))$ of one-dimensional bisections of the bisemisheaf of rings $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$ over $\mathrm{GL}_2(L_{\bar{v}} \times L_v)$.

Similarly, the set $\{\phi_R(M_{\bar{\omega}, \mu}) \otimes \phi_L(M_{\omega, \mu})\}_\mu$ of two-dimensional differentiable bifunctions constitutes the set $\Gamma(\phi_R(M_R(L_{\bar{\omega}})) \otimes \phi_L(M_L(L_\omega)))$ of complex bisections of the bisemisheaf of rings $\phi_R(M_R(L_{\bar{\omega}})) \otimes \phi_L(M_L(L_\omega))$ over $\mathrm{GL}_1(L_{\bar{\omega}} \times L_\omega)$.

2.3 Proposition (Langlands global correspondence: two-dimensional real case)

Let

$$\begin{aligned} \sigma_{\mu, m_\mu}(W_{\tilde{L}_{\bar{v}, m_\mu}} \times W_{\tilde{L}_{v, m_\mu}}) &= G^{(2)}(L_{\bar{v}, m_\mu} \times L_{v, m_\mu}) \\ &= M_{\bar{v}, m_\mu} \otimes M_{v, m_\mu} \end{aligned}$$

be the **2-dimensional representation subspace** $(M_{\bar{v}, m_\mu} \otimes M_{v, m_\mu})$ of the product, right by left, of the global Weil subgroups $W_{\tilde{L}_{\bar{v}, m_\mu}} \times W_{\tilde{L}_{v, m_\mu}}$ restricted to the μ -th real extensions.

Let

$$\Pi_{\mu, m_\mu}(\mathrm{GL}_2(L_{\bar{v}, m_\mu} \times L_{v, m_\mu})) = \Pi_{\mu, m_\mu}^\vee(T_2^t(L_{\bar{v}, m_\mu})) \times \Pi_{\mu, m_\mu}(T_2(L_{v, m_\mu}))$$

be the **cuspidal representation of the algebraic bilinear subsemigroup** $\mathrm{GL}_2(L_{\bar{v}, m_\mu} \times L_{v, m_\mu})$ in such a way that $\Pi_{\mu, m_\mu}^\vee(T_2^t(L_{\bar{v}, m_\mu}))$ be the contragredient cuspidal subrepresentation of $T_2^t(L_{\bar{v}, m_\mu})$ with respect to $T_2(L_{v, m_\mu})$.

Then, there exists the bijective morphism:

$$\mathrm{LGC}_{\mu, m_\mu} : \sigma_{\mu, m_\mu}(W_{\tilde{L}_{\bar{v}, m_\mu}} \times W_{\tilde{L}_{v, m_\mu}}) \longrightarrow \Pi_{\mu, m_\mu}(\mathrm{GL}_2(L_{\bar{v}, m_\mu} \times L_{v, m_\mu}))$$

between the (μ, m_μ) -th representative subspace of the product, right by left, of Weil global subgroups given by the $\mathrm{GL}_2(L_{\bar{v}, m_\mu} \times L_{v, m_\mu})$ -subbimodule $(M_{\bar{v}, m_\mu} \otimes M_{v, m_\mu})$ and the corresponding cuspidal class representative $\Pi_{\mu, m_\mu}(\mathrm{GL}_2(L_{\bar{v}, m_\mu} \times L_{v, m_\mu}))$.

This leads to the **Langlands global correspondence**:

$$\text{LGC} : \quad \sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v}) \longrightarrow \Pi(\text{GL}_2(L_{\bar{v}} \times L_v))$$

between the set $\sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v})$ of 2-dimensional representative subspaces of the Weil global subgroups given by the algebraic bilinear semigroup $(M_R(L_{\bar{v}}) \otimes M_L(L_v)) = G^{(2)}(L_{\bar{v}} \times L_v)$ and its cuspidal representation given by $\Pi(\text{GL}_2(L_{\bar{v}} \times L_v))$ [Pie6].

Proof. Let

$$\text{TC}_{L_{\mu, m_\mu}} : \quad \tilde{L}_{v_{\mu, m_\mu}} \longrightarrow L_{v_{\mu, m_\mu}}^T \quad (\text{resp.} \quad \text{TC}_{R_{\mu, m_\mu}} : \quad \tilde{L}_{\bar{v}_{\mu, m_\mu}} \longrightarrow L_{\bar{v}_{\mu, m_\mu}}^T)$$

be the **toroidal compactification of the extension $\tilde{L}_{v_{\mu, m_\mu}}$ (resp. $\tilde{L}_{\bar{v}_{\mu, m_\mu}}$) into a semi-circle $\tilde{L}_{v_{\mu, m_\mu}}^T$ (resp. $\tilde{L}_{\bar{v}_{\mu, m_\mu}}^T$)** localized in the upper (resp. lower) half space.

Then, by this mapping $\text{TC}_{L_{\mu, m_\mu}}$ (resp. $\text{TC}_{R_{\mu, m_\mu}}$), the complex-valued differentiable function $\phi_L(M_{v_{\mu, m_\mu}})$ (resp. $\phi_R(M_{\bar{v}_{\mu, m_\mu}})$) over $T_2(L_{v_{\mu, m_\mu}})$ (resp. $T_2^t(L_{\bar{v}_{\mu, m_\mu}})$) is transformed into:

$$\begin{aligned} \text{TC}_{L_{\mu, m_\mu}} : \quad & \phi_L(M_{v_{\mu, m_\mu}}) \longrightarrow \phi_L(M_{v_{\mu, m_\mu}^T}) \\ (\text{resp.} \quad \text{TC}_{R_{\mu, m_\mu}} : \quad & \phi_R(M_{v_{\mu, m_\mu}}) \longrightarrow \phi_R(M_{\bar{v}_{\mu, m_\mu}^T}) \end{aligned}$$

in such a way that:

$$\Pi_{\mu, m_\mu}(T_2(L_{v_{\mu, m_\mu}})) = \phi_L(M_{v_{\mu, m_\mu}^T}) \quad (\text{resp.} \quad \Pi_{\mu, m_\mu}^\vee(T_2^t(L_{\bar{v}_{\mu, m_\mu}})) = \phi_R(M_{\bar{v}_{\mu, m_\mu}^T}))$$

implying that the differentiable function $\phi_L(M_{v_{\mu, m_\mu}^T})$ (resp. $\phi_R(M_{\bar{v}_{\mu, m_\mu}^T})$) over the (μ, m_μ) -th toroidal compactified conjugacy class representative $T_2(L_{v_{\mu, m_\mu}}^T)$ (resp. $T_2^t(L_{\bar{v}_{\mu, m_\mu}}^T)$) of the algebraic linear semigroup $T_2(L_v^T)$ (resp. $T_2^t(L_{\bar{v}}^T)$) is the cuspidal representation $\Pi_{\mu, m_\mu}(T_2(L_{v_{\mu, m_\mu}}))$ (resp. contragradient cuspidal representation $\Pi_{\mu, m_\mu}^\vee(T_2^t(L_{\bar{v}_{\mu, m_\mu}}))$) of $T_2(L_{v_{\mu, m_\mu}})$ (resp. $T_2^t(L_{\bar{v}_{\mu, m_\mu}})$), where $L_v^T = \{L_{v_{\mu, m_\mu}}^T\}_{\mu, m_\mu}$.

Indeed, by summing over all μ and m_μ , we get the global elliptic semimodule [Pie5]:

$$\begin{aligned} \phi_L(M_L(L_v^T)) &= \sum_{\mu, m_\mu} \phi_L(M_{v_{\mu, m_\mu}^T}) \\ &= \sum_{\mu, m_\mu} r(\mu, m_\mu) e^{\pi i \mu x_L}, \quad x_L \in L_v, \\ (\text{resp.} \quad \phi_R(M_R(L_{\bar{v}}^T)) &= \sum_{\mu, m_\mu} \phi_R(M_{\bar{v}_{\mu, m_\mu}^T}) \\ &= \sum_{\mu, m_\mu} r(\mu, m_\mu) e^{-\pi i \mu x_L}), \end{aligned}$$

where $r(\mu, m_\mu) e^{\pi i \mu x_L}$ (resp. $r(\mu, m_\mu) e^{-\pi i \mu x_L}$) is a semicircle localized in the upper (resp. lower) half plane.

According to [Pie5], the **global elliptic semimodule $\phi_L(M_L(L_v^T))$ (resp. $\phi_R(M_R(L_{\bar{v}}^T))$) over $T_2(L_v^T)$ (resp. $T_2^t(L_{\bar{v}}^T)$) covers (and is thus isomorphic to) the corresponding cuspidal form over $T_1(L_\omega)$ (resp. $T_1^t(L_{\bar{\omega}})$)**, where $GL_1(L_{\bar{\omega}} \times L_\omega) = T_1^t(L_{\bar{\omega}}) \times T_1(L_\omega)$.

Thus, $\phi_L(M_{v^T, m_\mu})$ (resp. $\phi_R(M_{\bar{v}^T, m_\mu})$) is a cuspidal representation restricted to the (μ, m_μ) -th conjugacy class representative of $T_2(L_v)$ (resp. $T_2^t(L_{\bar{v}})$).

So, by considering the toroidal compactification of all (μ, m_μ) extensions or completions, we get the Langlands bilinear global correspondence

$$\text{LGC} : \quad \sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v}) \longrightarrow \Pi(\text{GL}_2(L_{\bar{v}} \times L_v))$$

equivalent to the isomorphism:

$$\text{LGC} : \quad \phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v)) \longrightarrow \Pi(\text{GL}_2(L_{\bar{v}} \times L_v))$$

where $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$, which is a bisemisheaf of rings over the algebraic bilinear semigroup $\text{GL}_2(L_{\bar{v}} \times L_v)$, constitutes the two dimensional functional representation space $\sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v})$ of the product, right by left, of global Weil groups. \blacksquare

2.4 Proposition

The bisemisheaf of rings $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$ is a physical bosonic quantum string field of an elementary particle.

Proof. Let $L_{\bar{v}}^T = \{L_{\bar{v}\mu, m_\mu}^T\}_{\mu, m_\mu}$ (resp. $L_v^T = \{L_{v\mu, m_\mu}^T\}_{\mu, m_\mu}$) be the set of right (resp. left) real pseudoramified **toroidal completions** obtained from $L_{\bar{v}}$ (resp. L_v) by a toroidal compactification of the corresponding completions $L_{\bar{v}\mu, m_\mu}$ (resp. $L_{v\mu, m_\mu}$) [Pie3].

The set $\{L_{\bar{v}\mu, m_\mu}^T\}_{m_\mu}$ (resp. $\{L_{v\mu, m_\mu}^T\}_{m_\mu}$) of μ -th completions are then semicircles covering a 2-dimensional right (resp. left) semitorus $T_R^2(\mu)$ (resp. $T_L^2(\mu)$), localized in the lower (resp. upper) half space.

Assume that the degree of $L_{\bar{v}\mu, m_\mu}^T$ and of $L_{v\mu, m_\mu}^T$ is given by

$$[L_{\bar{v}\mu, m_\mu}^T] \equiv [L_{v\mu, m_\mu}^T] = \mu N .$$

Then, $L_{\bar{v}\mu, m_\mu}^T$ and $L_{v\mu, m_\mu}^T$ are toroidal completions or semicircles at μ quanta, a quantum being an irreducible completion of degree N .

As we are essentially interested in circles, toroidal completions $L_{\bar{v}2\mu, m_{2\mu}}^T$ and $L_{v2\mu, m_{2\mu}}^T$ characterized by degrees:

$$[L_{\bar{v}2\mu, m_{2\mu}}^T] \equiv [L_{v2\mu, m_{2\mu}}^T] = 2 \mu N$$

will be taken into account.

By this way, the corresponding completions $L_{\bar{v}2\mu, m_{2\mu}}$ and $L_{v2\mu, m_{2\mu}}$ are closed paths or closed strings.

Now, each product $\{L_{\bar{v}2\mu, m_{2\mu}}^T \otimes L_{v2\mu, m_{2\mu}}^T\}$ of symmetric circles rotating in opposite senses according to [Pie2] is the representation of an harmonic oscillator. This is also the case for the product

$\{L_{\bar{v}2\mu, m_{2\mu}} \otimes L_{v2\mu, m_{2\mu}}\}$ of corresponding completions homeomorphic to $\{L_{\bar{v}2\mu, m_{2\mu}}^T \otimes L_{v2\mu, m_{2\mu}}^T\}$ and for the product $\{\phi_R(M_{\bar{v}2\mu, m_{2\mu}}) \otimes \phi_L(M_{v2\mu, m_{2\mu}})\}$ of \mathcal{C} -valued differentiable functions.

Consequently, the set of packets $\{\phi_R(M_{\bar{v}2\mu, m_{2\mu}}) \otimes \phi_L(M_{v2\mu, m_{2\mu}})\}_{2\mu, m_{2\mu}}$ of bifunctions on $\{L_{\bar{v}2\mu, m_{2\mu}} \otimes L_{v2\mu, m_{2\mu}}\}_{2\mu, m_{2\mu}}$ behaves like a set of packets of harmonic oscillators characterized by increasing integers 2μ .

Thus, the bisemisheaf of rings

$$\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v)) = \{\phi_R(M_{\bar{v}2\mu, m_{2\mu}}) \otimes \phi_L(M_{v2\mu, m_{2\mu}})\}_{2\mu, m_{2\mu}}$$

is a physical string field.

It is a quantum string field because the set of sections of the bisemisheaf $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$ is a tower of increasing bistrings, i.e. products of symmetric right and left strings, behaving like harmonic oscillators and characterized by a number of increasing biquanta, $2\mu \leq \infty$, corresponding to the normal modes of the string field.

This quantum string field is a bosonic field because biquanta can be added to (i.e. created) or removed (i.e. annihilated) from these bistrings by Galois automorphisms or antiautomorphisms as developed in [Pie1] and because each bistring with degree 2μ was interpreted as a ‘‘bound bisemiphoton’’ at 2μ biquanta. ■

2.5 Properties of the bisemisheaf $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$

The representation semispace $\text{Repsp}(T_2(L_v))$ (resp. $\text{Repsp}(T_2^t(L_{\bar{v}}))$) of the linear algebraic semigroup $T_2(L_v)$ (resp. $T_2^t(L_{\bar{v}})$) is the unitary left (resp. right) L_v -semimodule $M_L(L_v)$ (resp. $L_{\bar{v}}$ -semimodule $M_R(L_{\bar{v}})$), i.e. a left vector L_v -semispace (resp. a right vector $L_{\bar{v}}$ -semispace).

And the left (resp. right) semisheaf $\phi_L(M_L(L_v))$ (resp. $\phi_R(M_R(L_{\bar{v}}))$) is also a left (resp. right) vector L_v -semispace (resp. $L_{\bar{v}}$ -semispace) implying that the bisemisheaf $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$ is a vector $L_{\bar{v}} \times L_v$ -bisemispace as developed in [Pie4].

This vector $L_{\bar{v}} \times L_v$ -bisemispace $\phi_R(M_R(L_{\bar{v}})) \otimes_{L_{\bar{v}} \times L_v} \phi_L(M_L(L_v))$ splits naturally into:

$$\begin{aligned} \phi_R(M_R(L_{\bar{v}})) \otimes_{L_{\bar{v}} \times L_v} \phi_L(M_L(L_v)) \\ = (\phi_R(M_R(L_{\bar{v}})) \otimes_D \phi_L(M_L(L_v))) \oplus (\phi_R(M_R(L_{\bar{v}})) \otimes_{OD} \phi_L(M_L(L_v))) \end{aligned}$$

where:

- $\phi_R(M_R(L_{\bar{v}})) \otimes_D \phi_L(M_L(L_v))$ is a diagonal vector $L_{\bar{v}} \times L_v$ -bisemispace characterized by a bilinear diagonal basis $\{e_\alpha \otimes f_\alpha\}_{\alpha=1}^2$ of dimension 2;

- $(\phi_R(M_R(L_{\bar{v}})) \otimes_{OD} \phi_L(M_L(L_v)))$ is an off-diagonal vector $L_{\bar{v}} \times L_v$ -bisespace of dimension 2 characterized by a bilinear off-diagonal basis $\{e_\alpha \otimes f_\beta\}_{\alpha \neq \beta=1}^2$.

As it was seen in [Pie4], the vector $L_{\bar{v}} \times L_v$ -bisespace $\phi_R(M_R(L_{\bar{v}})) \otimes_D \phi_L(M_L(L_v))$, endowed with a suitable inner product at the condition that the right (resp. left) vector $L_{\bar{v}}$ -semispace $\phi_R(M_R(L_{\bar{v}}))$ (resp. $\phi_L(M_L(L_v))$) be projected onto its symmetric left (resp. right) equivalent $\phi_L(M_L(L_v))$ (resp. $\phi_R(M_R(L_{\bar{v}}))$), can give rise to an inner product bisemisphere.

Consequently, **this inner product bisemisphere can generate an orthogonal complement bisemisphere** as it is developed in the next section.

2.6 Endomorphisms E_L and E_R based on Galois antiautomorphisms

The representation semispace $M_L(L_v) = \text{Repsp}(T_2(L_v))$ (resp. $M_R(L_{\bar{v}}) = \text{Repsp}(T_2^t(L_{\bar{v}}))$) of the linear left (resp. right) algebraic semigroup $T_2(L_v)$ (resp. $T_2^t(L_{\bar{v}})$) is noetherian or solvable in the sense that it is composed of the set

$$\begin{aligned} M_L(L_{v_1}) &\subset \cdots \subset M_L(L_{v_\mu}) \subset \cdots \subset M_L(L_{v_t}) \\ (\text{resp. } M_R(L_{\bar{v}_1}) &\subset \cdots \subset M_R(L_{\bar{v}_\mu}) \subset \cdots \subset M_R(L_{\bar{v}_t})) \end{aligned}$$

of embedded increasing representation subsemispaces.

So, we can define the smooth endomorphism:

$$\begin{aligned} E_L : \quad M_L(L_v) &\xrightarrow{\sim} M_L^{(r)}(L_v) \oplus M_L^{(l)}(L_v) \\ (\text{resp. } E_R : \quad M_R(L_{\bar{v}}) &\xrightarrow{\sim} M_R^{(r)}(L_{\bar{v}}) \oplus M_R^{(l)}(L_{\bar{v}})) \end{aligned}$$

decomposing $M_L(L_v)$ (resp. $M_R(L_{\bar{v}})$) into the direct sum of the reduced representation semispace $M_L^{(r)}(L_v)$ (resp. $M_R^{(r)}(L_{\bar{v}})$), **submitted to Galois antiautomorphisms**, and of the complementary representation semispace $M_L^{(l)}(L_v)$ (resp. $M_R^{(l)}(L_{\bar{v}})$), submitted to Galois automorphisms.

Similarly, the semisheaf $\phi_L(M_L(L_v))$ (resp. $\phi_R(M_R(L_{\bar{v}}))$) can be submitted to the same endomorphism E_L (resp. E_R) transforming it into:

$$\begin{aligned} E_L(\phi_L(M_L(L_v))) &= \phi_L(M_L^{(r)}(L_v)) \oplus \phi_L(M_L^{(l)}(L_v)) \\ (\text{resp. } E_R(\phi_R(M_R(L_{\bar{v}}))) &= \phi_R(M_R^{(r)}(L_{\bar{v}})) \oplus \phi_R(M_R^{(l)}(L_{\bar{v}})) \end{aligned}$$

2.7 Proposition (Generation of an orthogonal complement (semi)sheaf)

The semisheaf $\phi_L(M_L(L_v))$ (resp. $\phi_R(M_R(L_{\bar{v}}))$) on the representation semispace $M_L(L_v) = \text{Repsp}(T_2(L_v))$ (resp. $M_R(L_{\bar{v}}) = \text{Repsp}(T_2^t(L_{\bar{v}}))$) of the algebraic semigroup $T_2(L_v)$ (resp.

$T_2^t(L_{\bar{v}})$ can generate **the orthogonal complement semisheaf** $\phi_L^\perp(\text{Repsp } T_2(L_v))$ (resp. $\phi_R^\perp(\text{Repsp } T_2^t(L_{\bar{v}}))$) **under the composition of morphisms**

$$\begin{aligned} \gamma_{t \rightarrow r} \circ E_L : \quad & \phi_L(M_L(L_v)) \longrightarrow \phi_L(M_L^{(r)}(L_v)) \oplus \phi_L^\perp(\text{Repsp } T_2(L_v)) \\ (\text{resp. } \gamma_{t \rightarrow r} \circ E_R : \quad & \phi_R(M_R(L_{\bar{v}})) \longrightarrow \phi_R(M_R^{(r)}(L_{\bar{v}})) \oplus \phi_R^\perp(\text{Repsp } T_2^t(L_{\bar{v}})) \end{aligned}$$

where $\gamma_{t \rightarrow r}$ is the emergent morphism [Pie8] mapping the complementary semisheaf $\phi_L^{(I)}(M_L(L_v))$ (resp. $\phi_R^{(I)}(M_R(L_{\bar{v}}))$) throughout the origin into the orthogonal complement semisheaf $\phi_L^\perp(\text{Repsp } T_2(L_v))$ (resp. $\phi_R^\perp(\text{Repsp } T_2^t(L_{\bar{v}}))$).

Proof. In fact, the endomorphism E_L (resp. E_R), acting by means of Galois antiautomorphisms, generates the reduced semisheaf $\phi_L(M_L^{(r)}(L_v))$ (resp. $\phi_R(M_R^{(r)}(L_{\bar{v}}))$) and the complementary disconnected semisheaf $\phi_L(M_L^{(I)}(L_v))$ (resp. $\phi_R(M_R^{(I)}(L_{\bar{v}}))$) as developed in [Pie8].

As $(\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v)))$ can give rise to an inner product bisemispaces [Pie4] due to the symmetry between the right and left semisheaves $\phi_R(M_R(L_{\bar{v}}))$ and $\phi_L(M_L(L_v))$, the disconnected complementary semisheaf $\phi_L^{(I)}(M_L(L_v))$ (resp. $\phi_R^{(I)}(M_R(L_{\bar{v}}))$) can be projected by the emergent morphism $\gamma_{t \rightarrow r}$ into an orthogonal complement (semi)space throughout the origin, generating then the orthogonal complement semisheaf $\phi_L^\perp(\text{Repsp}(T_2(L_v)))$ (resp. $\phi_R^\perp(\text{Repsp}(T_2^t(L_{\bar{v}})))$). ■

2.8 Proposition (Langlands general global correspondence)

Taking into account the existence of an orthogonal complement bisemispaces, the following Langlands general global correspondence can be stated:

$$\text{LGGC}_{\text{ST}} : \quad \sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v}) \xrightarrow{\sim} \Pi(\text{GL}_2^{(r)}(L_{\bar{v}} \times L_v)) + \Pi(\text{GL}_2^\perp(L_{\bar{v}} \times L_v))$$

where:

- $\Pi(\text{GL}_2^{(r)}(L_{\bar{v}} \times L_v))$ is the cuspidal representation on the reduced algebraic bilinear semigroup $\text{GL}_2^{(r)}(L_{\bar{v}} \times L_v)$;
- $\Pi(\text{GL}_2^\perp(L_{\bar{v}} \times L_v))$ is the cuspidal representation on the orthogonal complement algebraic bilinear semigroup $\text{GL}_2^\perp(L_{\bar{v}} \times L_v)$.

Proof. Under the toroidal compactification **TC** (see proposition 2.3), the semisheaf $\phi_L(M_L(L_v))$ (resp. $\phi_R(M_R(L_{\bar{v}}))$) is transformed into the cuspidal representation $\Pi(T_2(L_v))$ (resp. $\Pi(T_2^t(L_{\bar{v}}))$) of the algebraic semigroup $T_2(L_v)$ (resp. $T_2^t(L_{\bar{v}})$) and the bisemisheaf $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$ is transformed by:

$$\text{TC}_R \times \text{TC}_L : \quad \phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v)) \longrightarrow \Pi(\text{GL}_2(L_{\bar{v}} \times L_v))$$

into the cuspidal representation $\Pi(\mathrm{GL}_2(L_{\bar{v}} \times L_v))$ of the algebraic bilinear semigroup $\mathrm{GL}_2(L_{\bar{v}} \times L_v)$. So, we get the commutative diagram:

$$\begin{array}{ccc}
\mathrm{LGC} : & \sigma(W_{\tilde{L}_{\bar{v}}}) \times \sigma(W_{\tilde{L}_v}) & \xrightarrow[\mathrm{TC}_R \times \mathrm{TC}_L]{\sim} & \Pi(\mathrm{GL}_2(L_{\bar{v}} \times L_v)) \\
\downarrow & \downarrow \begin{array}{c} (\gamma_{t \rightarrow r} \times \gamma_{t \rightarrow r}) \\ \circ (E_R \times E_L) \end{array} & & \downarrow \begin{array}{c} (\gamma_{t \rightarrow r} \times \gamma_{t \rightarrow r}) \\ \circ (E_R \times E_L) \end{array} \\
\mathrm{LGGC}_{\mathrm{ST}} : & \begin{array}{c} \sigma^{(r)}(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v}) \\ \oplus \sigma^\perp(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v}) \end{array} & \xrightarrow{\sim} & \begin{array}{c} \Pi(\mathrm{GL}_2^{(r)}(L_{\bar{v}} \times L_v)) \\ \oplus \Pi(\mathrm{GL}_2^\perp(L_{\bar{v}} \times L_v)) \end{array}
\end{array}$$

leading to the Langlands general global correspondence $\mathrm{LGGC}_{\mathrm{ST}}$ where $\sigma^\perp(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v}) = \phi_R^\perp(M_R^\perp(L_{\bar{v}})) \oplus \phi_L^\perp(M_L^\perp(L_v))$ is the orthogonal complement representation bisemispaces of the product, right by left, of global Weil groups. \blacksquare

2.9 Proposition (Nonsingular universal $\mathrm{GL}(2)$ -structures)

- 1) *The functional representation bisemispaces $\sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v})$ of the product, right by left, of global Weil groups, given by*

$$\sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v}) = (\phi_R(M_R^{(r)}(L_{\bar{v}})) \otimes (\phi_L(M_L^{(r)}(L_v)) \oplus (\phi_R^\perp(M_R^\perp(L_{\bar{v}})) \otimes (\phi_L^\perp(M_L^\perp(L_v)))$$

the direct sum of the reduced bisemisheaf over the reduced algebraic bilinear semigroup $\mathrm{GL}_2^{(r)}(L_{\bar{v}} \times L_v)$ and of the orthogonal complement bisemisheaf over $\mathrm{GL}_2^\perp(L_{\bar{v}} \times L_v)$, is a nonsingular universal mathematical structure.

- 2) *It is also a nonsingular universal physical structure because it corresponds to the space-time string fields of the dark energy structure of an elementary particle.*

Proof. 1) The reduced bisemisheaf $\phi_R(M_R^{(r)}(L_{\bar{v}})) \otimes (\phi_L(M_L^{(r)}(L_v))$ and the orthogonal complement bisemisheaf $\phi_R^\perp(M_R^\perp(L_{\bar{v}})) \otimes (\phi_L^\perp(M_L^\perp(L_v))$ constitute a universal mathematical structure because:

- a) they are generated from the product, right by left, of global Weil groups $(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v})$;
- b) they are in one-to-one correspondence with the holomorphic and cuspidal representations of $\mathrm{GL}_2^{(r)}(L_{\bar{v}} \times L_v) + \mathrm{GL}_2^\perp(L_{\bar{v}} \times L_v)$ by means of the Langlands general global correspondence $\mathrm{LGGC}_{\mathrm{ST}}$.

- 2) The reduced bisemisheaf $\phi_R(M_R^{(r)}(L_{\bar{v}})) \otimes (\phi_L(M_L^{(r)}(L_v))$, as well as the bisemisheaf $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$, has one-dimensional bisections according to section 2.2. It is thus a time string field of an elementary particle. On the other hand, the ‘‘diagonal’’ orthogonal complement bisemisheaf $\phi_R^\perp(M_R^\perp(L_{\bar{v}})) \otimes_D (\phi_L^\perp(M_L^\perp(L_v))$, generated from the

reduced bisemisheaf by the $(\gamma_{t \rightarrow r} \times \gamma_{t \rightarrow r}) \circ (E_R \times E_L)$ morphisms, may be two- or three-dimensional according to [Pie8].

Consequently, this orthogonal complement bisemisheaf corresponds to the space string field of an elementary particle generated from the reduced time string field.

Let us abbreviate our notations:

- the time reduced bisemisheaf will be rewritten

$$\widetilde{M}_{\text{ST}_R}^T \otimes \widetilde{M}_{\text{ST}_L}^T, \quad \text{“}T\text{” for time, “}ST\text{” for space-time;}$$

- the space orthogonal complement bisemisheaf will be rewritten

$$\widetilde{M}_{\text{ST}_R}^S \otimes \widetilde{M}_{\text{ST}_L}^S, \quad \text{“}S\text{” for space.}$$

This space-time bisemisheaf

$$(\widetilde{M}_{\text{ST}_R}^T \otimes \widetilde{M}_{\text{ST}_L}^T) \oplus (\widetilde{M}_{\text{ST}_R}^S \otimes \widetilde{M}_{\text{ST}_L}^S)$$

constitutes the space-time string fields of the dark energy structure of an elementary fermion (lepton, quark or neutrino).

Indeed, the Zel’dovich’s idea of connecting the vacuum energy density of quantum field theories (QFT) with the cosmological constant Λ of general relativity (GR) led the author to propose a unification of these two theories by a new interpretation of the equations of GR being then in one-to-one correspondence with the equations of the internal dynamics of the vacuum and mass structures of a set of interacting elementary particles. In this context, the cosmological constant would deal with the internal vacuum substructure “ST” of the elementary particles.

The dark energy structure of an elementary particle then refers to the space-time fields of its internal vacuum structure “ST”.

As these fields are in fact bisemifields, i.e. products of right semifields by their symmetric left correspondents, an elementary particle at this dark energy level is a bisemiparticle composed of the product of a left semiparticle, localized in the upper half space, by its symmetric right (co)semiparticle, localized in the lower half space. ■

2.10 Shifted bisemisheaves

According to proposition 2.7, the “space” bisemisheaf $\widetilde{M}_{\text{ST}_R}^S \otimes \widetilde{M}_{\text{ST}_L}^S$ (and the “time” bisemisheaf $\widetilde{M}_{\text{ST}_R}^T \otimes \widetilde{M}_{\text{ST}_L}^T$) constitutes the functional representation space $\text{FRepsp}(\text{GL}_2(L_{\bar{v}} \times L_v))$ of the algebraic bilinear semigroup $\text{GL}_2(L_{\bar{v}} \times L_v)$.

The dynamics of the bisemisheaf $\widetilde{M}_{\text{ST}_R}^S \otimes \widetilde{M}_{\text{ST}_L}^S$ is obtained by the action of the elliptic differential bioperator $(D_R \otimes D_L)$ mapping $\widetilde{M}_{\text{ST}_R}^S \otimes \widetilde{M}_{\text{ST}_L}^S$ into its shifted equivalent according to:

$$D_R \otimes D_L : \quad \widetilde{M}_{\text{ST}_R}^S \otimes \widetilde{M}_{\text{ST}_L}^S \longrightarrow \widetilde{M}_{\text{ST}_R}^{S_p} \otimes \widetilde{M}_{\text{ST}_L}^{S_p}$$

where the bisemisheaf $\widetilde{M}_{ST_R}^{S_p} \otimes \widetilde{M}_{ST_L}^{S_p}$ is the functional representation space $\text{FRepsp}(\text{GL}_2((L_{\bar{v}} \otimes \mathbb{R})) \times (L_v \otimes \mathbb{R}))$ of the shifted bilinear algebraic semigroup $\text{GL}_2((L_{\bar{v}} \otimes \mathbb{R})) \times (L_v \otimes \mathbb{R})$ as it is developed in [Pie3].

Physically, $(\widetilde{M}_{ST_R}^{S_p} \otimes \widetilde{M}_{ST_L}^{S_p})$ is an operator valued space string field if it is referred to proposition 2.9. Similarly, the “time” bisemisheaf $\widetilde{M}_{ST_R}^T \otimes \widetilde{M}_{ST_L}^T$ is sent by the action of an elliptic differential bioperator into its shifted equivalent $(\widetilde{M}_{ST_R}^{T_p} \otimes \widetilde{M}_{ST_L}^{T_p})$ responsible for its dynamics.

3 Singular universal $GL(2)$ -structures

In this chapter, it will be shown that **degenerate singularities on the functional representation spaces $\mathbf{FRep}(\mathbf{GL}_2((L_{\bar{v}} \otimes \mathbb{R}) \times (L_v \otimes \mathbb{R})))$** , given by the space or time bisemisheaves $(\widetilde{M}_{ST_R}^{S_p} \otimes \widetilde{M}_{ST_L}^{S_p})$ or $(\widetilde{M}_{ST_R}^{T_p} \otimes \widetilde{M}_{ST_L}^{T_p})$, **can give rise, by versal deformations and blowups of these, to one or two new covering functional representation spaces of $\mathbf{GL}_2((L_{\bar{v}} \otimes \mathbb{R}) \times (L_v \otimes \mathbb{R})))$ according to the kind of considered degenerate singularities.**

3.1 One- and two-dimensional sections of the bisemisheaves

Let $(\widetilde{M}_{ST_R}^{S_p} \otimes \widetilde{M}_{ST_L}^{S_p})$ denote the space bisemisheaf introduced in section 2.10.

Let $\{\phi_R^{S_p}(M_{\bar{v}_{2\mu, m_{2\mu}}}) \otimes \phi_L^{S_p}(M_{v_{2\mu, m_{2\mu}}})\}_{\mu, m_{2\mu}}$ denote the set of bisections of $(\widetilde{M}_{ST_R}^{S_p} \otimes \widetilde{M}_{ST_L}^{S_p})$ as introduced in proposition 2.4 and noted there $\phi_R^{S_p}(M_R(L_{\bar{v}})) \otimes \phi_L^{S_p}(M_L(L_v))$.

Then, $\{\phi_L^{S_p}(M_{v_{2\mu, m_{2\mu}}})\}_{\mu, m_{2\mu}}$ (resp. $\{\phi_R^{S_p}(M_{\bar{v}_{2\mu, m_{2\mu}}})\}_{\mu, m_{2\mu}}$) will be the set $\Gamma(\phi_L^{S_p}(M_L(L_v)))$ (resp. $\Gamma(\phi_R^{S_p}(M_R(L_{\bar{v}})))$) of left (resp. right) one-dimensional sections of the left (resp. right) semisheaf $\phi_L^{S_p}(M_L(L_v))$ (resp. $\phi_R^{S_p}(M_R(L_{\bar{v}}))$) [Har].

Assume that the “ $m_{2\mu}$ ” one-dimensional sections $\{\phi_L^{S_p}(M_{v_{2\mu, m_{2\mu}}})\}_{m_{2\mu}}$ (resp. $\{\phi_R^{S_p}(M_{\bar{v}_{2\mu, m_{2\mu}}})\}_{m_{2\mu}}$) of each packet “ μ ” are glued together under a compactification map “ c ” in order to generate a surface $\phi_L^{S_p}(M_{v_{2\mu}(c)})$ (resp. $\phi_R^{S_p}(M_{\bar{v}_{2\mu}(c)})$) as described in [Pie1] and in [Pie7].

Then, $\{\phi_L^{S_p}(M_{v_{2\mu}(c)})\}_{\mu}$ (resp. $\{\phi_R^{S_p}(M_{\bar{v}_{2\mu}(c)})\}_{\mu}$) will denote the set $\Gamma(\phi_L^{S_p}(M_L(L_{v(c)})))$ (resp. $\Gamma(\phi_R^{S_p}(M_R(L_{\bar{v}(c)})))$) of left (resp. right) two-dimensional sections of the left (resp. right) semisheaf $\phi_L^{S_p}(M_L(L_{v(c)}))$ (resp. $\phi_R^{S_p}(M_R(L_{\bar{v}(c)}))$).

3.2 Degenerate singularities on the sections of the space bisemisheaves

Under external perturbations due to the strong fluctuations at this length scale, degenerate singularities are produced on the left and right sections of the above mentioned semisheaves. Furthermore, it is assumed that a same kind of degenerate singularities is generated on each left (resp. right) section which is a left (resp. right) differentiable function (resp. cofunction).

On one- or two-dimensional sections, **the simple germs $f(x) = x^{k+1}$, $1 \leq k \leq 3$, which are singular points of corank 1 and multiplicity “ $k - 1$ ”, can be produced in a 3-dimensional (semi)space by singularization morphisms** which are defined as sets of contracting surjective morphisms of normal crossing divisors as developed in [Pie2].

On a two-dimensional section, **the possible degenerate singular points are essentially the following germs of corank 2 and multiplicity inferior or equal to 3** [Tho], [Arn]:

$$f(x, y) = x^3 - 3y^2x, \quad f(x, y) = x^3 + y^3.$$

They are also produced by a set of contracting surjective morphisms of normal crossing divisors.

3.3 Versal deformations of degenerate singularities

Under the same kind of external perturbations, these degenerate singular germs are submitted to **versal deformations interpreted as extensions of the contracting surjective morphisms of singularizations** as proved in [Pie2].

The versal deformation or unfolding of a germ $f(x) = x^{k+1}$ of corank 1 and multiplicity “ $k - 1$ ” is given, in the frame of the Malgrange preparation theorem, by [Mal], [Mat]:

$$F(x, a(y, z)) = f(x) + \sum_{i=1}^{k-1} a_i(y, z) x^i .$$

$R(x, a_i(y, z)) = \sum_{i=1}^{k-1} a_i(y, z) x^i$ is the polynomial of the quotient algebra of the versal unfolding of the degenerate germ $f(x)$ with:

- $\{x^1, \dots, x^i, \dots, x^{k-1}\}$ being the basis of this quotient algebra of dimension $(k - 1)$, which is thus finitely generated;
- $a_i(y, z)$ being a function of two variables on which is mapped the respective generator x^i as developed in [Tho] and [Mal].

This function $a_i(y, z)$ belonging to a section of the space semisheaf is two-dimensional because the set of sections of the left (resp. right) semisheaf $\phi_L^{Sp}(M_L(L_v))$ (resp. $\phi_R^{Sp}(M_R(L_{\bar{v}}))$) is assumed to be compactified in a three-dimensional spatial semispace as developed in [Pie8].

More concretely, **the versal unfolding of the degenerate germs $f(x) = x^{k+1}$ in codimension inferior or equal to thee are** (to a translation):

1) **the fold:** $f(x) = x^3$

of which versal unfolding in codimension 1 is

$$F(x, a_1) = x^3 + a_1 x^1 ;$$

2) **the cusp:** $f(x) = x^4$

of which versal unfolding in codimension 2 is

$$F(x, a_1, a_2) = x^4 + a_1 x^1 + a_2 x^2 ;$$

3) **the swallowtail:** $f(x) = x^5$

of which versal unfolding in codimension 3 is

$$F(x, a_1, a_2, a_3) = x^5 + a_1 x^1 + a_2 x^2 + a_3 x^3 ;$$

where a_i is a contracted notation for $a_i(y, z)$, $1 \leq i \leq 3$.

The degenerate germs of corank 2 and multiplicity inferior of equal to 3 are [Tho] (to a translation):

- 1) **the elliptic umbilic:** $f(x, y) = x^3 - 3xy^2$
of which versal unfolding in codimension 3 is

$$F(x, y, b_1, b_2) = x^3 - 3xy^2 + b_1(x^2 + y^2) - b_2y ;$$

- 2) **the hyperbolic umbilic:** $f(x, y) = x^3 + y^3$
of which versal unfolding in codimension 3 is

$$F(x, y, b_2, b_3, b_4) = x^3 + y^3 - b_2y - b_3x + b_4xy$$

where b_2 and b_3 are two variable functions and b_1 and b_4 are one variable functions on a section of the space semisheaf.

More generally, let $\phi_L^{\star S_p}(M_L)$ (resp. $\phi_R^{\star S_p}(M_R)$) denote the left (resp. right) semisheaf of which sections:

- 1) are one- or two-dimensional;
- 2) are affected by the same kind of degenerate singularities of corank 1 or 2 as developed in sections 3.1 and 3.2.

Then, **the versal deformation** of the semisheaf $\phi_L^{\star S_p}(M_L)$ (resp. $\phi_R^{\star S_p}(M_R)$) of differentiable functions (resp. cofunctions) endowed with singular germs of corank 1 or 2 **is given by the contracting fiber bundle:**

$$\begin{aligned} D_{S_L} : \quad & \phi_L^{\star S_p}(M_L) \times \phi_{S_L} \longrightarrow \phi_L^{\star S_p}(M_L) \\ \text{(resp. } D_{S_R} : \quad & \phi_R^{\star S_p}(M_R) \times \phi_{S_R} \longrightarrow \phi_R^{\star S_p}(M_R) \end{aligned}$$

in such a way that the fiber $\phi_{S_L} = \{\dots, \phi_L(x^i), \dots\}_{i=1}^{k-1}$ (resp. $\phi_{S_R} = \{\dots, \phi_R(x^i), \dots\}_{i=1}^{k-1}$) is given by the set of $(k-1)$ semisheaves of the base S_L (resp. S_R) of the considered versal deformation where $\phi_L(x^i)$ (resp. $\phi_R(x^i)$) denotes the semisheaf of monomials “ x^i ” with respect to all the sections of $\phi_L^{\star S_p}(M_L)$ (resp. $\phi_R^{\star S_p}(M_R)$).

These semisheaves $\phi_L(x^i)$ (resp. $\phi_R(x^i)$) of monomials are projected on the respective coefficient semisheaves $\phi_L(a_i)$ (resp. $\phi_R(a_i)$) or $\phi_L(b_i)$ (resp. $\phi_R(b_i)$) of which sections are respectively the coefficient functions a_i or b_i .

As the semisheaf $\phi_L^{\star S_p}(M_L)$ (resp. $\phi_R^{\star S_p}(M_R)$) is defined on the algebraic semigroup M_L (resp. M_R) according to section 2.2, the semisheaves of monomials $\phi_L(x^i)$ (resp. $\phi_R(x^i)$) and the semisheaves of coefficients are also algebraic and characterized by a same set of increasing ranks, being algebraic dimensions defined from global residue degrees as developed in [Pie2].

3.4 Blowups of the versal deformations

A blowup of the versal deformation D_{S_L} (resp. D_{S_R}) can be envisaged: it consists in the extension of the quotient algebra of the versal deformation and corresponds to the inverse versal deformation $D_{S_L}^{-1}$ (resp. $D_{S_R}^{-1}$). It is based on the following smooth endomorphism:

$$\begin{aligned} E_{x^i}[\phi_L(x^i)] &= \phi_L(x^i)_r \oplus \phi_L(x^i)_I, & 1 \leq i \leq k-1, \\ \text{(resp. } E_{x^i}[\phi_R(x^i)] &= \phi_R(x^i)_r \oplus \phi_R(x^i)_I), \end{aligned}$$

based on Galois antiautomorphisms [Pie2], [Pie8], where:

- $\phi_L(x^i)_r$ (resp. $\phi_R(x^i)_r$) is the residual monomial semisheaf on the respective coefficient semisheaf;
- $\phi_L(x^i)_I$ (resp. $\phi_R(x^i)_I$) is the complementary monomial semisheaf disconnected from the respective coefficient semisheaf on which it was projected.

Let Π_{S_L} (resp. Π_{S_R}) denote the set

$$\{E_{x^i}[\phi_L(x^i)]\}_{i=1}^{k-1} \quad \text{(resp. } \{E_{x^i}[\phi_R(x^i)]\}_{i=1}^{k-1} \text{)}$$

of smooth endomorphisms disconnecting totally the monomial semisheaves $\phi_L(x^i)$ (resp. $\phi_R(x^i)$) from the respective coefficient semisheaves.

Let $T_{V_{x_L}} = \{\dots, T_{V_{x^i}}, \dots\}$ (resp. $T_{V_{x_R}} = \{\dots, T_{V_{x^i}}, \dots\}$) denote the set of tangent bundles obtained by projecting all the disconnected monomial base semisheaves $\phi_L(x^i)_I$ (resp. $\phi_R(x^i)_I$) in the vertical tangent space.

Then, **the extension of the quotient algebra of the versal deformation of the singular semisheaf** $\phi_L^{\star S_p}(M_L)$ (resp. $\phi_R^{\star S_p}(M_R)$), having an isolated degenerate singularity on each section, **is realized by the spreading out isomorphism**

$$(\text{SOT})_L = (T_{V_{x_L}} \circ \Pi_{S_L}) \quad \text{(resp. } (\text{SOT})_R = (T_{V_{x_R}} \circ \Pi_{S_R}) \text{)}.$$

as developed in [Pie2] and [Pie8].

Let $\phi_L(x^i)$ (resp. $\phi_R(x^i)$) and $\phi_L(x^j)$ (resp. $\phi_R(x^j)$) be two monomial base semisheaves of the base ϕ_{S_L} of the versal deformation.

Then, $\phi_L(Dx_L^{i-j})$ (resp. $\phi_R(Dx_R^{i-j})$) will denote the gluing up of these two monomial semisheaves on a connected domain Dx_L^{i-j} (resp. Dx_R^{i-j}).

Let $\phi_{\text{SOT}(1)_L}^{S_p}$ (resp. $\phi_{\text{SOT}(1)_R}^{S_p}$) be the set ϕ_{S_L} (resp. ϕ_{S_R}) of **the base monomial semisheaves totally disconnected** from $\phi_L^{\star S_p}$ (resp. $\phi_R^{\star S_p}$) **by the blowup** $D_{S_L}^{-1}$ (resp. $D_{S_R}^{-1}$) **of the versal deformation** in such a way that these monomial semisheaves:

- a) **are glued together section by section;**
- b) **cover partially the residue singular semisheaf $\phi_L^{\star S_p}$ (resp. $\phi_R^{\star S_p}$),** in the sense that each section of $\phi_L^{\star S_p}$ (resp. $\phi_R^{\star S_p}$) is totally or partially covered by the corresponding section of $\phi_{\text{SOT}(1)L}^{S_p}$ (resp. $\phi_{\text{SOT}(1)R}^{S_p}$) obtained by gluing up the base monomials of the versal deformation.

Remark that the **blowup $D_{S_L}^{-1}$ (resp. $D_{S_R}^{-1}$)** of the versal deformation **has been envisaged as being maximal**, i.e. when the base monomial semisheaves are totally disconnected from the singular semisheaf $\phi_L^{\star S_p}$ (resp. $\phi_R^{\star S_p}$). The intermediate cases, i.e. when the monomial semisheaves are partially disconnected from $\phi_L^{\star S_p}$ (resp. $\phi_R^{\star S_p}$), are studied in [Pie2] and in [Pie8].

3.5 Sequence of blowups of versal deformations

Referring to section 3.3, it appears that the blowup of the versal deformation of the swallowtail $F(x, a_1, a_2, a_3) = x^5 + a_1x^1 + a_2x^2 + a_3x^3$ generates especially the singular base monomial $f(x) = x^3$. Consequently, **the blowup of the versal deformation of the singular semisheaf $\phi_L^{\star S_p}$ (resp. $\phi_R^{\star S_p}$) of which sections are affected by degenerate singularities of type swallowtail generates monomial base semisheaves $\phi_{\text{SOT}(1)L}^{S_p}$ (resp. $\phi_{\text{SOT}(1)R}^{S_p}$) of which $\phi_L^{\star}(x^3)$ (resp. $\phi_R^{\star}(x^3)$), case $i = 3$, is again a singular semisheaf noted $\phi_L^{\star S_p}(x^3)$ (resp. $\phi_R^{\star S_p}(x^3)$).**

This singular semisheaf $\phi_L^{\star S_p}(x^3)$ (resp. $\phi_R^{\star S_p}(x^3)$) can then be submitted to a versal deformation and a blowup of it generating the monomial base semisheaf $\phi_{L(2)}(x)$ (resp. $\phi_{R(2)}(x)$) covering partially by patches the semisheaf $\phi_{\text{SOT}(1)L}^{S_p}$ (resp. $\phi_{\text{SOT}(1)R}^{S_p}$).

Thus, **in the case of a singular semisheaf $\phi_L^{\star S_p}(M_L)$ (resp. $\phi_R^{\star S_p}(M_R)$) of swallowtail type, the two semisheaves**

$$\begin{aligned} & \phi_{\text{SOT}(1)L}^{S_p} \quad (\text{resp. } \phi_{\text{SOT}(1)R}^{S_p}) \\ \text{and } & \phi_{\text{SOT}(2)L}^{S_p} \equiv \phi_{L(2)}(x) \quad (\text{resp. } \phi_{\text{SOT}(2)R}^{S_p} \equiv \phi_{R(2)}(x)), \end{aligned}$$

generated by versal deformation and blowup of this one, **cover partially the residual singular semisheaf $\phi_L^{\star S_p}(M_L)$ (resp. $\phi_R^{\star S_p}(M_R)$) leading to the embedding:**

$$\begin{aligned} & \phi_L^{\star S_p}(M_L) \subset \phi_{\text{SOT}(1)L}^{S_p} \subset \phi_{\text{SOT}(2)L}^{S_p} \\ (\text{resp. } & \phi_R^{\star S_p}(M_R) \subset \phi_{\text{SOT}(1)R}^{S_p} \subset \phi_{\text{SOT}(2)R}^{S_p}). \end{aligned}$$

Referring to section 3.1, the semisheaf $\phi_L^{\star S_p}(M_L)$ (resp. $\phi_R^{\star S_p}(M_R)$) is a space semifield. The covering semifields $\phi_{\text{SOT}(1)L}^{S_p}$ (resp. $\phi_{\text{SOT}(1)R}^{S_p}$) and $\phi_{\text{SOT}(2)L}^{S_p}$ (resp. $\phi_{\text{SOT}(2)R}^{S_p}$) are thus also of space nature.

The corresponding semifields of “time” type can be generated by a composition of morphisms $\gamma_{r \rightarrow t} \circ E_L$ (resp. $\gamma_{r \rightarrow t} \circ E_R$) as described in proposition 2.7:

$$\begin{aligned} \gamma_{r \rightarrow t} \circ E_L : \quad & \phi_L^{\star S_p}(M_L) \longrightarrow \phi_L^{T_p}(M_L) \oplus \phi_L^{\star S_p}(M_L) \\ (\text{resp. } \gamma_{r \rightarrow t} \circ E_R : \quad & \phi_R^{\star S_p}(M_R) \longrightarrow \phi_R^{T_p}(M_R) \oplus \phi_R^{\star S_p}(M_R)). \end{aligned}$$

Let us use the more condensed notation $\widetilde{M}_{ST_L}^{T_p-S_p}$ (resp. $\widetilde{M}_{ST_R}^{T_p-S_p}$) for $\phi_L^{T_p}(M_L) \oplus \phi_L^{\star S_p}(M_L)$ (resp. $\phi_R^{T_p}(M_R) \oplus \phi_R^{\star S_p}(M_R)$).

Similarly, time semisheaves can be generated by $(\gamma_{r \rightarrow t} \circ E_L)$ (resp. $(\gamma_{r \rightarrow t} \circ E_R)$) morphisms from the space semisheaves $\phi_{\text{SOT}(1)_L}^{S_p}$ (resp. $\phi_{\text{SOT}(1)_R}^{S_p}$) and $\phi_{\text{SOT}(2)_L}^{S_p}$ (resp. $\phi_{\text{SOT}(2)_R}^{S_p}$) leading to covering space-time semisheaves $\phi_{\text{SOT}(1)_L}^{T_p-S_p}$ (resp. $\phi_{\text{SOT}(1)_R}^{T_p-S_p}$) and $\phi_{\text{SOT}(2)_L}^{T_p-S_p}$ (resp. $\phi_{\text{SOT}(2)_R}^{T_p-S_p}$).

These two covering space-time semifields will be respectively labeled “MG” and “M”, as introduced in [Pie8], and will be noted according to:

$$\begin{aligned} \widetilde{M}_{\text{MGL}}^{T_p-S_p} &\equiv \phi_{\text{SOT}(1)_L}^{T_p-S_p} & (\text{resp. } \widetilde{M}_{\text{MGR}}^{T_p-S_p} &\equiv \phi_{\text{SOT}(1)_R}^{T_p-S_p}) \\ \text{and } \widetilde{M}_{\text{ML}}^{T_p-S_p} &\equiv \phi_{\text{SOT}(2)_L}^{T_p-S_p} & (\text{resp. } \widetilde{M}_{\text{MR}}^{T_p-S_p} &\equiv \phi_{\text{SOT}(2)_R}^{T_p-S_p}). \end{aligned}$$

Thus, **degenerate singularities of corank 1 and codimension 3 on the semisheaf $\widetilde{M}_{ST_L}^{S_p} \equiv \phi_L^{\star S_p}(M_L)$ (resp. $\widetilde{M}_{ST_R}^{S_p} \equiv \phi_R^{\star S_p}(M_R)$) are able to generate the three embedded semisheaves:**

$$\widetilde{M}_{ST_L}^{T_p-S_p} \subset \widetilde{M}_{\text{MGL}}^{T_p-S_p} \subset \widetilde{M}_{\text{ML}}^{T_p-S_p} \quad (\text{resp. } \widetilde{M}_{ST_R}^{T_p-S_p} \subset \widetilde{M}_{\text{MGR}}^{T_p-S_p} \subset \widetilde{M}_{\text{MR}}^{T_p-S_p}).$$

Remark that instead of considering degenerate singularities on a semifield of “space type” $\widetilde{M}_{ST_L}^{S_p} \equiv \phi_L^{\star S_p}(M_L)$, we could have envisaged them on the corresponding semifield of “time type” $\widetilde{M}_{ST_L}^{T_p} \equiv \phi_L^{T_p}(M_L)$ leading similarly to the same three embedded semisheaves of type $\text{ST} \subset \text{MG} \subset \text{M}$ as described explicitly in [Pie8].

3.6 Proposition (Langlands general global correspondences)

Let

$$\sigma(W_{\widetilde{L}_{\overline{v}}} \times W_{\widetilde{L}_v})_{\text{ST}} = \text{Repsp}((\text{GL}_2^{(r)}(L_{\overline{v}} \times L_v)) + (\text{GL}_2^\perp(L_{\overline{v}} \times L_v)))_{\text{ST}}$$

be the representation space of the product, right by left, of global Weil groups given by the time and space orthogonal bilinear algebraic semigroups “ST” $\text{GL}_2^{(r)}(L_{\overline{v}} \times L_v)_{\text{ST}}$ and $\text{GL}_2^\perp(L_{\overline{v}} \times L_v)_{\text{ST}}$.

Let

$$\begin{aligned} & \sigma(W_{\widetilde{L}_{\overline{\text{SOT}(1)}}} \times W_{\widetilde{L}_{v\text{SOT}(1)}})_{\text{MG}} \\ &= \text{Repsp}((\text{GL}_2^{(r)}(L_{\overline{\text{cov}(1)}} \times L_{v\text{cov}(1)}) + (\text{GL}_2^\perp(L_{\overline{\text{cov}(1)}} \times L_{v\text{cov}(1)})))_{\text{MG}} \\ (\text{resp. } & \sigma(W_{\widetilde{L}_{\overline{\text{SOT}(2)}}} \times W_{\widetilde{L}_{v\text{SOT}(2)}})_{\text{M}} \\ &= \text{Repsp}((\text{GL}_2^{(r)}(L_{\overline{\text{cov}(2)}} \times L_{v\text{cov}(2)}) + (\text{GL}_2^\perp(L_{\overline{\text{cov}(2)}} \times L_{v\text{cov}(2)})))_{\text{M}}) \end{aligned}$$

be the representation space of the product, right by left, of covering global Weil groups given by the covering time and space bilinear algebraic semigroups “MG ” (resp. “M ”), where $L_{\tilde{v}_{\text{cov}(1)}}$ (resp. $L_{\tilde{v}_{\text{cov}(2)}}$) is the set of completions covering L_v and corresponding to the set of extensions $\tilde{L}_{\tilde{v}_{\text{SOT}(1)}}$ (resp. $\tilde{L}_{\tilde{v}_{\text{SOT}(2)}}$) of which degrees are inferior or equal to that of L_v [Pie2].

Let $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$, $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$ and $(\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{M}_L}^{T_p-S_p})$ denote respectively the “ST”, “MG” and “M” bisemisheaves of differentiable bifunctions on the time and space bilinear algebraic semigroups “ST”, “MG” and “M” defined above.

After desingularization [Abh], [DeJ] of the bisemisheaves $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$ and $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$ and toroidal compactification of the bisemisheaves $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$, $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$ and $(\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{M}_L}^{T_p-S_p})$, we get the following general global correspondences of Langlands:

$$\begin{aligned} \text{LGGC}_{\text{ST}} : & \sigma(W_{\tilde{L}_{\tilde{v}}} \otimes W_{\tilde{L}_v}) \\ & \xrightarrow{\sim} \Pi(\text{GL}_2^{(r)}(L_{\tilde{v}} \times L_v)) + \Pi(\text{GL}_2^\perp(L_{\tilde{v}} \times L_v)) \\ \text{LGGC}_{\text{MG}} : & \sigma(W_{\tilde{L}_{\tilde{v}_{\text{SOT}(1)}}} \otimes W_{\tilde{L}_{v_{\text{SOT}(1)}}}) \\ & \longrightarrow \Pi(\text{GL}_2^{(r)}(L_{\tilde{v}_{\text{cov}(1)}} \times L_{v_{\text{cov}(1)}})) + \Pi(\text{GL}_2^\perp(L_{\tilde{v}_{\text{cov}(1)}} \times L_{v_{\text{cov}(1)}})) \\ \text{LGGC}_{\text{M}} : & \sigma(W_{\tilde{L}_{\tilde{v}_{\text{SOT}(2)}}} \otimes W_{\tilde{L}_{v_{\text{SOT}(2)}}}) \\ & \longrightarrow \Pi(\text{GL}_2^{(r)}(L_{\tilde{v}_{\text{cov}(2)}} \times L_{v_{\text{cov}(2)}})) + \Pi(\text{GL}_2^\perp(L_{\tilde{v}_{\text{cov}(2)}} \times L_{v_{\text{cov}(2)}})) \end{aligned}$$

where $\Pi(\text{GL}_2^{(r)}(\dots \times \dots))$ denotes the cuspidal representation of the considered bilinear algebraic semigroup.

Proof. We refer to the notations of proposition 2.8 where the Langlands general global correspondence LGGC_{ST} was already introduced.

And, more particularly, we have that:

- the representation space $\sigma(W_{\tilde{L}_{\tilde{v}}} \otimes W_{\tilde{L}_v})$ of the product, right by left, of global Weil groups decomposes into a reduced and an orthogonal part according to:

$$\begin{aligned} & \sigma(W_{\tilde{L}_{\tilde{v}}} \otimes W_{\tilde{L}_v}) \\ & \quad \left(\begin{array}{c} \text{SOT}(1) \\ \text{SOT}(2) \end{array} \right) \otimes \left(\begin{array}{c} \text{SOT}(1) \\ \text{SOT}(2) \end{array} \right) \\ & = \sigma^{(r)}(W_{\tilde{L}_{\tilde{v}}} \otimes W_{\tilde{L}_v}) \oplus \sigma^\perp(W_{\tilde{L}_{\tilde{v}}} \otimes W_{\tilde{L}_v}); \end{aligned}$$

- the bisemisheaves $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$, $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$ and $(\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{M}_L}^{T_p-S_p})$ are functional representation spaces of the corresponding bilinear algebraic semigroups.

As it results from section 3.5, the bisemisheaves $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$ and $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$ may be affected by residual singularities after the envisaged versal deformations and blowups

of these. Consequently, these bisemisheaves must be desingularized according to the classical procedure recalled in [Pie2] in order to envisage a cuspidal representation of these.

Furthermore, a toroidal compactification of the bisemisheaves $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$, $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$ and $(\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{M}_L}^{T_p-S_p})$ must be undertaken on these, as it was done in proposition 2.8, to reach the searched cuspidal representations. ■

3.7 Proposition (The three singular and nonsingular universal GL_2 -structures)

- 1) *The singular bisemisheaves $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$ and $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$ and the non-singular bisemisheaf $(\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{M}_L}^{T_p-S_p})$ are universal mathematical structures.*
- 2)
 - a) *The direct sum of the “ST and “MG ” bisemisheaves $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \oplus \widetilde{M}_{\text{MG}_R}^{T_p-S_p}) \otimes (\widetilde{M}_{\text{ST}_L}^{T_p-S_p} \oplus \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$, including their interactions, is a two-level universal physical structure corresponding to the space-time string fields of the dark matter structure of an elementary particle.*
 - b) *The direct sum of the “ST , “MG ” and “M ” bisemisheaves $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \oplus \widetilde{M}_{\text{MG}_R}^{T_p-S_p} \oplus \widetilde{M}_{\text{M}_R}^{T_p-S_p}) \otimes (\widetilde{M}_{\text{ST}_L}^{T_p-S_p} \oplus \widetilde{M}_{\text{MG}_L}^{T_p-S_p} \oplus \widetilde{M}_{\text{M}_L}^{T_p-S_p})$, including their interactions, is a three-level universal physical structure corresponding to the space-time string fields of the visible matter structure of an elementary particle.*

Proof. 1) Taking into account the desingularization of the “ST and “MG ” bisemisheaves $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$ and $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$, they are, as the nonsingular “M bisemisheaf $(\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{M}_L}^{T_p-S_p})$, universal mathematical structures as developed in proposition 2.9.

Indeed, these three (desingularized) bisemisheaves, generated from the products, right by left, of global Weil groups, are in one-to-one correspondence with the holomorphic and cuspidal representations of the associated GL_2 -bilinear algebraic semigroups by means of the Langlands general global correspondences LGGC_{ST} , LGGC_{MG} and LGGC_{M} of proposition 3.6.

In fact, the sums of the bisections of these bisemisheaves on the toroidal compactified conjugacy class representatives of the considered bilinear algebraic semigroups cover the products, right by left, of cusp forms as developed in [Pie5]: in this sense, these bisemisheaves are cuspidal representations.

- 2)
 - a) Let the “MG bisemisheaf $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$ be generated from the “ST bisemisheaf $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$ by versal deformations and blowups of degenerate singularities of corank 1 and codimension 1 and 2 or of degenerate singularities of corank 2 and codimension 3 on it.

Then,

$$\begin{aligned}
\text{DMS}_R(F_R) \otimes \text{DMS}_L(F_L) &= (\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \oplus \widetilde{M}_{\text{MG}_R}^{T_p-S_p}) \otimes (\widetilde{M}_{\text{ST}_L}^{T_p-S_p} \oplus \widetilde{M}_{\text{MG}_L}^{T_p-S_p}) \\
&= [(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p}) \oplus (\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})] \\
&\quad \oplus [(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p}) \oplus (\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})]_{\text{int}}
\end{aligned}$$

corresponds to the dark matter (string) fields of an elementary bisemifermion $(F_R \otimes F_L)$ where the bisemisheaves inside $[\cdot]_{\text{int}}$ are interaction semifields between mixed right and left “ST” and “MG” semisheaves.

The bisemisheaf $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$ corresponds to space-time “MG” internal fields of an elementary (bisemi)fermion for the same reasons for which the “ST” bisemisheaves $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$ are internal fields as developed in proposition 2.9.

This “MG” bisemisheaf is a “mass” structure field because it is of contracting nature limiting the expansion of the internal vacuum dark energy “ST” substructure $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$.

As this “MG” bisemisheaf is a mass field localized in the own vacuum of an elementary (bisemi)fermion, it must correspond to a dark mass substructure covering the dark energy substructure $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$ of an elementary bisemifermion.

Thus, the direct sum of the “ST” and “MG” bisemisheaves (or fields) corresponds to the dark matter structure of an elementary (bisemi)fermion.

- b) Let the “MG” and “M” bisemisheaves $(\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})$ and $(\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{M}_L}^{T_p-S_p})$ be generated from the “ST” bisemisheaf $(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p})$ by versal deformations and blowups of degenerate singularities of corank 1 and codimension 3 on it.

Then,

$$\begin{aligned}
\text{VMS}_R(F_R) \otimes \text{VMS}_L(F_L) &= (\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \oplus \widetilde{M}_{\text{MG}_R}^{T_p-S_p} \oplus \widetilde{M}_{\text{M}_R}^{T_p-S_p}) \otimes (\widetilde{M}_{\text{ST}_L}^{T_p-S_p} \oplus \widetilde{M}_{\text{MG}_L}^{T_p-S_p} \oplus \widetilde{M}_{\text{M}_L}^{T_p-S_p}) \\
&= (\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{ST}_L}^{T_p-S_p}) \oplus (\widetilde{M}_{\text{MG}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p}) \oplus (\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{M}_L}^{T_p-S_p}) \\
&\quad \oplus [(\widetilde{M}_{\text{ST}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p}) \oplus \dots \oplus (\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{MG}_L}^{T_p-S_p})]_{\text{int}}
\end{aligned}$$

corresponds to the visible matter stringfields of an elementary bisemifermion $(F_R \otimes F_L)$ where:

- $(\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{M}_L}^{T_p-S_p})$ denotes the visible mass covering substructure “M” of this bisemifermion;
- the bisemisheaves inside $[\cdot]_{\text{int}}$ are interaction fields between the three different right and left semifields “ST”, “MG” and “M”.

This visible mass substructure $(\widetilde{M}_{\text{M}_R}^{T_p-S_p} \otimes \widetilde{M}_{\text{M}_L}^{T_p-S_p})$ is a “mass” structure field because it is of contracting nature stabilizing the internal vacuum substructure, i.e. the dark matter structure $\text{DMS}_R(F_R) \otimes \text{DMS}_L(F_L)$.

This mass substructure “M” is visible because the frequency (of rotation) of its (bi)sections is inferior to that of the dark energy and dark mass substructures taking into account that the number of (bi)quanta on the (bi)sections of the bisemisheaf “M” is inferior to the number of (bi)quanta on the respective (bi)sections of the “MG” and “ST” semisheaves.

- c) The visible matter structure of elementary particles is composed of the three-embedded shells $ST \subset MG \subset M$ in such a way that the external shell “M” is generated from the middle ground shell “MG” on the basis of degenerate singularities of corank 1 and codimension 3 on the vacuum “ST” shell, when “vacuum” dark matter structure is composed of two embedded shell $ST \subset MG$ of which “MG” shell is generated from degenerate singularities of corank 1 or 2 and codimension < 3 on the “ST” shell. The generation of the visible mass shell “M” needs thus much more energy than the generation of the dark mass shell “MG”: this explains the preponderance of dark matter over ordinary visible matter in our universe. ■

3.8 Proposition (Sets of Langlands general global correspondences)

According to the existence or absence of degenerate singularities on the basic space-time bisemisheaf $(\widetilde{M}_{ST_R}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p})$, there are three sets of Langlands general global correspondences (LGGC).

- a) If there are no degenerate singularities on $(\widetilde{M}_{ST_R}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p})$, we get the one level LGGC_{ST} :

$$\text{LGGC}_{\text{ST}} : \quad \sigma(W_{\widetilde{L}_{\bar{v}}} \times W_{\widetilde{L}_v}) \longrightarrow \Pi(\text{GL}_2^{(r)}(L_{\bar{v}} \times L_v)) + \Pi(\text{GL}_2^\perp(L_{\bar{v}} \times L_v)) .$$

- b) If there are degenerate singularities of corank 1 and codimension inferior to 3 or degenerate singularities of corank 2 and codimension inferior or equal to 3 on $(\widetilde{M}_{ST_R}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p})$, we get the two-level LGGC_{ST-MG} :

$$\begin{aligned} \text{LGGC}_{\text{ST-MG}} : \quad & \sigma(W_{\widetilde{L}_{\bar{v}}} \times W_{\widetilde{L}_v}) + \sigma(W_{\widetilde{L}_{\bar{v}\text{SOT}(1)}} \times W_{\widetilde{L}_{v\text{SOT}(1)}}) \\ & \longrightarrow \Pi(\text{GL}_2^{(r)}(L_{\bar{v}} \times L_v)) + \Pi(\text{GL}_2^\perp(L_{\bar{v}} \times L_v)) \\ & \quad + \Pi(\text{GL}_2^{(r)}(L_{\bar{v}\text{cov}(1)} \times L_{v\text{cov}(1)})) + \Pi(\text{GL}_2^\perp(L_{\bar{v}\text{cov}(1)} \times L_{v\text{cov}(1)})) . \end{aligned}$$

- c) If there are degenerate singularities of corank 1 and codimension 3 on $(\widetilde{M}_{ST_R}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p})$,

we get the three level $\text{LGGC}_{\text{ST-MG-M}}$:

$$\begin{aligned}
\text{LGGC}_{\text{ST-MG-M}} : \quad & \sigma(W_{\tilde{L}_{\bar{v}}} \times W_{\tilde{L}_v}) + \sigma(W_{\tilde{L}_{\bar{v}_{\text{SOT}(1)}}} \times W_{\tilde{L}_{v_{\text{SOT}(1)}}}) + \sigma(W_{\tilde{L}_{\bar{v}_{\text{SOT}(2)}}} \times W_{\tilde{L}_{v_{\text{SOT}(2)}}}) \\
\longrightarrow & \Pi(\text{GL}_2^{(r)}(L_{\bar{v}} \times L_v)) + \Pi(\text{GL}_2^\perp(L_{\bar{v}} \times L_v)) \\
& + \Pi(\text{GL}_2^{(r)}(L_{\bar{v}_{\text{cov}(1)}} \times L_{v_{\text{cov}(1)}})) + \Pi(\text{GL}_2^\perp(L_{\bar{v}_{\text{cov}(1)}} \times L_{v_{\text{cov}(1)}})) \\
& + \Pi(\text{GL}_2^{(r)}(L_{\bar{v}_{\text{cov}(2)}} \times L_{v_{\text{cov}(2)}})) + \Pi(\text{GL}_2^\perp(L_{\bar{v}_{\text{cov}(2)}} \times L_{v_{\text{cov}(2)}})) .
\end{aligned}$$

Proof. These three sets of Langlands general global correspondences directly result from propositions 3.6 and 3.7 in such a way that the one-level correspondence LGGC_{ST} is related to a non-singular universal $\text{GL}(2)$ -structure while the two- and three-level correspondences $\text{LGGC}_{\text{ST-MG}}$ and $\text{LGGC}_{\text{ST-MG-M}}$ are related to singular universal GL_2 -structures. \blacksquare

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C. Pierre
 Université de Louvain
 Chemin du cyclotron, 2
 B-1348 Louvain-la-Neuve, Belgium
 pierre.math.be@gmail.com