The Energy Transformation Limit Theorem for Gas Flow Systems

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Abstract

The limit energy theorem which determines the possibility of transformation the energy flow in power systems in the absence of technical work is investigated and proved for such systems as gas lasers and plasmatrons, chemical gas reactors, vortex tubes, gas-acoustic and other systems, as well as a system of close stars. In the case of the same name ideal gas in the system the maximum ratio of energy conversion effectiveness is linked to the Carnot theorem, which in its turn is connected with the Nernst theorem. However, numerical analyses show that the class of flow energy systems is non-carnot one. The ratio of energy conversion effectiveness depends on the properties of the working medium; a conventional cycle in open-circuit is essentially irreversible. The proved theorem gives a more strongly worded II law of thermodynamics for the selected class of flow energy systems. Implications for astrophysical thermodynamic systems and the theory of a strong shock wave are discussed.

1 Introduction

There is a wide class of flow energy systems (FES) with a fast-flow gas where technical work is absent ($L_{tech} = 0$). These energy systems include: gas lasers and plasmatrons; chemical gas reactors, vortex tubes, ejectors, mixers, acoustic devices, and some astrophysical objects, such as systems of close stars.

The common property of these systems is that their energy efficiency is higher, the higher is the degree of transformation of kinetic energy of the flow into potential energy of pressure at the outlet of FES. However, there was no assessment of the maximum efficiency of energy transformation in the given class of energy systems in scientific literature so far. And as a rule, assessment of the effectiveness of these systems was carried out when compared with the Carnot cycle.

To determine the maximum efficiency of energy transformation in FES we will prove the following theorem.

2 The theorem

The efficiency ratio of energy transformation $\eta^{ideal}_{G>0}$ in an ideal flow energy system can not exceed the value

$$\Delta \overline{N}_{max} = \frac{\sum\limits_{i=1}^{m} \frac{\gamma_1 - 1}{\gamma_1} \mu_i \overline{R}_i \Theta_i \left(1 + \frac{\lambda_i^2}{\gamma_\mathrm{i} + 1}\right) + \overline{\dot{Q}}}{\sum\limits_{i=1}^{m} \mu_i \overline{C}_{p_i} \cdot \Theta_i + \overline{\dot{Q}}} - \frac{\gamma_1 - 1}{\gamma_1} \cdot \frac{\overline{R}_m}{\overline{C}_{p_m}}, \tag{2.1}$$

where $\mu_{i(j)} = \dfrac{G_{i(j)}}{G_1}$, $\Theta_i = \dfrac{T_i^*}{T_1^*}$, $\overline{C}_{P_m} = \dfrac{C_{P_m}}{C_1} = \dfrac{\sum\limits_{j=1}^{l} C_{P_j} \theta_j \mu_j}{\sum\limits_{j=1}^{l} \theta_j \mu_j}$, $\overline{C}_{v_i} = \dfrac{C_{v_i}}{C_1}$, $\overline{C}_{v_m} = \dfrac{\sum\limits_{j=1}^{l} C_{v_m} \theta_j \mu_j}{\sum\limits_{j=1}^{l} \theta_j \mu_j}$, $\overline{R}_m = \dfrac{R_m}{R_1} = \dfrac{\sum\limits_{j=1}^{l} R_j \theta_j \mu_j}{\sum\limits_{j=1}^{l} \theta_j \mu_j}$, $R_m = \dfrac{\sum\limits_{j=1}^{l} R_j G_j}{\sum\limits_{j=1}^{l} G_j}$, $\overline{\dot{Q}} = \dfrac{\dot{Q}}{C_{p_1} T_1^* G_1}$, $\gamma_1 = \dfrac{C_{p_1}}{C_{v_i}}$, $\overline{R}_i = \dfrac{R_i}{R_1}$, R_i, R_m– gas constant of i-inlet $(i = 1, 2, \ldots, m)$ and gas constant of the mixture respectively FES; C_{P_j}, T_j^* – heat capacity at constant pressure and total temperature of gas j- outlet respectively; T_i^*, C_{P_i} $(i = 1, 2, \ldots, m)$ – total temperature and heat capacity at constant gas pressure of i- inlet FES; C_{P_m}– heat capacity of gas at constant mixture pressure;G_i $(i = 1, 2, \ldots, m)$ – consumption of the medium through the i-inlet; G_j $(j = 1, 2, \ldots, k)$ – consumption through the j-outlet; $a_{crit(j)}$, $V_{i(j)}$, $\lambda_{i(j)} = \dfrac{V_{i(j)}}{a_{crit(j)}}$ – the critical speed and gas velocity at the i-inlet (j-outlet) in the working chamber of FES and speed coefficient respectively; $\dot{Q}$ – non-mechanical energy (power), carried to or out of FES.

3 Theorem proving

To determine the maximum evaluation of the effectiveness of energy conversion in FES, we consider gas as ideal and compressed. In addition, for heterogeneous gases entering the m-inlets of FES, we consider the process of mixing in the working chamber of FES completed (Fig. 1).

Total mechanical energy (power) of flow at the inlet to the FES and the energy (power) delivered from outside are

$$N_{ex} = \sum_{i=1}^{m} \left(\frac{P_i}{\rho_i} + \frac{\mathrm{v}_i^2}{2}\right) G_i + \dot{Q} = \sum_{i=1}^{m} G_i R_i T_i^* \left[\left(1 - \frac{\gamma_i - 1}{\gamma_i + 1}\lambda_i^2 + \frac{\gamma_i}{\gamma_i + 1}\lambda_i^2\right)\right] + \dot{Q}$$
$$= \sum_{i=1}^{m} G_i R_i T_i^* \left(1 + \frac{\lambda_i^2}{\gamma_i + 1}\right) + \dot{Q}, \tag{3.1}$$

where P_i, ρ_i, v_i – static pressure, density and velocity at the i-inlet to the heat engine, correspondingly. The total energy (power) delivered to FES is

$$N_\Sigma = \sum_{i=1}^{m} G_i i_i^* + \dot{Q} = \sum_{i=1}^{m} C_{P_i} T_i^* G_i + \dot{Q}. \tag{3.2}$$

The total energy (power) of gas flows at the outlet of FES diffuser is determined by

$$N_{out} = R_m \sum_{j=1}^{k} T_j^* G_j \left(1 + \frac{1}{\gamma_j + 1}\lambda_j^2\right). \tag{3.3}$$

For the stationary case the nondimensional equations of continuity and energy in the absence of technical work ($\mathrm{L}_{tech}= 0$) take the form

$$1 + \sum_{i=2}^{m} \mu_i = \sum_{j=1}^{\kappa} \mu_j;\ 1 + \sum_{i=2}^{m} \overline{C}_{P_i} \Theta_i \mu_i + \overline{Q} = \overline{C}_{P_m} \sum_{j=1}^{n} \Theta_m \mu_j. \tag{3.4}$$

We define the limit of the relative share of mechanical power flow utilized in FES taking into account (3.4) at $\lambda_j \to 0$ which corresponds to an infinite broadening of the diffuser $S \to \infty$ (Fig. 1).

$$\Delta\overline{N}_{max} = \lim_{\lambda_j \to 0} \Delta\overline{N}$$

$$= \frac{\sum_{i=1}^{m} \mu_i \overline{R}_i \Theta_i \left(1 + \frac{\lambda_i^2}{\gamma_i + 1}\right) \frac{\gamma_1 - 1}{\gamma_1} + \overline{\dot{Q}} - \frac{\overline{R}_m}{\overline{C}_{P_m}} \left[\sum_{i=1}^{m} \overline{C}_{P_i} \Theta_i \mu_i + \overline{\dot{Q}}\right] \frac{\gamma_1 - 1}{\gamma_1}}{\sum_{i=1}^{m} \overline{C}_{Pi} \Theta_i \mu_i + \overline{\dot{Q}}}$$

$$= \frac{\sum_{i=1}^{m} \mu_i \overline{R}_i \Theta_i \left(1 + \frac{\lambda_i^2}{\gamma_i + 1}\right) \frac{\gamma_1 - 1}{\gamma_1} + \overline{\dot{Q}}}{\sum_{i=1}^{m} \overline{C}_{P_i i} \Theta_i \mu_i + \overline{\dot{Q}}} - \frac{\overline{R}_m}{\overline{C}_{P_m}} \frac{\gamma_1 - 1}{\gamma_1}. \tag{3.5}$$

Since the rate of flow at the outlet of FES is nonzero, then the efficiency of energy transformation in ideal FES is less $\eta_{G>0}^{ideal} < \Delta\overline{N}_{max}$ Q.E.D which was to be proved.

4 Consequences

A global maximum of utilized mechanical power flow, as well as non-mechanical power carried out outside the working chamber in the FES equals

$$\Delta\overline{N}_{max}^{Global} = \lim_{\substack{\lambda_i \to 0 \\ \lambda_j \to \lambda_{max}}} \Delta\overline{N}_{max} = \frac{\sum_{i=1}^{m} \mu_i \overline{R}_i \Theta_i \frac{\gamma_i}{\gamma_i - 1} \cdot \frac{\gamma_1 - 1}{\gamma_1} + \overline{\dot{Q}}}{\sum_{i=1}^{m} \overline{C}_{P_i} \Theta_i \mu_i + \overline{\dot{Q}}} - \frac{\gamma_1 - 1}{\gamma_1} \cdot \frac{\overline{R}_m}{\overline{C}_{P_m}}. \tag{4.1}$$

In case of temperatures and the same name gases consumption being equal at m-inlets into the working chamber of FES equation (3.5) takes the form

$$\Delta\overline{N}_{max} = \frac{1}{\gamma}, \text{ where } \gamma = \frac{c_P}{c_v}. \tag{4.2}$$

Expression (4.1) for the same name gases with significant contribution of energy in FES (4.2) has the form $\overline{Q} \to \infty$ (4.2).

Ratio of efficiency of energy conversion in an ideal FES can not exceed the efficiency factor of ideal Carnot cycle.

From (2.1) for the same name gases at m-inlets at open-circuit $\overline{Q} \to 0$ we obtain:

$$\Delta\overline{N}_{max} = \frac{\frac{1}{\gamma}\eta^{ideal}_{Carnot} + \frac{1}{\gamma}\overline{\dot{Q}}}{1+\overline{\dot{Q}}}, \tag{4.3}$$

where

$$\eta_{Carnot} = 1 - \frac{T_1}{T_1^*} = \frac{\gamma-1}{\gamma+1}\lambda_1^2.$$

From (4.3) it follows that the ratio of energy conversion efficiency of FESη^{ideal}_{max} increases with gas velocity at the inlet. In addition, the formula (8) gives the relation of the three theorems: the proved theorem, and theorems of S. Carnot [1] and W. Nernst [2]. Ratio of energy conversion efficiency in a gas machine ($\eta^{ideal}_{G>0}$, $\dot{Q} = 0$) will be ã times less than gas dynamic efficiency of Carnot cycle ($\frac{1}{\gamma}\eta^{ideal}_{Carnot}$). At the same time even for ideal gas due to inaccessibility of absolute zero $T_2 > 0$ (Nernst theorem), Carnot efficiency is less than one, therefore, for the ratio of energy conversion efficiency in a flow machine we get

$$\eta^{ideal}_{G>0} = \frac{1}{\gamma}\eta^{ideal}_{Carnot} < \frac{1}{\gamma}. \tag{4.4}$$

5 Discussion of results

Theorem gives a more strongly worded II law of thermodynamics for the selected class of flow energy systems: only part of the energy of gas flow in an ideal FES can be converted into useful work (the effect). For the case of the same name gas at m-inlets of FES, this part will be $\frac{1}{\gamma}$ of the total energy of the gas flow.

Let us consider non mixing gases at the gas flow machine outlet. In this case, the estimate of the marginal efficiency of energy conversion in the gas flow machine is just the summation of solution (8) for m-inputs and n-outputs

$$\Delta\overline{N}_i = \frac{\frac{1}{\gamma_i}\eta_i + \frac{1}{\gamma_i}\cdot\overline{\dot{Q}}_i}{1+\overline{\dot{Q}}_i} \Rightarrow \Delta\overline{N}_i = \frac{\Delta N_i}{cC_{P_i}T_i^* G_i + \dot{Q}_i}, \tag{5.1}$$

hence

$$\Delta N_i = \Delta\overline{N}_i C_{P_i} T_i^* G_i (1 + \overline{\dot{Q}}_i).$$

The total value of utilized mechanical energy flow at the m-inputs is equal to

$$\Delta N_\Sigma = \sum_{i=1}^{m} C_{P_i} T_i^* \overline{\dot{Q}}_i (1 + \overline{\dot{Q}}_i) \Delta\overline{N}_i,$$

hence

$$\Delta\overline{N}_\Sigma = \frac{\Delta N_\Sigma}{\Delta N} = \frac{\sum\limits_{i=1}^{m} C_{P_i} T_i^* G_i (1 + \overline{\dot{Q}}_i) \Delta\overline{N}_i}{i_\Sigma^* + \dot{Q}_\Sigma}$$

$$= \frac{\sum\limits_{i=1}^{m} i_i^* (1 + \overline{\dot{Q}}_i) \Delta\overline{N}_i}{i_\Sigma^* (1 + \overline{\dot{Q}}_i) + \dot{Q}_i}. \tag{5.2}$$

In accordance with the average theorem we obtain

$$\eta_{G>0}^{ideal} = \Delta\overline{N}_\Sigma = \langle \Delta\overline{N}_i \rangle. \tag{5.3}$$

For open-circuit ($\overline{\dot{Q}} = O$) and non mixing gas flows we get

$$\eta_{G>0}^{ideal} = \langle \frac{1}{\gamma_i} \frac{\gamma_i - 1}{\gamma_i + 1} \lambda_i^2 \rangle = \langle \frac{1}{\gamma_i} \eta_i^{Carnot} \rangle \tag{5.4}$$

and for maximum input speed ($\lambda_i = \sqrt{\dfrac{\gamma_i + 1}{\gamma_i - 1}}$) we get

$$\eta_{G\rangle>0}^{ideal} = \langle \frac{1}{\gamma_i} \rangle. \tag{5.5}$$

As opposed to the consequences of Carnot theorem, the proved theorem (4.4) implies that the efficiency of energy conversion in an ideal FES depends on the properties of the working mass ($\gamma = C_p/C_v$). In this case the conditional cycle in FES is essentially irreversible (Fig. 2, 3).

The ideal conditional cycle in P-V coordinates presented in Fig. 2, for the case of open-circuit $\overline{Q} \to 0$, consists of one ideal isotherm (H-K), two ideal adiabats (K-1, 1'-H) and one percussive adiabat of Hugoniot (1-1').

It should be noted that in the supersonic flow regime ($\lambda_1 > 1$) there is always a shock wave, since the flow regime in the nozzle is off calculation. Especially clear the distinction of conditional cycle in FES from the Carnot cycle can be seen in T-S coordinates (Fig. 3). The difference between areas in Carnot cycle and PES (curvilinear triangle 1-1'-2-1) represents the energy required for pumping the environment in FES.

The crosshatched area of HK-1-1'-H, referred to the total area of the curve KH-1'-1, represents the relative share of lost or utilized energy in FES:

$$\eta_{G>0}^{ideal} = \Delta\overline{N} = 1 - \frac{\int\limits_{V_H}^{V_K} PdV}{\int\limits_{V_R}^{V_1} PdV + \int\limits_{V_1}^{V_{1'}} PdV + \int\limits_{V_{1'}}^{V_H} PdV}. \tag{5.6}$$

According to the theorem, this value can not exceed the corresponding maximum value determined by the formula

$$\eta_{G>0}^{ideal} \leq \eta_{max}^{ideal} = \Delta\overline{N}_{max}.$$

It should be noted that in P-V coordinates in general case the trajectory of the Hugoniot shock adiabat is not defined, there are only the initial (P_1, V_1) and final (P_1', V_1') values of the trajectory. For the case of confluent shock wave in L.D.Landau's work [3] there obtained a solution to the problem of the curvature trajectory of Hugoniot adiabat ($\partial^2 V/\partial P^2 > 0$). However, extrapolation of given result in the sphere of strong shock waves is not argued, and the concavity of the curve ($\partial^2 P/\partial^2 V > 0$) is taken as a hypothesis.

To determine the shape of the curve at the section (1-1') as a criterion value $\eta_{max}^{ideal} \leq \Delta\overline{N}$is used while varying the pressure at outlet from the system (P_H)) and fixed value $P_K = P_K^*$. In the first approximation at the area 1-1', a linear relationship between P and V was used. In this case, the area under the curve 1-1' is defined as the area of a trapezoid.

Numerical calculations have shown that up to values of the velocity coefficient at the entrance to the chamber $\lambda_1 = 2.307$ ($\gamma = 1.4$), criterion estimation obtained in the theorem will be satisfied on condition that

$$\int\limits_1^{1^1} PdV \left\langle \frac{(P_1 + P_{1^1})}{2}(V_1 - V_{1^1}), \right\rangle \tag{5.7}$$

i.e. the conditional trajectory 1-1' should be concave (Fig. 2).

Thus, the theorem without any additional hypotheses makes it possible to determine the shape of the conditional process trajectory at the site of the shock adiabat in P-V coordinates for the case of strong shock waves.

The table presents the classification of energy systems (machines) depending upon the directions of energy utilization used in them.

If the total energy is converted into mechanical work, then we deal with machines running on the Otto, Diesel, Rankine, Stirling cycles, etc., then a limit cycle in this direction of energy transformation is the Carnot cycle. The opposite direction is a cycle of refrigerating machines. When converting the total energy into kinetic energy of the gas flow at the outlet of FES, we deal with jet and air-breathing jet engines and, accordingly, with the conventional cycles of Brayton, Humphrey, pulsating air-breathing jet engine, where the limit cycle is also Carnot's. In the case of energy flow conversion in FES in the potential energy of pressure the limit cycle will be cycle FES (Fig. 2, 3).

Table 1: Classification of energy systems according to the method of the working body total energy conversion

Class FES	**I**	**II**	**III**
Consumption of gaseous working mass	$G = 0$	$G > 0$ $G \to G_{max}$	$G > 0$ $(G \to G_{max})$
Gas velocity at the outlet of FES	$V_{out} \ll a_{sound}$	$(V_{out} \geq a_{sound})$	$V_{out} \to 0$ $(S_{out} \to \infty)$
Direction of conversion of gas flow total energy	Total (internal) energy is converted into mechanical work $E_{tot} = E_{int} \leftrightarrow A_{mec}$	Total energy is converted into kinetic energy $E_{tot} \to E_{kin}$	Total energy is converted into potential energy of pressure $E_{tot} \to E_{pot}$
Efficiency	$\eta^{\text{ideal}}_{Carnot} = 1 - \dfrac{T_1}{T_k}$ $\eta_{Stirling}$ ets.	$\eta = 1 - \left(\dfrac{P_0}{P_1}\right)^{\dfrac{n-1}{n}}$ $\lim\limits_{P_1 \to \infty} \eta \to \eta^{\text{ideal}}_{Carnot}$	$L_{tech} = 0$ $\overline{Q} \to 0$ $\eta^{ideal}_{max} = \dfrac{1}{\gamma}\eta^{ideal}_{Carnot}$
Cycle of heat machine	Carnot cycle Stirling Otto, Diesel, etc.	Cycle Brighton, Humphrey, etc.	Cycle PES

From Fig. 4, which shows a spatial illustration of the given classification of energy systems, it follows that the proved theorem that has given the limit estimation of energy utilization in FES, allowed to move from flat space heat systems (machines) to three-dimensional, where the coordinates of any vector $\overrightarrow{M}$ represent the characteristics of a some combined flow energy system.

The effectiveness of energy systems FES can be defined by the indicator of flow process quality I, which equals to the ratio of useful utilized energy to the maximum possible energy that can be utilized in this device

$$I = \frac{E_{ut}}{\Delta N_{max}}, \tag{5.8}$$

where E_{ut} - utilized (useful) energy in FES.

Full effectiveness of this class machines is defined as follows

$$\eta^{\Sigma} = \prod_{i=1}^{N} \eta_i,\ i = 1, \ldots, n. \tag{5.9}$$

For example, for a gas laser (with pumping gas medium) electrically pumped full effectiveness (or efficiency) can be written as

$$\eta_{laser} = \eta_{gas}\eta_{serv}\eta_{eo}\eta_{quant}, \tag{5.10}$$

where $\eta_{gas} = I \cdot \eta_{max}^{ideal}$; η_{serv} – is the effectiveness of the gas path, the effectiveness of service laser; η_{eo}, η_{quant} – is electro-optical and quantum efficiency of gas laser.

So for CO_2 laser $\eta_{quant} \sim 0.4$, and for CO laser $\eta_{quant} \sim 0.8$ respectively, i.e. full efficiency of laser system, even in the ideal case ($\eta_{eo} = \eta_{serv} = 1$) can not exceed for CO_2 and CO lasers, correspondingly, values

$$\eta_{CO_2}^{\Sigma,ideal} = \frac{1}{\gamma_m}\eta_{quant}^{CO_2} \approx 0.25,\ \eta_{CO}^{\Sigma,ideal} = \frac{1}{\gamma_m}\eta_{quant}^{CO} \approx 0.5,\ \gamma_m = C_{p_m}/C_{v_{cm}}.$$

The results of the theorem can be used to estimate the energy release in astrophysical thermodynamic systems (in disks of close stars systems). In these energy systems the conditions of the theorem are ideally suited - the technical work is absent ($L_{tex} = 0$), gas velocity at the outlet from the system is zero $\lambda_1 = 0$. For the case of weak gravitational fields and small relativistic gas velocity $v < c$, we obtain on the basis of Consequence 1 of the theorem ($\overline{Q} \to \infty$), estimation of share energy radiation in close stars from Gc^2 $E_{rel} < \frac{1}{\gamma}Gc^2$.

Acknowledgements

I am very grateful to Prof. A.I. Leontiev, Prof. V.C. Shakhov, Dr. A.P. Zubarev for the useful advice concerning the theorem proving, to Dr. A.P.Zubarev and T.A. Bazhenova for assistance in preparing the article.

References

[1] Carnot S. Reflection on the Motive Power of Fire on Machines That Can Develop This Power. – In: The Second Law of Thermodynamics. M., 1934.

[2] Nernst W. Theoretical and Experimental Foundation of the New Thermal Law. / Trans. edited by academician Ioffe A.F. M. L., 1929.

[3] Landau L.D., Lifshitz M.E. Hydrodynamics. M.: Nauka, 1983. 871 pp.

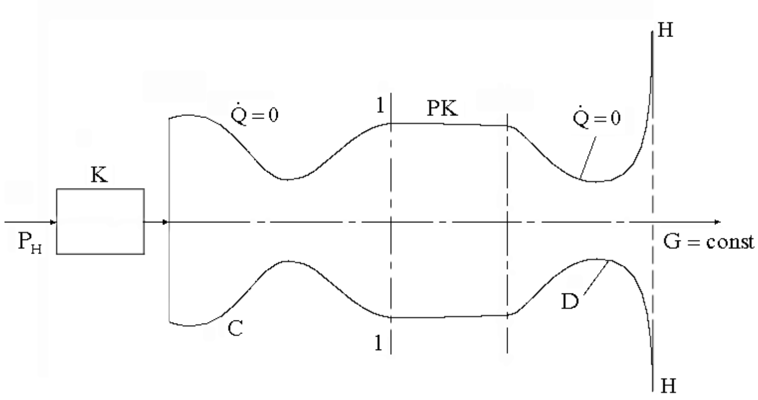

Figure 1: Fundamental diagram of the FES ($L_{tech} = 0$) with one outlet ($i = 1$) and one outlet ($j = 1$), where K - compressor; PK – working chamber; D – diffuser; P_H, P_K – pressure at the inlet and outlet of the compressor correspondingly, and C - nozzle. Section 1 – inlet into the working chamber, H – outlet of FES.

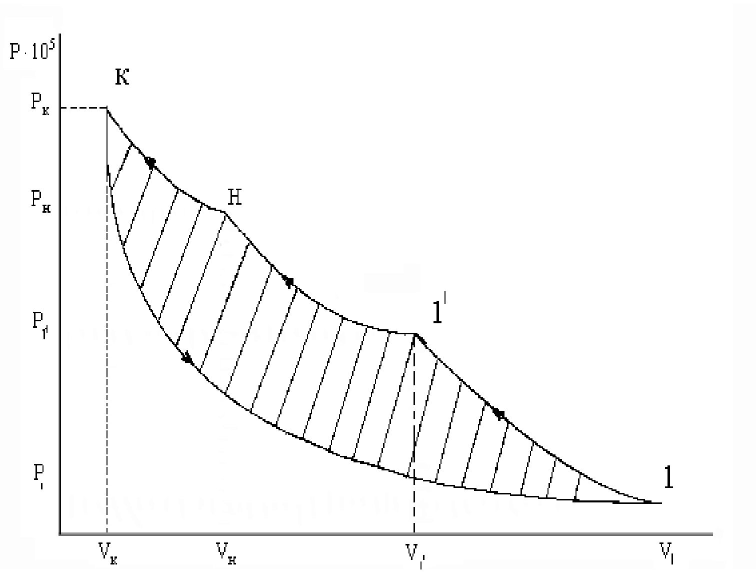

Figure 2: Conditional limit cycle of open-circuit G”C ($\dot{\bar{L}}_{tech} = 0$). HK – isothermal compression, for example, in the compressor; K-1 –adiabatic expansion in the nozzle; 1-1 ’ – Hugoniot shock adiabat; 1’-H – adiabatic compression in the diffuser.

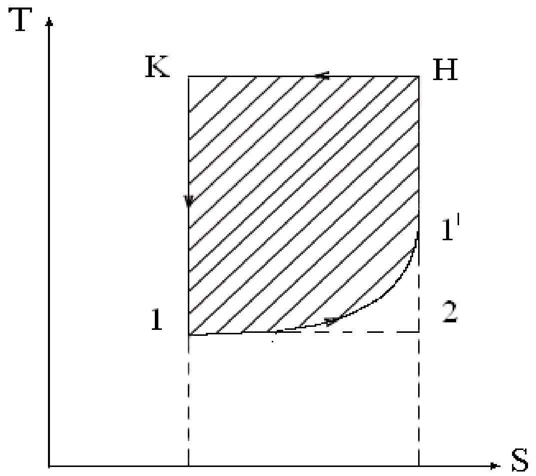

Figure 3: Limit cycle FES in T-S coordinates. HK – isothermal compression, for example, in the compressor, K-1 – adiabatic expansion in the nozzle; 1-1 ' – shock wave (Hugoniot percussive adiabat); 1'-H – adiabatic compression in the diffuser. K-1-2-H-K – Carnot cycle.

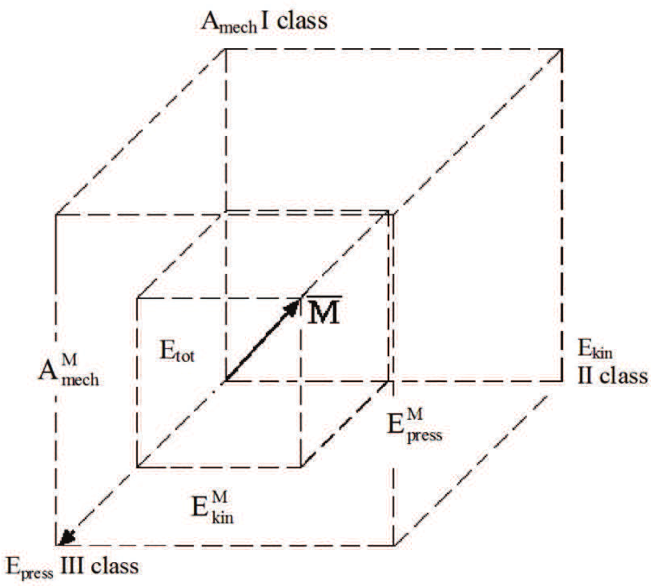

Figure 4: The space of flow energy systems.