Electromagnetic gravity with spin 3

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Abstract

We consider here the simple derivation of the Einstein equations by Fock. Then, we approach the way from the spin 1 fields to the spin 2 fields for massive and massless particles and we derive the gravity equations from this base. Then, we approach the spin 3 gravity called by us **electromagnetic gravity**.

1 Introduction

While the electromagnetic field was determined from the motion of charges and currents, the Einstein-Hilbert theory of gravity being the spacetime geometry was determined from presence of mass-energy and linear momentum. The corresponding equations - Einstein-Hilbert equations (EHE) - determine the metric tensor of spacetime for a given arrangement of stress-energy in the spacetime. The relationship between the metric tensor and the Einstein tensor allows the EHE to be written as a set of non-linear partial differential equations. The solutions of the EHE are the components of the metric tensor. The inertial trajectories of particles and radiation (geodesics) in the resulting geometry are then calculated using the geodesic equations. As well as obeying local energy-momentum conservation, the EHE reduce to Newtons law of gravitation, where the gravitational field is weak and velocities are much less than the speed of light.

We consider here the simple derivation of the Einstein equations by Fock. Then, we approach the way from the spin 1 fields to the spin 2 fields for massive and massless particles and we derive the corresponding action for spin 2 gravity from this base. Then, we approach the spin 3 gravity called by us **electromagnetic-gravity**.

2 The Einstein equations derived by Fock

There is the simple derivation of the EHE given by Fock (1964). The similar derivation was performed by Chandrasekhar (1972), Kenyon (1996), Landau et al. (1987), Rindler

(2003) and others. Source theory derivation of Einstein equations was performed by Schwinger (1970).

It is well known that the gravity mass M_G of some body is equal to the its inertial mass M_I, where gravity mass is a measure of a massive body to create the gravity field (or, gravity force) and the inertial mass of a massive body is a measure of the ability of the resistance of the body when it is accelerated. At present time we know, that if components of elementary particles have the same gravity and inertial masses, the body composed with such elementary particles has the identical gravity and inertial mass. There is no need to perform experimental verification. So, particle physics brilliantly confirms the identity of the inertial and gravity masses.

According to the Newton theory, the gravity potential is given by the equation

$$U(r) = -\kappa\frac{M}{r}, \tag{1}$$

where r is a distance from the center of mass of a body, κ is the gravitational constant and its numerical value is in SI units $6.67430(15)10^{-11}m^3.kg^{-1}.s^{-2}$ (CODATA, 2018).

The potential U is, as it is well known, the solution of the Poisson equation:

$$\Delta U(r) = -4\pi\kappa\varrho, \tag{2}$$

where ϱ is the density of the distributed masses.

The problem is, what is the geometrical formulation of gravity equation (2) following from the space-time element ds, which has the Minkowski form in case of the special theory of relativity.

Let us postulate that the motion of a body moving in the g-field is determined by the variational principle

$$\delta\int ds = 0. \tag{3}$$

In order to get the Newton equation of motion, we are forced to perform the following identity:

$$g_{00} = c^2 - 2U = -4\pi\kappa\varrho. \tag{4}$$

The second mathematical requirement, which has also the physical meaning is the covariance of the derived equation. It means that the necessary mathematical operation are the following replacing of original symbols:

$$U \to g_{\mu\nu} \tag{5}$$

with

$$\Delta U \to Tensor\ equation \tag{6}$$

and

$$\varrho \to T_{\mu\nu}, \tag{7}$$

where $T_{\mu\nu}$ is the tensor of energy and momentum.

In order to get the tensor generalization of eq. (2) it is necessary to construct new tensor $R_{\mu\nu}$, which is linear combination of the more complicated tensor $R_{\alpha\beta,\mu\nu}$, or

$$R_{\mu\nu} = g^{\alpha\beta}R_{\mu\alpha,\beta\nu} \tag{8}$$

and the scalar quantity R, which is defined by equation

$$R = g^{\lambda\mu} R_{\lambda\mu} \tag{9}$$

and construct the combination tensor $G_{\lambda\mu}$ of the form

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{10}$$

which has the mathematical property, that the covariant divergence of this tensor is zero, or,

$$\nabla^{\lambda} G_{\lambda\mu} = 0. \tag{11}$$

.

With regard to the fact that also the energy-momentum tensor $T_{\mu\nu}$ has the zero divergence, we can identify eq. (10) with the tensor $T_{\mu\nu}$, or

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi\kappa}{c^2} T_{\mu\nu}, \tag{12}$$

where the appeared constant in the last equation is introduce to get the classical limit of the equation.

The approximate solution of the last equation is as follows

$$ds^2 = (c^2 - 2U)dt^2 - \left(1 + \frac{2U}{c^2}(dx^2 + dy^2 + dz^2)\right). \tag{13}$$

The space-time element (13) is able to explain the shift of the frequency of light in gravitational field and the deflection of light in the gravitational field of massive body with mass M.

So, we have seen that the basic mathematical form of the Einstein general relativity is the Riemann manifold specified by the metric with the physical meaning. The crucial principle is the equality of the inertial and gravitational masses.

While the derivation of the EHE is elementary, Feynman wrote that the derivation of EHE by Einstein is difficult to understand. Namely:

Einstein himself, of course, arrived at the same Lagrangian but without the help of a developed field theory, and I must admit that I have no idea how he guessed the final result. We have had troubles enough arriving at the theory - but I feel as though he had done it while swimming underwater, blindfolded, and with his hands tied behind his back! (Feynman et al., 1995).

Now the question arises, what is the the force acting on the point moving in the homogenous gravitational field. It was calculated in the 3-form as follows (Landau, et al., 1987):

$$\mathbf{f} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \left\{ \mathbf{grad} \ln \sqrt{h} + \sqrt{h} \left[\frac{\mathbf{v}}{c} \mathbf{rot}\ \mathbf{g} \right] \right\}, \tag{14}$$

with (Landau, et al., 1987).

$$h = 1 + \frac{2\varphi}{c^2}, \tag{15}$$

where φ s gravitational potential generating the acceleration $\mathbf{g}$. So, we see that it is not in the simple Newton form.

Let us still remark that the derived Einstein equations (12) can be generalized to form the Einstein equations with the cosmological constant, or, (Einstein, 1917)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{8\pi\kappa}{c^2} T_{\lambda\mu}, \tag{16}$$

where Λ is the new cosmological constant introduced formally, with the goal to find new form of the cosmological model and their solutions in the mathematical form. In addition to that the last equation can be still derived in order to involve so cosmological matrix (Pardy, 2018b). The new form of such equations is as follows:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + (\Lambda_{\alpha\beta} g^{\alpha\beta}) g_{\mu\nu} = -\frac{8\pi\kappa}{c^2} T_{\mu\nu}, \tag{17}$$

where

$$\Lambda_{\alpha\beta} = \begin{pmatrix} \Lambda_{00} & \Lambda_{01} & \Lambda_{02} & \Lambda_{03} \\ \Lambda_{10} & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{20} & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{30} & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{pmatrix}. \tag{18}$$

The generalization of cosmology and the new deal of cosmology is then based on the Einstein-Pardy gravity equations (17) (Pardy, 2018b).

3 The spin 1 field equations

Spin was originally introduced as the rotation with an angular momentum of a particle around some axis. On the other hand, spin has some peculiar properties that distinguish it from orbital angular momenta: a) Spin quantum numbers may take half-integer values. b) Although the direction of its spin can be changed, an elementary particle cannot be made to spin faster or slower. c) The spin of a charged particle is associated with a magnetic dipole moment with a g-factor differing from 1. This could only occur classically if the internal charge of the particle were distributed differently from its mass.

The conventional definition of the spin quantum number, s, is s = n/2, where n can be any non-negative integer. So, the allowed values of s are 0, 1/2, 1, 3/2, 2, etc. The value of s for an elementary particle depends only on the type of particle, and cannot be altered in any known way (in contrast to the spin direction).

We show that the natural construction of the field of the particles with spin 1 is presented in source theory method. The relation

$$|\langle 0_+|0_-\rangle|^2 = \exp\{-2\mathrm{ImW}\} \leq 1 \tag{19}$$

is postulated to be valid for all spin fields. Let us show here the construction of action and field equations concerning spin one.

If spin zero particles and fields are described by the scalar source, then a vector source denoted here as $J^\mu(x)$ can be considered as a candidate for the description of the spin 1 fields and particles. However, there exist some obstacles because source $J^\mu(x)$ has four components and spin one particles have only three spin possibilities. Nevertheless first, let us investigate by analogy with the spin zero fields the following form of the action for the unit spin fields:

$$W(J) = \frac{1}{2} \int (dx)(dx') J^\mu(x) \Delta_+(x-x') J_\mu(x'). \tag{20}$$

Then,

$$|\langle 0_+|0_-\rangle|^2 = e^{iW} e^{iW^*} = \exp\left\{-\int d\omega_p J^{*\mu}(p) J_\mu(p)\right\}. \tag{21}$$

However,

$$J^{*\mu}(p) J_\mu(p) = |\mathbf{J}(p)|^2 - |J^0(p)|^2 \leq 0, \quad \text{or,} > 0 \tag{22}$$

and it means that the quantity defined by eq. (21) cannot be considered as the probability of the persistence of vacuum.

The difficulty can be overcome by replacing the original form $J^{*\mu}(x) J_\mu(x)$ by the following invariant structure:

$$J^{*\mu}(p) \left[g_{\mu\nu} + \frac{1}{m^2} p_\mu p_\nu\right] J^\nu(p), \tag{23}$$

which can be with regard to its invariance, determined in the rest frame of the time-like vector p^μ, where $p^\mu = (m, 0, 0, 0)$ in the rest frame. Then, with $g_{\alpha\alpha} = (-1, 1, 1, 1)$ we have

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{m^2} p_\mu p_\nu = \begin{cases} \delta_{kl}; & \mu = k; \quad \nu = l \\ 0; & \mu = 0; \quad \nu = 0 \\ 0; & \mu = k; \quad \nu = 0 \end{cases} \tag{24}$$

and

$$J^{*\mu}(p) \bar{g}_{\mu\nu} J^\nu(p) \equiv |\mathbf{J}|^2, \tag{25}$$

and now the quantity $|\langle 0_+|0_-\rangle|^2$ can be interpreted as the vacuum persistence probability.

At the same time $|\mathbf{J}|^2$ contains three independent source components, transforming among themselves under spatial rotation, as it is appropriate to unit spin.

After using eq. (23) it may be easy to get $W(J)$ in the space-time representation by the Fourier transformation, as it follows

$$W(J) = \frac{1}{2} \int (dx)(dx') \quad \times$$

$$\left\{J_\mu(x) \Delta_+(x - x') J^\mu(x') + \frac{1}{m^2} \partial_\mu J^\mu \Delta_+(x - x') \partial'_\nu J^\nu(x')\right\}. \tag{26}$$

The field of spin one particles can be defined using the definition of the test source $\delta J^\mu(x)$ by the relation

$$\delta W(J) = \int (dx) \delta J^\mu(x) \varphi_\mu(x), \tag{27}$$

where φ_μ is the field of particles with spin 1. After performing variation of the formula (26) and comparison with eq. (27) we get the equation for field of spin 1 in the following form:

$$\varphi_\mu(x) = \int (dx') \Delta_+(x - x') J_\mu(x') - \frac{1}{m^2} \partial_\mu \int (dx') \Delta_+(x - x') \partial'_\nu J^\nu(x'). \tag{28}$$

The divergence of the vector field $\varphi_\mu(x)$ is given by the relation

$$\partial_\mu \varphi^\mu(x) = \int (dx') \Delta_+(x - x') \partial'_\mu J^\mu(x') \quad -$$

$$\frac{1}{m^2} \partial^2 \Delta_+(x - x') \partial'_\nu J^\nu(x') = \frac{1}{m^2} \partial_\mu J^\nu(x), \tag{29}$$

where we used relation $-\partial^2\Delta_+ = \delta(x-x') - m^2\Delta_+$.

Further, we have after applying operator $(-\partial^2+m^2)$ on the equation (28) the following equation:

$$\left(-\partial^2 + m^2\right)\varphi_\mu(x) = J_\mu(x) - \frac{1}{m^2}\partial_\mu\partial_\nu J^\nu(x) \tag{30}$$

$$\left(-\partial^2 + m^2\right)\varphi_\mu(x) + \partial_\mu\partial_\nu\varphi^\nu(x) = J_\mu(x) \tag{31}$$

as a consequence of eq. (29).

It may be easy to cast the last equation into the following form

$$\partial^\nu G_{\mu\nu} + m^2\varphi_\mu = J_\mu, \tag{32}$$

where

$$G_{\mu\nu}(x) = -G_{\nu\mu}(x) = \partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu. \tag{33}$$

Identifying $G_{\mu\nu}$ with $F_{\mu\nu}$ of the electromagnetic field we get instead of eq. (32) so called the Proca equation for the electromagnetic field with the massive photon.

It is evident that the zero mass limit does not exist for $\partial_\mu J^\mu(x) \neq 0$. In such a way we are forced to redefine action $W(J)$. One of the possibilities is to put

$$\partial_\mu J^\mu(x) = mK(x) \tag{34}$$

and identify $K(x)$ in the limit $m \to 0$ with the source of massless spin zero particles.

Since the zero mass particles with zero spin are experimentally unknown in any event, we take $K(x) = 0$ and we write

$$W_{[m=0]}(J) = \frac{1}{2}\int (dx)(dx')J_\mu(x)D_+(x-x')J^\mu(x'), \tag{35}$$

where

$$\partial_\mu J^\mu(x) = 0 \tag{36}$$

and

$$D_+(x-x') = \Delta_+(x-x'; m=0). \tag{37}$$

The detail discussion concerning helicity, angular momentum, etc. of this new particle (photon) can be found in the Schwinger book (Schwinger, 1970).

4 Spin 2 fields

Exploiting the experience with spin 1 fields we form the combinations

$$T_{\mu\nu}(x), \tag{38}$$

$$\partial_\mu T^{\mu\nu}(x), \tag{39}$$

$$\partial_\mu\partial_\nu T^{\mu\nu}(x) \tag{40}$$

and $T(x)$ with appropriate coefficients in order to get the plausible form of action $W(T)$ for particles with spin 2.

While the spin one particles are described by the four-vector fields and sources the possible mathematical object describing particles with spin 2 should be the tensor field $\varphi_{\mu\nu}$ and the tensor source $T_{\mu\nu}$. Let us suppose that the tensor source is symmetrical, or, $T_{\mu\nu} = T_{\nu\mu}$.

Then, it has ten independent components. The vector source

$$\partial_\mu T^{\mu\nu}(x) \tag{41}$$

has 3 + 1 components and the scalar source

$$T(x) = g_{\mu\nu} T^{\mu\nu}(x) \tag{42}$$

is the one-component object. If we eliminate them, then the multiplicity of the system will be equal to five and this situation corresponds to the particle with spin 2.

Now, the question arises, what is the mathematical structure of the action $W(T)$ for particles with spin 2. Exploiting the experience with spin 1 fields we observe that $W(J)$ as the scalar quantity is formed by suitable combinations of sources and their derivatives. Similarly, in case with spin 2 particles we use the combinations of $T_{\mu\nu}(x), \partial_\mu T^{\mu\nu}(x), \partial_\mu\partial_\nu T^{\mu\nu}(x)$ and $T(x)$ with appropriate coefficients. The plausible form of $W(T)$ for particles with spin 2 is as follows:

$$W(T) = \frac{1}{2}\int (dx)(dx')\,\{T^{\mu\nu}(x)\Delta_+(x-x')T_{\mu\nu}(x) \quad +$$

$$\frac{2}{m^2}\partial_\nu T^{\mu\nu}(x)\Delta_+(x-x')\partial'^{\lambda}T_{\mu\lambda}(x') \quad +$$

$$\frac{1}{m^4}\partial_\mu\partial_\nu T^{\mu\nu}(x)\Delta_+(x-x')\partial'_\alpha\partial'_\beta T^{\alpha\beta}(x') \quad -$$

$$\frac{1}{3}\left(T(x) - \frac{1}{m^2}\partial_\mu\partial_\nu T^{\mu\nu}(x)\right)\Delta_+(x-x')\left(T(x') - \frac{1}{m^2}\partial'_\alpha\partial'_\beta T^{\alpha\beta}(x')\right)\Big\}, \tag{43}$$

where the coefficients follow (as we will see in the next text) from the probability condition $|\langle 0_+|0_-\rangle|^2 \le 1$.

The probability of the vacuum persistence generated by action (43) calculated in the momentum space is of the form:

$$|\langle 0_+|0_-\rangle|^2 = \exp\left\{-\int d\omega_p T^{*\mu\nu}(p)\Pi_{\mu\nu,\alpha\beta}(p)T^{\alpha\beta}(p)\right\}, \tag{44}$$

where (Schwinger, 1970):

$$\Pi_{\mu\nu,\alpha\beta}(p) = \frac{1}{2}\left[\bar{g}_{\mu\alpha}\bar{g}_{\nu\beta} + \bar{g}_{\nu\alpha}\bar{g}_{\mu\beta}\right] - \frac{1}{3}\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta} \tag{45}$$

with

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{m^2}p_{mu}p_{nu} \tag{46}$$

being the invariant tensor.

It may be easy to calculate $T^*\Pi T$ in the rest frame of p_μ where

$$\bar{g}_{\mu\nu} \to \delta_{kl}. \tag{47}$$

Under this condition it is

$$T^* \Pi T \to \bar{T}^{*kl} \bar{T}^{kl} \geq 0; \quad k, l \neq 0, \tag{48}$$

where

$$\bar{T}^{\mu\nu} = T^{\mu\nu} - \frac{1}{3} g^{\mu\nu} \bar{g}_{\varrho\sigma} T^{\varrho\sigma} \tag{49}$$

and the following relations have been used:

$$\bar{g}_{\mu\nu} \bar{T}^{\mu\nu} = 0 \tag{50}$$

$$p^\mu \bar{g}_{\mu\nu} = 0 \tag{51}$$

$$\bar{g}^{\mu\nu} \bar{g}_{\mu\nu} = 0. \tag{52}$$

By analogy with spin 1 fields we define the spin 2 field $\varphi_{\mu\nu}(x)$ by relation

$$\delta W(T) = \int (dx) \delta T^{\mu\nu}(x) \varphi_{\mu\nu}(x), \tag{53}$$

which can be easily transformed into momentum space as

$$\delta W(T) = \int \frac{(dp)}{(2\pi)^4} \delta T^{\mu\nu}(-p) \varphi_{\mu\nu}(p). \tag{54}$$

The symmetrical field $\varphi_{\mu\nu}(x)$ following from eq. (54) has the following form (Schwinger, 1970)

$$\begin{aligned}
\varphi_{\mu\nu}(x) = \int (dx') \Delta_+(x - x') T_{\mu\nu}(x') \quad & - \\
\frac{1}{m^2} \partial_\mu \int (dx') \Delta_+(x - x') \partial'^\lambda T_{\lambda\nu}(x') \quad & - \\
\frac{1}{m^2} \partial_\nu \int (dx') \Delta_+(x - x') \partial'^\lambda T_{\mu\lambda}(x') \quad & + \\
\frac{1}{m^4} \partial_\mu \partial_\nu \int (dx') \Delta_+(x - x') \partial'_\kappa \partial'_\lambda T^{\kappa\lambda}(x') \quad & - \\
\frac{1}{3} (g_{\mu\nu} - \frac{1}{m^2} \partial_\mu \partial_\nu) \int (dx') \Delta_+(x - x') \quad & \times \\
(T(x') - \frac{1}{m^2} \partial'_\kappa \partial'_\lambda T^{\kappa\lambda}(x')). &
\end{aligned} \tag{55}$$

From this equation follows immediately the divergence of $\varphi_{\mu\nu}(x)$:

$$\partial^\mu \varphi_{\mu\nu}(x) = \frac{1}{m^2} \partial^\mu T_{\mu\nu}(x) - \frac{1}{3m^2} \partial_\nu [T(x) - \frac{2}{m^2} \partial_\kappa \partial_\lambda T^{\kappa\lambda}(x)] \tag{56}$$

and

$$\varphi = g_{\mu\nu}\varphi^{\mu\nu} = -\frac{1}{3m^2}[T(x) + \frac{2}{m^2}\partial_\kappa\partial_\lambda T^{\kappa\lambda}(x)]. \tag{57}$$

The combination of eq. (56) and (57) forms

$$\partial^\mu\varphi_{\mu\nu}(x) - \partial_\nu\varphi = \frac{1}{m^2}\partial^\mu T_{\mu\nu}(x). \tag{58}$$

The differential equation following from eq. (55) is of the form:

$$(-\partial^2 + m^2)\varphi_{\mu\nu}(x) = T_{\mu\nu}(x) \quad -$$

$$\frac{1}{m^2}\left[\partial_\mu\partial^\lambda T_{\lambda\nu}(x) + \partial_\nu\partial^\lambda T_{\mu\lambda}(x)\right] + g_{\mu\nu}\frac{1}{m^2}\partial_\kappa\partial_\lambda T^{\kappa\lambda}(x) \quad -$$

$$\frac{1}{3}(g_{\mu\nu} - \frac{1}{m^2}\partial_\mu\partial_\nu)\left[T(x) + \frac{2}{m^2}\partial_\kappa\partial_\lambda T^{\kappa\lambda}(x)\right]. \tag{59}$$

If we replace the scalar and vector combinations of sources in eq. (59) by the field objects from eqs. (56)–(58), we get the following differential equation for $\varphi_{\mu\nu}$:

$$(-\partial^2 + m^2)\varphi_{\mu\nu} + \partial_\mu\partial^\lambda\varphi_{\lambda\nu}(x) + \partial_\nu\partial^\lambda\varphi_{\mu\lambda}(x) - \partial_\mu\partial_\nu\varphi(x) \quad -$$

$$g_{\mu\nu}\left[(-\partial^2 + m^2)\varphi(x) + \partial_\kappa\partial_\lambda\varphi^{\kappa\lambda}(x)\right] = T_{\mu\nu}(x), \tag{60}$$

which also follows from $\delta W(T) = 0$, where

$$W(T) = \int (dx)\left[T^{\mu\nu}(x)\varphi_{\mu\nu}(x) + £\right], \tag{61}$$

where £ is the Lagrange function and it has the following mathematical structure:

$$£ = -\frac{1}{2}\left[\partial^\alpha\varphi^{\mu\nu}(x)\partial_\alpha\varphi_{\mu\nu}(x) + m^2\varphi^{\mu\nu}(x)\varphi_{\mu\nu}(x) \quad -\right.$$

$$\left.\partial^\alpha\varphi(x)\partial_\alpha\varphi(x) - m^2\varphi^2(x)\right] \quad -$$

$$\partial_\mu\varphi^{\mu\nu}(x)\partial_\nu\varphi(x) + \partial_\mu\varphi^{\mu\nu}(x) + \partial_\mu\varphi^{\mu\nu}(x)\partial^\alpha\varphi_{\alpha\nu}(x). \tag{62}$$

If we take the spur of eq. (60), we get

$$(-\partial^2 + m^2)\varphi(x) + \partial_\mu\partial_\nu\varphi^{\mu\nu}(x) = -\frac{1}{2}\left[T(x) + m^2\varphi(x)\right]. \tag{63}$$

After inserting eq. (63) into eq. (60), we get

$$(-\partial^2 + m^2)\varphi_{\mu\nu}(x) + \partial_\mu\partial^\lambda\varphi_{\lambda\nu}(x) + \partial_\nu\partial^\lambda\varphi_{\mu\lambda}(x) \quad -$$

$$\partial_\mu\partial_\nu\varphi(x) + g_{\mu\nu}\frac{m^2}{2}\varphi(x) = T_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}T(x). \tag{64}$$

Introducing

$$G_{\mu\lambda\nu}(x) = -G_{\nu\lambda\mu}(x) = \partial_\mu\varphi_{\lambda\nu}(x) - \partial_\nu\varphi_{\lambda\mu}(x) \tag{65}$$

we can write eq. (64) in the following form:

$$\partial^\lambda G_{\mu\nu\lambda}(x) - \partial_\nu G^\lambda_{\mu\lambda}(x) + m^2\left[\varphi_{mu\nu}(x) + \frac{1}{2}g_{\mu\nu}\varphi(x)\right] \quad =$$

$$T_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}T(x). \tag{66}$$

Further relations concerning spin 2 fields can be found in monograph by Schwinger (1970).

5 The massless limit of the spin 2 theory

It is evident that in order action (43) continue to exist in the limit $m \to 0$ it is natural to put

$$\partial_\nu T^{\mu\nu}(x) = \frac{m}{\sqrt{2}}J^\mu(x) \tag{67}$$

and

$$\partial_\mu J^\mu(x) = m\left[\sqrt{3}K(x) - \frac{1}{\sqrt{2}}T(x)\right], \tag{68}$$

where $J^\mu(x)$ and $K(x)$ are independent sources. The independence of them is expressed by constants $\sqrt{3}$ and $1/\sqrt{2}$ which are chosen to eliminate any coupling between sources $K(x)$ and $T(x)$. After insertion of eqs. (67) and (68) into action (43), we get for $m \to 0$:

$$W(T) = \frac{1}{2}\int (dx)(dx')\Big\{T^{\mu\nu}(x)D_+(x-x')T_{\mu\nu}(x) \quad -$$

$$\frac{1}{2}T(x)D_+(x-x')T(x') + J^\mu(x)D_+(x-x')J_\mu(x') \quad +$$

$$K(x)D_+(x-x')K(x')\Big\}, \tag{69}$$

where $D_+(x-x') = \Delta_+(x-x'; m=0)$ and for $m = 0$

$$\partial_\mu T^{\mu\nu}(x) = 0 \tag{70}$$

$$\partial_\mu J^\mu(x) = 0. \tag{71}$$

The formula (69) represents the invariant decomposition by means of sources of massless particles with spin 2, 1 and 0. The massless particles of helicity ± 2 is called graviton and the action which corresponds to this particle follows from action (69) in the form

$$W(T) = \frac{1}{2}\int (dx)(dx')\left\{T^{\mu\nu}(x)D_+(x-x')T_{\mu\nu}(x) \quad -\right.$$

$$\frac{1}{2}T(x)D_+(x-x')T(x')\Big\} \tag{72}$$

with

$$\partial_\mu T^{\mu\nu}(x) = 0. \tag{73}$$

The graviton is the particle which was not hitherto experimentally discovered. Nevertheless, we can suppose it is the mediate boson which initiates the gravitational phenomena, just as the photon initiates the electromagnetic ones.

The source restriction $\partial_\mu T^{\mu\nu}(x) = 0$ for the graviton source states the existence of a conservation law of the vector

$$p^\nu = \int d\sigma_\mu T^{\mu\nu}(x), \tag{74}$$

where $d\sigma_\mu$ is an invariant element of area, which can be identified with the energy-momentum vector. The connection between the mechanical tensor $T^{\mu\nu}_{mech}$ and the gravitational tensor $T^{\mu\nu}_{grav}$ is postulated by relation

$$T^{\mu\nu}_{grav} = \kappa^{1/2} T^{\mu\nu}_{mech}, \tag{75}$$

where κ is the gravitational constant of the magnitude

$$\kappa = 8\pi G, \tag{76}$$

and $= \kappa = 6.67430(15)10^{-11} m^3.kg^{-1}.s^{-2}$ (CODATA, 2018).

The action corresponding to the gravitational field initiated by the mechanical tensor T^{μ}_{mech} is now

$$W(T) = \frac{\kappa}{2}\int (dx)(dx')\left\{T^{\mu\nu}(x)D_+(x-x')T^{\mu\nu}(x) \quad - \right.$$

$$\left.\frac{1}{2}T(x)D_+(x-x')T(x')\right\} \tag{77}$$

with

$$\partial_\mu T^{\mu\nu}(x) = 0. \tag{78}$$

The corresponding gravitational field definition is

$$\delta W(T) = \int (dx)\delta T^{\mu\nu} h_{\mu\nu}, \tag{79}$$

which implies the field equation for $h_{\mu\nu}(x)$ in the following form:

$$-\partial^2 h_{\mu\nu}(x) + \partial_\mu \partial^\alpha h_{\alpha\nu}(x) + \partial_\nu \partial^\alpha h_{\mu\alpha}(x) - \partial_\mu \partial_\nu h(x) \quad =$$

$$\kappa(T_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}T(x)) \tag{80}$$

with

$$T(x) = g_{\mu\nu}T^{\mu\nu}(x) \tag{81}$$

$$h(x) = g_{\mu\nu}h^{\mu\nu}(x). \tag{82}$$

Let us remark that action (77) was used by author to determination of the spectral form of the emitted gravitons by the binary system and by the related systems (Pardy, 1983; 1994a; 1994b; 1994c; 1994d; 2011; 2018a; 2019). Action (77) enables also the derivation of the Newton gravity potential and Einstein gravity field equations (Schwinger, 1970; Pardy, 1984). After the verbal investigation of the Schwinger book involving theory of gravity (Schwinger, 1970), we can honestly say that the words principle of equivalence are not involved in his book.

6 Spin 3 fields

The spin 3 fields is discussed as the analogue of the spin 2 situation. The spin 3 action is derived together with the spin 3 field equation. The massles situation is also discussed.

It may be easy to show that in case of the spin 2 fields the action can be expressed in the following form:

$$W(T) = \frac{1}{2}\int \frac{(dp)}{(2\pi)^4}T^{\mu\nu}(-p)\frac{\Pi_{\mu\nu,\alpha\beta}}{p^2+m^2-i\varepsilon}T^{\alpha\beta}(p) \tag{83}$$

where $\Pi_{\mu\nu,\alpha\beta}$ is given by eq. (45) of the spin 2 discussion. By analogy with spin 2 situation we can postulate the form of the action for fields with spin 3 as follows:

$$W(S) = \frac{1}{2}\int \frac{(dp)}{(2\pi)^4}S^{\lambda\mu\nu}(-p)\frac{\Pi_{\lambda\mu\nu,\alpha\beta\gamma}}{p^2+m^2-i\varepsilon}S^{\alpha'\beta'\gamma'}(p) \tag{84}$$

where function $\Pi_{\lambda\mu\nu,\alpha\beta\gamma}$ it is easy to determine. While in case of spin 0, 1 and 2 fields the determination of Π is not very difficult, the situation changes with higher spins. Nevertheless, there is the general method how to determine function Π for all integer spins as it will be shown in the next text and we here write down the derived result for spin 3 situation (Schwinger, 1970).

$$\Pi_{\alpha\beta\gamma,\alpha'\beta'\gamma'} = \bar{g}_{\alpha\alpha'}\bar{g}_{\beta\beta'}\bar{g}_{\gamma\gamma'} - \frac{1}{5}\left[\bar{g}_{\alpha\beta}\bar{g}_{\gamma\gamma'}\bar{g}_{\alpha'\beta'} + \bar{g}_{\beta\gamma}\bar{g}_{\alpha\alpha'}\bar{g}_{\beta'\gamma'} + \bar{g}_{\gamma\alpha}\bar{g}_{\beta\beta'}\bar{g}_{\alpha'\gamma'}\right] \tag{85}$$

where

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{m^2}p_\mu p_\nu \tag{86}$$

Let us write down some properties of tensor $\Pi_{\alpha\beta\gamma,\alpha'\beta'\gamma'}$.

$$p^\alpha\Pi_{\alpha\beta\gamma,\alpha'\beta'\gamma'} = \frac{p^2+m^2}{m^2} \quad \times$$

$$\left[p_{\alpha'}\bar{g}_{\beta\beta'}\bar{g}_{\gamma\gamma} - \frac{1}{5}\left(p_\beta\bar{g}_{\gamma\gamma'}\bar{g}_{\alpha'\beta'} + p_{\alpha'}\bar{g}_{\beta\gamma}\bar{g}_{\beta'\gamma'} + p_\gamma\bar{g}_{\alpha\alpha'}\bar{g}_{\alpha'\gamma'}\right)\right] \tag{87}$$

and

$$g^{\beta\gamma}\Pi_{\alpha\beta\gamma,\alpha'\beta'\gamma'} = \frac{p^2+m^2}{m^2} \quad \times$$

$$\left[\frac{p_{\beta'}p_{\gamma'}}{m^2}\bar{g}_{\alpha\alpha'} - \frac{1}{5}\left(\frac{p_\alpha p_{\gamma'}}{m^2}\bar{g}_{\alpha'\beta'} + \bar{g}_{\alpha\alpha'}\bar{g}_{\beta'\gamma'} + \frac{p_\alpha p_{\beta'}}{m^2}\bar{g}_{\gamma'\alpha'}\right)\right] \tag{88}$$

The field of spin 3, $\varphi_{\alpha\beta\gamma}(p)$, in the momentum representation is defined in analogy with spin 0, 1 and 2 fields, or

$$\delta W(S) = \int \frac{(dp)}{(2\pi)^4}\delta S^{\alpha\beta\gamma}(-p)\varphi_{\alpha\beta\gamma}(p) \tag{89}$$

and it is obtained as

$$\varphi_{\alpha\beta\gamma}(p) = \frac{1}{p^2+m^2-i\varepsilon}\Pi_{\alpha\beta\gamma,\alpha'\beta'\gamma'}(p)S^{\alpha'\beta'\gamma'}(p) \tag{90}$$

We know from the experience with spin 0, 1, and 2 particles that action W can be cast into various forms. If we take the obligate form, we write for the spin 3 action:

$$W(S) = \int (dx)\left[S^{\alpha\beta\gamma}(x)\varphi_{\alpha\beta\gamma}(x) + \mathcal{L}(x)\right] \tag{91}$$

where $\mathcal{L}$ (x) is the Lagrange function of the spin 3 fields and it can be shown that its mmathematical structure is as follows (Schwinger, 1970):

$$\mathcal{L} = -\frac{1}{2}\left[\partial^\kappa\varphi^{\alpha\beta\gamma}\partial_\kappa\varphi_{\alpha\beta\gamma} + m^2\varphi^{\alpha\beta\gamma}\varphi_{\alpha\beta\gamma} - 3\partial_\kappa\varphi^{\kappa\beta\gamma}\partial^\alpha\varphi_{\alpha\beta\gamma} \quad + \right.$$

$$\left. 6\,\partial_\beta\varphi^{\alpha\beta\gamma}\partial_\gamma\varphi_\alpha - 3\partial^\alpha\varphi^\gamma\partial_\alpha\varphi_\gamma - 3m^2\varphi^\kappa\varphi_\kappa - \frac{3}{2}(\partial_\kappa\varphi^\kappa)^2\right] \quad +$$

$$\frac{1}{2}m\left(\varphi^\kappa\partial_\kappa\Phi - \Phi\partial_\kappa\varphi^\kappa\right) + \partial^\kappa\Phi\partial_\kappa\Phi + 4m^2\Phi \tag{92}$$

where the auxiliary function Φ is to receive independent variation in the stationary action principle and

$$\varphi_\lambda(x) = \varphi^\kappa_{\lambda\kappa}(x) \tag{93}$$

6.1 The massless fields with helicity 3

It is possible to show that from the massive field action for spin 3 generate the massless limit with the following action (Schwinger, 1970):

$$W(S, m=0) = \frac{1}{2}\int (dx)(dx')$$

$$\left[S^{\lambda\mu\nu}(x)D_+(x-x')S_{\lambda\mu\nu} - \frac{3}{4}S^\lambda(x)D_+(x-x')S_\lambda(x')\right] \tag{94}$$

where

$$S^\lambda(x) = g_{\mu\nu}S^{\lambda\mu\nu}(x) \tag{95}$$

and

$$\partial_\lambda S^{\lambda\mu\nu}(x) = 0 \tag{96}$$

The momentum space transformation of eqs. (94)–(96) is

$$W(S) = \frac{1}{2}\int \frac{(dp)}{(2\pi)^4}\frac{1}{p^2 - i\varepsilon} \quad \times$$

$$\left[S^{\lambda\mu\nu}(-p)S_{\lambda\mu\nu}(p) - \frac{3}{4}S^\lambda(-p)S_\lambda(p)\right] \tag{97}$$

with

$$S^\lambda(p) = S_\nu^{\lambda\nu}(p) \tag{98}$$

and

$$p_\lambda S^{\lambda\mu\nu}(p) = 0 \tag{99}$$

The field is defined as usually

$$\delta W(S) = \int \frac{(dp)}{(2\pi)^4}\delta S^{\lambda\mu\nu}(-p)S_{\lambda\mu\nu}(p) \tag{100}$$

and any additional term containing p_λ, p_μ or p_ν as factors will not contribute in eq. (100) because of the source restriction (17). Then, the general form of the field with mass $m = 0$ is

$$\varphi_{\lambda\mu\nu}(p) = \frac{1}{p^2 - i\varepsilon}\left[S_{\lambda\mu\nu}(p) - \frac{1}{4}(g_{\mu\nu}S_\lambda(p) + g_{\nu\lambda}S_\mu(p) + g_{\lambda\mu}S_\nu(p)\right] \quad +$$

$$p_\lambda\varphi_{\mu\nu}(p) + p_\mu\varphi_{\nu\lambda}(p) + p_\nu\varphi_{\lambda\mu}(p) \tag{101}$$

where the cyclically related terms are required by the total symmetry of the tensor $\varphi_{\lambda\mu\nu}$ and $\varphi_{\mu\nu}(p)$ is the new symmetrical tensor determined by the source restriction. In order to use the tensor $\varphi_{\mu\nu}(p)$, first, let us note that

$$\varphi_\lambda = \varphi_{\lambda\nu}^{\nu}(p) = \frac{1}{p^2 - i\varepsilon}\left(-\frac{1}{2}\right)S_\lambda(p) + p_\lambda\varphi(p) + 2p^\nu\varphi_{\lambda\nu}(p) \tag{102}$$

where

$$\varphi(p) = \varphi_\nu^\nu(p) \tag{103}$$

Multiplication of eq. (19) by p^λ then introduces just the combination equal to $\frac{1}{2}(\varphi_\lambda - p_\lambda\varphi)$ and we get:

$$p^2\varphi_{\mu\nu} = p^\lambda\varphi_{\lambda\mu\nu} - \frac{1}{2}(p_\mu\varphi_\nu + p_\nu\varphi_\mu) + p_\mu p_\nu\varphi \tag{104}$$

An equation for $\varphi(p)$ is produced by combination

$$p^\lambda p^\mu p^\nu\varphi_{\lambda\mu\nu} = 3p^2p^\mu p^\nu\varphi_{\mu\nu} \tag{105}$$

with

$$p^2 p^\mu p^\nu \varphi_{\mu\nu} = p^\lambda p^\mu p^\nu \varphi_{\lambda\mu\nu} - p^2 p^\lambda \varphi_\lambda + (p)^2 \varphi \tag{106}$$

namely

$$(p^2)^2 \varphi = p^2 p^\lambda \varphi_\lambda - \frac{2}{3} p^\lambda p^\mu p^\nu \varphi_{\lambda\mu\nu} \tag{107}$$

The momentum space version of the field equation for $\varphi_{\lambda\mu\nu}(p)$ is now derived as

$$\begin{aligned} p^2 \varphi_{\lambda\mu\nu} - p_\lambda p^{\lambda'} \varphi_{\lambda'\mu\nu} - p_\mu p^{\mu'} \varphi_{\lambda\mu'\nu} - p_\nu p^{\nu'} \varphi_{\lambda\mu\nu'} &+ \\ p_\mu p_\nu \varphi_\lambda + p_\nu p_\lambda \varphi_\mu + p_\lambda p_\mu \varphi_\nu - 3 p_\lambda p_\mu p_\nu \varphi &= \\ S_{\lambda\mu\nu} - \frac{1}{4}(g_{\mu\nu} S_\lambda + g_{\nu\lambda} S_\mu + g_{\lambda\mu} S_\nu) & \end{aligned} \tag{108}$$

From this equation we can derive eq. (25) by multiplication with $p^\lambda p^\mu p^\nu$. The field equation involves only the combination

$$\varphi_{\lambda\mu\nu}(p) - 3 \frac{p_\lambda p_\mu p_\nu}{p^2 - i\varepsilon} \varphi(p) \tag{109}$$

since

$$p_\lambda p^{\lambda'} (p_{\lambda'} p_\mu p_\nu) - (p^2) p_\mu p_\nu p_\lambda = 0 \tag{110}$$

and it means that this combination is redefinition of $\varphi_{\lambda\mu\nu}$ which means that $\varphi(p)$ can be transformed away. Thus the final set of the field equations for spin 3 and $m = 0$ is with $\varphi = 0$:

$$\begin{aligned} p^2 \varphi_{\lambda\mu\nu} - p_\lambda p^{\lambda'} \varphi_{\lambda'\mu\nu} - \dots + p_m p_\nu \varphi_\lambda + \dots &- \\ g_{\mu\nu}(p^2 \varphi_\lambda + \frac{1}{2} p_\lambda p^{\lambda'} \varphi_{\lambda'} - p^{\mu'} p^{\nu'} \varphi_{\lambda\mu'\nu'}) - \dots = S_{\lambda\mu\nu} & \end{aligned} \tag{111}$$

where dots represents the terms that are generated by the cyclic permutation from the given ones.

The algebraic consequence of this equation is as follows:

$$p^\lambda S_{\lambda\mu\nu}(p) = g_{\mu\nu} \frac{1}{4} p^\lambda S_\lambda(p) \tag{112}$$

and it is consistent with the vanishing divergence of the source, but does not imply it.

7 Spin 3 electromagnetic gravity

We have seen that from the massive field action for spin 3, the massless limit is generated with the following action (Schwinger, 1970):

$$W(S, m = 0) = \frac{1}{2} \int (dx)(dx')$$

$$\left[S^{\lambda\mu\nu}(x)D_+(x-x')S_{\lambda\mu\nu} - \frac{3}{4}S^{\lambda}(x)D_+(x-x')S_{\lambda}(x')\right] \quad (113)$$

where

$$S^{\lambda}(x) = g_{\mu\nu}S^{\lambda\mu\nu}(x) \quad (114)$$

and

$$\partial_{\lambda}S^{\lambda\mu\nu}(x) = 0 \quad (115)$$

According to Schwinger (1970) - *Ordinary matter possesses no conserved physical properties that could be identified with the ones described by the local conservation law (115), or indeed for any* $n \geq 3$. *The inability to construct their sources strongly affirms the empirical absence of the particles. But perhaps one should not reject totally the possibility of eventually encountering such properties, and the associated particles, under circumstances that are presently unattainable.*

Here we think that matter with the spin 3 is the electromagnetic-gravity bacause it involves the equation

$$electromagnetism(spin1) + gravity(spin2) = electromagnetic - gravity(spin3), \quad (116)$$

which is the scientific physical transformation of the arithmetic equation 1+2=3.

8 Discussion

We have seen that the Einstein equivalence principle was not used in the Schwinger spin 2 gravity. Einstein formulated this principle with two reference frames, K and K' where K is a uniform gravitational field, whereas K' has no gravitational field. It is uniformly accelerated in such a way that objects in the two frames experience identical forces. According to Einstein systems K and K' are physically exactly equivalent. This assumption of exact physical equivalence makes it impossible to speak of the absolute acceleration of the system of reference, just as the usual theory of relativity forbids to talk of the absolute velocity of a system. It makes the equal falling of all bodies in a gravitational field (Einstein, 1911).

Or, *Inertia and gravity are identical; hence and from the results of special relativity theory it inevitably follows that the symmetric fundamental tensor* $g_{\mu\nu}$ *determines the metric properties of space, of the motion of bodies due to inertia in it, and, also, the influence of gravity* (Einstein, 1918).

According to Fock (1964), *principle of equivalence is understood to be the statement that in some sense a field of acceleration is equivalent to a gravitational field. It means that by introducing a suitable system of coordinates (which is usually interpreted as an accelerated frame of reference) one can so transform the equations of motion of a mass point in a gravitational field that in this new system they will have the appearance of equations of motion of a free mass point. Thus a gravitational field can, so to speak, be replaced, or rather imitated, by a field of acceleration. Owing to the equality of inertial and gravitational mass such a transformation is the same for any value of the mass of* *the particle. But it will succeed in its purpose only in an infinitesimal region of space, i.e. it will be strictly local. In the general case the transformation described corresponds mathematically to passing to a locally geodesic system of coordinates.*

The principle of equivalence states that it is impossible to distinguish between the action on a particle of matter of a constant acceleration, or, of static support in a gravitational field (Lyle, 2008). We have seen that Schwinger theory does not use the principle of equivalence.

The controversy between different opinions on the principle of equivalence can be easily solved by the physical definition of gravity and inertia. Namely: gravity is the specific form of matter, or, form of vacuum. And inertia is the interaction of the massive body with vacuum which is the physical medium. So, Gravity is form of matter and inertia is form of interaction.

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