The magnetic moment for pedestrians

Miroslav Pardy
Department of Physical Electronics
and
The Laboratory of Plasma Physics
Faculty of Science, Masaryk University
Kotlářská 2, 611 37 Brno
Czech Republic
e-mail:pamir@physics.muni.cz

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Abstract

The causal arrangement of the two extended emission and detection sources and the photon source is considered. The emission source emitts the charged particle and photon. The photon source which is located betwen emission and detection sources is capable of the transmitting of space-like momenta. The photon propagates undisturbed between emission and detection sources, while the charged particle is reflected by the extended photon source. The result of such process is the modification of the vacumm-to-vacuum amplitude in form of the additional term which can be interpreted as the additional magnetic moment of electron. The derivation is realized with the mathemathical simplicity and the Schwinger pedagogical clarity.

We consider the causal arrangement of the two extended emission and detection sources sources K_2, K_1 and the photon source J (Schwinger, 1973). The source K_2 emitts the charged particle (electron) and photon, K_1 is the detection source of those particles. The photon source J which is located betwen sources K_1 and K_2 is capable of the transmitting of space-like momenta. The photon propagates undisturbed between K_2 and K_1, while the charged particle is reflected by the extended photon source. The result of such process is the modification of the vacumm-to-vacuum amplitude in form of the additional term which can be interpreted as the additional magnetic moment of electron. The described situation can be graphically expressed (Schwinger, 1973).

We have seen (Schwinger, 1973) that the vacuum amplitude of the two-particle exchange betwen sources, when one particle is photon and the other particle is electron, is as follows:

$$\langle 0_+|0_-\rangle = \int (dx)(dx')(d\xi)(d\xi') \quad \times$$

$$iJ_1^\mu(\xi)\eta_1(x)\gamma^0|_{eff}D_+(\xi-\xi')G_+(x-x')iJ_{2\mu}(\xi')\eta_2(x')|_{eff}, \qquad (1)$$

which after the specification of the effective sources and transformation $G_+ \to G_+^A$, can be transcribed as

$$\langle 0_+|0_-\rangle = e^2 \int (dx)(dx')\psi_1(x)\gamma^0\gamma^\mu D_+(x-x')G_+^A(x,x')\gamma_\mu\psi_2(x'). \qquad (2)$$

Using the symbolic equation for the Green function G_+^A

$$(\gamma\Pi + m)G_+^A = 1, \qquad (3)$$

where

$$\Pi = p - eqA, \qquad (4)$$

we can transcribe eq. (2) in the momentum representation as the following amplitude (Schwinger, 1973):

$$\langle 0_+|0_-\rangle = e^2 \int \frac{(dk)}{(2\pi)^4}\psi\gamma^0\gamma^\mu\frac{1}{k^2}\,\frac{m-\gamma(\Pi-k)}{(\Pi-k)^2 - eq\sigma F + m^2}\gamma_\mu\psi, \qquad (5)$$

where it is supposed the presence of the term $-i\varepsilon$ and where we have dropped the indeces 1 and 2.

Using the well known relations

$$\frac{1}{(\Pi-k)^2 - eq\sigma F + m^2} = i\int_0^\infty ds e^{-is[(\Pi-k)^2 - eq\sigma F + m^2]} \qquad (6)$$

and

$$\frac{1}{k^2} = i\int_0^\infty ds e^{-ik^2 s}, \tag{7}$$

which can be combined as

$$\frac{1}{k^2}\,\frac{1}{(\Pi-k)^2 - eq\sigma F + m^2} = \int_0^\infty ds_1 ds_2 e^{-is_1[(\Pi-k)^2 - eq\sigma F + m^2] - is_2 k^2}, \tag{8}$$

we get, using transformations

$$s_1 u = su; \quad s_2 = s(1-u); \quad ds_1 ds_2 = s\; ds\; du \tag{9}$$

the following identity:

$$\frac{1}{k^2}\frac{1}{(\Pi-k)^2 - eq\sigma F + m^2} = -\int_0^\infty ds\; s\int_0^1 du e^{-is\chi(u)}, \tag{10}$$

where

$$\chi(u) = u\left[(\Pi-k)^2 - eq\sigma F + m^2\right] + (1-u)k^2 \quad =$$

$$(k - u\Pi)^2 + u(1-u)(\Pi^2 - eq\sigma F + m^2) + u^2(m^2 - eq\sigma F). \tag{11}$$

The vacuum amplitude is then

$$\langle 0_+|0_-\rangle =$$

$$-e^2\int \frac{(dk)}{(2\pi)^4}\psi\gamma^0\gamma^\mu\int_0^\infty sds\int_0^1 du\;\frac{1}{k^2}\left(m - \gamma(\Pi-k)\right)e^{-is\chi(u)}\gamma_\mu\psi. \tag{12}$$

In the absence of the electromagnetic field, we can write (Schwinger, 1973):

$$e^2\int\frac{(dk)}{(2\pi)^4}e^{-is\chi(u)} = -i\frac{\alpha}{4\pi}\;\frac{1}{s^2}e^{-ism^2u^2}e^{-is\mathcal{H}}, \tag{13}$$

where

$$\mathcal{H} = u(1-u)\left(m^2 - (\gamma\Pi)^2\right) \tag{14}$$

and therefore

$$-\int_0^\infty sds \int_0^1 due^2 \int \frac{(dk)}{(2\pi)^4} \left(m - \gamma(\Pi - k)\right) e^{-is\chi(u)} \quad =$$

$$-i\frac{\alpha}{2\pi} m \int_0^1 du(1+u) \int_0^\infty \frac{ds}{s} e^{-is(m^2u^2+\mathcal{H})}. \tag{15}$$

After including the spin term in case of the weak electromagnetic field, we write

$$e^2 \int \frac{(dk)}{(2\pi)^4} e^{-is\chi(u)} \quad =$$

$$-i\frac{\alpha}{4\pi}\frac{1}{s^2} e^{-ism^2u^2} e^{-is\mathcal{H}}(1 + isu^2 eq\sigma F). \tag{16}$$

After the same mathematical manipulations, we can write for $\mathcal{H} \to 0$ (Schwinger, 1973):

$$e^2 \int \frac{(dk)}{(2\pi)^4} \gamma^\mu \left(m - \gamma(\Pi - k)\right) e^{-is\chi} \gamma_\mu \quad \to$$

$$-\frac{\alpha}{2\pi}\frac{m}{s} u^2(1+u) e^{-ism^2u^2} e^{eq\sigma F} + \frac{\alpha}{\pi}\frac{m}{s} e^{-ism^2u^2} eq\sigma F \quad =$$

$$\frac{\alpha}{2\pi}\frac{m}{s} u^2(1-u) e^{-ism^2u^2} eq\sigma F. \tag{17}$$

The integrals indicated in eq. (15) then give

$$i\frac{\alpha}{2\pi}\frac{1}{m}\int_0^1 du(1-u) eq\sigma F = i\frac{\alpha}{2\pi}\frac{eq}{2m}\sigma F \tag{18}$$

and the additional term obtained by the space-time extrapolation

$$\int (dx) \frac{1}{2} \psi(x) \gamma^0 \frac{\alpha}{2\pi}\frac{eq}{2m} \sigma F \psi(x) \tag{19}$$

states the $\alpha/2\pi$ supplement to the magnetic moment.

So, using the derivation of the process with the mathematical simplicity, we can say that the magnetic moment theory is for pedestrians.

Discussion

We have conisedered the situation of motion of electron in electromagnetic effeld. Deviations from the primitive electromagnetic interaction appeared. This effect is caused by the existence of the subsequent interaction which is not present in case of the primitive electromagnetic interactions. In such a way such interpretation imply modifications of the efective electromagnetic coupling. We expressed the experimental consequence of this fact by a simple modification of algorithm for calculation of the charged particle propagation function. We use here the Schwinger source method of quantum field theory (Schwinger, 1970, 1973, 1989; Dittrich, 1978). The derivation is performed with the mathemathical simplicity and the Schwinger pedagogical clarity. The standard calculation of the magnetic moment of electron can be seen, for instance, in the classical textbooks (Itzykson et al., 1980)

The magnetic moment of the Lee model was calculated by author at the different article with the interesting results. (Pardy, 1979). The magnetic moment investigation can be extended to further physical objects defined as Nobelian problems. Namely, the amomalous magnetic moment of proton and antiproton, neutron and antineutron, neutrino and antineutrino, omega-meson and omega-antimeson and so on. The actual problem is the anomalous magnetic moment of muon (Jegerlehner, 2008, 2017), the anomalous magnetic moments of all chemical elements, and, 30.000.000 organic compounds. So, the goals of the particle physics of anomalous magnetic moment are great.

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