A New Principle of Least Action

Alejandro A. Torassa

(2014) Buenos Aires, Argentina
atorassa@gmail.com

Abstract

In classical mechanics, this paper presents a new principle of least action which is invariant under transformations between reference frames and which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

The New Principle of Least Action

If we consider two particles i and j then the new principle of least action is given by:

$$\delta \int_{t_1}^{t_2} L_{ij}\, dt = 0$$

$$\delta \int_{t_1}^{t_2} (T_{ij} - V_{ij})\, dt = 0$$

$$T_{ij} = +\,{}^1\!/_2\, m_i\, m_j\, [(\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{v}_i - \mathbf{v}_j) + (\mathbf{a}_i - \mathbf{a}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j)]$$

$$V_{ij} = -\,{}^1\!/_2\, m_i\, m_j \left[2 \int \left(\frac{\mathbf{F}_i}{m_i} - \frac{\mathbf{F}_j}{m_j} \right) \cdot d(\mathbf{r}_i - \mathbf{r}_j) + \left(\frac{\mathbf{F}_i}{m_i} - \frac{\mathbf{F}_j}{m_j} \right) \cdot (\mathbf{r}_i - \mathbf{r}_j) \right]$$

where m_i and m_j are the masses of particles i and j, $\mathbf{r}_i$, $\mathbf{r}_j$, $\mathbf{v}_i$, $\mathbf{v}_j$, $\mathbf{a}_i$ and $\mathbf{a}_j$ are the positions, the velocities and the accelerations of particles i and j, and $\mathbf{F}_i$ and $\mathbf{F}_j$ are the net (conservative) forces acting on particles i and j.

The Lagrangian L_{ij} is invariant under transformations between reference frames.

The Lagrangian L_{ij} can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.