

From Multifractal Analysis to Quantum Field Theory

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Abstract

The multifractal formalism and the Euclidean Path-Integral formalism of quantum field theory (QFT) share an identical mathematical structure; both are built from the duality between a “free energy” and a conjugate “order parameter”. We make this correspondence explicit, deriving the propagator, the vertex functions, the perturbative (Feynman-diagram) expansion, and the Callan–Symanzik equation from the invariant measure properties of chaotic attractors. We identify where the correspondence is exact (the algebraic structure of Legendre duality, cumulant expansions, and RG fixed points) and where it is a physically motivated analogy requiring further justification (the identification of a specific attractor with the UV completion of a specific QFT, e.g. the Standard Model). A worked example using the Feigenbaum attractor of the logistic map illustrates the dictionary numerically. We close by outlining the conditions under which the correspondence would constitute a genuine non-perturbative definition of a QFT, rather than a suggestive isomorphism of formalisms.

Keywords: multifractals; quantum field theory; Renormalization Group (RG); Legendre transform; singularity spectrum; Feigenbaum universality; Standard Model; anomalous dimension.

Author’s note. This paper develops our previous work entitled “On the Geometric Foundations of Complex Dynamics” [15]. It expands each step of the construction sketched in Section 4.5 of that paper, supplies the intermediate derivations that were compressed there, adds a worked numerical example, and discusses the scope and limits of the correspondence. Equation numbers and section labels are internal to this paper; where a result is inherited directly from the parent paper it is marked with a citation to Ref. [15].

1 Introduction

The paper is motivated by a couple of well-known facts.

A. Thermodynamic interpretation of multifractals. According to Ruelle, the thermodynamic description of a chaotic attractor and its invariant measure proceeds as follows: a “free energy” $\tau(q)$ is defined from the scaling of moments of the measure, and its Legendre transform is the singularity spectrum $f(\alpha)$.

B. Wick rotation. The Euclidean path integral of a QFT is formally a partition function: $Z_{\text{QFT}} = \int \mathcal{D}\phi e^{-S_E[\phi]/\hbar}$, structurally identical to $Z_{\text{stat}} = \int \mathcal{D}\phi e^{-\beta H[\phi]}$, with $\beta H \leftrightarrow S_E/\hbar$ [15].

Combining these two facts yields a dictionary between multifractal geometry and Euclidean QFT (Table 2 of the parent paper). The purpose of the present paper is to take that dictionary seriously as a *derivation engine*: given only the multifractal partition sum $Z_q(\varepsilon)$ and its scaling exponent $\tau(q)$, we reconstruct, term by term, the standard construction of perturbative QFT — the generating functional, the propagator, the vertex functions, the Feynman expansion, and the Renormalization Group (RG). We then ask what has actually been derived and what has merely been relabeled.

The paper is organized as follows. Section 2 recalls the minimal multifractal formalism needed. Section 3 restates the Wick-rotation connection. Sections 4–8 carry out the five-step derivation — generating functional, propagator, vertex functions, Feynman expansion, and RG — with more intermediate algebra

than in the original presentation. Section 9 works a fully numerical example on the Feigenbaum attractor. Section 10 is a critical discussion of the correspondence’s status: exact isomorphism versus physical connection. Section 11 lists the conditions under which the program would become predictive.

2 Minimal Multifractal Apparatus

Let Λ be a strange attractor supporting a Sinai–Ruelle–Bowen (SRB) measure μ [10, 9, 11]. Cover Λ with boxes of size ε , indexed by i , with box measure $\mu_i(\varepsilon)$. Define the partition sum

$$Z_q(\varepsilon) = \sum_i \mu_i(\varepsilon)^q \sim \varepsilon^{\tau(q)}, \quad \tau(q) = (q-1)D_q, \quad (1)$$

where D_q is the generalized Rényi dimension. The Legendre-dual pair is

$$\alpha(q) = \frac{d\tau}{dq}, \quad f(\alpha) = q\alpha(q) - \tau(q), \quad (2)$$

with $f(\alpha)$ the Hausdorff dimension of the exponent- α layer $E_\alpha = \{x \in \Lambda : \alpha(x) = \alpha\}$ [15]. The function $\tau(q)$ plays the role of a (negative) free energy, q the role of inverse temperature, and α the role of specific energy, exactly as in the Ruelle thermodynamic formalism [15].

Two structural facts about (1)–(2) will do all the work below:

- (A) **Convexity.** $\tau(q)$ is convex in q (equivalently $f(\alpha)$ is concave in α), so the Legendre transform in (2) is invertible and involutive on its domain.
- (B) **Cumulant generation.** $\tau(q)$ is, up to normalization, a cumulant generating function: writing $\mu_i = e^{-s_i}$ for “local actions” $s_i = -\log \mu_i$, the moments $\langle e^{-(q-1)s} \rangle$ generate $\tau(q)$, so derivatives $d^n \tau / dq^n$ are cumulants of s under the q -tilted measure.

Both facts are inherited unchanged by any function built from a convex Legendre pair — which is precisely why the QFT correspondence below is not a coincidence of notation but a consequence of (A)–(B) alone.

3 The Wick-Rotation Connection, Restated

Analytic continuation of time, $t \rightarrow -i\tau$, turns the Feynman weight $e^{iS_M[\phi]/\hbar}$ into the Euclidean weight $e^{-S_E[\phi]/\hbar}$, giving

$$Z_{\text{QFT}}[J] = \int \mathcal{D}\phi \, e^{-S_E[\phi]/\hbar + \int J\phi}, \quad (3)$$

formally identical to a classical partition function at temperature $\hbar$ with source J conjugate to the field ϕ [15]. Combining (1)–(2) with (3) via the identification $\beta H \leftrightarrow S_E/\hbar$ produces the dictionary of Table 2 in the parent paper: $q \leftrightarrow 1/\hbar$, $\alpha \leftrightarrow \Delta_\phi$ (a scaling dimension), $\tau(q) \leftrightarrow -\hbar \ln Z_{\text{QFT}}$, and $f(\alpha) \leftrightarrow \rho(\Delta)$, a density of operator dimensions.

The physical content of the bridge is a single observation: **the Hölder exponent is a scaling dimension.** From $\mu(B(x, \varepsilon)) \sim \varepsilon^{\alpha(x)}$ and the conformal field theory (CFT) transformation law $\langle \mathcal{O}(\lambda x) \rangle = \lambda^{-\Delta_\mathcal{O}} \langle \mathcal{O}(x) \rangle$, both objects measure how a local density (of measure, or of a field’s correlations) responds to a rescaling of the observation window. Everything that follows is this one identification, propagated through the construction of QFT.

4 Step 1 — From Partition Sum to Generating Functional

Rewrite (1) as a sum over field configurations rather than boxes. Each box i at resolution ε is reinterpreted as a discretized field configuration ϕ_i on a lattice of spacing $\varepsilon = \Lambda_{UV}^{-1}$, with

$$\mu_i(\varepsilon) \longrightarrow e^{-S_E[\phi_i]}. \quad (4)$$

The moment order q is fixed to $1/\hbar$ (restoring units, $q\hbar = 1$), and the sum over boxes becomes a functional integral over configurations:

$$Z_{1/\hbar}(\varepsilon) = \sum_i e^{-S_E[\phi_i]/\hbar} \longrightarrow \int \mathcal{D}\phi e^{-S_E[\phi]/\hbar}. \quad (5)$$

To obtain the *generating* functional rather than the bare partition function, perturb the measure: $\mu_i \rightarrow \mu_i(1 + \delta_i)$ for a small deformation δ_i localized near box i . In the continuum limit this deformation is exactly a source term,

$$\delta_i \longrightarrow \int J(x) \phi(x) d^d x, \quad (6)$$

which multiplies the weight by $e^{\int J\phi}$. This gives

$$Z[J] = \int \mathcal{D}\phi \exp\left(-S_E[\phi] + \int J(x) \phi(x) d^d x\right), \quad (7)$$

matching (3), with $Z[0] = e^{\tau(q)}$ identifying the multifractal free energy as the QFT vacuum energy $-\ln Z_{\text{QFT}}$.

What this step establishes. The passage from (1) to (7) is a relabeling: “box index” becomes “field configuration,” “box measure” becomes “Boltzmann weight of a configuration,” and a measure perturbation becomes a source term. No dynamical content has been added — this step shows that the two formalisms use the *same combinatorial object* (a weighted sum/integral over a configuration space with a linear source coupling), not that one implies the other.

5 Step 2 — The Propagator from Two-Point Scaling

The pair-correlation integral of the SRB measure,

$$C(r) = \iint \Theta(r - |x - y|) d\mu(x) d\mu(y) \sim r^{D_2}, \quad (8)$$

is the multifractal analogue of a two-point function [15]. Differentiating $C(r)$ with respect to r gives the density–density correlation

$$\langle \mu(x) \mu(y) \rangle \sim |x - y|^{D_2 - d} \quad (9)$$

in d ambient dimensions. For a (locally) monofractal measure with common exponent α , the correlation dimension obeys $D_2 = 2\alpha - d$ — this is the statement that the measure of a ball of radius r scales as r^α , so the *joint* probability of two points within r of each other scales as the product of two such factors relative to the ambient volume, $r^{2\alpha} r^{-d} \cdot r^d = r^{2\alpha - d}$ after the appropriate normalization. Substituting into (9),

$$\langle \mu(x) \mu(y) \rangle \sim |x - y|^{2\alpha - 2d} \cdot |x - y|^d = |x - y|^{-(d - 2\alpha)}. \quad (10)$$

Under the dictionary $\alpha \leftrightarrow \Delta_\phi$, and identifying $\langle \mu(x)\mu(y) \rangle$ with $\langle \phi(x)\phi(y) \rangle$, this is exactly the CFT two-point function of a primary field of dimension Δ_ϕ :

$$\langle \phi(x)\phi(y) \rangle_0 = \frac{\delta^2 \ln Z[J]}{\delta J(x) \delta J(y)} \Big|_{J=0} \sim |x-y|^{-(d-2\Delta_\phi)}. \quad (11)$$

Fourier transforming to momentum space, $|x-y|^{-(d-2\Delta_\phi)} \rightarrow p^{2\Delta_\phi-d}$, and writing $\Delta_\phi = \frac{d-2}{2} + \frac{\eta}{2}$ for an anomalous dimension η , gives the dressed propagator

$$G(p) = \frac{1}{p^{2+\eta}} \approx \frac{1}{p^2} \left(1 + \eta \ln \frac{p^2}{\mu^2} \right), \quad (12)$$

the standard one-loop-resummed form. The multifractal correction $\delta\alpha = \alpha - \alpha_{\text{free}}$, where $\alpha_{\text{free}} = (d-2)/2$ is the exponent of a monofractal (free-field) measure, is thus the *geometric* origin of the anomalous dimension:

$$\eta = 2\delta\alpha. \quad (13)$$

Physical interpretation. Equation (13) says: a chaotic attractor whose local Hölder exponent deviates from the free-field value α_{free} behaves, under the dictionary, exactly as an interacting field whose propagator acquires anomalous scaling. The *width* of the singularity spectrum, $\Delta\alpha = \alpha_{\text{max}} - \alpha_{\text{min}}$ [15], is therefore a direct analog for the spread of anomalous dimensions across the operator content of the QFT.

6 Step 3 — Vertex Functions from the Legendre Duality

The 1PI (proper) vertex functional is the Legendre transform of $W[J] = \ln Z[J]$:

$$\Gamma[\phi_c] = W[J] - \int J(x)\phi_c(x) d^d x, \quad \phi_c(x) = \frac{\delta W}{\delta J(x)}. \quad (14)$$

Compare with (2): $f(\alpha) = q\alpha - \tau(q)$, $\alpha = d\tau/dq$. These are the *same transform*, term for term, under

$$\tau(q) \leftrightarrow -W[J], \quad q \leftrightarrow -J, \quad \alpha \leftrightarrow \phi_c, \quad f(\alpha) \leftrightarrow \Gamma[\phi_c]. \quad (15)$$

This is the strongest and most robust of the correspondences in this paper, because it rests only on fact (A) of Section 2: both $\tau(q)$ and $W[J]$ are convex generating functions of a linear source, so their Legendre duals ($f(\alpha)$ and $\Gamma[\phi_c]$) obey identical algebra irrespective of the physical system generating them.

Differentiating (14) twice gives the inverse propagator (the two-point 1PI function),

$$\Gamma^{(2)}(p) = p^2 + m^2 - \Pi(p^2), \quad (16)$$

with $\Pi(p^2)$ the self-energy. On the multifractal side, the analogous object is minus the curvature of the singularity spectrum, $-f''(\alpha)$, which measures the inverse *width* of the exponent distribution at fixed α : a sharply peaked (near-monofractal) $f(\alpha)$ has large $|f''|$, corresponding to a weakly-interacting theory with small self-energy Π ; a broad, strongly multifractal $f(\alpha)$ corresponds to large Π and strong coupling.

7 Step 4 — Feynman Diagrams from the Cumulant Expansion

Expand (7) in powers of the interaction part of S_E and in powers of J . This is the standard perturbative construction:

$$Z[J] = Z_0[J] \sum_{n=0}^{\infty} \frac{(-\lambda)^n}{n!} \left(\int \frac{\delta^4}{\delta J^4} \right)^n Z_0[J], \quad Z_0[J] = e^{\frac{1}{2} \int J G_0 J}. \quad (17)$$

On the multifractal side, $Z_q(\varepsilon)$ admits a cumulant expansion in $\log \varepsilon$:

$$Z_q(\varepsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} \kappa_n(q) [\log \varepsilon]^n, \quad \kappa_n(q) = \frac{d^n \tau}{dq^n}, \quad (18)$$

by fact (B) of Section 2: $\tau(q)$ is a cumulant generating function of the local actions $s_i = -\log \mu_i$ under the q -tilted measure. Matching order by order,

$$\kappa_1(q) \leftrightarrow \text{vacuum energy (tadpole)}, \quad \kappa_2(q) \leftrightarrow G_0(p), \quad \kappa_3(q) \leftrightarrow \lambda \Gamma^{(3)}, \quad \kappa_4(q) \leftrightarrow \lambda \Gamma^{(4)}, \dots \quad (19)$$

Each Feynman diagram at order λ^n corresponds to a specific product of cumulants $\kappa_{n_1} \cdots \kappa_{n_k}$ with $\sum n_i = 2n + 2$; the combinatorial (symmetry) factors of the diagrams arise from the same multinomial coefficients that appear in (18), just as Wick's theorem generates them in the standard derivation.

A word of caution. The *combinatorics* of (17) and (18) are genuinely the same — any cumulant expansion of a convex generating function produces this diagrammatic bookkeeping, whether the underlying object is a field theory or a dynamical system's measure. What is *not* automatic is the convergence radius and physical interpretation of the expansion. In QFT, λ is a fixed coupling constant with a well-defined perturbative regime. In the multifractal picture, the expansion parameter is effectively $\Delta\alpha$ (the departure from monofractality); the expansion is under control only when the attractor is *weakly* multifractal. Section 9 quantifies this for the Feigenbaum attractor, whose $\Delta\alpha \approx 0.57$ is not small — signaling that the “QFT” dual to the Feigenbaum attractor, if the correspondence is taken literally, is intrinsically strongly coupled and outside the reach of the diagrammatic expansion.

8 Step 5 — Renormalization Group from Dimensional Flow

UV divergences in QFT arise as loop momenta $p \rightarrow \infty$; in the multifractal picture this is the resolution limit $\varepsilon \rightarrow 0$. Dimensional regularization works in $d = 4 - \varepsilon_{\text{reg}}$ dimensions with renormalization scale μ , subject to the Callan–Symanzik equation

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - n\gamma(g) \right) G^{(n)} = 0. \quad (20)$$

The multifractal statement that the free energy $\tau(q)$ is scale-independent in the $\varepsilon \rightarrow 0$ limit,

$$\varepsilon \frac{\partial \tau}{\partial \varepsilon} = 0 \iff \mu \frac{\partial}{\partial \mu} (-\ln Z_{\text{QFT}}) = 0, \quad (21)$$

is the analogue of (20) at $n = 0$. If the effective (generalized Rényi) dimension flows from $D_{q,n}$ to $D_{q,n+1}$ across successive levels of a hierarchical decomposition of the attractor [15], the beta function is identified with the discrete derivative of $\tau(q)$ with respect to resolution:

$$\beta(g) \leftrightarrow - \frac{d\tau}{d \log \varepsilon} \Big|_q = (q-1) \frac{dD_q}{d \log \varepsilon}, \quad (22)$$

and the anomalous dimension is the running of the Hölder exponent itself,

$$\gamma(g) \leftrightarrow \frac{d\alpha}{d \log \varepsilon}. \quad (23)$$

A fixed point of the multifractal RG, $d\alpha/d \log \varepsilon = 0$ together with $\beta = 0$, is simultaneously a zero of the QFT beta function and — by the standard equivalence between scale invariance and conformal invariance in unitary local theories — a candidate conformal fixed point. The Feigenbaum RG fixed point g^* [15] is proposed as the prototype: the doubling operator's single relevant eigenvalue $\delta = 4.6692 \dots$ plays the role of the leading relevant RG eigenvalue $y_t = 1/\nu$ at a Wilson–Fisher-type fixed point.

9 Worked Example: The Feigenbaum Attractor as a Toy “QFT”

To make Sections 4–8 concrete, we evaluate the dictionary numerically on the period-doubling attractor of the logistic map $f_r(x) = rx(1 - x)$ at $r = r_\infty \approx 3.56995$, using the data already reported in the parent paper [15].

Input data.

- Hausdorff dimension of the attractor: $\dim_H(\Lambda_\infty) \approx 0.5388$.
- Support of the singularity spectrum: $\alpha \in [\alpha_{\min}, \alpha_{\max}] \approx [0.3995, 0.9685]$, mean $\bar{\alpha} \approx 0.67$.
- Spread: $\Delta\alpha = \alpha_{\max} - \alpha_{\min} \approx 0.569$.
- Feigenbaum constant: $\delta = 4.669201 \dots$

Step 2 applied (propagator). Taking the ambient embedding dimension for the toy correspondence as $d = 1$ (the attractor lives on the interval), and identifying the free-field exponent as $\alpha_{\text{free}} = (d - 2)/2 = -1/2$ (formally, since $d = 1$ is below the physical range where this identification is standard, this is a purely illustrative bookkeeping choice), the multifractal deviation at the mean exponent is

$$\delta\alpha = \bar{\alpha} - \alpha_{\text{free}} \approx 0.67 - (-0.5) = 1.17 \implies \eta = 2\delta\alpha \approx 2.34.$$

An anomalous dimension of order unity is, by the standards of perturbative QFT, enormous — consistent with the qualitative point made in Section 7: the Feigenbaum attractor, if forced through the dictionary, corresponds to a maximally strongly-coupled dual theory, not a small perturbation of a free field. This is expected: r_∞ is the *accumulation point* of infinitely many bifurcations, i.e., the attractor is maximally non-uniform by construction.

Step 5 applied (beta function analog). Using the dimensional-flow relation (38) of the parent paper, $d_{\text{eff}}(n) = d_\infty + c_1\delta^{-n} + c_2\delta^{-2n} + \dots$ [15], the discrete beta function between hierarchy levels n and $n + 1$ is

$$\beta_n \equiv d_{\text{eff}}(n + 1) - d_{\text{eff}}(n) \approx c_1 \delta^{-n} (\delta^{-1} - 1),$$

which decays geometrically with rate $\delta^{-1} \approx 0.2141$ — the discrete analogue of a beta function approaching an IR-attractive fixed point at $d_\infty \approx 0.5388$ with universal approach rate fixed by δ . This is the cleanest and most defensible piece of the correspondence: it is a legitimate statement about the *convergence rate of a renormalization-group-like recursion* (the doubling operator), independent of whether one accepts the deeper QFT identification.

Summary of the example. The propagator/anomalous-dimension identification (Step 2) is numerically extreme at the Feigenbaum point, flagging that this particular attractor sits far from any weakly-coupled QFT regime. The RG/dimensional-flow identification (Step 5) is numerically well-behaved and reproduces, by construction, the known universal approach to the fixed point governed by δ . This asymmetry is instructive: the parts of the dictionary that rely only on the *algebra of RG fixed points* are robust; the parts that require a literal quantitative match to a specific d -dimensional QFT’s field content are not yet justified by the data given.

10 Discussion: Isomorphism of Formalisms versus Physical Connection

It is useful to separate three distinct claims embedded in Sections 4–8, of decreasing certainty.

Claim 1 (algebraic isomorphism). The Legendre-transform pair $(\tau(q), f(\alpha))$ and the pair $(W[J], \Gamma[\phi_c])$ are instances of the same mathematical structure: a convex generating function and its dual. This claim is a theorem, not a hypothesis — it follows from convexity (fact A, Section 2) alone, and holds for *any* system described by a Legendre-dual pair of convex functions, whether or not that system has anything to do with a dynamical attractor.

Claim 2 (combinatorial isomorphism). The cumulant expansion of $\tau(q)$ and the perturbative (Feynman) expansion of $\ln Z[J]$ share the same diagrammatic bookkeeping. This too is close to a theorem: any exponential generating function admits a cumulant/diagram expansion with universal combinatorial (symmetry-factor) structure. What is *not* guaranteed is that the multifractal cumulants $\kappa_n(q)$ correspond to a *convergent, unitary, causal* QFT with the field content of, say, the Standard Model. The example of Section 9 shows the expansion can be driven far outside its perturbative regime.

Claim 3 (physical identification). That a *specific* dynamical system — the RG flow of the Standard Model’s gauge and Yukawa couplings entering a chaotic (Feigenbaum-type) regime in the deep UV — is *literally* the microscopic origin of the Euclidean path integral of the Standard Model, with the singularity spectrum $f(\alpha)$ fixing the operator spectrum of the UV completion [15]. This is a substantive physical connection, advanced in the cited prior work of the parent paper’s author [1, 2, 4, 5, 6], and it is *not* established by Claims 1–2 alone. Claims 1–2 show that *if* such an identification held, the resulting theory would automatically reproduce the standard formalism of propagators, vertices, and an RG equation with the right formal properties. They do not show that the SM’s UV RG flow *does* become chaotic, nor that its would-be Feigenbaum attractor has the specific $f(\alpha)$ needed to reproduce the observed SM spectrum. That is an independent dynamical claim about the beta functions of the SM at very high μ , requiring separate justification (and is explicitly listed as an open problem in [15], items (2) and (3), and in Refs. [1, 2, 3] there).

The value of Sections 4–8, properly read, is therefore this: they show that *whatever* the correct microscopic theory of the SM’s UV completion turns out to be, if it is governed by a chaotic attractor with a well-defined multifractal spectrum, the resulting effective description is *guaranteed* — by Claims 1–2 — to look like a QFT with anomalous dimensions set by $\Delta\alpha$ and a beta function set by the flow of D_q . This is a strong constraint-generating framework even before Claim 3 is settled: it tells you what to compute ($f(\alpha)$ of the candidate attractor) and what observable it predicts (the spectrum of anomalous dimensions / operator content), turning “the UV RG flow is chaotic” from a qualitative slogan into a program with calculable outputs.

11 Conditions for a Predictive Program

For the correspondence developed here to graduate from formal dictionary to predictive physics, three things are needed, in order of increasing difficulty:

1. **An independent computation of $f(\alpha)$ for the actual SM beta functions** in the regime where they are conjectured to become chaotic, rather than assuming a Feigenbaum-type universality class by analogy with the logistic map. The SM beta functions are multidimensional and non-generic; establishing that their UV flow undergoes a period-doubling (or any universal) route to chaos is a numerical RG question that has not, to our knowledge, been settled from first principles.
2. **A check of unitarity and locality** for the “QFT.” Legendre duality and cumulant expansions are agnostic to unitarity; an attractor’s $f(\alpha)$ could produce a $\Gamma[\phi_c]$ with no consistent unitary completion. This must be checked, not assumed.

3. **A quantitative match** between the predicted anomalous dimensions $\eta = 2\delta\alpha$ (Section 5) — or the predicted mass ratios via the Feigenbaum-scaling argument of the parent paper’s Appendix A [15] — and measured SM parameters, beyond the leading-order geometric-mean fit shown in the parent paper’s Figure 8(b). A statistically meaningful fit, with quantified uncertainties and a falsifiable prediction for at least one not-yet-precisely-measured SM parameter, would move Claim 3 from hypothesis toward evidence.

Until these are done, the results of this paper should be read as establishing the *formal compatibility* of multifractal dynamics with the mathematical structure of QFT — a non-trivial and, we think, useful fact — rather than as a derivation of the Standard Model from chaos.

12 Summary Table

Multifractal object	QFT object	Status of correspondence
Partition sum $Z_q(\varepsilon)$, eq. (1)	Generating functional $Z[J]$, eq. (7)	Relabeling (combinatorial)
Two-point orbit correlation $C(r) \sim r^{D_2}$, eq. (8)	Propagator $G(p) \sim p^{-(2+\eta)}$, eq. (12)	Physical connection ($\alpha \leftrightarrow \Delta_\phi$)
Legendre transform $f(\alpha) = q\alpha - \tau(q)$, eq. (2)	Effective action $\Gamma[\phi_c] = W[J] - \int J\phi_c$, eq. (14)	Exact algebraic isomorphism
Cumulants $\kappa_n(q) = d^n\tau/dq^n$, eq. (18)	n -point vertices $\Gamma^{(n)}$, eq. (19)	Exact combinatorial isomorphism; convergence not guaranteed
Curvature $-f''(\alpha)$	Inverse self-energy $[\Gamma^{(2)}]^{-1}$, eq. (16)	Physical connection
Scale-invariance of $\tau(q)$, eq. (21)	Callan–Symanzik equation, eq. (20)	Structural analogy
Dimensional flow $dD_q/d\log \varepsilon$, eq. (22)	Beta function $\beta(g)$	Physical connection, numerically well-behaved at Feigenbaum point (§9)
Running of α , eq. (23)	Anomalous dimension $\gamma(g)$	Physical connection

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- [15] E. Goldfain, *On the Geometric Foundations of Complex Dynamics*, GIREs preprint (2026) — parent paper, Section 4.5, from which the present derivation is developed.