Coherence Frame, Entanglement Conservation, and Einselection

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In this paper, the theory of coherence frame is developed. Two kinds of coherence frame are classified. Under coherence frame, the entanglement is conserved in the entanglement swapping process, without entanglement sudden death and birth. The einselection method for the preferred basis problem in the entangle process is shown as incomplete.

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Entanglement, the foundation of quantum dynamics and quantum information processing (QIP), is realized as a special kind of quantum correlation [1]. Many seminal arguments and concepts have been induced by entanglement in various contexts, e.g., the EPR paradox with hidden variable theory [2, 3], the collapse for measurement [4], the superselection rule (SSR) [5] etc. Recently, the method of coherence frame (CF) (or quantum reference frame) [6] is widely concerned in QIP, e.g., clock synchronization, phase reference etc. It proves that the method, einselection [7, 8], to resolve the preferred basis problem, and the method, the Aharonov-Susskind experiment [9], to challenge the SSR are based on the method of CF. In this Letter, we focus on the theory of CF relating to entanglement transfer and conservation.

The CF problem rises when the representation can not be easily decided, which is traditionally seldom analyzed [10]. For single-body system, e.g., spin $S_{1/2}$, only one representation can exist in reality at one time. If under the representation of $\hat{S}_x$, the measurement on $\hat{S}_y$ or $\hat{S}_z$ is meaningless. Yet, for double $S_{1/2}$ system, any component of the first spin $\hat{S}_1$ commutates with that of the second spin $\hat{S}_2$ [10]. Thus, $\{\hat{S}_1, \hat{S}_2\}$ forms the complete set of commuting observable (CSCO). In the entangled state $|\Phi\rangle = (|\uparrow_z\downarrow_x\rangle + |\downarrow_z\uparrow_x\rangle)/\sqrt{2}$, spin $\hat{S}_1$ ($\hat{S}_2$) is under the $\hat{S}_z$ ($\hat{S}_x$) representation, the z-component of $\hat{S}_1$ and x-component of $\hat{S}_2$ can be simultaneously defined. Another way to extend representation is to consider the time evolution, leading to the method of decohered history, i.e., framework [11]. Under one framework, the representation may differ in various sections of the history, e.g., $\hat{S}_x$ and $\hat{S}_z$ can be measured successively for single spin.

The theory of CF mainly involves the many-body and macroscopic superposition and dynamics, for which, the SSR was developed earlier [5]. Lacking one CF, leading to decoherence, is equivalent to SSR. Also, the method of SSR is shown similar with density matrix for mixed state [5]. Note that there are two cases for SSR. Case (1): SSR for isotropic particles, e.g., electrons, which is the ensemble. Case (2): SSR for particles not isotropic, e.g., electrons and protons, which is the mixture. The Hilbert space is the direct sum of that of each kind of particle, and the density matrix is block-diagonal. As shown in Ref. [9], it is possible to introduce coherence to the mixture. Thus, the difference of the two cases is merely apparent, and we need only refer to the first case for simplicity. Below, we discuss the relation of CF with SSR and density matrix in detail.

The density matrix is $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$, $\sum_i p_i = 1$, $|\psi_i\rangle$ is the state vector for the subsystem. In the spirit of SSR, the state vector of the whole ensemble can be [12]

$$|\Psi\rangle = \sum_i \gamma_i |\psi_i\rangle, \tag{1}$$

where the parameters γ_i are complex, with $|\gamma_i|^2 = p_i$.

The density matrix can be re-written as

$$\rho \equiv |\Psi\rangle\langle\Psi| = \sum_i |\gamma_i|^2 |\psi_i\rangle\langle\psi_i| \tag{2a}$$

$$\neq \sum_{i,j} \gamma_i\gamma_j |\psi_i\rangle\langle\psi_j|, \tag{2b}$$

with $|\langle\psi_i|\psi_j\rangle| \geq 0$, the nonorthogonality condition.

The illegality of the form in Eq. (2b) is ensured by the SSR; if not, the coherence in the ensemble will be considered redundantly. Physically, the form in Eq. (2a) is a kind of coarse-graining, since each sector $|\psi_i\rangle$ can be further decomposed as the superposition of orthogonal eignstates [12]. This indicates that the coherence due to nonorthogonality is localized and *globally inaccessible*. To surpass SSR, the CF and special interactions are needed to entangle the sectors together, as a result, the localized coherence will transfer globally and realize the coherence delocalization [13]. In the entangled state, parties can be the CF of each other, respectively. Tracing out one party, equivalent to SSR physically, leads to the apparent decoherence and classical state. Note that when SSR applies, the state is not necessarily classical.

To develop the complete theory of CF, primarily, we propose that CF should be clarified into several categories since there exist different quantum coherence and entangle processes. Briefly, we separate two kinds of CF:

(I). *Primary coherence frame*: quantum field, vacuum, environment (like noise, phonon, etc).

The role of the primary CF in some cases may be trivial then can be ignored. This type plays the similar role as space-time in classical mechanics.

(II). *Measurement-type coherence frame*: various interacting systems under specific conditions.

This type includes all kinds of CF beyond the primary type, in principle. In practice, this type mainly refers to the parties in the entangled or nonlocal states.

For clarity, we take the well known double-slit interference experiment for example. The electron ensemble shows fringes under the structure of double-slit. The primary CF of electron is the electromagnetic field, which, together with electron, form the entangled state

$$|\psi\rangle = \alpha_1|l\rangle|f_l\rangle + \alpha_2|r\rangle|f_r\rangle, \tag{3a}$$

where $|l(r)\rangle$ ($|f_{l(r)}\rangle$) stand for the eignstates of electron (field) at the left and right slits, $\alpha_1^2 + \alpha_2^2 = 1$. Physically, the electromagnetic field is the continuous variable system, i.e., $|f_l\rangle \approx |f_r\rangle \equiv |f\rangle$, thus, the state reduces to

$$|\psi\rangle = (\alpha_1|l\rangle + \alpha_2|r\rangle)|f\rangle, \tag{3b}$$

it is clear that the state of electron is superposed leading to the interference. The CF in this case is *trivial*, that it does not delocalize the coherence of electron to the field.

If we introduce measurement, which will interact with the electron thus become one of the parties of the global state. The state under measurement-type CF is

$$|\phi\rangle = (\beta_1|l\rangle|M_l\rangle + \beta_2|r\rangle|M_r\rangle)|f\rangle, \tag{4}$$

where $|M_{l(r)}\rangle$ are the states of the measurement, $\beta_1^2 + \beta_2^2 = 1$. The interfere process depends on the type of measurement, e.g., weak measurement [14], under which both the wave and particle properties can be observed.

The phenomenon for macroscopic objects, e.g., the ball, however, is different [15]. Note that the CF for this case should be the gravitational field relating to the classical dynamics. The entangled state reads

$$|\chi\rangle = \gamma_1|l\rangle|c_l\rangle + \gamma_2|r\rangle|c_r\rangle, \tag{5a}$$

where $|c_{l(r)}\rangle$ stand for the eignstates of field at the left and right slits, $\gamma_1^2 + \gamma_2^2 = 1$. Since states $|c_l\rangle$ and $|c_r\rangle$ can be distinguished via Newtonian mechanics, i.e., $\langle c_l|c_r\rangle = 0$, the state of the ball should be density matrix

$$\rho_b = \mathrm{tr}_c(|\chi\rangle\langle\chi|) = \gamma_1^2|l\rangle\langle l| + \gamma_2^2|r\rangle\langle r|, \tag{5b}$$

which brings the classical results and the absence of interference. In addition, this does not mean the movement in gravitational field is classical, in contrast, the corresponding effects of quantum coherence can be significant on the larger scale in Quantum Cosmology [16].

Entanglement transfer under CF. Next, we explore the basic entanglement transfer (re-distribution) process, a kind of coherence delocalization, under CF. It is reasonable to assume that for one system there exists at least one CF, which leads to the bi-party entanglement. This is similar with the so-called "pure universe" model for other studies [17, 18]. Under the Schmidt representation, the entangled state is

$$|\Psi\rangle = \sum_i^n \lambda_i|s_i^A\rangle|e_i^A\rangle, \tag{6a}$$

where $|s_i^A\rangle$ ($|e_i^A\rangle$) is the basis for the system (referred as) A (CF). If $|\langle e_i|e_j\rangle| \approx 1$, i.e., the disturbance and reference effect of the CF to the system is trivial, thus, the CF can be treated classically, which is a kind of classical limit satisfying the correspondence principle [19].

Suppose, there exists another system (referred as B) which will be entangled together with the system A. The state of system B with its CF is expressed as

$$|\Phi\rangle = \sum_j^m \gamma_j|s_j^B\rangle|e_j^B\rangle, \tag{6b}$$

where $|s_j^B\rangle$ ($|e_j^B\rangle$) is the basis for the system B (CF). The general bi-party entangle process under CF takes as

$$|\Psi\rangle|\Phi\rangle \to \sum_k^w \alpha_k|s_k^A\rangle|s_k^B\rangle \sum_k^w \beta_k|e_k^A\rangle|e_k^B\rangle, \tag{7}$$

with $w \le \min(n, m)$, the coefficients λ_i, γ_j, α_k, β_k guarantee the normalization rule. This process is similar with the entanglement swapping process firstly realized by Bell-type states [20]. The von Neumann premeasurement for the collapse model [4] ignores the roles of environment and quantum openness [18, 21], i.e., ignoring the CF, which can be deduced by taking the classical limit of the swapping process.

Further, we study the entanglement transfer quantitatively. One central problem is that too many entanglement measures exist especially for mixed state [22]. The main physical reason is there does exist the mismatch between entanglement and density matrix. Entanglement, as the many-body property, is defined referring to the number of parties, while density matrix is defined referring to state instead of parties. As a result, the expression *entanglement in density matrix* is not complete. Below, we present our method of entanglement measure. Generally, density matrix can be two types: ensemble of single-body system and ensemble of many-party system (with or without noise), only the later type contains entanglement. For the bi-party system, the ensemble-entangled qudit [12] is written as

$$\begin{aligned} \rho_{eE} &= \sum_\xi p_\xi|\psi_\xi\rangle\langle\psi_\xi| \\ &= \sum_\xi p_\xi \sum_{i,j} \lambda_i^\xi \lambda_j^\xi |A_i\rangle\langle A_j| \otimes |B_i\rangle\langle B_j|, \end{aligned} \tag{8}$$

with $\sum_i(\lambda_i^\xi)^2 = 1$, $\sum_\xi p_\xi = 1$, $|A_i\rangle$ ($|B_i\rangle$) is the local eigenstate of subsystem A (B), λ_i^ξ is Schmidt coefficient.

When $\xi = 1$, the state reduces to the pure entangled qudit. The degree of entanglement [12] for the pure entangled qudit state is defined as

$$E \equiv \sum_{i<j}^n |\lambda_i||\lambda_j|, \tag{9}$$

which is the entanglement monotone [22]. This entanglement measure characterizes the *distributed coherence*, different from information (entropy).

For the general mixed state ρ, the method of decomposition is employed to extract its entanglement

$$\rho = \pi\rho_{eE} + (1-\pi)\rho', \tag{10}$$

with $\pi \in [0,1]$. The state ρ can be viewed as the result of the initial state ρ_{eE} disturbed by ρ', thus, the entanglement in ρ is defined as $E(\rho) = \pi E(\rho_{eE})$. We name state ρ_{eE} as the *natural point* of state ρ for convenience.

Physically, there exist problems of the entanglement of formation E_f [23], which relies on the decomposition to entangled qudit. However, from Eq. (10), there are situations noise or product states (ρ') are added, so that E_f and the related concurrence do not directly quantify entanglement. The detailed properties of this measure have partly been discussed in Ref. [12].

For example, Werner state $|\rho_w\rangle = \frac{1-z}{4}\mathrm{I} + z|\Psi^-\rangle\langle\Psi^-|$ is naturally the mixture of noise and singlet, the entanglement is just $z/2$. For the X-type state with $\rho_{14} = \rho_{41}^* = w$, $\rho_{23} = \rho_{23}^* = z$, the entanglement equals $|w| + |z|$.

For convenience, we model system A and B also their CF (environment) α and β as qubit. Based on Eq. (6), the initial state for the global four-party system is set as

$$\begin{aligned} |\Upsilon\rangle &= |\psi_{A\alpha}\rangle \otimes |\psi_{B\beta}\rangle \qquad (11)\\ &= (a_1|s_1^A e_1^A\rangle + a_2|s_2^A e_2^A\rangle) \otimes (b_1|s_1^B e_1^B\rangle + b_2|s_2^B e_2^B\rangle), \end{aligned}$$

after swapping, which can also be expressed as

$$|\Upsilon\rangle = |\psi_{AB}^+\rangle|\psi_{\alpha\beta}^+\rangle + |\psi_{AB}^-\rangle|\psi_{\alpha\beta}^-\rangle + |\phi_{AB}^+\rangle|\phi_{\alpha\beta}^+\rangle + |\phi_{AB}^-\rangle|\phi_{\alpha\beta}^-\rangle, \tag{12}$$

where

$$\begin{aligned} |\psi_{AB}^\pm\rangle &= s_1|s_1^A s_1^B\rangle \pm s_2|s_2^A s_2^B\rangle, \qquad (13)\\ |\phi_{AB}^\pm\rangle &= t_1|s_1^A s_2^B\rangle \pm t_2|s_2^A s_1^B\rangle,\\ |\psi_{\alpha\beta}^\pm\rangle &= x_1|e_1^A e_1^B\rangle \pm x_2|e_2^A e_2^B\rangle,\\ |\phi_{\alpha\beta}^\pm\rangle &= y_1|e_1^A e_2^B\rangle \pm y_2|e_2^A e_1^B\rangle, \end{aligned}$$

the coefficients a_i, b_i, s_i, t_i, x_i, and y_i $(i = 1, 2)$ satisfy the normalization rule, also, $a_1b_1 = 2s_1x_1$, $a_2b_2 = 2s_2x_2$, $a_1b_2 = 2t_1y_1$, $a_2b_1 = 2t_2y_2$. Under symmetrical condition, it reduces to the swapping for Bell's state [20].

From the definition in Eq. (9), the entanglement of state, e.g., $|\psi_{A\alpha}\rangle$ is $|a_1a_2|$. It is easy to find that the entanglement during the swapping satisfies

$$\begin{aligned} &E(|\psi_{A\alpha}\rangle)E(|\psi_{B\beta}\rangle) = \qquad (14)\\ &\quad E(|\psi_{AB}^+\rangle)E(|\psi_{\alpha\beta}^+\rangle) + E(|\psi_{AB}^-\rangle)E(|\psi_{\alpha\beta}^-\rangle)\\ &\quad + E(|\phi_{AB}^+\rangle)E(|\phi_{\alpha\beta}^+\rangle) + E(|\phi_{AB}^-\rangle)E(|\phi_{\alpha\beta}^-\rangle), \end{aligned}$$

calculated as $|a_1a_2b_1b_2| = 4|t_1t_2y_1y_2| = 4|s_1s_2x_1x_2|$, which can be verified by the coefficients relation above. If we view the entanglement of the four-party state $|\Upsilon\rangle$ as the product of the entanglement of the bi-party state $|\psi_{A\alpha}\rangle$ and $|\psi_{B\beta}\rangle$, the relation in Eq. (14) stands for a kind of *conservation of entanglement* during the swapping. If the initial state of the global state is other types, it is easy to check that the conservation still exists. For the general mixed state, based on the decomposition in Eq. (10), the entanglement is also conserved.

As the example, we study one actual system from Ref. [24] relating to the entanglement sudden death and birth (ESDB) [25]. The model contains the entangled cavity photons affected by dissipation (i.e., CF). The initial state is set as $|\Phi_0\rangle = (\alpha|0\rangle_{c_1}|0\rangle_{c_2} + \beta|1\rangle_{c_1}|1\rangle_{c_2})|\bar{\mathbf{0}}\rangle_{r_1}|\bar{\mathbf{0}}\rangle_{r_2}$, the entanglement, amount to $|\alpha\beta|$, is distributed between the two cavity photons. The reduced density matrix of the two-cavity is $\rho_{c_1c_2}$ [24], from Eq. (10), it is direct to get its natural point σ, written by elements, $\sigma_{11} = \alpha^2/\pi$, $\sigma_{44} = \beta^2\xi^4/\pi$, $\sigma_{14} = \sigma_{41} = \alpha\beta\xi^2/\pi$, others zero. The parameter $\pi = \alpha^2 + \beta^2\xi^4$, $\xi = \sqrt{1-\chi^2} = \exp(-\kappa t/2)$, κ is the decay constant. Thus, the entanglement of state $\rho_{c_1c_2}$ is $E(\rho_{c_1c_2}) = |\alpha\beta|\xi^2$. Following the similar method, the entanglement for the state of the two-reservoir is $E(\rho_{r_1r_2}) = |\alpha\beta|\chi^2$. Thus, the conservation reads

$$E(|\Phi_0\rangle) = E(\rho_{c_1c_2}) + E(\rho_{r_1r_2}) = |\alpha\beta|. \tag{15}$$

As time evolves, $E(\rho_{c_1c_2})$ of the two-cavity decreases exponentially, while $E(\rho_{r_1r_2})$ of the two-reservoir increases exponentially. There is no ESDB, i.e., the localized and distributed coherence translate mutually.

Preferred basis problem. From the method of CF, we discuss the preferred basis problem (PBP), which has been studied via the einselection approach [7, 8]. We will show, yet, the method of einselection is incomplete.

This method is described via the Stern-Gerlach experiment, as shown in the Fig. 1 of Ref. [7]. The system is represented by the spin states (up and down) along some directions. One atom is put near one channel to serve as the apparatus to interact with the spin, causing entanglement. In this measurement, the PBP means that there is no physical difference between states

$$\begin{aligned} |\psi_1\rangle &= (|d\rangle|U\rangle - i|u\rangle|D\rangle)/\sqrt{2}, \qquad (16)\\ |\psi_2\rangle &= (|S^+\rangle|A^+\rangle - |S^-\rangle|A^-\rangle)/\sqrt{2}, \end{aligned}$$

where $|S^\pm\rangle = (|u\rangle \pm i|d\rangle)/\sqrt{2}$, $|A^\pm\rangle = (|U\rangle \pm |D\rangle)/\sqrt{2}$, $|u\rangle$ (up), $|d\rangle$ (down), $|S^\pm\rangle$ are the states of the spin S, $|U\rangle$, $|D\rangle$, $|A^\pm\rangle$ are the states of the atom A.

The main ideas of einselection are as follows. Suppose the pre-measurement between S and A, and one special interaction between A and E, the global state vector is

$$|\phi\rangle = (-ic|uD\mu\rangle - s|uD\delta\rangle + c|dU\mu\rangle + is|dU\delta\rangle)/\sqrt{2}, \tag{17}$$

with $|\mu\rangle$, $|\delta\rangle$ the states of E, $c \equiv \cos A(t)$, $s \equiv \sin A(t)$, $A(t)$ depends on the coupling periodically.

Introduce $|E^\pm\rangle = (|\mu\rangle \pm i|\delta\rangle)/\sqrt{2}$, for the special case $A(t) = \pi/4$, the state reduces to the GHZ-type

$$|\phi\rangle = (|dUE^-\rangle - i|uDE^+\rangle)/\sqrt{2}. \tag{18}$$

Operators $|U\rangle\langle U|$, $|D\rangle\langle D|$ can project out the two branches of $|\phi\rangle$, yet, not the case for $|A^{\pm}\rangle\langle A^{\pm}|$. Thus, states $|U\rangle$ and $|D\rangle$ instead of $|A^{\pm}\rangle$ are the *pointer states*, i.e., the environment (E) of A serves to singlet out the pointer state of A to entangle with S.

The einselection method is flawed, however. One reason is that the demonstration of pointer state depends on the special case $A(t) = \pi/4$; in contrast, pointer state, robust to the noise, should not depend on time, i.e, the other basis, $|A^{\pm}\rangle$, of the apparatus cannot act due to the action of E. Yet, it is obvious to check that when $A(t) = 0$ or $\frac{\pi}{2}$, A can be written in $|A^{\pm}\rangle$ basis, which is in conflict with the spirit of einselection.

Another reason is the collapse in the bi-party model for measurement [4] is not physical, i.e., the branches in the entangled state only manifest the potential results of S corresponding to different states of A. The apparatus is a kind of *inner observer*. By introducing E, similar with the physical collapse theory [26], einselection improperly changes entanglement from bi-party to tri-party type.

Indeed, the PBP can be resolved by the CF method. We re-express the PBP into two aspects:

(I). *Whether the entangled state between one system S and another system A can be written in infinite ways as*

$$|\psi\rangle = \sum_i a_i |S_i\rangle|A_i\rangle = \sum_j b_j |S'_j\rangle|A'_j\rangle = \cdots, \qquad (19a)$$

(II). *Whether the entangled state can be written as*

$$|\psi\rangle = \sum_i \alpha_i |S_i\rangle|A'_i\rangle = \sum_j \beta_j |S'_j\rangle|A''_j\rangle = \cdots, \qquad (19b)$$

where $|S_i\rangle$, $|A_i\rangle$, $\cdots$ satisfy the superposition principle, and a_i, b_j, α_i, β_j satisfy the normalization rule.

PBP(I) concerns the problem of the definite form of the actual basis, which is similar with the traditional version of PBP; while PBP(II) concerns the problem of the exact structure of the entangled state, i.e., how the basis of S and A correlate with each other one-to-one.

Different forms in PBP(I) have special CF, relating to different physical conditions. The states $|\psi_1\rangle$ and $|\psi_2\rangle$ in Eq. (16) correspond to different states of CF (see Eq. (7)). The configuration space, whose effect to the coherence is trivial though, plays the role of primary CF for the spin state (up and down), and decides the representation of the system. As a result, PBP(I) is only one *pseudo* problem. Note that it indicates when the CF is trivial, the entangled state shows symmetry under rotation.

For PBP(II), the reason for the non-equivalence of various forms is that the parties in the entangled state play the CF of each other, respectively. The rotation of the basis of one party will change the structure of the state, thus change the reference relation between S and A. The detailed form of the entangled state also depends on the interaction within and the practical arrangement, e.g., the generations of the two type Bell's basis are different.

Last, we note that our method of CF is consistent with the recently developed method of *relational* Hilbert space [6], according to which, there exist external (also classical) and internal CF. Under the internal CF, to extract the coherence of the system only, the Hilbert space is mapped onto $\mathcal{H}_{gl} \otimes \mathcal{H}_{rel}$, where $\mathcal{H}_{gl}$ plays the role of the classical CF, and $\mathcal{H}_{rel}$, the relational Hilbert space, is the space for the system with a slight difference from $\mathcal{H}_S$, which can be ignored in the classical limit. That is to say, the CF should always be treated internally, while in practice, some CF is trivial then can be ignored.

In conclusion, we mainly developed the theory of coherence frame in this paper. We showed that the entanglement is conserved in the swapping process, and the entanglement sudden death and birth does not exist. We also demonstrated that the preferred basis problem can be resolved more naturally by the method of coherence frame than the einselection method.

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