A New System of Equations in Classical Mechanics

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Abstract

In classical mechanics, this paper presents a new system of equations which is invariant under transformations between reference frames and which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

Definitions

If we consider a system of N particles then the total moment of inertia I of the system of particles, the total kinetic liveliness Y of the system of particles, and the total kinetic energy K of the system of particles, are as follows:

$$I = \sum_{i=1}^{N} \left(m_i \, \bar{\mathbf{r}}_i \cdot \bar{\mathbf{r}}_i \right)$$

$$Y = \sum_{i=1}^{N} \left(m_i \, \bar{\mathbf{r}}_i \cdot \bar{\mathbf{v}}_i \right)$$

$$K = \sum_{i=1}^{N} \left(m_i \, \bar{\mathbf{v}}_i \cdot \bar{\mathbf{v}}_i + m_i \, \bar{\mathbf{a}}_i \cdot \bar{\mathbf{r}}_i \right)$$

where $\bar{\mathbf{r}}_i = \mathbf{r}_i - \mathbf{r}_{cm}$, $\bar{\mathbf{v}}_i = \mathbf{v}_i - \mathbf{v}_{cm}$, $\bar{\mathbf{a}}_i = \mathbf{a}_i - \mathbf{a}_{cm}$, $\mathbf{r}_i$, $\mathbf{v}_i$ and $\mathbf{a}_i$ are the position, the velocity and the acceleration of the i-th particle, $\mathbf{r}_{cm}$, $\mathbf{v}_{cm}$ and $\mathbf{a}_{cm}$ are the position, the velocity and the acceleration of the center of mass of the system of particles, and m_i is the mass of the i-th particle.

If we consider a system of N particles then the total push P done by the forces acting on the system of particles, the total work W done by the forces acting on the system of particles, and the total potential energy U of the system of particles, are as follows:

$$P = \sum_{i=1}^{N} + \left(\bar{\mathbf{r}}_i \cdot \int_1^2 \mathbf{F}_i \; dt \right)$$

$$W = \sum_{i=1}^{N} + \left(2 \int_1^2 \mathbf{F}_i \cdot d\bar{\mathbf{r}}_i + \Delta \, \mathbf{F}_i \cdot \bar{\mathbf{r}}_i \right)$$

$$\Delta U = \sum_{i=1}^{N} - \left(2 \int_1^2 \mathbf{F}_i \cdot d\bar{\mathbf{r}}_i + \Delta \, \mathbf{F}_i \cdot \bar{\mathbf{r}}_i \right)$$

where $\bar{\mathbf{r}}_i = \mathbf{r}_i - \mathbf{r}_{cm}$, $\mathbf{r}_i$ is the position of the i-th particle, $\mathbf{r}_{cm}$ is the position of the center of mass of the system of particles, $\mathbf{F}_i$ is the net force acting on the i-th particle, and t is the time.

Theorems

In a system of N particles, the total push P done by the forces acting on the system of particles is equal to the change in the total kinetic liveliness Y of the system of particles.

$$P = +\Delta Y$$

In a system of N particles, the total work W done by the forces acting on the system of particles is equal to the change in the total kinetic energy K of the system of particles.

$$W = +\Delta K$$

In a system of N particles, the total work W done by the conservative forces acting on the system of particles is equal and opposite in sign to the change in the total potential energy U of the system of particles.

$$W = -\Delta U$$

Conservations

In a system of N particles, if the forces acting on the system of particles do not perform push then the total kinetic liveliness of the system of particles remains constant.

$$Y = constant$$

In a system of N particles, if the non-conservative forces acting on the system of particles do not perform work then the total (mechanical) energy of the system of particles remains constant.

$$K + U = constant$$

Observations

The new system of equations is invariant under transformations between reference frames.

The new system of equations can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

The new system of equations would be valid even if Newton's third law of motion were false in an inertial reference frame.

The new system of equations would be valid even if Newton's three laws of motion were false in a non-inertial reference frame.

On the other hand, the new system of equations can be obtained from the general equation of motion (**A. Torassa**, General Equation of Motion)

Annex

If we consider an isolated system of N particles and if Newton's third law of motion is valid then the total push P done by the forces acting on the system of particles, the total work W done by the forces acting on the system of particles, and the total potential energy U of the system of particles, are as follows:

$$P = \sum_{i=1}^{N} + \left(\mathbf{r}_i \cdot \int_1^2 \mathbf{F}_i \; dt \right)$$

$$W = \sum_{i=1}^{N} + \left(2 \int_1^2 \mathbf{F}_i \cdot d\mathbf{r}_i + \Delta\, \mathbf{F}_i \cdot \mathbf{r}_i \right)$$

$$\Delta U = \sum_{i=1}^{N} - \left(2 \int_1^2 \mathbf{F}_i \cdot d\mathbf{r}_i + \Delta\, \mathbf{F}_i \cdot \mathbf{r}_i \right)$$

where $\mathbf{r}_i$ is the position of the i-th particle, $\mathbf{F}_i$ is the net force acting on the i-th particle, and t is the time.

If we consider a system of N particles then the total moment of inertia I of the system of particles, the total kinetic liveliness Y of the system of particles, and the total kinetic energy K of the system of particles, can also be expressed as follows:

$$I = \sum_{i=1}^{N} \left(m_i \, \mathbf{r}_i \cdot \mathbf{r}_i \right) - m_{cm} \, \mathbf{r}_{cm} \cdot \mathbf{r}_{cm}$$

$$Y = \sum_{i=1}^{N} \left(m_i \, \mathbf{r}_i \cdot \mathbf{v}_i \right) - m_{cm} \, \mathbf{r}_{cm} \cdot \mathbf{v}_{cm}$$

$$K = \sum_{i=1}^{N} \left(m_i \, \mathbf{v}_i \cdot \mathbf{v}_i + m_i \, \mathbf{a}_i \cdot \mathbf{r}_i \right) - m_{cm} \, \mathbf{v}_{cm} \cdot \mathbf{v}_{cm} - m_{cm} \, \mathbf{a}_{cm} \cdot \mathbf{r}_{cm}$$

$$I = \sum_{i=1}^{N} \sum_{j>i}^{N} \left(\frac{m_i \, m_j}{m_{cm}} \, (\mathbf{r}_i - \mathbf{r}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j) \right)$$

$$Y = \sum_{i=1}^{N} \sum_{j>i}^{N} \left(\frac{m_i \, m_j}{m_{cm}} \, (\mathbf{r}_i - \mathbf{r}_j) \cdot (\mathbf{v}_i - \mathbf{v}_j) \right)$$

$$K = \sum_{i=1}^{N} \sum_{j>i}^{N} \left(\frac{m_i \, m_j}{m_{cm}} \, (\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{v}_i - \mathbf{v}_j) + \frac{m_i \, m_j}{m_{cm}} \, (\mathbf{a}_i - \mathbf{a}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j) \right)$$

where $\mathbf{r}_i$, $\mathbf{v}_i$, $\mathbf{a}_i$, $\mathbf{r}_j$, $\mathbf{v}_j$, $\mathbf{a}_j$, $\mathbf{r}_{cm}$, $\mathbf{v}_{cm}$, $\mathbf{a}_{cm}$ are the positions, the velocities and the accelerations of the i-th particle, of the j-th particle and of the center of mass of the system of particles, and m_i, m_j, m_{cm} are the masses of the i-th particle, of the j-th particle and of the center of mass of the system of particles.