Regularization of the vacuum fluctuation energy in quantum electrodynamics

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The total energy of vacuum fluctuations of electron-positron and electromagnetic fields in quantum electrodynamics is examined and is proven to vanish if the value of the "bare" electron charge is chosen in a proper way. The "bare" charge value calculated in this work is in a qualitative agreement with the independently obtained one from the solution of the "physical electron" problem in [1].

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I. INTRODUCTION

The quantum electrodynamics (QED) nowadays can be considered without any doubts as a part of the general gauge theory [2]. At the same time QED as an isolated system remains the most successful quantum field model that allows one to calculate the observable characteristics of the electromagnetic processes with the unique accuracy using only two parameters - the mass m and the charge e of the "physical" electron [3–5]. However, there are two "dark spots" in the theoretical construction of QED which do not permit to consider this model as the mathematically perfect theory [6, §81], [7].

One of them is referred to the renormalization procedure where the connection between characteristics of the "bare" electron with the mass m_0 and the charge e_0 and the parameters m and e of the "physical" one is defined by the divergent integrals. The second one is the infinite vacuum energy of both electron-positron and electromagnetic fields which is considered as the zero point energy. In spite of its infinite value the vacuum energy does not effect the transition amplitudes of the quantum electrodynamical processes, nevertheless, it is revealed in the Casimir effect [8] (see, however, [9]).

In a series of our publications [1, 10, 11] was shown that it is possible to renormalize a mass and a charge with the finite values and the connection between characteristics of the "bare" and the "physical" electron is defined by the formulas in Ref. [1]:

$$\alpha_0 = \frac{a_0^2}{\alpha} \approx \frac{12.47}{\alpha} \approx 1708; \quad \alpha_0 = \frac{e_0^2}{4\pi}; \quad \alpha = \frac{e^2}{4\pi};$$
$$m_0 = m\frac{2|a_0|}{T\alpha} \approx \frac{9.43}{\alpha} m \approx 1292m. \tag{1}$$

Here α is the observable fine structure constant, a_0, T are the constants which are calculated numerically from the equations that define one-particle excitation of the electron-positron field in QED and $\hbar = c = 1$. It was also shown that the perturbation theory built in the physical value of the parameter α corresponds to the strong coupling series in α_0^{-1}.

In the present paper the ground state (vacuum) energy $E_0(\alpha_0)$ of the QED Hamiltonian is calculated and shown to vanish when the coupling constant α_0 equals to $\tilde{\alpha}_0$ ($E_0(\tilde{\alpha}_0) = 0$). The value $\tilde{\alpha}_0 \gg 1$ which we obtained is in a qualitative agreement with the coupling constant in equation (1).

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II. ONE-PARTICLE EXCITATION IN QED

Let us consider the QED Hamiltonian in the Coulomb gauge [5]:

$$\hat{H} = \int d\vec{r}\hat{H}(\vec{r}) = \int d\vec{r}\{\hat{\psi}^{+}(\vec{r})[\vec{\alpha}(\hat{\vec{p}} - e_0\hat{\vec{A}}(\vec{r})) + \beta m_0]\hat{\psi}(\vec{r}) + \\ + \sum_{\vec{k}\lambda}\omega(\vec{k})(c^{+}_{\vec{k}\lambda}c_{\vec{k}\lambda} + \frac{1}{2}) + \frac{\alpha_0}{2}\int d\vec{r}'\frac{\hat{\rho}(\vec{r})\hat{\rho}(\vec{r}')}{|\vec{r}-\vec{r}'|}\}; \tag{2}$$

$$\hat{\rho}(\vec{r}) = \frac{1}{2}[\hat{\psi}^{+}(\vec{r})\hat{\psi}(\vec{r}) - \hat{\psi}(\vec{r})\hat{\psi}^{+}(\vec{r})].$$

The field operators are given in the Schrödinger representation, the operator of an electromagnetic field and the spinor components of the electron-positron operators are defined in a standard way [3]:

$$\hat{\vec{A}}(\vec{r})) = \sum_{\vec{k}\lambda}\frac{\vec{e}^{(\lambda)}}{\sqrt{2kV}}[c_{\vec{k}\lambda}e^{i\vec{k}\vec{r}} + c^{+}_{\vec{k}\lambda}e^{-\vec{k}\vec{r}}];$$

$$\hat{\psi}_{\nu}(\vec{r},t) = \sum_{s}\int\frac{d\vec{p}}{(2\pi)^{3/2}}\{a_{\vec{p}s}u_{\vec{p}s\nu}e^{i\vec{p}\vec{r}} + b^{+}_{\vec{p}s}v_{-\vec{p}-s\nu}e^{-i\vec{p}\vec{r}}\}, \tag{3}$$

where V is the normalization volume.

Firstly we briefly revise our consideration of the one-particle excitation [1]. In the zero-order of the conventional perturbation theory (PT) the one electron ($1e$) and one positron ($1p$) excitations are defined by the state vectors:

$$|\Phi^{(PT)}_{1e}> = a^{+}_{\vec{p}s}|0;0;0>; \quad |\Phi^{(PT)}_{1p}> = b^{+}_{\vec{p}s}|0;0;0>, \tag{4}$$

with $|\Phi_0> \equiv |0;0;0>$ being the vacuum state vector.

In our approach more complicated trial state vectors were used for a one-particle excitation. They were represented by the wave packet corresponding to the "physical" electron or positron:

$$|\Phi^{(0)}_1> = \sum_{s}\int d\vec{q}\{U_{\vec{q}s}a^{+}_{\vec{q}s} + V_{\vec{q}s}b^{+}_{\vec{q}s}\}|0;0;0> . \tag{5}$$

Here $U_{\vec{q}s}$ and $V_{\vec{q}s}$ are variational parameters. For the "physical" electron with the zero total momentum they should satisfy the additional conditions:

$$< \Phi^{(0)}_1|\hat{\vec{P}}|\Phi^{(0)}_1> = \sum_{s}\vec{q}\left[|U_{\vec{q}s}|^2 + |V_{\vec{q}s}|^2\right]d\vec{q} = \vec{P} = 0;$$

$$< \Phi^{()}_1|\Phi^{(0)}_1> = \sum_{s}\left[|U_{qs}|^2 + |V_{qs}|^2\right]d\vec{q} = 1,$$

$$< \Phi^{(0)}_1|\hat{Q}|\Phi^{(0)}_1> = e_0\sum_{s}\left[|V_{qs}|^2 - |U_{qs}|^2\right]d\vec{q} = e. \tag{6}$$

where e is the "physical" electron charge.

Then we have calculated the one-particle excitation energy, defined as:

$$E_1(0) \simeq E^{(0)}_1(U_{qs};V_{qs}) - E_0 = < \Phi^{(0)}_1|\hat{H}|\Phi^{(0)}_1> - < \Phi_0|\hat{H}|\Phi_0>, \tag{7}$$

where the averages are calculated with the total Hamiltonian (2).

The functions $U_{qs};V_{qs}$ were found as solutions of the variational equations:

$$\frac{\partial E^{(0)}_1(U_{qs};V_{qs})}{\partial U_{qs}} = \frac{\partial E^{(0)}_1(U_{qs};V_{qs})}{\partial V_{qs}} = 0 \tag{8}$$

with the additional conditions (6).

Using the coordinate representation

$$\Psi_{\nu}(\vec{r}) = \int\frac{d\vec{q}}{(2\pi)^{3/2}}\sum_{s}U_{qs}u_{\vec{q}s\nu}e^{i\vec{q}\vec{r}}; \ \Psi^{c}_{\nu}(\vec{r}) = \int\frac{d\vec{q}}{(2\pi)^{3/2}}\sum_{s}V^{*}_{qs}v_{\vec{q}s\nu}e^{i\vec{q}\vec{r}}, \tag{9}$$

we have found

$$E_1(0) = \int d\vec{r}\{\Psi^+(\vec{r})[(-i\vec{\alpha}\vec{\nabla} + \beta m_0) + \frac{1}{2}\varphi(\vec{r})]\Psi(\vec{r}) - \Psi^{+c}(\vec{r})[(-i\vec{\alpha}\vec{\nabla} + \beta m_0) + \frac{1}{2}e_0\varphi(\vec{r})]\Psi^c(\vec{r});$$

$$\int d\vec{r}[\Psi^+(\vec{r})\Psi(\vec{r}) + \Psi^{+c}(\vec{r}')\Psi^c(\vec{r}')] = 1; \tag{10}$$

$$\varphi(\vec{r}) = \alpha_0 \int \frac{d\vec{r}'}{|\vec{r} - \vec{r}'|}[\Psi^+(\vec{r}')\Psi(\vec{r}') - \Psi^{+c}(\vec{r}')\Psi^c(\vec{r}')]. \tag{11}$$

The extremum of this functional for the state with the orbital momentum $l = 0$ has led to the system of equations in the dimensionless variables and functions:

$$x = rm_0; \quad E = \epsilon m_0; \quad e_0\varphi(r) = m_0\phi(x); \quad \frac{e_0^2}{4\pi} = \alpha_0;$$
$$u(x)\sqrt{m_0} = rg(r); \quad v(x)\sqrt{m_0} = rf(r); \quad u_1(x)\sqrt{m_0} = rg_1(r); \quad v_1(x)\sqrt{m_0} = rf_1(r). \tag{12}$$

$$\frac{du}{dx} - \frac{1}{x}u - (1 - \phi(x))v = 0; \ \frac{dv}{dx} + \frac{1}{x}v - (1 + \phi(x))u = 0;$$
$$\frac{du_1}{dx} + \frac{1}{x}u_1 - (1 + \phi(x))v_1 = 0; \ \frac{dv_1}{dx} - \frac{1}{x}v_1 - (\ 1 - \phi(x))u_1 = 0;$$
$$\phi(x) = \alpha_0[\int_x^\infty dy\frac{\rho(y)}{y} + \frac{1}{x}\int_0^x dy\rho(y)];$$
$$\rho(x) = [u^2(x) + v^2(x) - u_1^2(x) - v_1^2(x)]. \tag{13}$$

A solution of this equations exists when the following conditions are fulfilled:

$$a = \alpha_0\frac{1 - C}{1 + C} = a_0 \approx -3.531... \ \int_0^\infty dx(u^2 + v^2) = \frac{1}{1 + C}; \ \int_0^\infty dx(u_1^2 + v_1^2) = \frac{C}{1 + C}. \tag{14}$$

Next by exploiting the connection (6) between the "bare" and the "physical" charges, the relation (1) between the "bare" coupling constant α_0 and the observed value of the fine structure constant α has been obtained.

When the analogous calculations have been performed for the state corresponding to the "physical" electron with the total momentum $\vec{P}$ the spectrum of the one-particle excitations has been found [1]:

$$E_1(P) = \sqrt{P^2 + m^2}; \quad m = \frac{m_0\alpha T}{|a_0|}. \tag{15}$$

III. VACUUM ENERGY IN QED

In this section we will apply the same approach to the calculation of the vacuum energy E_0, with the trial state vector $|\Phi_0 >\equiv |0;0;0>\equiv |0>$:

$$E_0(\alpha_0) =< 0|\int d\vec{r}\hat{H}(\vec{r})|0> . \tag{16}$$

By using the definition of the Hamiltonian (2) in equation (16) three terms are obtained [5]:
1) The zero energy of the Dirac field:

$$< 0|\hat{H}_D|0 >= -2\sum_{\vec{p}}\sqrt{p^2 + m_0^2} = -2\frac{V}{(2\pi)^3}\int\sqrt{p^2 + m_0^2}d\vec{p}; \tag{17}$$

2) The vacuum energy of an electromagnetic field:

$$< 0|\hat{H}_E|0 >= \sum_{\vec{k}\lambda}\omega(\vec{k})\frac{1}{2} = \frac{V}{(2\pi)^3}\int d\vec{k}\ k; \tag{18}$$

3) The vacuum energy of the Coulomb field:

$$< 0|\hat{H}_C|0 >= \frac{\alpha_0}{2}\int \frac{d\vec{r}d\vec{r}'}{|\vec{r}-\vec{r}'|} < 0|\rho(\vec{r})\rho(\vec{r}')|0 > . \tag{19}$$

It is important that among these three terms one is negative and the other two are positive, while all of them are equal to infinity. From a mathematical perspective we deal with the indefinite limit for E_0. In order to investigate this limit let us formally introduce a spherically symmetrical regularizing function in the integrand for the integration in a momentum space:

$$\int d\vec{p} \Rightarrow \int d\vec{p} f\left(\frac{p}{L}\right); \quad f(0) = 1; \quad f(\infty) = 0, \tag{20}$$

with L as the regularization parameter ($L \to \infty$).

Consequently we can rewrite the Dirac (17) and the electromagnetic (18) energies as

$$< 0|\hat{H}_D|0 >= -2\frac{V}{(2\pi)^3}\int d\vec{p} f\left(\frac{p}{L}\right)\sqrt{p^2+m_0^2} = -2\frac{V}{(2\pi)^3}L^4\int d\vec{u} f(u)\sqrt{u^2+\frac{m_0^2}{L^2}}, \tag{21}$$

$$< 0|\hat{H}_E|0 >= \frac{V}{(2\pi)^3}L^4\int d\vec{u} f(u)u, \tag{22}$$

and the last term can be transformed into the momentum space:

$$\begin{aligned} < 0|\hat{H}_C|0 > &= 2\alpha_0\int\int d\vec{r}d\vec{r}'\int\frac{d\vec{q}}{2\pi^2q^2}\int\int\frac{d\vec{p}d\vec{p}'}{(2\pi)^6}f\left(\frac{p}{L}\right)f\left(\frac{p'}{L}\right)e^{i(\vec{p}+\vec{p}'+\vec{q})(\vec{r}-\vec{r}')} \\ &= 2\alpha_0 V\int\frac{d\vec{q}}{2\pi^2q^2}\int\int\frac{d\vec{p}d\vec{p}'}{(2\pi)^3}f\left(\frac{p}{L}\right)f\left(\frac{p'}{L}\right)\delta(\vec{p}+\vec{p}'+\vec{q}) \\ &= 2\alpha_0 V\int\frac{d\vec{q}}{2\pi^2q^2}\int\frac{d\vec{p}}{(2\pi)^3}f\left(\frac{p}{L}\right)f\left(\frac{|\vec{p}+\vec{q}|}{L}\right) \\ &= 2\alpha_0 VL^4\int\frac{d\vec{u}}{2\pi^2u^2}\int\frac{d\vec{v}}{(2\pi)^3}f(v)f(|\vec{v}+\vec{u}|), \end{aligned} \tag{23}$$

where $\vec{u}$ and $\vec{v}$ are the dimensionless variables.

Therefore, it is possible to turn the vacuum energy into zero if the "bare" coupling constant is chosen as the solution of the equation:

$$\begin{aligned} &\alpha_0(\frac{m_0}{L}) = 2\pi^2\frac{\int d\vec{u}f(u)[2\sqrt{u^2+m_0^2/L^2}-u]}{\int\frac{d\vec{u}}{u^2}\int d\vec{v}f(v)f(|\vec{v}+\vec{u}|)} \approx \tilde{\alpha}_0 + O\left[\frac{m_0^2}{L^2}\right]; \\ &L\to\infty, \quad \tilde{\alpha}_0 = \pi^2\frac{\int d\vec{u}f(u)u}{\int\frac{d\vec{u}}{u^2}\int d\vec{v}f(v)f(|\vec{v}+\vec{u}|)} \equiv I[f]. \end{aligned} \tag{24}$$

IV. CALCULATION OF $\tilde{\alpha}_0$.

A concrete form of the dimensionless regularizing (cut-off) function with the boundary condition (20) is still not specified. Therefore, we suggest to select this function such that the value of $\tilde{\alpha}_0$ in (24) depends on its form in the least possible way. The most general form of f is a series with N indefinite coefficients:

$$f(u) = e^{-u^2}\sum_{l=0}^{N}C_l u^l; \quad C_0 = 1. \tag{25}$$

Insertion of (25) into equation (24) and integration over the angular coordinates yields for $\tilde{\alpha}_0$:

$$\int d\vec{u} f(u) u = 4\pi \int_0^\infty du f(u) u^3 \equiv 4\pi J_1;$$
$$\int \frac{d\vec{u}}{u^2} \int d\vec{v} f(v) f(|\vec{v}+\vec{u}|) = \int \frac{d\vec{u}}{(\vec{u}-\vec{v})^2} \int d\vec{v} f(v) f(u)$$
$$= 4\pi^2 \int_0^\infty du \int_0^\infty dv u v f(v) f(u) \ln \frac{(u+v)^2}{(u-v)^2} \equiv 4\pi^2 J_2;$$
$$\tilde{\alpha}_0 = \pi \frac{J_1}{J_2}. \tag{26}$$

Let us now define two sets of integrals: the vector set

$$I_l = \int_0^\infty du e^{-u^2} u^{l+3}; \tag{27}$$

and the tensor set

$$M_{k,r} = \int_0^\infty du \int_0^\infty dv u v e^{-u^2-v^2} u^k v^r \ln \frac{(u+v)^2}{(u-v)^2+\delta^2}; \tag{28}$$

where the indices l, k and r run the values $0, 1, ...N$.

By using the integrals (27) and (28) the coupling constant $\tilde{\alpha}_0$ can be rewritten as:

$$\tilde{\alpha}_0 = \pi \frac{I_0 + A}{M_{0,0} + 2A_1 + B}$$
$$A = \sum_{l\neq 0}^N C_l I_l; \quad A_1 = \sum_{l\neq 0}^N C_l M_{0,l}; \quad B = \sum_{k\neq 0}^N \sum_{r\neq 0}^N M_{k,r} C_k C_r. \tag{29}$$

As we stated above the value of $\tilde{\alpha}_0$ must depend as least as possible on the form of f. This gives the additional conditions on the coefficients C_l:

$$\frac{\partial \tilde{\alpha}_0}{\partial C_l} = 0; \quad l \neq 0,$$
$$I_l(M_{0,} + 2A_1 + B) = 2(I_0 + A)(M_{0,l} + \sum_{k\neq 0} M_{l,k} C_k). \tag{30}$$

The solution of the system of equations (30) can be written in the general form as:

$$C_l = K \sum_{k\neq 0}^N (M^{-1})_{l,k} I_k - \sum_{k\neq 0}^N (M^{-1})_{l,k} M_{0,k}; \quad K = \frac{(M_{0,0} + 2A_1 + B)}{2(I_0 + A)}. \tag{31}$$

These solutions will be consistent if the coefficients A, A_1, B in equation (29) are also determined in the same way. Plugging equation (31) into (29) one obtains

$$A = K \sum_{k\neq 0} \sum_{l\neq 0} I_l (M^{-1})_{l,k} I_k - \sum_{k\neq 0} \sum_{l\neq 0} I_l (M^{-1})_{l,k} M_{0,k} \equiv KT - T_1;$$
$$A_1 = K \sum_{l\neq 0} \sum_{l\neq 0} M_{0,l} (M^{-1})_{l,k} I_k - \sum_{l\neq 0} \sum_{l\neq 0} M_{0,l} (M^{-1})_{l,k} M_{0,k} \equiv KT_1 - T_2;$$
$$B = K^2 \sum_{k\neq 0} \sum_{r\neq 0} \sum_{l\neq 0} \sum_{m\neq 0} M_{k,r} (M^{-1})_{l,k} I_l (M^{-1})_{r,m} I_m -$$
$$- 2K \sum_{r\neq 0} \sum_{l\neq 0} \sum_{m\neq 0} M_{k,r} (M^{-1})_{l,k} M_{0,l} (M^{-1})_{r,m} I_m +$$
$$+ \sum_{k\neq 0} \sum_{r\neq 0} \sum_{l\neq 0} \sum_{m\neq 0} M_{k,r} (M^{-1})_{l,k} M_{0,l} (M^{-1})_{r,m} M_{0,m} = K^2 T - 2KT_1 + T_2;$$
$$T_2 = \sum_{k\neq 0} \sum_{l\neq 0} M_{0,k} (M^{-1})_{l,k} M_{0,l}. \tag{32}$$

Then equation (32) leads us to the system of algebraic equations

$$2(A+T_1)(I_0+A)-[(M_{0,0}+2A_1+B)]T=0;$$
$$(A+T_1)T_1-(A_1+T_2)T=0;$$
$$(B-T_2)T-(A+T_1)^2+(A_1+T_2)T=BT-(A+T_1)^2+A_1T=0. \quad (33)$$

Next, we introduce a new unknown variable x as $(A+T_1)=x$, for which the quadratic equation is obtained and can be easily solved:

$$x^2+2x(I_0-T_1)-(M_{0,0}-T_2)T=0$$
$$x_{1,2}=-(I_0-T_1)\pm\sqrt{(I_0-T_1)^2+(M_{0,0}-T_2)T}. \quad (34)$$

Returning to the old variables we obtain the unknowns A, A_1 and B

$$A=x-T_1;\ A_1=-T_2+\frac{xT_1}{T};\ B=\frac{x^2}{T}+T_2-\frac{xT_1}{T}. \quad (35)$$

Consequently this brings us to the equation for the determination of $\tilde{\alpha}_0$:

$$\tilde{\alpha}_0=\pi\frac{I_0+A}{M_{0,0}+2A_1+B} \quad (36)$$

V. NUMERICAL RESULTS AND DISCUSSION

It is evident that the numerical value $\tilde{\alpha}_0$ depends on the number N in the series (25) which defines the dimension of the matrices in (32). We have found numerically that for the fixed N the first root of equation (34) corresponds to the minimal value $\tilde{\alpha}_0$ and the second to the maximal $\tilde{\alpha}_0$. Both values become closer to each other when N increases. It was proved that the positive solution $\tilde{\alpha}_0>0$ could be found for $N\leq 12$ and the maximal value of $\tilde{\alpha}_0$ corresponds to $N=12$. In this case the following values were calculated:

$$I_0=0.5000;\ M_0=0.768306;\ T=29.1837;\ T_1=0.500496;\ T_2=0.768195;$$
$$\tilde{\alpha}_0\approx 239.21. \quad (37)$$

One can see that the condition $\tilde{\alpha}_0\gg 1$ is in a qualitative agreement with the value α_0 from (1). There is no surprise that these parameters do not coincide exactly. In the present analysis the vacuum energy was calculated for the only one fermion and the one boson fields referred to QED. Let us suppose that there is an additional set of quantum fields, which includes $N_F^{(c)}$ charged and $N_F^{(0)}$ neutral fermion fields with degeneracies $g_F^{(c)}$ and $g_F^{(0)}$ respectively. Analogous values $N_B^{(c)}, g_B^{(c)}, N_B^{(0)}, g_B^{(0)}$ could be defined for the charged and neutral boson fields. If the vacuum energy of all these fields are taken into account by the same way as in the QED case the value $\tilde{\alpha}_0$ can be recalculated as follows:

$$\tilde{\alpha}_0\Rightarrow\tilde{\alpha}_0\frac{1+g_F^{(c)}N_F^{(c)}+g_F^{(0)}N_F^{(0)}-1/2(g_B^{(c)}N_B^{(c)}+g_B^{(0)}N_B^{(0)})}{1+1/4([g_F^{(c)}]^2N_F^{(c)}+[g_B^{(c)}]^2N_B^{(c)})}. \quad (38)$$

For example, if one includes the contribution of the vacuum energy from neutrino (ν_e) the value (37) of $\tilde{\alpha}_0$ changes to

$$\tilde{\alpha}_0\approx 717.63, \quad (39)$$

which is much closer to (1).

Concluding, we have shown that the two independent ways of the definition of the "bare" coupling constant between the electromagnetic and the matter fields are qualitatively agree with each other, that can be considered as the basis for further analysis of this problem.

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