Modified nonlinear model of arcsin-electrodynamics

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Abstract

A new modified model of nonlinear arcsin-electrodynamics with two parameters is suggested and analyzed. The effect of vacuum birefringence takes place when the external constant magnetic field is present. We calculate indices of refraction for two perpendicular polarizations of electromagnetic waves and estimate the parameter γ from the Biréfringence Magnétique du Vide (BMV) experiment. It is shown that the electric field of a point-like charge is finite at the origin. We calculate the finite static electric energy of point-like particles and demonstrate that the electron mass can have the pure electromagnetic nature. The canonical and symmetrical Belinfante energy-momentum tensors and dilatation current are found. We show that the dilatation symmetry and dual symmetry are broken in the model suggested.

1 Introduction

In Maxwell's electrodynamics a point-like charge possesses an infinite electromagnetic energy but in Born-Infeld (BI) electrodynamics [1], [2], [3], where there is a new parameter with the dimension of the length, that problem of singularity is solved. In BI electrodynamics the dimensional parameter gives the maximum of the electric fields. In addition, non-linear electrodynamics may give a finite electromagnetic energy of a charged point-like particle. As a result, in such models the electron mass can have pure electromagnetic nature. It is known that in QED one-loop quantum corrections contribute to classical electrodynamics and give non-linear terms in the Lagrangian [4], [5], [6]. Different models of non-linear electrodynamics were investigated in [7], [8], [9], [10] and [11]. The non-linear effects should be taken into account for strong electromagnetic fields.

The structure of the paper is as follows. In section 2, we propose the Lagrangian of the new model of nonlinear electrodynamics. The field equations

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are written in the form of Maxwell's equations where the electric permittivity ε_{ij} and magnetic permeability μ_{ij} tensors depend on electromagnetic fields. In section 3 we show that the electric field of a point-like charge is not singular at the origin and have the finite value. The phenomenon of vacuum birefringence is investigated in section 4 and we estimate the parameter γ from BMV experiment. In section 5 we obtain the canonical and symmetrical Belinfante energy-momentum tensors, the dilatation current, and its non-zero divergence. The finite static electric energy of point-like particles is calculated and we demonstrate that the electron mass can be treated as the pure electromagnetic energy. We discuss the result obtained in section 6.

The Heaviside-Lorentz system with $\hbar = c = \varepsilon_0 = \mu_0 = 1$ and Euclidian metric are used. Greek letters run from 1 to 4 and Latin letters run from 1 to 3.

2 Field equations of the model

We suggest nonlinear electrodynamics with the Lagrangian density

$$\mathcal{L} = -\frac{1}{\beta}\arcsin\left(\beta\mathcal{F} - \frac{\beta\gamma}{2}\mathcal{G}^2\right), \tag{1}$$

where β, γ are dimensional parameters ($\beta\mathcal{F}$, $\beta\gamma\mathcal{G}^2$ are dimensionless). The Lorentz-invariants are defined by $\mathcal{F} = (1/4)F_{\mu\nu}^2 = (\mathbf{B}^2 - \mathbf{E}^2)/2$, $\mathcal{G} = (1/4)F_{\mu\nu}\tilde{F}_{\mu\nu} = \mathbf{E}\cdot\mathbf{B}$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength, $\tilde{F}_{\mu\nu} = (1/2)\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$ is a dual tensor ($\varepsilon_{1234} = -i$) and A_μ is the 4-vector-potential. The non-linear model of electrodynamics introduced can be considered as an effective model. At weak electromagnetic fields the model based on the Lagrangian density (1) approaches to classical electrodynamics, $\mathcal{L} \simeq -\mathcal{F}$, and the correspondence principle takes place [12]. The variables $\beta^{1/4}$, $\gamma^{1/4}$ possess the dimension of the length and can be considered as fundamental constants in the model.

Euler-Lagrange equations lead to the equations of motion

$$\partial_\mu\left(\frac{F_{\mu\nu} - \gamma\mathcal{G}\tilde{F}_{\mu\nu}}{\sqrt{1 - \left(\beta\mathcal{F} - \frac{\beta\gamma}{2}\mathcal{G}^2\right)^2}}\right) = 0. \tag{2}$$

The electric displacement field, $\mathbf{D} = \partial\mathcal{L}/\partial\mathbf{E}$ ($E_j = iF_{j4}$), is given by

$$\mathbf{D} = \frac{1}{\Pi}\left(\mathbf{E} + \gamma\mathcal{G}\mathbf{B}\right), \qquad \Pi = \sqrt{1 - \left(\beta\mathcal{F} - \frac{\beta\gamma}{2}\mathcal{G}^2\right)^2}. \tag{3}$$

Defining $D_i = \varepsilon_{ij}E_j$, we obtain the electric permittivity tensor ε_{ij}:

$$\varepsilon_{ij} = \frac{1}{\Pi}\left(\delta_{ij} + \gamma B_i B_j\right). \tag{4}$$

The magnetic field can be found from the relation $\mathbf{H} = -\partial\mathcal{L}/\partial\mathbf{B}$ ($B_j = (1/2)\varepsilon_{jik}F_{ik}$, $\varepsilon_{123} = 1$),

$$\mathbf{H} = \frac{1}{\Pi}\left(\mathbf{B} - \gamma\mathcal{G}\mathbf{E}\right). \tag{5}$$

The magnetic induction field is $\mathbf{B}_i = \mu_{ij}\mathbf{H}_j$, and we find the inverse magnetic permeability tensor $(\mu^{-1})_{ij}$:

$$(\mu^{-1})_{ij} = \frac{1}{\Pi}\left(\delta_{ij} - \gamma E_i E_j\right). \tag{6}$$

Equations of motion (2) may be written, with the help of Eqs. (3),(5), in the form of the first pair of the Maxwell equations

$$\nabla\cdot\mathbf{D} = 0, \qquad \frac{\partial\mathbf{D}}{\partial t} - \nabla\times\mathbf{H} = 0. \tag{7}$$

The second pair of Maxwell's equation follows from the Bianchi identity $\partial_\mu\widetilde{F}_{\mu\nu} = 0$, and are given by

$$\nabla\cdot\mathbf{B} = 0, \qquad \frac{\partial\mathbf{B}}{\partial t} + \nabla\times\mathbf{E} = 0. \tag{8}$$

Eqs. (7),(8) are non-linear Maxwell equations because ε_{ij} and μ_{ij} depend on the electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$. From Eqs. (3),(5) we obtain the relation

$$\mathbf{D}\cdot\mathbf{H} = \frac{\mathbf{E}\cdot\mathbf{B}}{\Pi^2}\left(1 + 2\gamma\mathcal{F} - \gamma^2\mathcal{G}^2\right). \tag{9}$$

The dual symmetry is broken in this model because $\mathbf{D}\cdot\mathbf{H} \neq \mathbf{E}\cdot\mathbf{B}$ [13]. It should be noted that BI electrodynamics is dual symmetrical but in generalized BI electrodynamics [14] the dual symmetry is broken as well as in QED due to one loop quantum corrections.

3 The field of the point-like charged particles

From Maxwell's equation (7) in the presence of the point-like charge source, we obtain the equation

$$\nabla \cdot \mathbf{D}_0 = e\delta(\mathbf{r}) \tag{10}$$

having the solution

$$\mathbf{D}_0 = \frac{e}{4\pi r^3}\mathbf{r}. \tag{11}$$

Eq. (11) with the help of Eq. (4), and at $\mathbf{B} = 0$, becomes

$$E_0 \left(\frac{1}{\sqrt{1 - \beta^2 E_0^4/4}} \right) = \frac{e}{4\pi r^2}. \tag{12}$$

When $r \to 0$ the solution to Eq.(12) is

$$E_0 = \sqrt{\frac{2}{\beta}}. \tag{13}$$

Eq. (13) represents the maximum electric field at the origin of the charged point-like particle. This attribute is similar to BI electrodynamics, and is in contrast to linear electrodynamics where the electric field strength possesses the singularity. Let us to introduce unitless variables

$$x = \frac{4\pi r^2}{e\sqrt{\beta}}, \qquad y = \sqrt{\frac{\beta}{2}}E_0. \tag{14}$$

Eq. (12) using (14) becomes $y^4 + 2x^2y^2 - 1 = 0$ with the real solution

$$y = \sqrt{\sqrt{x^4 + 1} - x^2}. \tag{15}$$

From Eq. (15) we find the asymptotic value $y \to 1$ at $x \to 0$ ($r \to 0$) that is equal to Eq. (13). If $x \to \infty$ (distance r approaches to infinity), $y \to 0$. Therefore, the electric field of a point-like charged particle at the origin is finite and equals the value in Eq. (13).

4 Vacuum birefringence

It is known that vacuum birefringence takes place in QED due to one-loop quantum corrections [15], [16]. We note that in generalized BI electrodynamics [14] there is the effect of birefringence. Here we consider the superposition of the external constant and uniform magnetic induction field $\mathbf{B}_0 = B_0(1,0,0)$ and the plane electromagnetic wave $(\mathbf{e},\mathbf{b})$,

$$\mathbf{e} = \mathbf{e}_0 \exp\left[-i\left(\omega t - kz\right)\right], \quad \mathbf{b} = \mathbf{b}_0 \exp\left[-i\left(\omega t - kz\right)\right] \tag{16}$$

propagating in z-direction. Let consider the strong magnetic induction field $\mathbf{B}$ so that the resultant electromagnetic fields are given by $\mathbf{E} = \mathbf{e}$, $\mathbf{B} = \mathbf{b}+\mathbf{B}_0$. We assume that amplitudes of the electromagnetic wave, e_0, b_0, are much less than the magnetic induction field, $e_0, b_0 \ll B_0$. From Eq. (1) after linearizing on fields (see [17]), we obtain the electric permittivity tensor and magnetic permeability

$$\varepsilon_{ij} = \frac{1}{\mu}\left(\delta_{ij} + \gamma B_{0i}B_{0j}\right), \quad \mu_{ij} = \mu\delta_{ij}, \quad \mu = \frac{1}{\sqrt{1-\beta^2 B_0^4/4}}. \tag{17}$$

When the polarization of the electromagnetic wave is parallel to external magnetic field, $\mathbf{e} = e_0(1,0,0)$, we find from Maxwell's equations (7), (8) the equation $\mu\varepsilon_{11}\omega^2 = k^2$, and the index of refraction is given by

$$n_\parallel = \sqrt{\mu\varepsilon_{11}} = \sqrt{1+\gamma B_0^2}. \tag{18}$$

If the polarization of the electromagnetic wave is perpendicular to external induction magnetic field, $\mathbf{e} = e_0(0,1,0)$, we have $\mu\varepsilon_{22}\omega^2 = k^2$. As a result, the index of refraction is

$$n_\perp = \sqrt{\varepsilon_{22}\mu} = 1. \tag{19}$$

The phase velocity depends on the polarization of the electromagnetic wave, and the effect of vacuum birefringence holds. The speed of electromagnetic wave is $v = 1/n_\parallel \neq 1$ when the polarization of the electromagnetic wave is parallel to external magnetic field, $\mathbf{e}_0 \parallel \mathbf{B}_0$. In the case $\mathbf{e} \perp \mathbf{B}_0$, the speed of the electromagnetic wave coincides with the speed of light, $v = 1/n_\perp = 1$.

According to the Cotton-Mouton (CM) effect [18] the coefficient k_{CM} is defined as

$$\triangle n_{CM} = n_\parallel - n_\perp = k_{CM} B_0^2. \tag{20}$$

From Eqs. (18)-(20), using the approximation $\gamma B_0^2 \ll 1$, we obtain

$$\triangle n_{CM} = \sqrt{1+\gamma B_0^2} - 1 \approx \frac{\gamma B_0^2}{2}, \qquad k_{CM} \approx \frac{1}{2}\gamma. \tag{21}$$

The value k_{CM}, found in the BMV experiment [19], for the magnetic induction field $B_0 = 6.5$ T is gives by

$$k_{CM} = (5.1 \pm 6.2) \times 10^{-21}\mathrm{T}^{-2}. \tag{22}$$

From Eqs. (21), (22), we evaluate the parameter of our model

$$\gamma \approx 10^{-20}\mathrm{T}. \tag{23}$$

It should be noted that the value of k_{CM} calculated from QED is much smaller than the experimental value (22) [19], $k_{CM}^{QCD} \approx 4.0 \times 10^{-24}\mathrm{T}^{-2}$. Therefore the model of non-linear electrodynamics under consideration with two free parameters are of interest.

5 The energy-momentum tensor and dilatation current

The symmetrical Belinfante tensor, obtained from Eq. (1) with the help of the method of [20], is

$$T_{\mu\nu}^B = -\frac{1}{\Pi} F_{\nu\alpha}\left(F_{\mu\alpha} - \gamma\mathcal{G}\tilde{F}_{\mu\alpha}\right) - \delta_{\mu\nu}\mathcal{L}, \tag{24}$$

where Π is given by Eq. (3). From Eq. (24) one finds the energy density

$$T_{44}^B = \frac{1}{\Pi}\left(\mathbf{E}^2 - \gamma\mathcal{G}^2\right) + \frac{1}{\beta}\arcsin\left(\beta\mathcal{F} - \frac{\beta\gamma}{2}\mathcal{G}^2\right). \tag{25}$$

We obtain the trace of the energy-momentum tensor (24):

$$T_{\mu\mu}^B = -\frac{4}{\Pi}\left(\mathcal{F} - \gamma\mathcal{G}^2\right) + \frac{4}{\beta}\arcsin\left(\beta\mathcal{F} - \frac{\beta\gamma}{2}\mathcal{G}^2\right). \tag{26}$$

Because the trace of the energy-momentum tensor is not zero [20], one finds the dilatation current

$$D_\mu^B = x_\alpha T_{\mu\alpha}^B. \tag{27}$$

Then the divergence of dilatation current is given by

$$\partial_\mu D^B_\mu = T^B_{\mu\mu}. \tag{28}$$

So, the scale (dilatation) symmetry is broken as we have introduced the dimensional parameters β, γ. The dilatation symmetry is also broken in BI electrodynamics [14] but in classical electrodynamics the dilatation symmetry is conserved.

5.1 Energy of the point-like charged particle

Now we calculate the total electric energy of charged point-like particle. In the case of electrostatics ($\mathbf{B} = 0$) the electric energy density (25) becomes

$$\rho_E = T^B_{44} = \frac{E^2}{\sqrt{1-\beta^2 E_0^4/4}} - \frac{1}{\beta}\arcsin\left(\frac{\beta E^2}{2}\right). \tag{29}$$

Defining the total energy $\mathcal{E} = \int \rho_E dV$, and using (14), (15), the value $\beta^{1/4}\mathcal{E}$ is given by

$$\beta^{1/4}\mathcal{E} = \frac{e^{3/2}}{4\sqrt{\pi}}\int_0^\infty \left[\frac{\sqrt{2(\sqrt{x^4+1}-x^2)}}{\sqrt{x}} - \sqrt{x}\arcsin\left(\sqrt{x^4+1}-x^2\right)\right]dx$$
$$\approx 0.071. \tag{30}$$

Implying that the electron mass equals the electromagnetic energy of the point-like charged particle, $\mathcal{E} = 0.51$ MeV, we obtain the parameter $l = \beta^{1/4} = 27.6$ fm. Thus, the old idea of the Abraham and Lorentz [21], [22], [23], about the electromagnetic nature of the electron is realized here for the model suggested.

6 Conclusion

We have proposed a new model of nonlinear electrodynamics possessing two dimensional parameters β, γ. There is the effect of vacuum birefringence if the external constant and uniform induction magnetic field is present. If $\gamma = 0$ the phenomenon of vacuum birefringence vanishes. The value γ was estimated from the data of BMV experiment, $\gamma \approx 10^{-20}$ T. We have calculated the indices of refraction for two polarizations of electromagnetic waves, parallel and perpendicular to the magnetic field so that phase velocities of electromagnetic waves depend on polarizations. The canonical and symmetrical Belinfante energy-momentum tensors and dilatation current were found and we show that the dilatation symmetry is violated. The scale symmetry is broken as the dimensional parameters β, γ are introduced. The electric field of a point-like charge is finite at the origin in the model considered. We have calculated the finite electromagnetic energy of point-like charged particles. For $l \equiv \beta^{1/4} = 27.6$ fm the electron mass equals the total electromagnetic energy and we can assume that the mass of the electron has a pure electromagnetic nature.

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