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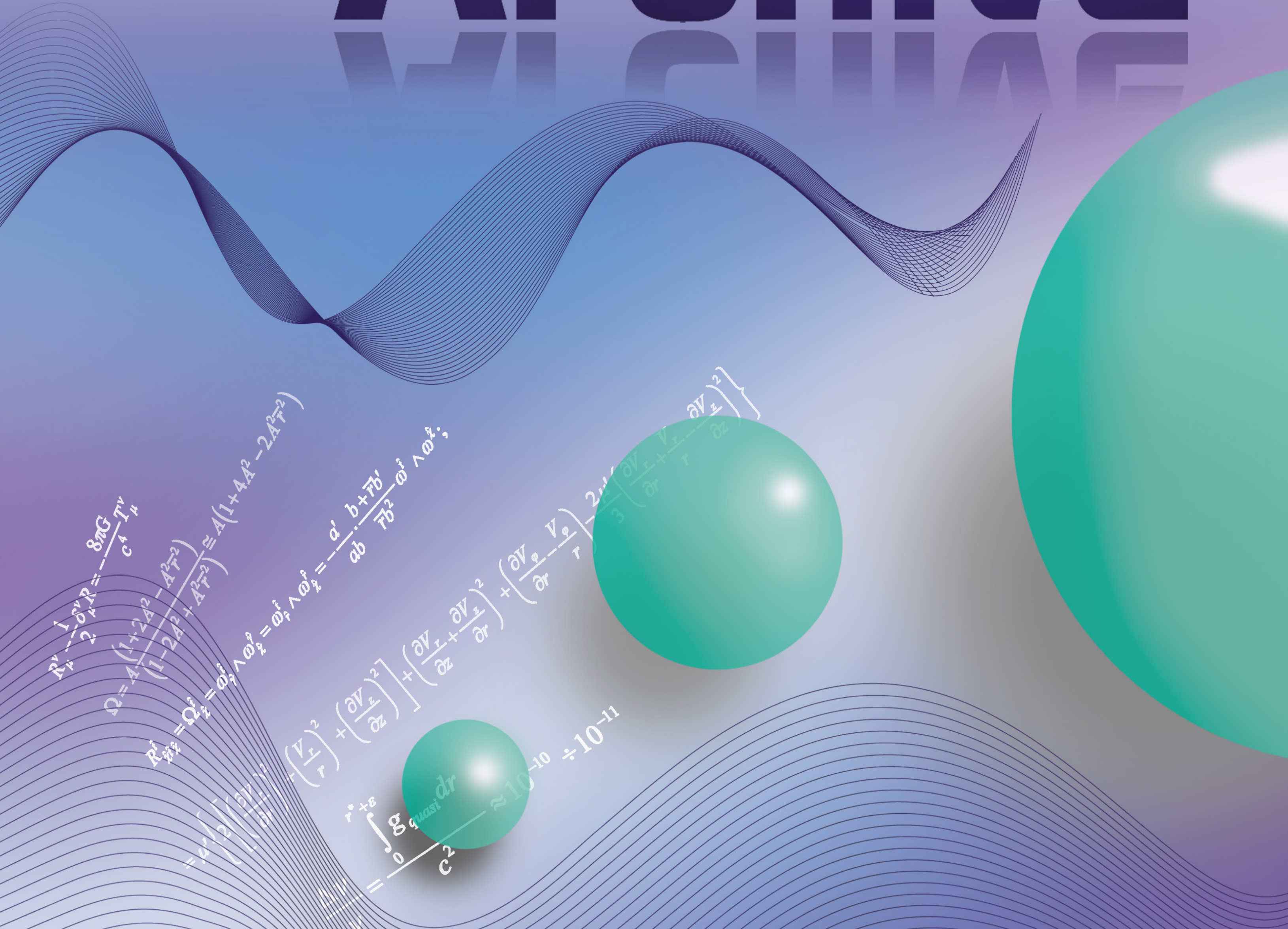

Volume 14 Number 1

January - March
2025

Intellectual Archive

Volume 14, Number 1

Publisher : Shiny World Corp.
Address : 9200 Dufferin Street
P.O. Box 20097
Concord, Ontario
L4K 0C0
Canada

E-mail : support@IntellectualArchive.com
Web Site : www.IntellectualArchive.com
Series : Journal
Frequency : Every 3 months
Month : January – March 2025
ISSN : 1929-4700
DOI : 10.32370/IA_2025_1
Trademark : **IntellectualArchive™**

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Volume 14 **Number 1** **January - March 2025**

Table of Contents

Toronto, January – March 2025

INTELLECTUAL ARCHIVE, Vol. 14, No. 1, 2025

DOI: 10.32370/IA_2025_01_1

Electron scattering with radiative corrections

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January 27, 2025

Abstract

The scattering process in the framework of the source theory is considered as the synergism of the elastic and the inelastic process. In this approach the infrared divergences never occur. The resulting differential cross section appears as a factor multiplying the lowest-order (Born) differential cross section plus a contribution referring to the magnetic form factor.

1 Introduction

The scattering process in the framework of the source theory is considered as the synergism of the elastic and the inelastic process. In this approach the infrared divergences never occur. The resulting differential cross section appears as a factor multiplying the lowest-order (Born) differential cross section plus a contribution referring to the magnetic form factor.

The problem of radiative corrections for particle scattering is presented in many articles and textbooks (Schwinger, 1949; Schwinger, 1973; Akchiezer et al., 1965) In this article we will solve this problem in the framework of the source theory and we will follow the articles of Lester de Raad et al. (1972) and Schwinger (1973).

At the previous papers there was an effort to separate the elastic process from the inelastic one. However, the realistic approach is to consider the scattering in its physical complexity i.e. to consider the elastic and inelastic scattering in a unified manner.

We will see that in this method of approach the infrared sensitivity never occurs. However, we will show, that there are also the infrared sensitive nonelastic processes that occur separately and have no associated elastic counterparts. The corresponding terms require to insert a photon mass, but the dependence on this mass vanishes after summing all such purely inelastic contributions.

Let us first remember the basic formalism. Consider a situation when an electron is moving in the time-independent electromagnetic field of the four-potential A_μ. The vacuum amplitude describing the propagation of electron from the source η_2 to source η_1 is as follows:

$$\langle 0_+|0_-\rangle = i\int (dx)(dx')\eta_1(x)\gamma^0 G_+(x-x')\eta_2(x') \quad + \quad i\int (dx)(dx')\psi_1(x)\gamma^0 Z(A,x,x')\psi_2(x') \tag{1}$$

where G_+ is the Green function of the free electron, ψ are fields associated with sources and $Z(A,x,x')$ is a functional of A. The first term describes the propagation of electron without interaction and the second term involves all interactions with the external field. The field ψ_2 is before any interaction and ψ_1 is after any interaction. The radiative corrections are obviously involved in $Z(A,x,x')$.

2 Radiative corrections to electron scattering

Our goal here is to determine the forward scattering amplitude $\langle p\sigma q|p\sigma q\rangle$, where the symbols refer to the momentum, spin and charge eigenvalues of the electron. It means that we must extract this amplitude from the vacuum amplitude $\langle 0_+|0_-\rangle$. The general treatment was described by textbooks of source theory (Schwinger, 1969; Schwinger; 1970; 1973; 1989) . In this article we briefly remark that the contribution of the first term in eq.(1) provides unity and we insert the following formulas into the second term of eq. (1):

$$\psi_1(x) \to (2md\omega_p)^{1/2}\, u^*_{p\sigma q}e^{-ipx} \tag{2a}$$

$$\psi_2(x) \to (2md\omega_p)^{1/2}\, u_{p\sigma q}e^{ipx} \tag{2b}$$

where $u_{p\sigma q}$ is the spinor with eigenvalues σ, q being the eigenvalues of the charge matrix q (Lester de Raad et al., 1972)

$$q = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{3}$$

and

$$(m+\gamma p)u_{p\sigma q} = 0 \tag{4}$$

$$u^*_{p\sigma q}\gamma^0 u_{p\sigma' q'} = \delta_{\sigma\sigma'}\delta_{qq'}. \tag{5}$$

The resulting formula after insertion is the forward-scattering probability amplitude

$$\langle p\sigma q|p\sigma q\rangle = 1 + i2md\omega_p\int(dx)(dx')e^{-ipx}u^*_{p\sigma q}\gamma^0 Z(A,x,x')u_{p\sigma q}e^{ipx'}. \qquad (6)$$

which in turn yields

$$1 = |\langle p\sigma q|p\sigma q\rangle|^2 + 4md\omega_p\mathrm{Im}\,\left\{\int(dx)(dx')e^{-ipx}u^*_{p\sigma q}\gamma^0 Z(A,x,x')u_{p\sigma q}e^{ipx'}\right\} \qquad (7)$$

The last formula is the sum of the probability that the initial electron goes to the state $\langle p\sigma q|$ and the probability that the initial electron goes to the state other than $\langle p\sigma q|$. From the later probability we can determine the total cross section which can be obtained by division by T and $2|\mathbf{p}|d\omega_p$, where T is the time during which the external field acts upon the electron and $2|\mathbf{p}|d\omega_p$ is the incident particle flux. The total cross section is then

$$\sigma = \frac{1}{T}\,\frac{2m}{|\mathbf{p}|}\mathrm{Im}\,\left\{\int(dx)(dx')e^{-ipx}u^*_{p\sigma q}\gamma^0 Z(A,x,x')u_{p\sigma q}e^{ipx'}\right\}. \qquad (8)$$

Let us first consider the simple example of an electron scattering without radiative corrections in the electromagnetic field, expanded to the second order in this field. The basic task is to determine $Z(A,x,x')$. We determine is as follows.

The single electron exchange between source η_2 and η_1 represents in the analogy with the non-interacting case the vacuum amplitude

$$\langle 0_+|0_-\rangle = i\int(dx)(dx')\eta_1(x)\gamma^0 G^A(x,x')\eta_2(x'), \qquad (9)$$

where

$$G^A(x,x') = G_+(x-x') + \int(dy)G_+(x-y)eq\gamma A(y)G_+(y-x') \quad +$$

$$\int(dy)(dy')G_+(x-y)eq\gamma A(y)G_+(y-y')eq\gamma A(y')G_+(y'-x') + ... \qquad (10)$$

The field is defined by the equation

$$\psi(x) = \int(dx')G_+(x-x')\eta(x'). \qquad (11)$$

After insertion of eq. (10) into eq. (11) we get the contribution

$$Z(A,x,x') = eq\gamma A(x)\delta(x-x') + eq\gamma A(x)G_+(x-x')eq\gamma A(x') \qquad (12)$$

and for σ we have

$$\sigma = \frac{2m}{|\mathbf{p}|}\,\mathrm{Im}\,\Big(u^*_{p\sigma q}\gamma^0 eq\gamma A(0)u_{p\sigma q} \quad +$$

$$\int\,\frac{(d\mathbf{p}')}{(2\pi)^3}u^*_{p\sigma q}\gamma^0 eq\gamma A(p-p')\frac{m-\gamma p'}{p'^2+m^2-i\varepsilon}eq\gamma A(p'-p)u_{p\sigma q}\Big) \quad =$$

$$\frac{m}{|\mathbf{p}|}\int\frac{(d\mathbf{p}')}{(2\pi)^2}\delta(p'^2+m^2)u^*_{p\sigma q}\gamma^0 eq\gamma A(p-p')(m-\gamma p')eq\gamma A(p'-p)u_{p\sigma q}, \tag{13}$$

where $p^0=p'^0$ since the potential is time-independent and the three-dimensional Fourier transforms have been employed.

Using the identity

$$\int\frac{(d\mathbf{p}')}{(2\pi)^2}\delta(p'^2+m^2)=\int d\Omega\frac{|\mathbf{p}'|}{8\pi^2}, \tag{14}$$

we get

$$\frac{d\sigma}{d\Omega}=\frac{m}{8\pi^2}u^*_{p\sigma q}eq\gamma A(p-p')(m-\gamma p')eq\gamma A(p'-p)u_{p\sigma q}, \tag{15}$$

which is the Born formula for the differential cross section in the potential scattering, $\mathbf{p}'$ being the momentum of the scattered electron.

3 Calculation of the radiative corrections

There are two general types of radiative corrections to the lowest order. 1) The causal exchange of an electron between sources η_2 and η_1, 2) the exchange of an electron-photon pair.

The first process with a local external potential gives rise to a Born approximation as we have seen yet with with the absence of radiative processes. The second process involves radiative corrections.

The action involving the interaction of an electron with the electromagnetic field is

$$W_{int}=\frac{1}{2}\int(dx)\psi(x)\gamma^0 eq\gamma^\mu A_\mu(x)\psi(x) \tag{16}$$

and the three-field analogy is called the primitive interaction.

The vacuum amplitude corresponding to W_{int} is

$$\langle 0_+|0_-\rangle=i\int(dx)A_{1\mu}(x)\psi_1(x)\gamma^0 eq\gamma^\mu\psi_2(x) \tag{17}$$

as a reslut of insertion of $\psi=\psi_1+\psi_2$ into W_{int} and expansion of exp iW_{int}. In analogy with the principle of superposition for ψ-decomposition we write for sources

$$\eta=\eta_1+\eta_2, \tag{18}$$

where we take η_2 for the emission source of the time-like virtual particle excitation and η_1 is real particle detection source. The virtual particle decays into an electron and photon and the pair propagates without further interaction. the photon is detected by the photon source J_2. The situation can be graphically pictured. The source η_2 can be identified with so called effective electron-photon source ηJ because it emits through the virtual particle the electron and photon. The $\eta - J$ structure of this source can be determined after expansion of

$$\langle 0_+|0_-\rangle^{\eta J} = \langle 0_+|0_-\rangle^{\eta}\langle 0_+|0_-\rangle^{J} \tag{19}$$

and by the extraction from it the term

$$\langle 0_+|0_-\rangle = i\int(dx)(d\xi)A_{1\mu}(\xi)\psi_1(x)\gamma^0 i\eta_2(x)J_2^{\mu}(\xi) \tag{20}$$

and after comparison of eq. (20) with eq. (16). The result is

$$i\eta_2(x)J_2^{\mu}(\xi)|_{eff} = \delta(x-\xi)eq\gamma^{\mu}\psi_2(x). \tag{21}$$

In a similar manner, considering the situation with source η_1 as a detection source of the virtual particle, we get the effective electron-photon detection source of the form:

$$i\eta_1(x)\gamma^0 J_1^{\mu}(\xi)|_{eff} = \psi_1(x)\delta(x-\xi)\gamma^0 eq\gamma^{\mu}. \tag{22}$$

The process involving the both partial processes i.e. emission and absorption is synthetized.

The corresponding vacuum amplitude describing the exchange of a noninteractig electron-positron pair is extracted from

$$\langle 0_+|0_-\rangle^{\eta J} = \langle 0_+|0_-\rangle^{\eta}\langle 0_+|0_-\rangle^{J} \tag{23}$$

or, from

$$\langle 0_+|0_-\rangle^{\eta J} = \int(dx)(dx')\eta_1(x)\gamma^0 G_+(x-x')\eta_2(x')i\int(d\xi)(d\xi')J_1^{\mu}(\xi)D_+(\xi-\xi')J_{2\mu}(\xi'). \tag{24}$$

Then, we insert eqs.(21) and (22) into (24) in order to get

$$\langle 0_+|0_-\rangle^{\eta J} = e^2\int(dx)(dx')\psi_1(x)\gamma^0\gamma^{\mu}G_+(x-x')D_+(x-x')\gamma_{\mu}\psi_2(x') \tag{25}$$

Since we are interested in the electron-photon process in the presence of the external electromagnetic field, we replace the electron fields ψ and the propagation function G_+ by ψ^A and G_+^A corresponding to situation with the presence of electromagnetic field. Then, we have:

$$\langle 0_+|0_-\rangle^{\eta J}|_{A\neq 0} = e^2\int(dx)(dx')\psi_1^A(x)\gamma^0\gamma^{\mu}G^A(x,x')D_+(x-x')\gamma_{\mu}\psi_2^A(x'). \tag{26}$$

The validity of eq. (26) is restricted to $x^0 > x'^0$ as a consequence of the causal situation i.e. the detection source is later than the emission source. The extension to the general situation is postulated by the space-time extrapolation.

When G^A in eq. (26) is replaced by expansion

$$G^A \approx G_+ + G_+ eq\gamma A G_+ + G_+ eq\gamma A G_+ eq\gamma A G_+, \tag{27}$$

which is a sufficient approximation for the propagation function of electron in the external field, we get three types of processes. 1) $G^A \to G_+$ implies the electron propagator modification, 2) $G^A \to G_+ eq\gamma A G_+$ is the linear term contribution , 3) $G^A \to G_+ eq\gamma A G_+ eq\gamma A G_+$ is the double scattering contribution.

Only the vacuum amplitude terms quadratic in A are retained and they are sufficient for an approximation. The diagram corresponding to the linear interaction can be graphically pictured.

Now, we approach to the discussion of of the contribution of vacuum polarization and double scattering.

4 Vacuum polarization calculation

After performing the Fourier transformation, the total external potential is written as

$$A^\mu(q) = D_+(q^2) J^\mu(q) \tag{28}$$

where J^μ is the associate source. We know that the vacuum polarization leads to the following modification of the photon Green function

$$\tilde{D}_+(q^2) = \frac{1}{q^2} + \int_{4m^2}^{\infty} dM^2 \frac{a(M^2)}{q^2 + M^2} \tag{29}$$

where

$$a(M^2) = \frac{\alpha}{3\pi} \frac{1}{M^2} \left(1 + \frac{2m^2}{M^2}\right) \left(1 - \frac{4m^2}{M^2}\right)^{1/2} \tag{30}$$

Using. eq. (28) it means that the Born cross section is multiplied by factor $(\tilde{D}_+/D_+)^2 \approx D_+^2(1 + 2\varepsilon/D_+)$, and it determines the parameter δ_1 in the corrected cross section

$$\frac{d\sigma}{d\Omega} = (1 - \delta_1) \left(\frac{d\sigma}{d\Omega}\right)_{Born} \tag{31}$$

is of the form

$$\delta_1 = -2q^2 \int_{4m^2}^{\infty} dM^2 \frac{a(M^2)}{q^2 + M^2}, \tag{32}$$

which can be expressed by change of variables

$$M^2 = \frac{4m^2}{1 - v^2} \tag{33}$$

as

$$\delta_1 = -\frac{\alpha}{6\pi}\frac{q^2}{m^2}\int_0^1 dv \frac{v^2(3-v^2)}{1-\frac{q^2}{4m^2}(1-v^2)}. \quad (34)$$

5 Propagator modifications

The vacuum amplitude corresponding to the transformation $G^A \to G_+$ in eq. (26) is

$$\langle 0_+|0_-\rangle = e^2 \int (dx)(dx')\psi_1^A(x)\gamma^0\gamma^\mu G_+(x,x')D_+(x-x')\gamma_\mu\psi_2^A(x') \quad (35)$$

where we use for G_+ and D_+ the causal representation

$$G_+(x-x') = \left(m - \frac{1}{i}\gamma^\mu\partial_\mu\right)\Delta_+(x-x') \quad (36)$$

with

$$\Delta_+(x-x') = i\int d\omega_p e^{ip(x-x')}; \quad x^0 > x'^0 \quad (37)$$

$$D_+(x-x') = \Delta_+(x-x'; m^2=0). \quad (38)$$

Using the Fourier transformation and the identity

$$1 = \frac{1}{2\pi}\int d\omega_P dM^2 (2\pi)^4\delta(P-p-k) \quad (39)$$

with $-P^2 = M^2$, we get with eq. (36), (37) and (38):

$$\langle 0_+|0_-\rangle = \frac{ie^2}{2\pi}\int id\omega_P dM^2\psi_1^A(-P)\gamma^0\int d\omega_p d\omega_k(2\pi)^4\delta(P-p-k)\gamma^\mu(m-\gamma p)\gamma_\mu\psi_2^A(P), \quad (40)$$

where we supposed the mas of photon is $\mu \ll m$.

The $p-k$ phase-space integral it is suitable to calculate in the P rest frame. Then after application of the space-time extrapolation, we have

$$\langle 0_+|0_-\rangle = i\frac{\alpha}{4\pi}\int \frac{(dP)}{(2\pi)^4}\int_{(m+\mu)^2}^\infty \frac{dM^2}{M^2}\left[(M^2-m^2)-4M^2\mu^2\right]^{1/2} \times$$

$$\psi_1^A(-P)\left\{\left(-4m^2 - \frac{M^2+m^2}{M^2}\gamma P\right)\frac{1}{P^2+M^2-i\varepsilon} + C.T.\right\}\psi_2^A(P), \quad (41)$$

where the space-time extrapolation was realized as

$$d\omega_P \to \frac{dP}{(2\pi)^4}\frac{1}{P^2+M^2-i\varepsilon}. \quad (42)$$

The contact term C.T. is introduced here as a necessity, because the causal process does not inform us about behavior of vacuum amplitude for $x^0 \approx x'^0$. This term is consequently proportional to $\delta(x-x')$ or derivatives thereof. When $\psi^A \to \psi$, the factor $\{...\}$ in eq. (41) and its first derivative with respect to γP must vanish for $\gamma P = -m$. This requirement has consequence to determine C.T. in such a way that

$$\{./.\} = \frac{(\gamma P + m)^2 \omega}{P^2 + M^2 - i\varepsilon}, \tag{43}$$

where

$$\omega = \frac{1}{2M^2}\left[\left(1 - \frac{2Mm}{(M-m)^2}\right)(M - \gamma P) + \left(1 + \frac{2Mm}{(M+m)^2}\right)(-M - \gamma P)\right]. \tag{44}$$

If we further retain only the infrared singular part of ω and appropriate ψ^A by

$$(\gamma P + m)\psi^A(P) = \int (dx) e^{-iPx} eq\gamma A(x) \equiv (eq\gamma A\psi)(P), \tag{45}$$

we get instead of (41)

$$\langle 0_+|0_-\rangle = i\frac{\alpha}{4\pi}\int \frac{(dP)}{(2\pi)^4}\int \frac{dM^2}{M^2}\left[(M^2 - m^2) - 4M^2\mu^2\right]^{1/2}(\psi_1\gamma^0 eq\gamma A)(-P) \quad \times$$
$$\left(-\frac{m}{M}\right)\frac{1}{(M-m)^2}\frac{m - \gamma P}{P^2 + M^2 - i\varepsilon}(eq\gamma A\psi_2)(P), \tag{46}$$

from which we extract the factor δ_2 of the correction to the Born cross section

$$\delta_2 = -\frac{\alpha}{4\pi}\int_{(m+\mu)^2}^{(m+\delta M)^2} \frac{dM^2}{M^2}\left[(M^2 - m^2) - 4M^2\mu^2\right]^{1/2}\left(-\frac{m}{M}\right)\frac{1}{(M-m)^2} \quad \approx$$
$$\frac{\alpha}{\pi}\int_{\mu}^{\delta M} d(M-m)\left[(M-m)^2 - \mu^2\right]^{1/2}(M-m)^{-2} \quad =$$
$$\frac{\alpha}{\pi}\left[\ln\frac{2\delta M}{\mu} - 1\right]. \tag{47}$$

where $m^2 \leq -p'^2 \leq (m + \delta M)^2$, $\delta M \ll m$.

6 Double scattering

The vacuum amplitude for double scattering is obtained by transformation

$$G^A \to G_+ A G_+ A G_+ \tag{48}$$

in the vacuum amplitude

$$\langle 0_+|0_-\rangle^{\eta J}_{A\neq 0} = e^2 \int (dx)(dx')\psi_1^A(x)\gamma^0\gamma^\mu G^A(x,x')D_+(x-x')\gamma_\mu\psi_2^A(x'). \tag{49}$$

The resulting amplitude for double scattering can be expressed after necessary calculations as

$$\langle 0_+|0_-\rangle = ie^2 \int \frac{(dP_1)}{(2\pi)^4} \int \frac{(dP_2)}{(2\pi)^4} \quad \times$$

$$\psi_1(-P_1)\gamma^0\gamma^\mu \frac{m-\gamma P_1+\gamma k}{(P_1-k)^2+m^2} eq\gamma A(P_1-P)dM^2 d\omega_P d\omega_k \quad \times$$

$$\delta((P-k)^2+m^2)(m-\gamma P+\gamma k)eq\gamma A(P-P_2)\frac{m-\gamma P_2+\gamma k}{(P_2-k)^2+m^2}\gamma_\mu\psi_2(P_2). \tag{50}$$

where the momentum of the exchanged electron has been substituted according to $p = P - k$

$$d\omega_P = \frac{(dP)}{(2\pi)^3}\delta((P-k)^2+m^2) = dM^2 d\omega_P \delta((P-k)^2+m^2) \tag{51}$$

Equation (50) leads to purely inelastic contribution and it means that only its infrared singular part need be retained, which means the γk factor in the numerator may be dropped. Upon rearrangement and operation of the projection on the fields, the vacuum amplitude reduces to

$$\langle 0_+|0_-\rangle = ie^2 \int \frac{(dP_1)}{(2\pi)^4} \int \frac{(dP_2)}{(2\pi)^4} i d\omega_P \int_{(m+\mu)^2}^{\infty} dM^2 \psi_1(-P)\gamma^0 eq\gamma A(P_1-P) \quad \times$$

$$\left(\int d\omega_k \delta((P-k)^2+m^2)\frac{P_2^2}{(P_2 k)^2}(m-\gamma P)\right) eq\gamma A(P-P_2)\psi_2(P_2). \tag{52}$$

After space-time extrapolation we get (the contact terms are not inserted since they are not physically required) for the correction δ_3:

$$\delta_3 = -e^2 \int_{(m+\mu)^2}^{(m+\delta M)^2} dM^2 d\omega_k \delta((P-k)^2+m^2)\frac{P_2^2}{(P_2 k)^2} \approx$$

$$\frac{\alpha}{\pi}\int_{\mu}^{\delta M} d(M-m)\left[(M-m)^2-\mu^2\right]^{1/2}\left[(M-m)^2+\mu^2\frac{q^2}{m^2}\left(1+\frac{q^2}{4m^2}\right)\right]^{-1} \quad =$$

$$\frac{\alpha}{\pi}\left(\ln\frac{2\delta M}{\mu} - 1 - \frac{q^2}{4m^2}\int_0^1 dv\,\frac{1+v^2}{1+\frac{q^2}{4m^2}(1-v^2)}\right). \tag{53}$$

where $q - P - P_2$ is the momentum transfer. The k-integral was evaluated in the P rest frame and using relations:

$$P_2^0 = -\frac{PP_2}{M} = \frac{1}{2M}(M^2 + m^2 + q^2) \tag{54}$$

and

$$|\mathbf{P}_2|^2 = \frac{1}{4M^2}(M^2 + m^2 + q^2)^2 - m^2 \approx q^2\left(1 + \frac{q^2}{4m^2}\right). \tag{55}$$

7 The linear term. The electric part

The diagram corresponding to the linear term $G^A \to G_+ A G_+$ follows from the formula

$$\langle 0_+|0_-\rangle = e^2 \int (dx)(dx')\psi_1^A(x)\gamma^0\gamma^\mu G_+^A(x,x')D_+(x-x')\gamma_\mu\psi_2^A(x') \tag{56}$$

Upon transformation of the linearized amplitude (56) into momentum space we get

$$\langle 0_+|0_-\rangle = -\int \frac{(dP_1)}{(2\pi)^4}\int \frac{(dP_2)}{(2\pi)^4}\psi_1(-P_1)\gamma^0 eqI^\mu A_\mu(q)\psi_2(P_2) \tag{57}$$

with

$$q = P_1 - P_2 \tag{58}$$

and

$$I^\mu = ie^2\int d\omega_k d\omega_p d\omega_{p'}(2\pi)^4\delta(p+k-P_2)(2\pi)^4\delta(p'+k-P_1)\gamma^\nu(m-\gamma p')\gamma^\mu(m-\gamma p)\gamma_\nu \tag{59}$$

which can be expressed in the general form as

$$I^\mu = i\alpha\pi\gamma^\mu f(M_1^2, M_2^2, q^2) + \frac{\alpha\pi}{2m}\sigma^{\mu\nu}q_\nu g(M_1^2, M_2^2, q^2), \tag{60}$$

where $-P_1^2 = M_1^2$ and functions f and g are to be determined. The eq.(57) contains no q^μ because we work in the Lorentz gauge. Upon contraction of I^μ with appropriate vector the functions f and g are isolated and expressed in terms of the known kinematic factors as follows (de Raad et al., 1972):

$$f = -2q^2\Delta^{-5/2}\left\{q^8 + q^6\left[3(M_1^2 + M_2^2) + 4m^2\right] \quad +\right.$$

$$q^4\left[3M_1^4 + 3M_1^2M_2^2 + 3M_2^2 + 9m^2(M_1^2 + M_2^2) + 5m^4\right] \quad +$$

$$q^2\left[M_1^6 - 2M_1^4M_2^2 - 2M_1^2M_4^2 + M_6^2 + 6m^2M_1^4 + 2m^2M_1^2M_2^2\right] \quad +$$

$$q^2\left[6m^2M_2^4 + 13m^4(M_1^2 + M_2^2) - 6m^6\right] \quad -$$

$$(M_1^2 - M_2^2)^2 \left[2M_1^2M_2^2 - m^2(M_1^2 + M_2^2) - 8m^4\right]\Big\} \tag{61}$$

and

$$g = -4m^2q^2\Delta^{-3/2}\times$$

$$\Big\{6\left[q^2(M_1^2 - m^2)(M_2^2 - m^2) - m^2(M_1^2 - M_2^2)^2\right]\Delta^{-1} - (M_1^2 + M_2^2 - 2m^2)\Big\} \tag{62}$$

with

$$\Delta = (q^2 + M_1^2 + M_2^2)^2 - 4M_1^2M_2^2. \tag{63}$$

Quantities M_1^2 and M_2^2 satisfy the relation:

$$q^2(M_1^2 - m^2)(M_2^2 - m^2) \geq m^2(M_1^2 - M_2^2)^2. \tag{64}$$

After space-time extrapolation, the vacuum amplitude of vertex is as

$$\langle 0_+|0_-\rangle = \frac{i\alpha}{4\pi}\int \frac{(dP_1)}{(2\pi)^4}\int \frac{(dP_2)}{(2\pi)^4} \quad \times$$

$$\psi_1(-P_1)\gamma^0\left(eq\gamma^\mu A_\mu(q)F(P_1,P_2) + \frac{eq}{2m}\,\frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}(q)G(P_1,P_2)\right)\psi_2(P_2) \tag{65}$$

with

$$F(P_1,P_2) = \int dM_1^2 dM_2^2 \frac{f(M_1^2, M_2^2, q^2)}{(P_1^2 + M_1^2 - i\varepsilon)(P_2^2 + M_2^2 - i\varepsilon)} \tag{66}$$

and

$$G(P_1,P_2) = \int dM_1^2 dM_2^2 \frac{g(M_1^2, M_2^2, q^2)}{(P_1^2 + M_1^2 - i\varepsilon)(P_2^2 + M_2^2 - i\varepsilon)}. \tag{67}$$

where the region of integration is determined by eq. (64).

Using variables x and v, defined by

$$\frac{1}{2}(M_1^2 + M_2^2) = m^2 + (m^2 + \frac{1}{4}q^2)2x \tag{68}$$

$$(M_1^2 - M_2^2) = \left[q^2(m^2 + \frac{1}{4}q^2)\right]^{1/2} 2xv \tag{69}$$

where $x \in (0,\infty)$ and $v \in (-1,1)$, we have:

$$\frac{dM_1^2 dM_2^2}{\Delta^{1/2}} = \frac{dv}{2}\,\frac{xdx}{\beta^{1/2}}(q^2 + 4m^2) \tag{70}$$

$$\Delta^{1/2} f = -\frac{1}{2\beta^2}\left[q^2(4+12x+9x^2+3x^2v^2-x^3+5x^3v^2-2x^4v^2+2x^4v^4) \quad + \right.$$

$$\left. 4m^2(2+6x+2x^2+2x^2v^2-x^3-x^3v^2-2x^4v^2)\right] \tag{71}$$

and

$$\Delta^{1/2} g = -\frac{1}{2}(q^2+4m^2)m^2x\left[3x(1-v^2)\beta^{-2}-2\beta^{-1}\right], \tag{72}$$

where

$$\beta = 1+2x+x^2v^2. \tag{73}$$

Now, the vacuum amplitude is of the form:

$$\langle 0_+|0_-\rangle = \frac{i\alpha}{4\pi}\int\frac{(dP_1)}{(2\pi)^4}\int\frac{(dP_2)}{(2\pi)^4}\int_{-1}^{1}\frac{1}{2}dv \quad \times$$

$$\int_{x_0}^{\infty}\frac{xdx}{\beta^{1/2}}\psi_1^A(-P_1)\gamma^0 eqM^\mu A_\mu(q)\psi_2^A(P_2), \tag{74}$$

where

$$x_0 = \mu(m^2+\frac{1}{4}q^2)^{-1/2}(1-v^2)^{-1/2} \tag{75}$$

with

$$M^\mu = \gamma^\mu\left[\frac{q^2(q^2+4m^2)}{(P_1^2+M_1^2-i\varepsilon)(P_2^2+M_2^2-i\varepsilon)}f_1+\frac{q^2/4m^2}{1+\frac{q^2}{4m^2}(1-v^2)} \quad + \right.$$

$$\left.\frac{4f_3}{x^2}\left(\frac{(M_1^2-m^2)(M_2^2-m^2)}{(P_1^2+M_1^2-i\varepsilon)(P_2^2+M_2^2-i\varepsilon)}-1\right)\right] \quad +$$

$$\{2mf_4\left[(m+\gamma P_1)\gamma^\mu+\gamma^\mu(m+\gamma P_2)\right]+f_5(m+\gamma P_1)\gamma^\mu(m+\gamma P_2)\} \quad \times$$

$$\frac{1}{(P_1^2+M_1^2-i\varepsilon)(P_2^2+M_2^2-i\varepsilon)}. \tag{76}$$

where f_1 nd f_3 are extracted form eq. (71). To find the extraction, we substitute the equality

$$4m^2 = \left[4m^2+q^2(1-v^2)\right]-q^2(1-v^2) \quad =$$

$$\frac{4}{x^2}(M_1^2-m^2)(M_2^2-m^2)(q^2-4m^2)^{-1}-q^2(1-v^2) \tag{77}$$

into eq. (71). Then f_1 is identified as the coefficient of q^2 in eq. (71) and f_3 as the coefficient of

$$\frac{4}{x^2}(M_1^2 - m^2)(M_2^2 - m^2)(q^2 - 4m^2)^{-1}. \tag{78}$$

Then,

$$f_1 = -(1+x)(1+v^2)\beta^{-1} - \frac{3}{2}x^2(1-v^2)(1+xv^2)\beta^{-2} \tag{79}$$

$$f_3 = -(1+x-x^2)\beta^{-1} - \frac{3}{2}x^3(1-v^2)\beta^{-2}. \tag{80}$$

Till this moment we do not discuss the contact terms. They are determined as it is known by the special physical conditions. Here the contact terms are $-4f_3/x^2$ and f_2, f_3 in the expression for M^μ. The former was determined by the physical situation of non external electromagnetic influence, i.e. $J = 0$ and zero vacuum amplitude (74). The contact term f_2 is determined by te requirement that for real external electrons $\gamma P_1 = \gamma P_2 = -m^2$, eq. (74) with $\psi^A \to \psi$ reproduces the ordinary electric form factor (f_3, f_4, f_5 terms vanish). The contact term f_3 was derived from identification of eq. (41) with eq. (43) with $P \to P - eqA$, by the identification of their linear parts in A. The explicit form of f_2 is found to be

$$f_2 = \frac{-6(1+x)v^2}{x\beta}. \tag{81}$$

The basic structure, which appears in the imaginary part calculation of the cross section is M^μ multiplied by the propagator $(m - \gamma p)(p^2 + m^2 - i\varepsilon)^{-1}$, where p is P_2 in one vacuum term and P_1 in the other term. Then,

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\,(G_+ M^\mu) \quad &= \\ \frac{m-\gamma P_1}{(M_1^2-m^2)(M_2^2-m^2)}\gamma^\mu q^2(q^2+4m^2) f_1 \left[\delta(P_1+m^2) - \delta(P_1^2+M^2)\right] \quad &+ \\ (m-\gamma P_1)\gamma^\mu \frac{q^2}{4m^2} f_2 \left(1 + \frac{q^2}{4m^2}(1-v^2)\right)^{-1} \delta(P_1^2+m^2) \quad &- \\ (m-\gamma P_1)\gamma^\mu 4 f_3 x^{-2} \delta(P_1^2+M_1^2) \quad &+ \\ 2m f_4 \gamma^\mu (M_2-m^2)^{-1}\delta(P_1^2+M_1^2) + (P_1 \leftrightarrow P_2, M_1 \leftrightarrow M_2)&\end{aligned} \tag{82}$$

The f_5 term in M^μ did not enter in (82) because of the projection factors associated with it. The f_4 term is not infrared singular, so, as a purely inelastic contribution, it may be dropped. The major contribution comes from f_1 which is unification of elastic-inelastic structure and it is not infrared sensitive.

The x-integration limits for the inelastic contribution are given by $(x_0 \le x \le x_1)$

$$x_0 = \frac{\mu}{m}\left[(1-v^2)\left(1+\frac{q^2}{4m^2}\right)\right]^{-1/2} \tag{83}$$

$$x_1 = \frac{\delta M}{m}\left\{1+\frac{q^2}{4m^2} - v\left[\frac{q^2}{4m^2}\left(1+\frac{q^2}{4m^2}\right)\right]^{1/2}\right\}^{-1}. \tag{84}$$

So, finally , if we express the contribution to the cross section by δ_4 in analogy with the previous text, we have:

$$\delta_4 = -\frac{\alpha}{2\pi}\int_{-1}^{1}\frac{dv}{2}\left(\int_{x_1}^{\infty}\frac{4q^2 f_1}{x^2[4m^2+q^2(1-v^2)]} + \int_0^{\infty}\frac{(q^2/4m^2)f_2}{1+\frac{q^2}{4m^2}(1-v^2)} - \int_{x_0}^{x_1}\frac{4f_3}{x^2}\right)\frac{xdx}{\beta^{1/2}} = -\frac{2\alpha}{\pi}\left\{\ln\frac{2\delta M}{\mu} - 1 + \frac{q^2}{4m^2}\int_0^1 dv\frac{1}{1+\frac{q^2}{4m^2}(1-v^2)} \times \left[(1+v^2)\ln\left(\frac{\delta M}{4m}\,\frac{(1-v^2)^{3/2}}{v^2}\right) + \frac{1}{2} + v^2\right]\right\} \tag{85}$$

where we have used the identity

$$\int_{-1}^{1}\frac{dv}{2}\frac{1+v^2}{1+\frac{q^2}{4m^2}(1-v^2)} \times \ln\left\{1+\frac{q^2}{4m^2} - v\left[\frac{q^2}{4m^2}\left(1+\frac{q^2}{4m^2}\right)\right]^{1/2}\right\} = \int_0^1 dv\frac{1}{1+\frac{q^2}{4m^2}(1-v^2)}\left((1+v^2)\ln\frac{2v^2(1+v)}{(1-v)^{3/2}} - v - v^2\right) \tag{86}$$

in the derivation of δ_4 and this identity obtained from (4-5.104) and (4-12.42) in text by Schwinger (1973).

8 The linear term. The magnetic part

The corresponding vertex correction in this case is the magnetic part of vacuum amplitude (65). The magnetic vertex is not infrared singular and therefore to one order of approximation in δM, the inelastic contributions may be neglected. The correction reduces to the ordinary magnetic form factor and the vacuum amplitude is of the form:

$$\langle 0_+|0_-\rangle = \frac{i\alpha}{2\pi}\int (dx)(dx')(dx'')$$

$$\Big\{\psi_1(x)\gamma^0 eq\gamma A(x)G_+(x-x')\frac{eq}{2m}\frac{1}{2}\,\sigma^{\mu\nu}\psi_2(x')F_2(x'-x'')F_{\mu\nu}(x'') \quad +$$

$$F_{\mu\nu}(x'')F_2(x''-x)\psi_1(x)\gamma^0\frac{eq}{2m}\frac{1}{2}\,\sigma^{\mu\nu}G_+(x-x')eq\gamma A(x')\psi_2(x')\Big\}, \tag{87}$$

where the magnetic form factor is given by de Raad et al.(1972)

$$F_2(q) = \int_0^1 dv \frac{1}{1+\frac{q^2}{4m^2}(1-v^2)}. \tag{88}$$

From the amplitude (87) then can be deduced the cross section of the form

$$\left(\frac{d\sigma}{d\Omega}\right)_{mag} = \frac{\alpha m}{32\pi^3}F_2(q)\left\{u^*_{p\sigma q}\gamma^0\frac{e}{2m}\sigma^{\mu\nu}F_{\mu\nu}(-q)(m-\gamma p')eq\gamma A(q)u_{p\sigma q} \quad + \right.$$

$$\left. u^*_{p\sigma q}\gamma^0 eq\gamma A(-q)(m-\gamma p')\frac{e}{2m}\sigma_{\mu\nu}F_{\mu\nu}(q)u_{p\sigma q}\right\} \tag{89}$$

9 Discussion

We have presented here some new methods for calculating the radiative corrections for the scattering of an electron by an external electromagnetic field. This work differs from previous efforts' on the subject because it is formulated within Schwinger's source theory. But an additional important difference is that the conventional separation of elastic and inelastic (electron plus soft photon) processes is avoided. In such a way we obtained a reduction of calculation, relative to the conventional approach, and some standard infrared divergences are absent.

Our method combines a given elastic contribution with a corresponding inelastic contribution, and in such sums infrared sensitivity never occurs.

But there are also infrared-sensitive inelastic contributions that occur separately. In these terms we must insert a photon mass, then vanishing when all such purely inelastic contributions are explicitly summed.

Let us summarize the final results. We have yet mentioned that the complete formula of the cross section is of the form

$$\frac{d\sigma}{d\Omega} = (1-\delta)\left(\frac{d\sigma}{d\Omega}\right)_{Born} + \left(\frac{d\sigma}{d\Omega}\right)_{mag}, \tag{90}$$

where $(d\sigma/d\Omega)_{mag}$ is given by eq. (84) and δ is the sum of $\delta_1, \delta_2, \delta_3$ and δ_4, and it is explicitly equal to

$$\delta = \frac{\alpha}{2\pi}\frac{q^2}{m^2}\int_0^1 dv\frac{1}{1+\frac{q^2}{4m^2}(1-v^2)} \quad \times$$

$$\left[(1+v^2)\ln\left(\frac{4m}{\delta M}\,\frac{v^2}{(1-v^2)^{3/2}}\right)-1-\frac{5}{2}v^2+\frac{1}{3}v^4\right]. \tag{91}$$

After evaluation of the v-integral we get (Schwinger, 1973).

$$\delta=-\frac{2\alpha}{\pi}\left\{-\frac{19}{18}+\frac{4}{3}\frac{m^2}{q^2}+\ln\frac{m}{2\delta M}\quad-\right.$$

$$\frac{2m^2}{q^2}\zeta\left[-\frac{4}{3}\frac{m^2}{q^2}+\frac{11}{6}+\frac{19}{6}\frac{q^2}{4m^2}-\left(1+\frac{q^2}{2m^2}\right)\ln\frac{m}{2\delta M}\right]\ln\frac{1-\zeta}{1+\zeta}\quad+$$

$$\left.\frac{3}{2}\left(\frac{2m^2}{q^2+1}\right)\zeta\left[-f\left(\frac{2\zeta}{1-\zeta}\right)+f\left(-\frac{2\zeta}{1-\zeta}\right)+\frac{4}{3}f(\zeta)-\frac{4}{3}f(-\zeta)\right]\right\}, \tag{92}$$

where

$$\zeta^2=\frac{q^2}{4m^2+q^2} \tag{93}$$

and $f(x)$ is the Spence function (Berestetskii et al, 1982) defined as

$$f(x)=-\int_0^x\frac{dt}{t}\ln|1-t| \tag{94}$$

The non relativistic and ultra-relativistic asymptotic forms are

$$\delta_{nonrel.}=-\frac{\alpha}{2\pi}\frac{q^2}{m^2}\left(\frac{4}{3}\ln\frac{2\delta M}{m}-\frac{31}{90}\right);\quad \frac{q^2}{m^2}\ll 1 \tag{95}$$

and

$$\delta_{ultra-rel.}=\frac{2\alpha}{\pi}\left[\left(\ln\frac{q^2}{m^2}-1\right)\left(\ln\frac{4m}{\delta M}-\frac{19}{12}\right)-\frac{19}{36}\quad+\right.$$

$$\left.\frac{3}{4}\left(\ln\frac{q^2}{4m^2}\right)^2+3(\ln 2)(1-\ln 2)\right];\quad \frac{q^2}{m^2}\gg 1. \tag{96}$$

In the non-relativistic limit the magnetic cross section reduces to the Born cross section multiplied by a factor

$$\delta+\delta_{mag}=-\frac{2\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{2\delta M}{m}-\frac{19}{30}\right), \tag{97}$$

where

$$\delta_{mag}=\frac{\alpha}{4\pi}\frac{q^2}{m^2}. \tag{98}$$

In this limit $\delta M=\delta E$. Let us remark that during calculation we have not considered such effect as recoil which requires together other effect more further work.

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INTELLECTUAL ARCHIVE, Vol. 14, No. 1, 2025
DOI: 10.32370/IA_2025_01_2

Electromagnetic gravity with spin 3

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February 25, 2025

Abstract

We consider here the simple derivation of the Einstein equations by Fock. Then, we approach the way from the spin 1 fields to the spin 2 fields for massive and massless particles and we derive the gravity equations from this base. Then, we approach the spin 3 gravity called by us **electromagnetic gravity**.

1 Introduction

While the electromagnetic field was determined from the motion of charges and currents, the Einstein-Hilbert theory of gravity being the spacetime geometry was determined from presence of mass-energy and linear momentum. The corresponding equations - Einstein-Hilbert equations (EHE) - determine the metric tensor of spacetime for a given arrangement of stress-energy in the spacetime. The relationship between the metric tensor and the Einstein tensor allows the EHE to be written as a set of non-linear partial differential equations. The solutions of the EHE are the components of the metric tensor. The inertial trajectories of particles and radiation (geodesics) in the resulting geometry are then calculated using the geodesic equations. As well as obeying local energy-momentum conservation, the EHE reduce to Newtons law of gravitation, where the gravitational field is weak and velocities are much less than the speed of light.

We consider here the simple derivation of the Einstein equations by Fock. Then, we approach the way from the spin 1 fields to the spin 2 fields for massive and massless particles and we derive the corresponding action for spin 2 gravity from this base. Then, we approach the spin 3 gravity called by us **electromagnetic-gravity**.

2 The Einstein equations derived by Fock

There is the simple derivation of the EHE given by Fock (1964). The similar derivation was performed by Chandrasekhar (1972), Kenyon (1996), Landau et al. (1987), Rindler

(2003) and others. Source theory derivation of Einstein equations was performed by Schwinger (1970).

It is well known that the gravity mass M_G of some body is equal to the its inertial mass M_I, where gravity mass is a measure of a massive body to create the gravity field (or, gravity force) and the inertial mass of a massive body is a measure of the ability of the resistance of the body when it is accelerated. At present time we know, that if components of elementary particles have the same gravity and inertial masses, the body composed with such elementary particles has the identical gravity and inertial mass. There is no need to perform experimental verification. So, particle physics brilliantly confirms the identity of the inertial and gravity masses.

According to the Newton theory, the gravity potential is given by the equation

$$U(r) = -\kappa \frac{M}{r}, \tag{1}$$

where r is a distance from the center of mass of a body, κ is the gravitational constant and its numerical value is in SI units $6.67430(15)10^{-11} m^3.kg^{-1}.s^{-2}$ (CODATA, 2018).

The potential U is, as it is well known, the solution of the Poisson equation:

$$\Delta U(r) = -4\pi\kappa\varrho, \tag{2}$$

where ϱ is the density of the distributed masses.

The problem is, what is the geometrical formulation of gravity equation (2) following from the space-time element ds, which has the Minkowski form in case of the special theory of relativity.

Let us postulate that the motion of a body moving in the g-field is determined by the variational principle

$$\delta \int ds = 0. \tag{3}$$

In order to get the Newton equation of motion, we are forced to perform the following identity:

$$g_{00} = c^2 - 2U = -4\pi\kappa\varrho. \tag{4}$$

The second mathematical requirement, which has also the physical meaning is the covariance of the derived equation. It means that the necessary mathematical operation are the following replacing of original symbols:

$$U \to g_{\mu\nu} \tag{5}$$

with

$$\Delta U \to Tensor\ equation \tag{6}$$

and

$$\varrho \to T_{\mu\nu}, \tag{7}$$

where $T_{\mu\nu}$ is the tensor of energy and momentum.

In order to get the tensor generalization of eq. (2) it is necessary to construct new tensor $R_{\mu\nu}$, which is linear combination of the more complicated tensor $R_{\alpha\beta,\mu\nu}$, or

$$R_{\mu\nu} = g^{\alpha\beta} R_{\mu\alpha,\beta\nu} \tag{8}$$

and the scalar quantity R, which is defined by equation

$$R = g^{\lambda\mu} R_{\lambda\mu} \tag{9}$$

and construct the combination tensor $G_{\lambda\mu}$ of the form

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{10}$$

which has the mathematical property, that the covariant divergence of this tensor is zero, or,

$$\nabla^\lambda G_{\lambda\mu} = 0. \tag{11}$$

.

With regard to the fact that also the energy-momentum tensor $T_{\mu\nu}$ has the zero divergence, we can identify eq. (10) with the tensor $T_{\mu\nu}$, or

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi\kappa}{c^2} T_{\mu\nu}, \tag{12}$$

where the appeared constant in the last equation is introduce to get the classical limit of the equation.

The approximate solution of the last equation is as follows

$$ds^2 = (c^2 - 2U)dt^2 - \left(1 + \frac{2U}{c^2}(dx^2 + dy^2 + dz^2)\right). \tag{13}$$

The space-time element (13) is able to explain the shift of the frequency of light in gravitational field and the deflection of light in the gravitational field of massive body with mass M.

So, we have seen that the basic mathematical form of the Einstein general relativity is the Riemann manifold specified by the metric with the physical meaning. The crucial principle is the equality of the inertial and gravitational masses.

While the derivation of the EHE is elementary, Feynman wrote that the derivation of EHE by Einstein is difficult to understand. Namely:

Einstein himself, of course, arrived at the same Lagrangian but without the help of a developed field theory, and I must admit that I have no idea how he guessed the final result. We have had troubles enough arriving at the theory - but I feel as though he had done it while swimming underwater, blindfolded, and with his hands tied behind his back! (Feynman et al., 1995).

Now the question arises, what is the the force acting on the point moving in the homogenous gravitational field. It was calculated in the 3-form as follows (Landau, et al., 1987):

$$\mathbf{f} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \left\{ \mathbf{grad} \ln \sqrt{h} + \sqrt{h} \left[\frac{\mathbf{v}}{c} \mathbf{rot\ g} \right] \right\}, \tag{14}$$

with (Landau, et al., 1987).

$$h = 1 + \frac{2\varphi}{c^2}, \tag{15}$$

where φ s gravitational potential generating the acceleration $\mathbf{g}$. So, we see that it is not in the simple Newton form.

Let us still remark that the derived Einstein equations (12) can be generalized to form the Einstein equations with the cosmological constant, or, (Einstein, 1917)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -\frac{8\pi\kappa}{c^2}T_{\lambda\mu}, \tag{16}$$

where Λ is the new cosmological constant introduced formally, with the goal to find new form of the cosmological model and their solutions in the mathematical form. In addition to that the last equation can be still derived in order to involve so cosmological matrix (Pardy, 2018b). The new form of such equations is as follows:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + (\Lambda_{\alpha\beta}g^{\alpha\beta})g_{\mu\nu} = -\frac{8\pi\kappa}{c^2}T_{\mu\nu}, \tag{17}$$

where

$$\Lambda_{\alpha\beta} = \begin{pmatrix} \Lambda_{00} & \Lambda_{01} & \Lambda_{02} & \Lambda_{03} \\ \Lambda_{10} & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{20} & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{30} & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{pmatrix}. \tag{18}$$

The generalization of cosmology and the new deal of cosmology is then based on the Einstein-Pardy gravity equations (17) (Pardy, 2018b).

3 The spin 1 field equations

Spin was originally introduced as the rotation with an angular momentum of a particle around some axis. On the other hand, spin has some peculiar properties that distinguish it from orbital angular momenta: a) Spin quantum numbers may take half-integer values. b) Although the direction of its spin can be changed, an elementary particle cannot be made to spin faster or slower. c) The spin of a charged particle is associated with a magnetic dipole moment with a g-factor differing from 1. This could only occur classically if the internal charge of the particle were distributed differently from its mass.

The conventional definition of the spin quantum number, s, is s = n/2, where n can be any non-negative integer. So, the allowed values of s are 0, 1/2, 1, 3/2, 2, etc. The value of s for an elementary particle depends only on the type of particle, and cannot be altered in any known way (in contrast to the spin direction).

We show that the natural construction of the field of the particles with spin 1 is presented in source theory method. The relation

$$|\langle 0_+|0_-\rangle|^2 = \exp\{-2\mathrm{ImW}\} \leq 1 \tag{19}$$

is postulated to be valid for all spin fields. Let us show here the construction of action and field equations concerning spin one.

If spin zero particles and fields are described by the scalar source, then a vector source denoted here as $J^\mu(x)$ can be considered as a candidate for the description of the spin 1 fields and particles. However, there exist some obstacles because source $J^\mu(x)$ has four components and spin one particles have only three spin possibilities. Nevertheless first, let us investigate by analogy with the spin zero fields the following form of the action for the unit spin fields:

$$W(J) = \frac{1}{2}\int (dx)(dx')J^\mu(x)\Delta_+(x-x')J_\mu(x'). \tag{20}$$

Then,

$$|\langle 0_+|0_-\rangle|^2 = e^{iW}e^{iW^*} = \exp\left\{-\int d\omega_p J^{*\mu}(p)J_\mu(p)\right\}. \tag{21}$$

However,

$$J^{*\mu}(p)J_\mu(p) = |\mathbf{J}(p)|^2 - |J^0(p)|^2 \leq 0, \quad \text{or,} \ > 0 \tag{22}$$

and it means that the quantity defined by eq. (21) cannot be considered as the probability of the persistence of vacuum.

The difficulty can be overcome by replacing the original form $J^{*\mu}(x)J_\mu(x)$ by the following invariant structure:

$$J^{*\mu}(p)\left[g_{\mu\nu} + \frac{1}{m^2}p_\mu p_\nu\right]J^\nu(p), \tag{23}$$

which can be with regard to its invariance, determined in the rest frame of the time-like vector p^μ, where $p^\mu = (m, 0, 0, 0)$ in the rest frame. Then, with $g_{\alpha\alpha} = (-1, 1, 1, 1)$ we have

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{m^2}p_\mu p_\nu = \begin{cases} \delta_{kl}; & \mu = k; \quad \nu = l \\ 0; & \mu = 0; \quad \nu = 0 \\ 0; & \mu = k; \quad \nu = 0 \end{cases} \tag{24}$$

and

$$J^{*\mu}(p)\bar{g}_{\mu\nu}J^\nu(p) \equiv |\mathbf{J}|^2, \tag{25}$$

and now the quantity $|\langle 0_+|0_-\rangle|^2$ can be interpreted as the vacuum persistence probability.

At the same time $|\mathbf{J}|^2$ contains three independent source components, transforming among themselves under spatial rotation, as it is appropriate to unit spin.

After using eq. (23) it may be easy to get $W(J)$ in the space-time representation by the Fourier transformation, as it follows

$$W(J) = \frac{1}{2}\int (dx)(dx') \quad \times$$
$$\left\{J_\mu(x)\Delta_+(x-x')J^\mu(x') + \frac{1}{m^2}\partial_\mu J^\mu \Delta_+(x-x')\partial'_\nu J^\nu(x')\right\}. \tag{26}$$

The field of spin one particles can be defined using the definition of the test source $\delta J^\mu(x)$ by the relation

$$\delta W(J) = \int (dx)\delta J^\mu(x)\varphi_\mu(x), \tag{27}$$

where φ_μ is the field of particles with spin 1. After performing variation of the formula (26) and comparison with eq. (27) we get the equation for field of spin 1 in the following form:

$$\varphi_\mu(x) = \int (dx')\Delta_+(x-x')J_\mu(x') - \frac{1}{m^2}\partial_\mu \int (dx')\Delta_+(x-x')\partial'_\nu J^\nu(x'). \tag{28}$$

The divergence of the vector field $\varphi_\mu(x)$ is given by the relation

$$\partial_\mu\varphi^\mu(x) = \int (dx')\Delta_+(x-x')\partial'_\mu J^\mu(x') \quad -$$
$$\frac{1}{m^2}\partial^2\Delta_+(x-x')\partial'_\nu J^\nu(x') = \frac{1}{m^2}\partial_\mu J^\nu(x), \tag{29}$$

where we used relation $-\partial^2\Delta_+ = \delta(x-x') - m^2\Delta_+$.

Further, we have after applying operator $(-\partial^2 + m^2)$ on the equation (28) the following equation:

$$\left(-\partial^2 + m^2\right)\varphi_\mu(x) = J_\mu(x) - \frac{1}{m^2}\partial_\mu\partial_\nu J^\nu(x) \tag{30}$$

$$\left(-\partial^2 + m^2\right)\varphi_\mu(x) + \partial_\mu\partial_\nu\varphi^\nu(x) = J_\mu(x) \tag{31}$$

as a consequence of eq. (29).

It may be easy to cast the last equation into the following form

$$\partial^\nu G_{\mu\nu} + m^2\varphi_\mu = J_\mu, \tag{32}$$

where

$$G_{\mu\nu}(x) = -G_{\nu\mu}(x) = \partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu. \tag{33}$$

Identifying $G_{\mu\nu}$ with $F_{\mu\nu}$ of the electromagnetic field we get instead of eq. (32) so called the Proca equation for the electromagnetic field with the massive photon.

It is evident that the zero mass limit does not exist for $\partial_\mu J^\mu(x) \neq 0$. In such a way we are forced to redefine action $W(J)$. One of the possibilities is to put

$$\partial_\mu J^\mu(x) = mK(x) \tag{34}$$

and identify $K(x)$ in the limit $m \to 0$ with the source of massless spin zero particles.

Since the zero mass particles with zero spin are experimentally unknown in any event, we take $K(x) = 0$ and we write

$$W_{[m=0]}(J) = \frac{1}{2}\int (dx)(dx')J_\mu(x)D_+(x-x')J^\mu(x'), \tag{35}$$

where

$$\partial_\mu J^\mu(x) = 0 \tag{36}$$

and

$$D_+(x-x') = \Delta_+(x-x'; m=0). \tag{37}$$

The detail discussion concerning helicity, angular momentum, etc. of this new particle (photon) can be found in the Schwinger book (Schwinger, 1970).

4 Spin 2 fields

Exploiting the experience with spin 1 fields we form the combinations

$$T_{\mu\nu}(x), \tag{38}$$

$$\partial_\mu T^{\mu\nu}(x), \tag{39}$$

$$\partial_\mu\partial_\nu T^{\mu\nu}(x) \tag{40}$$

and $T(x)$ with appropriate coefficients in order to get the plausible form of action $W(T)$ for particles with spin 2.

While the spin one particles are described by the four-vector fields and sources the possible mathematical object describing particles with spin 2 should be the tensor field $\varphi_{\mu\nu}$ and the tensor source $T_{\mu\nu}$. Let us suppose that the tensor source is symmetrical, or, $T_{\mu\nu} = T_{\nu\mu}$.

Then, it has ten independent components. The vector source

$$\partial_\mu T^{\mu\nu}(x) \tag{41}$$

has 3 + 1 components and the scalar source

$$T(x) = g_{\mu\nu} T^{\mu\nu}(x) \tag{42}$$

is the one-component object. If we eliminate them, then the multiplicity of the system will be equal to five and this situation corresponds to the particle with spin 2.

Now, the question arises, what is the mathematical structure of the action $W(T)$ for particles with spin 2. Exploiting the experience with spin 1 fields we observe that $W(J)$ as the scalar quantity is formed by suitable combinations of sources and their derivatives. Similarly, in case with spin 2 particles we use the combinations of $T_{\mu\nu}(x), \partial_\mu T^{\mu\nu}(x), \partial_\mu\partial_\nu T^{\mu\nu}(x)$ and $T(x)$ with appropriate coefficients. The plausible form of $W(T)$ for particles with spin 2 is as follows:

$$W(T) = \frac{1}{2}\int (dx)(dx')\left\{T^{\mu\nu}(x)\Delta_+(x-x')T_{\mu\nu}(x) \quad + \right.$$

$$\frac{2}{m^2}\partial_\nu T^{\mu\nu}(x)\Delta_+(x-x')\partial'^{\lambda}T_{\mu\lambda}(x') \quad +$$

$$\frac{1}{m^4}\partial_\mu\partial_\nu T^{\mu\nu}(x)\Delta_+(x-x')\partial'_\alpha\partial'_\beta T^{\alpha\beta}(x') \quad -$$

$$\left.\frac{1}{3}\left(T(x) - \frac{1}{m^2}\partial_\mu\partial_\nu T^{\mu\nu}(x)\right)\Delta_+(x-x')\left(T(x') - \frac{1}{m^2}\partial'_\alpha\partial'_\beta T^{\alpha\beta}(x')\right)\right\}, \tag{43}$$

where the coefficients follow (as we will see in the next text) from the probability condition $|\langle 0_+|0_-\rangle|^2 \leq 1$.

The probability of the vacuum persistence generated by action (43) calculated in the momentum space is of the form:

$$|\langle 0_+|0_-\rangle|^2 = \exp\left\{-\int d\omega_p T^{*\mu\nu}(p)\Pi_{\mu\nu,\alpha\beta}(p)T^{\alpha\beta}(p)\right\}, \tag{44}$$

where (Schwinger, 1970):

$$\Pi_{\mu\nu,\alpha\beta}(p) = \frac{1}{2}\left[\bar{g}_{\mu\alpha}\bar{g}_{\nu\beta} + \bar{g}_{\nu\alpha}\bar{g}_{\mu\beta}\right] - \frac{1}{3}\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta} \tag{45}$$

with

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{m^2}p_{mu}p_{nu} \tag{46}$$

being the invariant tensor.

It may be easy to calculate $T^*\Pi T$ in the rest frame of p_μ where

$$\bar{g}_{\mu\nu} \to \delta_{kl}. \tag{47}$$

Under this condition it is

$$T^* \Pi T \to \bar{T}^{*kl} \bar{T}^{kl} \geq 0; \quad k, l \neq 0, \tag{48}$$

where

$$\bar{T}^{\mu\nu} = T^{\mu\nu} - \frac{1}{3} g^{\mu\nu} \bar{g}_{\varrho\sigma} T^{\varrho\sigma} \tag{49}$$

and the following relations have been used:

$$\bar{g}_{\mu\nu} \bar{T}^{\mu\nu} = 0 \tag{50}$$

$$p^\mu \bar{g}_{\mu\nu} = 0 \tag{51}$$

$$\bar{g}^{\mu\nu} \bar{g}_{\mu\nu} = 0. \tag{52}$$

By analogy with spin 1 fields we define the spin 2 field $\varphi_{\mu\nu}(x)$ by relation

$$\delta W(T) = \int (dx) \delta T^{\mu\nu}(x) \varphi_{\mu\nu}(x), \tag{53}$$

which can be easily transformed into momentum space as

$$\delta W(T) = \int \frac{(dp)}{(2\pi)^4} \delta T^{\mu\nu}(-p) \varphi_{\mu\nu}(p). \tag{54}$$

The symmetrical field $\varphi_{\mu\nu}(x)$ following from eq. (54) has the following form (Schwinger, 1970)

$$\begin{aligned}
\varphi_{\mu\nu}(x) = \int (dx') \Delta_+(x - x') T_{\mu\nu}(x') \quad - \\
\frac{1}{m^2} \partial_\mu \int (dx') \Delta_+(x - x') \partial'^\lambda T_{\lambda\nu}(x') \quad - \\
\frac{1}{m^2} \partial_\nu \int (dx') \Delta_+(x - x') \partial'^\lambda T_{\mu\lambda}(x') \quad + \\
\frac{1}{m^4} \partial_\mu \partial_\nu \int (dx') \Delta_+(x - x') \partial'_\kappa \partial'_\lambda T^{\kappa\lambda}(x') \quad - \\
\frac{1}{3} (g_{\mu\nu} - \frac{1}{m^2} \partial_\mu \partial_\nu) \int (dx') \Delta_+(x - x') \quad \times \\
(T(x') - \frac{1}{m^2} \partial'_\kappa \partial'_\lambda T^{\kappa\lambda}(x')).
\end{aligned} \tag{55}$$

From this equation follows immediately the divergence of $\varphi_{\mu\nu}(x)$:

$$\partial^\mu \varphi_{\mu\nu}(x) = \frac{1}{m^2} \partial^\mu T_{\mu\nu}(x) - \frac{1}{3m^2} \partial_\nu [T(x) - \frac{2}{m^2} \partial_\kappa \partial_\lambda T^{\kappa\lambda}(x)] \tag{56}$$

and

$$\varphi = g_{\mu\nu}\varphi^{\mu\nu} = -\frac{1}{3m^2}[T(x) + \frac{2}{m^2}\partial_\kappa\partial_\lambda T^{\kappa\lambda}(x)]. \tag{57}$$

The combination of eq. (56) and (57) forms

$$\partial^\mu\varphi_{\mu\nu}(x) - \partial_\nu\varphi = \frac{1}{m^2}\partial^\mu T_{\mu\nu}(x). \tag{58}$$

The differential equation following from eq. (55) is of the form:

$$(-\partial^2 + m^2)\varphi_{\mu\nu}(x) = T_{\mu\nu}(x) \quad -$$

$$\frac{1}{m^2}\left[\partial_\mu\partial^\lambda T_{\lambda\nu}(x) + \partial_\nu\partial^\lambda T_{\mu\lambda}(x)\right] + g_{\mu\nu}\frac{1}{m^2}\partial_\kappa\partial_\lambda T^{\kappa\lambda}(x) \quad -$$

$$\frac{1}{3}(g_{\mu\nu} - \frac{1}{m^2}\partial_\mu\partial_\nu)\left[T(x) + \frac{2}{m^2}\partial_\kappa\partial_\lambda T^{\kappa\lambda}(x)\right]. \tag{59}$$

If we replace the scalar and vector combinations of sources in eq. (59) by the field objects from eqs. (56)–(58), we get the following differential equation for $\varphi_{\mu\nu}$:

$$(-\partial^2 + m^2)\varphi_{\mu\nu} + \partial_\mu\partial^\lambda\varphi_{\lambda\nu}(x) + \partial_\nu\partial^\lambda\varphi_{\mu\lambda}(x) - \partial_\mu\partial_\nu\varphi(x) \quad -$$

$$g_{\mu\nu}\left[(-\partial^2 + m^2)\varphi(x) + \partial_\kappa\partial_\lambda\varphi^{\kappa\lambda}(x)\right] = T_{\mu\nu}(x), \tag{60}$$

which also follows from $\delta W(T) = 0$, where

$$W(T) = \int (dx)\left[T^{\mu\nu}(x)\varphi_{\mu\nu}(x) + \pounds\right], \tag{61}$$

where £ is the Lagrange function and it has the following mathematical structure:

$$\pounds = -\frac{1}{2}\left[\partial^\alpha\varphi^{\mu\nu}(x)\partial_\alpha\varphi_{\mu\nu}(x) + m^2\varphi^{\mu\nu}(x)\varphi_{\mu\nu}(x) \quad -\right.$$

$$\left.\partial^\alpha\varphi(x)\partial_\alpha\varphi(x) - m^2\varphi^2(x)\right] \quad -$$

$$\partial_\mu\varphi^{\mu\nu}(x)\partial_\nu\varphi(x) + \partial_\mu\varphi^{\mu\nu}(x) + \partial_\mu\varphi^{\mu\nu}(x)\partial^\alpha\varphi_{\alpha\nu}(x). \tag{62}$$

If we take the spur of eq. (60), we get

$$(-\partial^2 + m^2)\varphi(x) + \partial_\mu\partial_\nu\varphi^{\mu\nu}(x) = -\frac{1}{2}\left[T(x) + m^2\varphi(x)\right]. \tag{63}$$

After inserting eq. (63) into eq. (60), we get

$$(-\partial^2 + m^2)\varphi_{\mu\nu}(x) + \partial_\mu\partial^\lambda\varphi_{\lambda\nu}(x) + \partial_\nu\partial^\lambda\varphi_{\mu\lambda}(x) \quad -$$

$$\partial_\mu\partial_\nu\varphi(x) + g_{\mu\nu}\frac{m^2}{2}\varphi(x) = T_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}T(x). \tag{64}$$

Introducing

$$G_{\mu\lambda\nu}(x) = -G_{\nu\lambda\mu}(x) = \partial_\mu\varphi_{\lambda\nu}(x) - \partial_\nu\varphi_{\lambda\mu}(x) \tag{65}$$

we can write eq. (64) in the following form:

$$\partial^\lambda G_{\mu\nu\lambda}(x) - \partial_\nu G^\lambda_{\mu\lambda}(x) + m^2\left[\varphi_{mu\nu}(x) + \frac{1}{2}g_{\mu\nu}\varphi(x)\right] \quad =$$

$$T_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}T(x). \tag{66}$$

Further relations concerning spin 2 fields can be found in monograph by Schwinger (1970).

5 The massless limit of the spin 2 theory

It is evident that in order action (43) continue to exist in the limit $m \to 0$ it is natural to put

$$\partial_\nu T^{\mu\nu}(x) = \frac{m}{\sqrt{2}}J^\mu(x) \tag{67}$$

and

$$\partial_\mu J^\mu(x) = m\left[\sqrt{3}K(x) - \frac{1}{\sqrt{2}}T(x)\right], \tag{68}$$

where $J^\mu(x)$ and $K(x)$ are independent sources. The independence of them is expressed by constants $\sqrt{3}$ and $1/\sqrt{2}$ which are chosen to eliminate any coupling between sources $K(x)$ and $T(x)$. After insertion of eqs. (67) and (68) into action (43), we get for $m \to 0$:

$$W(T) = \frac{1}{2}\int (dx)(dx')\Big\{T^{\mu\nu}(x)D_+(x-x')T_{\mu\nu}(x) \quad -$$

$$\frac{1}{2}T(x)D_+(x-x')T(x') + J^\mu(x)D_+(x-x')J_\mu(x') \quad +$$

$$K(x)D_+(x-x')K(x')\Big\}, \tag{69}$$

where $D_+(x-x') = \Delta_+(x-x'; m=0)$ and for $m = 0$

$$\partial_\mu T^{\mu\nu}(x) = 0 \tag{70}$$

$$\partial_\mu J^\mu(x) = 0. \tag{71}$$

The formula (69) represents the invariant decomposition by means of sources of massless particles with spin 2, 1 and 0. The massless particles of helicity ± 2 is called graviton and the action which corresponds to this particle follows from action (69) in the form

$$W(T) = \frac{1}{2}\int (dx)(dx')\left\{T^{\mu\nu}(x)D_+(x-x')T_{\mu\nu}(x) \quad -\right.$$

$$\frac{1}{2}T(x)D_+(x-x')T(x')\Big\} \tag{72}$$

with

$$\partial_\mu T^{\mu\nu}(x) = 0. \tag{73}$$

The graviton is the particle which was not hitherto experimentally discovered. Nevertheless, we can suppose it is the mediate boson which initiates the gravitational phenomena, just as the photon initiates the electromagnetic ones.

The source restriction $\partial_\mu T^{\mu\nu}(x) = 0$ for the graviton source states the existence of a conservation law of the vector

$$p^\nu = \int d\sigma_\mu T^{\mu\nu}(x), \tag{74}$$

where $d\sigma_\mu$ is an invariant element of area, which can be identified with the energy-momentum vector. The connection between the mechanical tensor $T^{\mu\nu}_{mech}$ and the gravitational tensor $T^{\mu\nu}_{grav}$ is postulated by relation

$$T^{\mu\nu}_{grav} = \kappa^{1/2} T^{\mu\nu}_{mech}, \tag{75}$$

where κ is the gravitational constant of the magnitude

$$\kappa = 8\pi G, \tag{76}$$

and $= \kappa = 6.67430(15)10^{-11} m^3.kg^{-1}.s^{-2}$ (CODATA, 2018).

The action corresponding to the gravitational field initiated by the mechanical tensor T^μ_{mech} is now

$$W(T) = \frac{\kappa}{2}\int (dx)(dx')\left\{T^{\mu\nu}(x)D_+(x-x')T^{\mu\nu}(x) \quad - \right.$$

$$\left.\frac{1}{2}T(x)D_+(x-x')T(x')\right\} \tag{77}$$

with

$$\partial_\mu T^{\mu\nu}(x) = 0. \tag{78}$$

The corresponding gravitational field definition is

$$\delta W(T) = \int (dx)\delta T^{\mu\nu}h_{\mu\nu}, \tag{79}$$

which implies the field equation for $h_{\mu\nu}(x)$ in the following form:

$$-\partial^2 h_{\mu\nu}(x) + \partial_\mu\partial^\alpha h_{\alpha\nu}(x) + \partial_\nu\partial^\alpha h_{\mu\alpha}(x) - \partial_\mu\partial_\nu h(x) \quad =$$

$$\kappa(T_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}T(x)) \tag{80}$$

with

$$T(x) = g_{\mu\nu}T^{\mu\nu}(x) \tag{81}$$

$$h(x) = g_{\mu\nu}h^{\mu\nu}(x). \tag{82}$$

Let us remark that action (77) was used by author to determination of the spectral form of the emitted gravitons by the binary system and by the related systems (Pardy, 1983; 1994a; 1994b; 1994c; 1994d; 2011; 2018a; 2019). Action (77) enables also the derivation of the Newton gravity potential and Einstein gravity field equations (Schwinger, 1970; Pardy, 1984). After the verbal investigation of the Schwinger book involving theory of gravity (Schwinger, 1970), we can honestly say that the words principle of equivalence are not involved in his book.

6 Spin 3 fields

The spin 3 fields is discussed as the analogue of the spin 2 situation. The spin 3 action is derived together with the spin 3 field equation. The massles situation is also discussed.

It may be easy to show that in case of the spin 2 fields the action can be expressed in the following form:

$$W(T) = \frac{1}{2}\int \frac{(dp)}{(2\pi)^4}T^{\mu\nu}(-p)\frac{\Pi_{\mu\nu,\alpha\beta}}{p^2+m^2-i\varepsilon}T^{\alpha\beta}(p) \tag{83}$$

where $\Pi_{\mu\nu,\alpha\beta}$ is given by eq. (45) of the spin 2 discussion. By analogy with spin 2 situation we can postulate the form of the action for fields with spin 3 as follows:

$$W(S) = \frac{1}{2}\int \frac{(dp)}{(2\pi)^4}S^{\lambda\mu\nu}(-p)\frac{\Pi_{\lambda\mu\nu,\alpha\beta\gamma}}{p^2+m^2-i\varepsilon}S^{\alpha'\beta'\gamma'}(p) \tag{84}$$

where function $\Pi_{\lambda\mu\nu,\alpha\beta\gamma}$ it is easy to determine. While in case of spin 0, 1 and 2 fields the determination of Π is not very difficult, the situation changes with higher spins. Nevertheless, there is the general method how to determine function Π for all integer spins as it will be shown in the next text and we here write down the derived result for spin 3 situation (Schwinger, 1970).

$$\Pi_{\alpha\beta\gamma,\alpha'\beta'\gamma'} = \bar{g}_{\alpha\alpha'}\bar{g}_{\beta\beta'}\bar{g}_{\gamma\gamma'} - \frac{1}{5}\left[\bar{g}_{\alpha\beta}\bar{g}_{\gamma\gamma'}\bar{g}_{\alpha'\beta'} + \bar{g}_{\beta\gamma}\bar{g}_{\alpha\alpha'}\bar{g}_{\beta'\gamma'} + \bar{g}_{\gamma\alpha}\bar{g}_{\beta\beta'}\bar{g}_{\alpha'\gamma'}\right] \tag{85}$$

where

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{m^2}p_\mu p_\nu \tag{86}$$

Let us write down some properties of tensor $\Pi_{\alpha\beta\gamma,\alpha'\beta'\gamma'}$.

$$p^\alpha\Pi_{\alpha\beta\gamma,\alpha'\beta'\gamma'} = \frac{p^2+m^2}{m^2} \quad \times$$

$$\left[p_{\alpha'}\bar{g}_{\beta\beta'}\bar{g}_{\gamma\gamma} - \frac{1}{5}\left(p_\beta\bar{g}_{\gamma\gamma'}\bar{g}_{\alpha'\beta'} + p_{\alpha'}\bar{g}_{\beta\gamma}\bar{g}_{\beta'\gamma'} + p_\gamma\bar{g}_{\alpha\alpha'}\bar{g}_{\alpha'\gamma'}\right)\right] \tag{87}$$

and

$$g^{\beta\gamma}\Pi_{\alpha\beta\gamma,\alpha'\beta'\gamma'} = \frac{p^2+m^2}{m^2} \quad \times$$

$$\left[\frac{p_{\beta'}p_{\gamma'}}{m^2}\bar{g}_{\alpha\alpha'} - \frac{1}{5}\left(\frac{p_\alpha p_{\gamma'}}{m^2}\bar{g}_{\alpha'\beta'} + \bar{g}_{\alpha\alpha'}\bar{g}_{\beta'\gamma'} + \frac{p_\alpha p_{\beta'}}{m^2}\bar{g}_{\gamma'\alpha'}\right)\right] \tag{88}$$

The field of spin 3, $\varphi_{\alpha\beta\gamma}(p)$, in the momentum representation is defined in analogy with spin 0, 1 and 2 fields, or

$$\delta W(S) = \int \frac{(dp)}{(2\pi)^4}\delta S^{\alpha\beta\gamma}(-p)\varphi_{\alpha\beta\gamma}(p) \tag{89}$$

and it is obtained as

$$\varphi_{\alpha\beta\gamma}(p) = \frac{1}{p^2+m^2-i\varepsilon}\Pi_{\alpha\beta\gamma,\alpha'\beta'\gamma'}(p)S^{\alpha'\beta'\gamma'}(p) \tag{90}$$

We know from the experience with spin 0, 1, and 2 particles that action W can be cast into various forms. If we take the obligate form, we write for the spin 3 action:

$$W(S) = \int (dx)\left[S^{\alpha\beta\gamma}(x)\varphi_{\alpha\beta\gamma}(x) + \mathcal{L}(x)\right] \tag{91}$$

where $\mathcal{L}$ (x) is the Lagrange function of the spin 3 fields and it can be shown that its mmathematical structure is as follows (Schwinger, 1970):

$$\mathcal{L} = -\frac{1}{2}\left[\partial^\kappa\varphi^{\alpha\beta\gamma}\partial_\kappa\varphi_{\alpha\beta\gamma} + m^2\varphi^{\alpha\beta\gamma}\varphi_{\alpha\beta\gamma} - 3\partial_\kappa\varphi^{\kappa\beta\gamma}\partial^\alpha\varphi_{\alpha\beta\gamma} \quad +\right.$$

$$\left.6\,\partial_\beta\varphi^{\alpha\beta\gamma}\partial_\gamma\varphi_\alpha - 3\partial^\alpha\varphi^\gamma\partial_\alpha\varphi_\gamma - 3m^2\varphi^\kappa\varphi_\kappa - \frac{3}{2}(\partial_\kappa\varphi^\kappa)^2\right] \quad +$$

$$\frac{1}{2}m\left(\varphi^\kappa\partial_\kappa\Phi - \Phi\partial_\kappa\varphi^\kappa\right) + \partial^\kappa\Phi\partial_\kappa\Phi + 4m^2\Phi \tag{92}$$

where the auxiliary function Φ is to receive independent variation in the stationary action principle and

$$\varphi_\lambda(x) = \varphi^\kappa_{\lambda\kappa}(x) \tag{93}$$

6.1 The massless fields with helicity 3

It is possible to show that from the massive field action for spin 3 generate the massless limit with the following action (Schwinger, 1970):

$$W(S, m=0) = \frac{1}{2}\int (dx)(dx')$$

$$\left[S^{\lambda\mu\nu}(x)D_+(x-x')S_{\lambda\mu\nu} - \frac{3}{4}S^\lambda(x)D_+(x-x')S_\lambda(x')\right] \tag{94}$$

where

$$S^\lambda(x) = g_{\mu\nu}S^{\lambda\mu\nu}(x) \tag{95}$$

and

$$\partial_\lambda S^{\lambda\mu\nu}(x) = 0 \tag{96}$$

The momentum space transformation of eqs. (94)–(96) is

$$W(S) = \frac{1}{2}\int \frac{(dp)}{(2\pi)^4}\frac{1}{p^2 - i\varepsilon} \quad \times$$

$$\left[S^{\lambda\mu\nu}(-p)S_{\lambda\mu\nu}(p) - \frac{3}{4}S^{\lambda}(-p)S_{\lambda}(p)\right] \tag{97}$$

with

$$S^{\lambda}(p) = S^{\lambda\nu}_{\nu}(p) \tag{98}$$

and

$$p_\lambda S^{\lambda\mu\nu}(p) = 0 \tag{99}$$

The field is defined as usually

$$\delta W(S) = \int \frac{(dp)}{(2\pi)^4}\delta S^{\lambda\mu\nu}(-p)S_{\lambda\mu\nu}(p) \tag{100}$$

and any additional term containing p_λ, p_μ or p_ν as factors will not contribute in eq. (100) because of the source restriction (17). Then, the general form of the field with mass $m = 0$ is

$$\varphi_{\lambda\mu\nu}(p) = \frac{1}{p^2 - i\varepsilon}\left[S_{\lambda\mu\nu}(p) - \frac{1}{4}(g_{\mu\nu}S_\lambda(p) + g_{\nu\lambda}S_\mu(p) + g_{\lambda\mu}S_\nu(p)\right] \quad +$$

$$p_\lambda\varphi_{\mu\nu}(p) + p_\mu\varphi_{\nu\lambda}(p) + p_\nu\varphi_{\lambda\mu}(p) \tag{101}$$

where the cyclically related terms are required by the total symmetry of the tensor $\varphi_{\lambda\mu\nu}$ and $\varphi_{\mu\nu}(p)$ is the new symmetrical tensor determined by the source restriction. In order to use the tensor $\varphi_{\mu\nu}(p)$, first, let us note that

$$\varphi_\lambda = \varphi^{\nu}_{\lambda\nu}(p) = \frac{1}{p^2 - i\varepsilon}\left(-\frac{1}{2}\right)S_\lambda(p) + p_\lambda\varphi(p) + 2p^\nu\varphi_{\lambda\nu}(p) \tag{102}$$

where

$$\varphi(p) = \varphi^{\nu}_{\nu}(p) \tag{103}$$

Multiplication of eq. (19) by p^λ then introduces just the combination equal to $\frac{1}{2}(\varphi_\lambda - p_\lambda\varphi)$ and we get:

$$p^2\varphi_{\mu\nu} = p^\lambda\varphi_{\lambda\mu\nu} - \frac{1}{2}(p_\mu\varphi_\nu + p_\nu\varphi_\mu) + p_\mu p_\nu\varphi \tag{104}$$

An equation for $\varphi(p)$ is produced by combination

$$p^\lambda p^\mu p^\nu\varphi_{\lambda\mu\nu} = 3p^2 p^\mu p^\nu\varphi_{\mu\nu} \tag{105}$$

with

$$p^2 p^\mu p^\nu \varphi_{\mu\nu} = p^\lambda p^\mu p^\nu \varphi_{\lambda\mu\nu} - p^2 p^\lambda \varphi_\lambda + (p)^2 \varphi \tag{106}$$

namely

$$(p^2)^2 \varphi = p^2 p^\lambda \varphi_\lambda - \frac{2}{3} p^\lambda p^\mu p^\nu \varphi_{\lambda\mu\nu} \tag{107}$$

The momentum space version of the field equation for $\varphi_{\lambda\mu\nu}(p)$ is now derived as

$$p^2 \varphi_{\lambda\mu\nu} - p_\lambda p^{\lambda'} \varphi_{\lambda'\mu\nu} - p_\mu p^{\mu'} \varphi_{\lambda\mu'\nu} - p_\nu p^{\nu'} \varphi_{\lambda\mu\nu'} \quad +$$

$$p_\mu p_\nu \varphi_\lambda + p_\nu p_\lambda \varphi_\mu + p_\lambda p_\mu \varphi_\nu - 3 p_\lambda p_\mu p_\nu \varphi \quad =$$

$$S_{\lambda\mu\nu} - \frac{1}{4}(g_{\mu\nu} S_\lambda + g_{\nu\lambda} S_\mu + g_{\lambda\mu} S_\nu) \tag{108}$$

From this equation we can derive eq. (25) by multiplication with $p^\lambda p^\mu p^\nu$. The field equation involves only the combination

$$\varphi_{\lambda\mu\nu}(p) - 3 \frac{p_\lambda p_\mu p_\nu}{p^2 - i\varepsilon} \varphi(p) \tag{109}$$

since

$$p_\lambda p^{\lambda'} (p_{\lambda'} p_\mu p_\nu) - (p^2) p_\mu p_\nu p_\lambda = 0 \tag{110}$$

and it means that this combination is redefinition of $\varphi_{\lambda\mu\nu}$ which means that $\varphi(p)$ can be transformed away. Thus the final set of the field equations for spin 3 and $m = 0$ is with $\varphi = 0$:

$$p^2 \varphi_{\lambda\mu\nu} - p_\lambda p^{\lambda'} \varphi_{\lambda'\mu\nu} - ... + p_m p_\nu \varphi_\lambda + ... \quad -$$

$$g_{\mu\nu}(p^2 \varphi_\lambda + \frac{1}{2} p_\lambda p^{\lambda'} \varphi_{\lambda'} - p^{\mu'} p^{\nu'} \varphi_{\lambda\mu'\nu'}) - ... = S_{\lambda\mu\nu} \tag{111}$$

where dots represents the terms that are generated by the cyclic permutation from the given ones.

The algebraic consequence of this equation is as follows:

$$p^\lambda S_{\lambda\mu\nu}(p) = g_{\mu\nu} \frac{1}{4} p^\lambda S_\lambda(p) \tag{112}$$

and it is consistent with the vanishing divergence of the source, but does not imply it.

7 Spin 3 electromagnetic gravity

We have seen that from the massive field action for spin 3, the massless limit is generated with the following action (Schwinger, 1970):

$$W(S, m = 0) = \frac{1}{2} \int (dx)(dx')$$

$$\left[S^{\lambda\mu\nu}(x)D_+(x-x')S_{\lambda\mu\nu} - \frac{3}{4}S^{\lambda}(x)D_+(x-x')S_{\lambda}(x')\right] \tag{113}$$

where

$$S^{\lambda}(x) = g_{\mu\nu}S^{\lambda\mu\nu}(x) \tag{114}$$

and

$$\partial_{\lambda}S^{\lambda\mu\nu}(x) = 0 \tag{115}$$

According to Schwinger (1970) - *Ordinary matter possesses no conserved physical properties that could be identified with the ones described by the local conservation law (115), or indeed for any* $n \geq 3$. *The inability to construct their sources strongly affirms the empirical absence of the particles. But perhaps one should not reject totally the possibility of eventually encountering such properties, and the associated particles, under circumstances that are presently unattainable.*

Here we think that matter with the spin 3 is the electromagnetic-gravity bacause it involves the equation

$$electromagnetism(spin1) + gravity(spin2) = electromagnetic - gravity(spin3), \tag{116}$$

which is the scientific physical transformation of the arithmetic equation 1+2=3.

8 Discussion

We have seen that the Einstein equivalence principle was not used in the Schwinger spin 2 gravity. Einstein formulated this principle with two reference frames, K and K' where K is a uniform gravitational field, whereas K' has no gravitational field. It is uniformly accelerated in such a way that objects in the two frames experience identical forces. According to Einstein systems K and K' are physically exactly equivalent. This assumption of exact physical equivalence makes it impossible to speak of the absolute acceleration of the system of reference, just as the usual theory of relativity forbids to talk of the absolute velocity of a system. It makes the equal falling of all bodies in a gravitational field (Einstein, 1911).

Or, *Inertia and gravity are identical; hence and from the results of special relativity theory it inevitably follows that the symmetric fundamental tensor* $g_{\mu\nu}$ *determines the metric properties of space, of the motion of bodies due to inertia in it, and, also, the influence of gravity* (Einstein, 1918).

According to Fock (1964), *principle of equivalence is understood to be the statement that in some sense a field of acceleration is equivalent to a gravitational field. It means that by introducing a suitable system of coordinates (which is usually interpreted as an accelerated frame of reference) one can so transform the equations of motion of a mass point in a gravitational field that in this new system they will have the appearance of equations of motion of a free mass point. Thus a gravitational field can, so to speak, be replaced, or rather imitated, by a field of acceleration. Owing to the equality of inertial and gravitational mass such a transformation is the same for any value of the mass of* *the particle. But it will succeed in its purpose only in an infinitesimal region of space, i.e. it will be strictly local. In the general case the transformation described corresponds mathematically to passing to a locally geodesic system of coordinates.*

The principle of equivalence states that it is impossible to distinguish between the action on a particle of matter of a constant acceleration, or, of static support in a gravitational field (Lyle, 2008). We have seen that Schwinger theory does not use the principle of equivalence.

The controversy between different opinions on the principle of equivalence can be easily solved by the physical definition of gravity and inertia. Namely: gravity is the specific form of matter, or, form of vacuum. And inertia is the interaction of the massive body with vacuum which is the physical medium. So, Gravity is form of matter and inertia is form of interaction.

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INTELLECTUAL ARCHIVE, Vol. 14, No. 1, 2025

DOI: 10.32370/IA_2025_01_3

THE SCATTERING OF LIGHT BY LIGHT

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March 23, 2025

Abstract

The scattering of light by light is considered in case where the internal particles of this process have spin 0. The first calculation of this process was performed by Karplus et al. (1950). The pedagogical explanation of this proces was realized for instance by Akhiezer et al. (1965), or, by Beresteteskii et al. (1982). We use here the model with the Green function for the spin 0 particles. The article is written with the mathematical simplicity and the Schwinger pedagogical clarity.

1 Introduction

The scattering of light by light which is considered here is in no case interference of light, or, quantum interfrence because interference is a phenomenon in which two waves superimpose to form a resultant wave of greater or lesser amplitude. Interference usually refers to the interaction of waves that are correlated (coherent) with each other because they originate from the same source, or they have the same or nearly the same frequency. When two or more waves are incident on the same point, the total displacement at that point is equal to the vector sum of the displacements of the individual waves. If a crest of one wave meets a crest of another wave of the same frequency at the same point, then the magnitude of the displacement is the sum of the individual magnitudes. This is constructive interference and occurs when the phase difference between the waves is a multiple of 2π. Destructive interference occurs when the crest of one wave meets a trough of another wave. In this case, the magnitude of the displacements is equal to the difference in the individual magnitudes, and occurs when this difference is an odd multiple of π.

We here do not consider the light interference but interaction of light by light. The first calculation of this process was performed by Karplus et al. (1950). The pedagogical explanation of this proces was realized, for instance, by Akhiezer et al. (1965), or, by Beresteteskii et al. (1982). We use here the model with the spin 0. The phenomenon scattering of light by light is problem of quantum electrodynamics and we use here the

Schwinger quantum field theory methods (Schwinger, 1969; 1970) at the calculation of this physical process.

2 Scattering of light by light

The scattering of light by light is considered where the internal particles of this process have spin 0. The only difference with the situation with spin 1/2 particles is in using the Green function for the spin 0 particles. Instead of the Green function G_+^A for spin 1/2 particles we work with the function Δ_+^A. We follow the Schwinger monograph (Schwinger, 1973).

The proceeds involving the two spin 0 particle exchange and various numbers of photons is contained in the coupling term

$$W_{2..} = \frac{1}{2}\int (dx)(dx')K(x)\Delta_+^A(x,x')K(x'). \tag{1}$$

This formula generates the photon sources in terms of an effective two-particle field in the form

$$i\varphi(x)\varphi(x')|_{eff} = \Delta_+^A(x,x'). \tag{2}$$

The effective two-particle source follows from comparison

$$\langle 0_+|0_-\rangle = i\int (dx)\varphi(x)eq\frac{1}{i}\partial^\mu\varphi(x)\delta A_\mu(x) \tag{3}$$

with

$$\langle 0_+|0_-\rangle = \frac{1}{2}\left[i\int (dx)K(x)\varphi(x)\right]^2 =$$

$$-\frac{1}{2}\int (dx)(dx')\varphi(x)K(x)K(x')\varphi(x'), \tag{4}$$

which gives

$$iK(x)K(x')|_{eff} = eq\left(\delta A^\mu(x) + \delta A^\mu(x')\right)\frac{1}{i}\partial_\mu\delta(x-x'), \tag{5}$$

or,

$$iK(x)K(x')|_{eff} = eq\left(p\delta A + \delta Ap\right)(x)\delta(x-x'). \tag{6}$$

The vacuum amplitude of causal coupling between two photon sources symbolized by δA and A is analogical to the spin 1/2 situation, or,

$$\langle 0_+|0_-\rangle = i\delta W(A) = \frac{1}{2}\int (dx)(dx')\mathrm{tr}\left[iK(x)K(x')|_{eff}\; i\varphi(x)\varphi(x')|_{eff}\right] =$$

$$\frac{1}{2}\mathrm{Tr}\left[eq(p\delta A + \delta Ap)\Delta_+^A\right], \tag{7}$$

where Tr is the more compact notation in which the space-time coordinates join spin and charge indices as matrix labels.

For Δ_+^A we can use the well known integral equation with the formal solution (Schwinger, 1970)

$$\Delta_+^A = \left[1 - \Delta_+ \left(eq(pA + Ap) - e^2A^2\right)\right]^{-1} \Delta_+ \quad =$$

$$\Delta_+ \left[1 - \left(eq(pA + Ap) - e^2A^2\right) \Delta_+\right]^{-1}. \tag{8}$$

Now, instead of eq. (7) we write

$$i\delta W(A) = \frac{1}{2}\mathrm{Tr}\left[\left(eq(p\delta A + \delta Ap) - 2e^2\delta AA\right) \Delta_+^A\right]. \tag{9}$$

because δA and A are disjoint and their product vanishes in the causal arrangement for which eq. (7) is derived.

We then write eq. (9) as

$$i\delta W(A) \quad =$$

$$\frac{1}{2}\mathrm{Tr}\left[\left(eq(p\delta A + \delta Ap) - 2e^2\delta AA\right) \left(1 - \Delta_+(eq(pA + Ap) - e^2A^2)\right)^{-1} \Delta_+\right] \quad =$$

$$-\frac{1}{2}\delta\mathrm{Tr}\ln\left[1 - \left(eq(pA + Ap) - 2e^2A^2\right) \Delta_+\right]. \tag{10}$$

There are other representation of $W(A)$ which are useful in special situations. For instance, we get the specific form of $W(A)$ if we use the proper-time representation of Δ_+^A (Schwinger, 1973):

$$\Delta_+^A = \frac{1}{\Pi^2 + m^2 - i\varepsilon} = i\int_0^\infty ds e^{-is(\Pi^2+m^2)}, \tag{11}$$

where $\varepsilon \to 0_+$ is implicit in the integral as a convergence factor $\exp(-\varepsilon s)$. After insertion of eq. (11) into eq. (9) we get:

$$\delta W(A) = -\frac{1}{2}i\int_0^\infty ds\mathrm{Tr}\left[\delta(\Pi)^2 e^{-is(\Pi^2+m^2)}\right], \tag{12}$$

or,

$$W(A) = -\frac{i}{2}\int \frac{ds}{s}\mathrm{Tr}e^{-is(\Pi^2+m^2)}. \tag{13}$$

Now, the goal is to evaluate the trace of of the corresponding term in (13) and to express $W(A)$ in terms of the effective Lagrange function, or in other words, to express it in the form

$$W(A) = \int (dx)\mathcal{L}(F), \tag{14}$$

where $\mathcal{L}(F)$ is the effective Lagrange function. First, let us try to find Tr of $\exp(-is(\Pi^2 + m^2))$.

Using the commutator

$$[\Pi_\mu, \Pi_\nu] = ieqF_{\mu\nu}, \tag{15}$$

we get

$$\left[\Pi_\mu, \Pi^2\right] = 2ieqF_{\mu\nu}\Pi^\nu \tag{16}$$

and therefore

$$\Pi_\mu(s) = e^{is\Pi^2}\Pi_\mu e^{-is\Pi^2}, \tag{17}$$

which can be transcribed in the equivalent form

$$\frac{d\Pi_\mu(s)}{ds} = 2eqF_{\mu\nu}\Pi^\nu(s) \tag{18}$$

with the matrix solution

$$\Pi(s) = e^{2eqFs}\Pi = \Pi e^{-2eqFs}, \tag{19}$$

because of the antisymmetry of $F_{\mu\nu}$.

Now, let us introduce the following tensor

$$T_{\mu\nu} = \mathrm{Tr}'\left[\Pi_\mu\Pi_\nu e^{-is\Pi^2}\right] = \mathrm{Tr}'\left[\Pi_\mu e^{-is\Pi^2}\Pi_\nu(s)\right] =$$

$$\mathrm{Tr}'\left[\Pi_\nu(s)\Pi_\mu e^{-is\Pi^2}\right], \tag{20}$$

where Tr' does not refer to charge space.

The equivalent form of $T_{\mu\nu}$ is as follows:

$$T_{\mu\nu} = \mathrm{Tr}'\left[\Pi_\mu\Pi_\nu(s)e^{-is\Pi^2}\right] - \mathrm{Tr}'\left[[\Pi_\mu, \Pi_\nu(s)]\, e^{-is\Pi^2}\right], \tag{21}$$

where for the commutator that appeared it is

$$[\Pi_\mu, \Pi_\nu(s)] = \left[\Pi_\mu, \Pi^\lambda(s)\left(e^{-2eqFs}\right)_{\lambda\nu}\right] = ieq\left\{Fe^{-2eqFs}\right\}_{\mu\nu}. \tag{22}$$

Now, using eqs. (19) and (22) we can express eq. (21) in the matrix form as follows:

$$T = Te^{-2eqFs} - ieqFe^{-2eqFs}\mathrm{Tr}'e^{-is\Pi^2}, \tag{23}$$

or,

$$T\left(1 - e^{-2eqFs}\right) = -ieqFe^{-2eqFs}\mathrm{Tr}'e^{-is\Pi^2}, \tag{24}$$

or,

$$\mathrm{Tr}'\Pi\Pi e^{-is\Pi^2} = -ieqF\frac{F}{e^{2eqFs} - 1}\mathrm{Tr}'e^{-i\Pi^2}. \tag{25}$$

We use this result to get

$$i\frac{d}{ds}\mathrm{Tr}'e^{-is\Pi^2} = \mathrm{Tr}'\Pi^2 e^{-is\Pi^2} = -ieq\mathrm{Tr}'\frac{F}{e^{2eqFs} - 1}\mathrm{Tr}'e^{-is\Pi^2}, \tag{26}$$

which is the differential equation for $\mathrm{Tr}'\exp(-is\Pi^2)$.

The solution of the derived differential equation (26) is possible express as follows (Schwinger, 1973):

$$\mathrm{Tr}' e^{-is\Pi^2} = C\exp\left\{-\frac{1}{2}\mathrm{Tr}'\ln\left(\frac{\sinh eqFs}{eqF}\right)\right\} = \frac{C}{s^2}\left[\det\frac{eFs}{\sinh eFs}\right]^{1/2}, \tag{27}$$

where we have used identity $\mathrm{Tr}\ln = \ln\det$, the dimensionality of space-time in the latter form and the fact that the sign of q is immaterial.

The constant C can be determined from considering the small s limit. This situation with the small s is dominated by large Π values and the non-commutativity of different Π components cases to be significant. Using four-dimensional forms of conventional quantum relations, we get

$$s\to 0: \quad \mathrm{Tr}' e^{-is\Pi^2} = \int (dx)\langle x|e^{-isp^2}|x\rangle = \int \frac{(dp)}{(2\pi)^4}(dx)e^{-isp^2}, \tag{28}$$

where

$$\int \frac{(dp)}{(2\pi)^4}e^{-isp^2} = \left(\int_{-\infty}^{\infty}\frac{(dp_1)}{(2\pi)}e^{-isp_1^2}\right)^3\int_{-\infty}^{\infty}\frac{(dp_0)}{(2\pi)}e^{-isp_0^2}. \tag{29}$$

With regard to Laplace relation

$$\int_{-\infty}^{\infty} dp_1 e^{-isp_1^2} = \left(\frac{\pi}{is}\right)^{1/2}; \quad s>0 \tag{30}$$

we have

$$\int \frac{(dp)}{(2\pi)^4} = \frac{1}{4\pi^2}\frac{1}{is^2}. \tag{31}$$

After insertion of eq. (31) into eq. (28) and then into eq. (27), we get

$$C = -\frac{1}{(4\pi)^2}i\int (dx). \tag{32}$$

Now, we can write for $W(A)$:

$$W(A) = -\int (dx)\frac{1}{(4\pi)^2}\int_0^{\infty}\frac{ds}{s^3}e^{-ism^2}\left[\det\frac{eFs}{\sinh eFs}\right]^{1/2} = \int (dx)\mathcal{L}(F), \tag{33}$$

where

$$\mathcal{L}_{spin\ 0}(F) = -\frac{1}{(4\pi)^2}\int_0^{\infty}\frac{ds}{s^3}e^{-ism^2}\left[\det\frac{eFs}{\sinh eFs}\right]^{1/2}. \tag{34}$$

The reality of the Lagrange function is obvious after deforming the path integration according to transformation $s\to is$. In other words

$$\mathcal{L}_{spin\,0}(F) = \frac{1}{(4\pi)^2}\int_0^\infty \frac{ds}{s^3} e^{-sm^2}\left\{\left[\det \frac{eFs}{\sinh eFs}\right]^{1/2} - 1 - \frac{1}{3}(es)^2\tilde{F}\right\}, \qquad (35)$$

where we have used notation

$$\tilde{F} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{H}^2) \qquad (36)$$

to which we add

$$\tilde{G} = -\frac{1}{4}\,{}^*F^{\mu\nu}F_{\mu\nu} = \mathbf{E}\cdot\mathbf{H} \qquad (37)$$

and

$$\tilde{H}_\pm = 2(\tilde{F} \pm i\tilde{G}) = (\mathbf{E} \pm i\mathbf{H})^2. \qquad (38)$$

The general evaluation of the determinant can be realized by means of the eigenvalues of tensor F. It is convenient to introduce the selfdual tensors as follows:

$$F_\pm = F \pm i\,{}^*F, \quad {}^*F_\pm = \mp iF_\pm. \qquad (39)$$

Considered as matrices, the two tensors commute, and the square of each is multiple of the unit matrix. It is possible to checked it by explicit use of the small number of independent components. The squares are

$$(F_\pm^2)_{\mu\nu} = g_{\mu\nu}\tilde{H}_\pm \qquad (40)$$

where the coefficients $\tilde{H}_\pm$ are found by forming the trace. Using the equivalent relations

$$\frac{1}{2}(F^2 - {}^*F^2) = g_{\mu\nu}\tilde{F}, \quad ({}^*FF)_{\mu\nu} = g_{\mu\nu}\tilde{G}, \qquad (41)$$

the eigenvalues appear in oppositely signet pairs, $\pm F', \pm F''$, where

$$F', F'' = \frac{1}{2}\left[\tilde{H}_+^{1/2} \pm \tilde{H}_-^{1/2}\right]. \qquad (42)$$

Accordingly

$$\left[\det \frac{eFs}{\sin eFs}\right]^{1/2} = \frac{eF's}{\sin eF's}\,\frac{eF''s}{\sin eF''s} =$$

$$\frac{2(es)^2 i\tilde{G}}{\cos(es\tilde{H}_-^{1/2}) - \cos(es\tilde{H}_+^{1/2})} = \frac{(es)^2\tilde{G}}{\operatorname{Im}\cos(es\tilde{H}^{1/2})}, \qquad (43)$$

where we have finally written just $\tilde{H}$ in place of $\tilde{H}_-$.

In such a way the spin 0 result Lagrangian is

$$\mathcal{L}_{spin\,0}(F) = \frac{1}{(4\pi)^2}\int_0^\infty \frac{ds}{s^3} e^{-sm^2}\left[\frac{(es)^2\tilde{G}}{\operatorname{Im}\cos(es\tilde{H}^{1/2})} - 1 - \frac{1}{3}(es)^2\tilde{F}\right]. \qquad (44)$$

The Lagrange function (35) can be expressed approximately in terms quartic in the fields. To obtain it we use the determinant expansion according to the algorithm:

$$\det(1+A) = 1 + \mathrm{tr}A + \frac{1}{2}\left((\mathrm{tr}A)^2 - \mathrm{tr}A^2\right) + ... \tag{45}$$

This gives

$$\left[\det \frac{eFs}{\sin eFs}\right]^{1/2} = 1 + \frac{1}{3}(es)^2\tilde{F} + \frac{1}{90}(es)^4(7\tilde{F}^2 + \tilde{G}^2) + ... \tag{46}$$

and

$$\mathcal{L}_{04;spin\ 0} = \frac{\alpha^2}{90}\frac{1}{m^4}(7\tilde{F}^2 + \tilde{G}^2) = \frac{\alpha^2}{90}\frac{1}{m^4}\left[\frac{7}{4}(\mathbf{E}^2 - \mathbf{H}^2)^2 + (\mathbf{E}\cdot\mathbf{H})^2\right]. \tag{47}$$

3 Discussion

In the preceding text we exhibited the space-time form of couplings that involve only the electromagnetic field, and we also used these forms directly for calculations, in the special circumstance of slowly varying fields. With more general situations, however, it is usually preferable to consider an appropriate causal arrangement and then perform the space-time extrapolation. We are recognizing now that source theory is flexible; it is not committed to any special calculational method and is free to choose the most convenient one. Indeed, it is the interplay and synthesis of various calculational devices, each adapted to specific circumstances, that constitutes the general source theory computational method

The arrangement is, two photons collide to create a charged particle pair, and then the two photons emitted in the subsequent annihilation of the particles are detected. For spin 0 particles, we can use the analogy with the preceding methods with some extensions (Schwinger, 1973) and considering the strong magnetic field situation (Schwinger, 1989).

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INTELLECTUAL ARCHIVE, Vol. 14, No. 1, 2025
DOI: 10.32370/IA_2025_01_4

Challenges of Radioactive Waste Management and Spent Nuclear Fuel Handling in the Context of Nuclear Energy Development

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Introduction

In recent decades, nuclear energy has become an integral part of the global energy system, providing a significant share of electricity for both developing and developed countries. With the growing energy demands driven by rapid industrial development, technological advancements, and population growth, nuclear power plants represent one of the most efficient and low-carbon energy sources. However, along with the advantages of nuclear energy a serious challenge arises—the management of radioactive waste (RW) and spent nuclear fuel (SNF).

To meet the increasing energy demands required for the growth of industries, artificial intelligence, cryptocurrency, and electric transport, the construction of new nuclear power plants and the modernization of existing ones are necessary. At the same time, it is essential to recognize that the volume of spent radioactive materials (waste) will only continue to grow.

Nuclear energy is an ideal option for sustainable power generation due to its long lifespan and ability to produce electricity with minimal greenhouse gas emissions. However, this form of energy generation produces radioactive waste, which must either be safely stored and disposed of or reprocessed. Long-term sustainable solutions for radioactive waste management require a combination of technical expertise, regulatory oversight, and continuous research to ensure safe containment and final disposal.

The generation of radioactive waste is an inevitable byproduct of nuclear energy, making its safe storage and disposal crucial for environmental protection and human health. As energy production and, consequently, waste volumes increase, the need for reliable and efficient waste management systems becomes more pressing. Improper handling of radioactive waste can lead to serious environmental disasters and health threats, emphasizing the significance of this issue in modern society.

Modern approaches to radioactive waste management require a comprehensive strategy that includes both scientific research and technological development. Currently, there are multiple classifications of radioactive waste, complicating their disposal and requiring specialized treatment and storage methods. Many countries have already developed and implemented effective systems for safe waste management. However, in Russia, this issue demands special attention due to the accumulated volumes of radioactive materials and the need for new infrastructure facilities.

Additionally, public perception of nuclear energy and its waste is often based on fear and distrust, highlighting the importance of transparency in waste management and public awareness efforts regarding safety measures. In an era of globalization and increasing international cooperation, sharing experiences and jointly addressing radioactive waste disposal challenges are crucial for the sustainable development of nuclear energy.

This article examines the pressing issues of radioactive waste and spent nuclear fuel management, analyzing existing approaches, challenges, and prospects, and emphasizing the significance of scientific research and technological innovations in solving these problems. Given the increasing reliance on nuclear energy, it is vital to ensure safe waste management, which will ultimately contribute to sustainable development both at the national and global levels.

Radioactive Waste Classification and Disposal Methods

Radioactive waste refers to materials and substances that can no longer be used, as well as equipment and products (including spent ionizing radiation sources) that contain radionuclides exceeding levels established by the Russian government.(1)

There are various classifications of radioactive waste. According to the classification of the Russian State Corporation “Rosatom,” there are six classes of radioactive waste based on their radionuclide composition. (2)

Table 1. Classification of Radioactive Waste

Class of RW	Type of RW	Composition of RW	Disposal Methods
1	SRW (Solid Radioactive Waste)	- Materials - Equipment - Products - Solidified LRW (Liquid Radioactive Waste) - High-heat-generating HLW (High-Level Waste)	Final isolation in deep geological repositories with preliminary storage
2	SRW	- Materials - Equipment - Products - Soil -Solidified LRW - Sealed sources of ionizing radiation (Categories 1 & 2) - Low-heat-generating HLW - Long-lived ILW (Intermediate-Level Waste)	Final isolation in deep geological repositories
3	SRW	- Materials - Equipment - Products - Soil - Solidified LRW - Sealed sources of ionizing radiation (Category 3)	Final isolation in near-surface disposal facilities at depths up to 100 meters

		- Short-lived ILW - Long-lived LLW (Low-Level Waste)	
4	SRW	- Materials - Equipment - Products - Soil - Solidified LRW - Biological objects - Sealed sources of ionizing radiation (Categories 4 & 5) - Short-lived LLW - Long-lived VLLW (Very Low-Level Waste)	Final isolation in near-surface disposal facilities at ground level
5	LRW	- Organic and inorganic liquids, slurries, sludges - Short-lived ILW - Long-lived LLW	Final isolation in existing deep geological repositories
6	-	Radioactive waste generated during uranium ore mining and processing, as well as mineral and organic raw materials with elevated natural radionuclide content	Final isolation in near-surface disposal facilities

Many people often confuse radioactive waste (RW) with spent nuclear fuel (SNF), which can still be reused if reprocessed. SNF contains significant amounts of uranium-235 and uranium-238, plutonium, and other isotopes valuable for medicine and science. Although reprocessing is a complex technology, many countries possess this capability. Thus, SNF is considered a valuable secondary resource. In the past, SNF was deemed useless and highly hazardous waste. However, the disposal or burial of radioactive waste remains a pressing issue due to the expansion and development of nuclear energy.

The most hazardous radioactive waste (RAW) is classified as Class 1-2 waste, which includes materials and equipment from nuclear facilities that have reached the end of their service life, solidified liquid radioactive waste (LRW), and similar materials. These must be permanently disposed of at great depths, as the half-life of the isotopes they contain can reach hundreds of thousands of years. Class 3-4 waste consists of low- and intermediate-level short-lived radioactive waste (which will become non-hazardous in approximately 300 years), such as protective clothing and radionuclide-contaminated debris. These can be disposed of in near-surface disposal facilities (NSDF) at depths of up to 100 meters. Class 5 waste includes liquid waste generated during the operation of nuclear facilities, such as water used for cleaning floors at nuclear power plants. Class 6 waste consists of radioactive waste produced during the extraction of uranium and other minerals with naturally elevated radiation levels, making them relatively the least hazardous. (3)

Global and Russian Approaches to Radioactive Waste Management

Worldwide, companies operating nuclear power plants face daily challenges in disposing of spent fuel and radioactive waste. In the United States alone, there are more than 90,000 metric tons of high-level radioactive waste stored near nuclear power plants and weapons production facilities, emitting radiation that poses a serious risk to human health and the environment. Most high-level radioactive waste is awaiting permanent disposal in deep geological repositories; however, not a single such repository is currently operational in the U.S. This is confirmed by publications from the National Nuclear Security Administration (NNSA) and reports from the U.S. Department of Energy (DOE). (4)

In the Russian Federation, approximately 500 million cubic meters of radioactive waste (RAW) have been accumulated since the commissioning of the first nuclear power plant.

The majority of this volume is located in the Chelyabinsk region, in the bottom sediments of the Techa Cascade Reservoirs, as well as in the Ulyanovsk region, the Udmurt Republic, and the Krasnoyarsk and Zabaykalsky territories. These are restricted-access areas where a barrier system has been established to prevent radiation from escaping into the environment, according to official sources.

It is also worth noting that radioactive waste (RAW) is present in many other regions across our vast country.

Additionally, radioactive waste (RAW) is stored at Rosatom company sites in temporary storage facilities, which also maintain safety conditions. However, the operational lifespan of these facilities (approximately 70 years) is nearing its end. Therefore, it is time to address this issue decisively by creating a fully reliable system for the final isolation of hazardous waste.

Several European countries, including France, Hungary, and Sweden, such facilities have existed for a long time. Some have even been decommissioned and converted into meadows and hills covered with grass.

But this is only a half-measure: a truly reliable disposal site must be underground. In this case, its protection would be ensured not only by engineered systems but also by geological conditions—hundreds of meters of stable, preferably impermeable, rock or clay formations.

Since 2015, such an underground dry storage facility has been in use and is simultaneously under construction in Finland. At Onkalo, high-level radioactive waste (HLW) and spent nuclear fuel (SNF) will be sealed in granite bedrock at a depth of approximately 440 meters, placed in copper canisters further isolated by bentonite clay, ensuring containment for at least 100,000 years. In 2017, Swedish energy company SKB announced that they would adopt this method and construct their own repository near Forsmark. In the United States, debates continue over the construction of the Yucca Mountain repository in the Nevada desert, which would extend hundreds of meters into a volcanic mountain range.

In Russia, after studying the experience of these nations, a new strategy for radioactive waste (RAW) management was also initiated. In 2011, a special law was adopted, and in 2012, a federal unitary state enterprise was established as the sole legally authorized entity responsible for carrying out this work.

The first region to develop next-generation storage facilities was the Middle Urals, where a near-surface disposal facility (NSDF) for Class 3-4 radioactive waste is already operational. In 2022, the second phase was commissioned. The first phase began receiving packaged radioactive waste in 2016 and is now fully filled—its construction was initially started by the Ural Electrochemical Combine (UECC) in Novouralsk before the facility was transferred to the designated operator. As for the second phase, it is a completely independent project by the National Operator for Radioactive Waste Management (NO RAO), from design to full completion.

A near-surface disposal facility (NSDF) for Class 3-4 radioactive waste is structured as a complex of buildings, including a reception building for transport and packaging containers (TUKs) and an entry control facility, which conducts both radiation monitoring and visual inspections of the containers' condition. The core of the facility consists of massive concrete storage structures divided into cells. A gantry crane lowers the TUKs into these cells after thorough inspections, including spectrometric analysis, ensuring no radiation leaks. Unlike older designs, the concrete structures of the second phase are housed inside hangars rather than being exposed to the open air, allowing waste to be loaded in any weather, year-round. The spaces between containers are filled with inert materials such as bentonite or clay. Once the storage facility is full, the concrete structures will be sealed with a waterproof concrete slab to prevent water infiltration and corrosion of the waste packaging, thereby eliminating the risk of radiation leakage. Finally, the entire facility will be covered with soil and grass will be planted, integrating it into the surrounding landscape.

The experience gained in Novouralsk is aiding in the design and construction of similar facilities elsewhere. Work is already underway in Ozersk (Chelyabinsk region) and Seversk (Tomsk region), with storage capacities of 225,000 and 142,000 cubic meters, respectively. Another facility with a capacity of 200,000 cubic meters is planned for Dimitrovgrad (Ulyanovsk region), though investment justification is still pending. Thus, in a few years, Russia will have four final disposal sites for short-lived radioactive waste, with a total capacity exceeding 620,000 cubic meters. For comparison, the French operator ANDRA's disposal facility in Champagne is designed to store one million cubic meters.

The isolation of liquid radioactive waste (LRW), of which a significant amount is also formed, is done in a completely different way: they are simply pumped into the bowels of the earth, where they will remain until the total attenuation of activity. There are no concrete storage facilities and thick–walled steel barrels here - these are just very deep wells. According to experts, it is completely safe, the waste there is hermetically sealed in special geological formations separated by a water-resistant layer of clay rocks. Thus, the penetration of radiation into water sources is excluded: the injection depth of LRW is from 250 to 450 meters, while aquifers are no deeper than 50 meters. And, of course, there is constant monitoring of underground storage facilities. For this purpose, observation wells have been created – there are 404 of them on an area of about 32 square kilometers at the Seversk deep injection site (PGZ LRW).

This site was established in 1963 and was managed by the Siberian Chemical Combine (SCC) until 2012. It is now operated by the Federal State Unitary Enterprise "NO RAO." Over nearly 60 years, there has not been a single emergency, including radiation leaks to the surface, either at this site or at other Russian deep injection facilities for liquid radioactive waste (PGZ LRW). Currently, the underground reservoirs in Seversk are only about one-third full, so plans are in place to extend the facility's operation until 2033. Additionally, there will be a gradual transition to solidifying liquid waste, vitrifying it, and transferring it to near-surface disposal sites. Other PGZ LRW facilities in Russia follow a similar operational model.

When it comes to the most hazardous radioactive waste—solid Class 1-2 RAW—their disposal requires the most rigorous approach. Although the volume of such waste is relatively small, the costs of final isolation are significantly higher than for other classes. This is reflected in disposal tariffs: for example, in 2022, the state-approved rate for Class 4 RAW disposal was 55,200 rubles per cubic meter, whereas for Class 1 waste, it was nearly 1.57 million rubles per cubic meter.

Currently, there is an insufficient number of storage facilities for Class 1-2 radioactive waste (RAW) both in Russia and worldwide. To address this issue, a repository is being constructed in Krasnoyarsk Krai, in the depths of the Nizhnekansk rock massif, near Zheleznogorsk. The project consists of two phases. The first stage is the construction of a special underground research laboratory, where more than a hundred experiments will be conducted to refute or confirm the feasibility of environmentally safe burial of high- and medium-active long-lived radioactive waste in this area. The second stage is the construction of the storage facility itself. The facility has been under construction since 2018, most of the infrastructure is already ready, in 2022 they began to make the mouths of the ventilation duct, in 2023 they began to drill vertical shafts with a depth of 550 meters (there will be three of them), and then horizontal workings will be drilled into the rock.

If laboratory studies confirm that this method of disposal is safe, the most hazardous radioactive waste will be transported to this site. Vitrified high-level radioactive waste will be placed in special containers and lowered to depths of 450–525 meters into the underground excavations, where they will be sealed for millennia. (3)

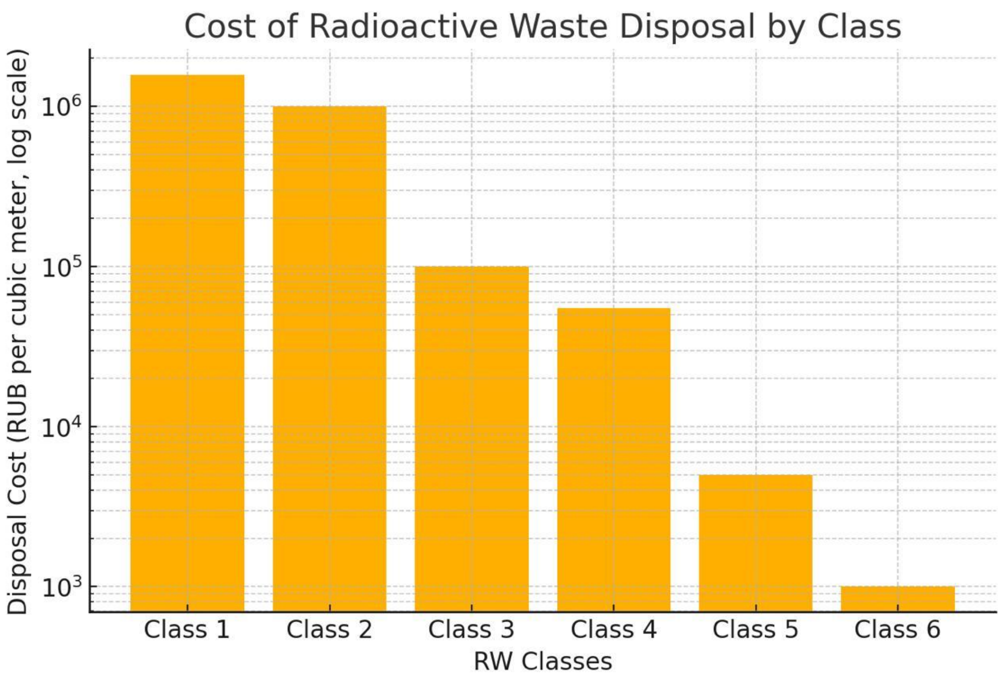
At the same time, one unresolved issue remains: the final decision on the materials for the containers intended for the transportation and storage of radioactive waste (RAW) has not yet been made. This choice depends on several key factors, primarily the material's corrosion resistance, weight, and structural strength.

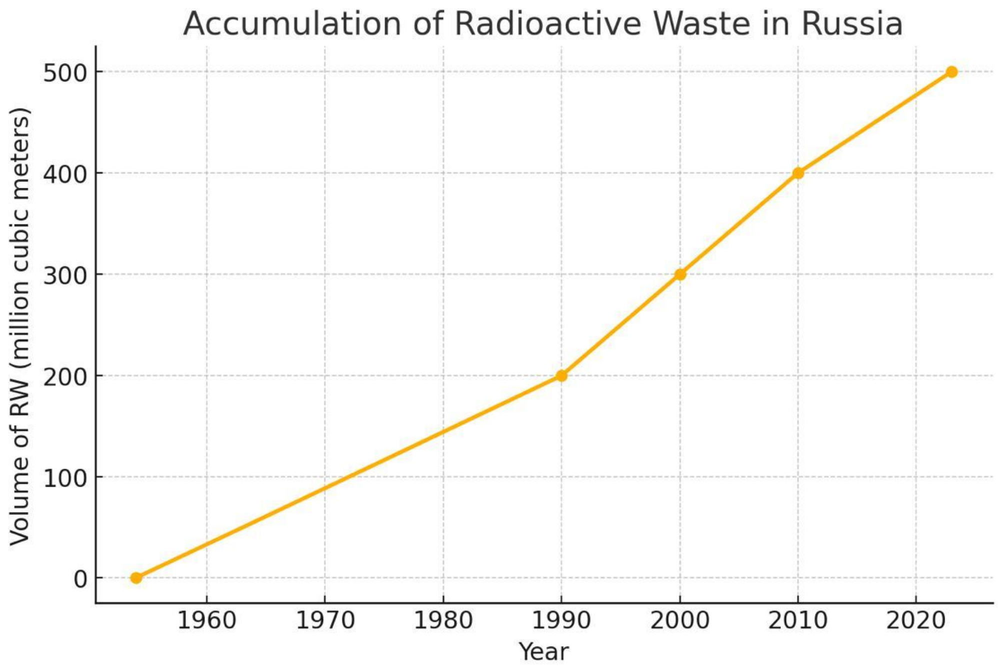
The relevance of radioactive waste (RAW) disposal and spent nuclear fuel (SNF) management is driven by several key factors that underscore its importance in modern society.

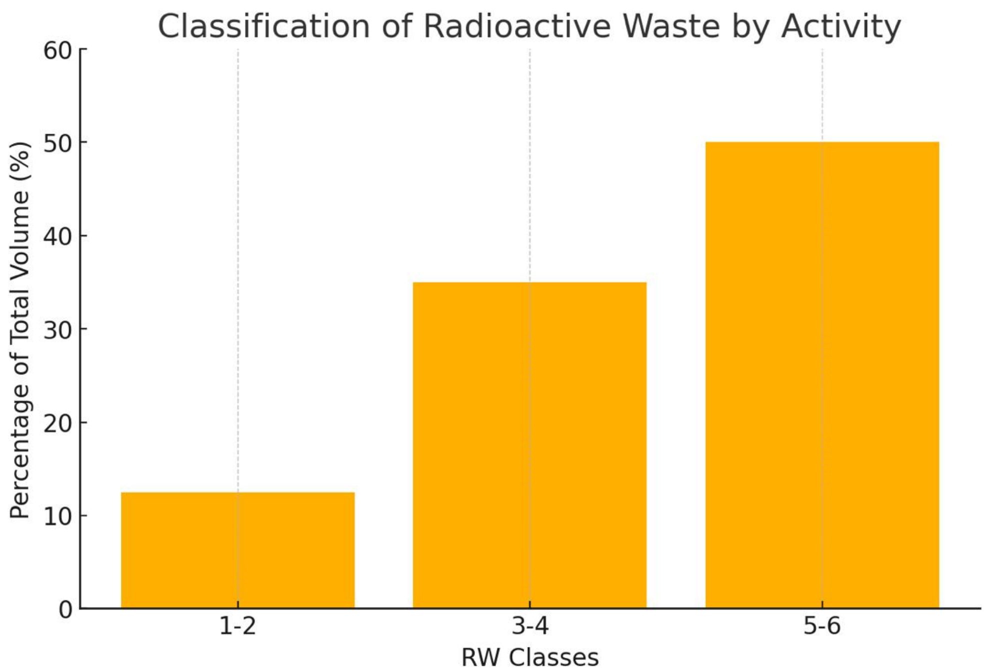
1. Increasing Energy Consumption: With global economic growth and rapid technological advancement, the demand for energy continues to rise. Nuclear energy, as one of the key sources of electricity, offers an efficient solution to meet these needs. However, this also brings the challenge of safely and effectively disposing of the resulting waste, making the issue of radioactive waste (RAW) management particularly relevant.
2. Environmental and Health Risks: Radioactive waste poses a significant threat to both the environment and human health. Improper handling can lead to contamination of water bodies, soil, and air, potentially causing diseases and long-term ecological damage. The relevance of this issue lies in the need to develop reliable storage and disposal methods that minimize these risks and ensure environmental safety.
3. Technical and Scientific Challenges: The management of radioactive waste requires advanced technologies and scientific expertise. Developing efficient methods for processing and safely disposing of RAW is a complex challenge for scientists and engineers. This issue is relevant not only from an environmental perspective but also as a driver of scientific research and technological progress.
4. Social Perception and Regulatory Framework: Public opinion on nuclear energy and its waste is often influenced by concerns and skepticism. The need for a transparent radioactive waste management system, along with efforts to educate the public on safety measures, makes this issue crucial for the sustainable development of nuclear

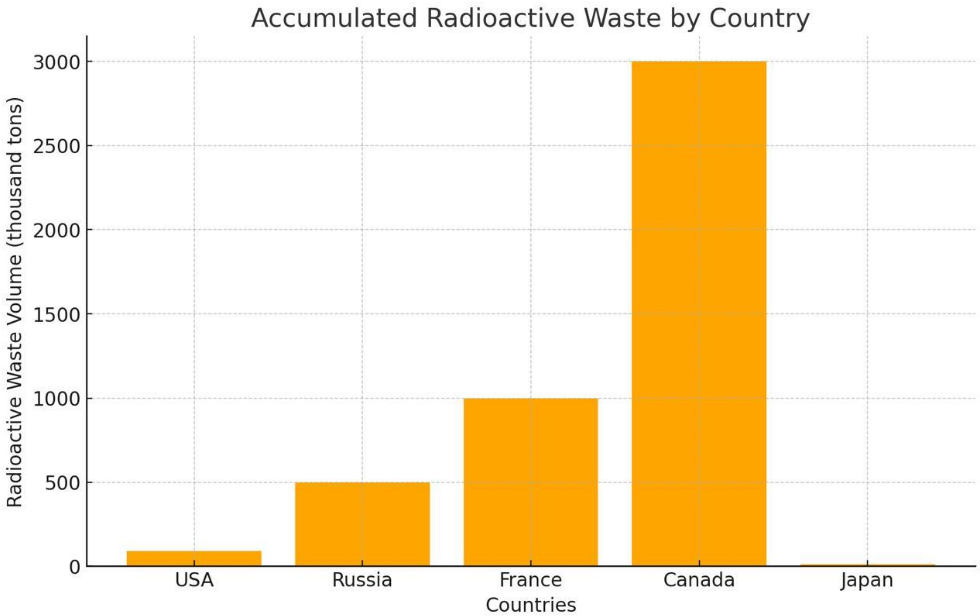
energy. Establishing clear regulations and fostering public trust are essential to ensuring long-term acceptance and responsible handling of nuclear waste.

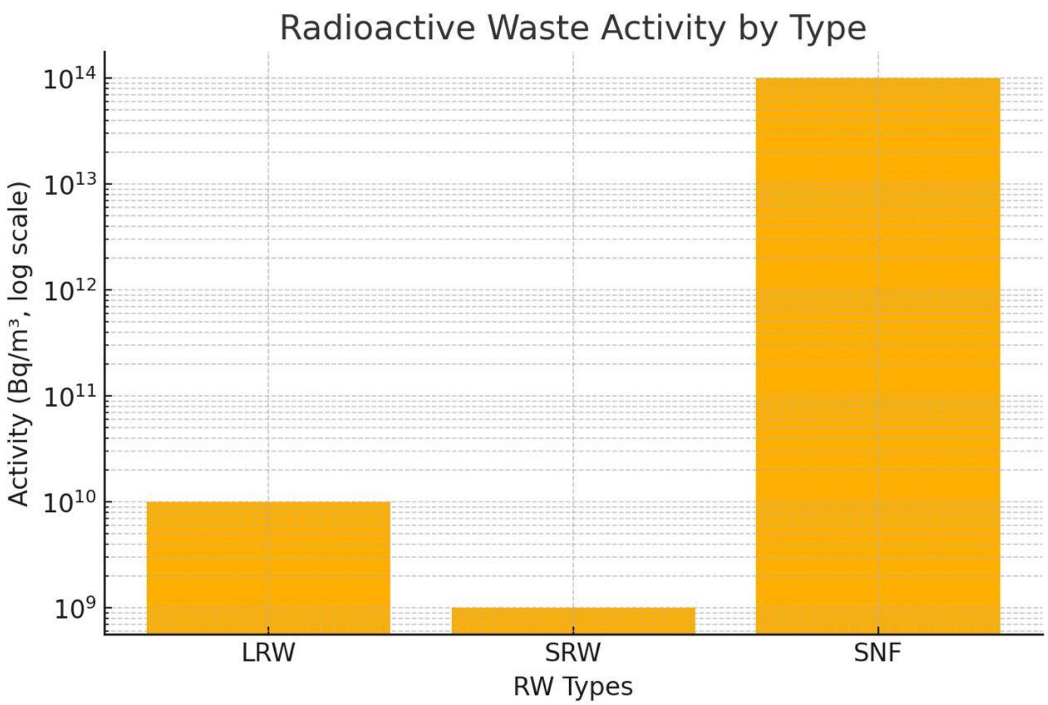
5. International Cooperation and Knowledge Sharing: The issue of radioactive waste disposal is a global challenge. Countries utilizing nuclear energy face similar difficulties, creating opportunities for international collaboration, experience exchange, and the implementation of best practices. The relevance of this topic lies in the necessity of joint efforts at the international level to develop effective and sustainable solutions for radioactive waste management.

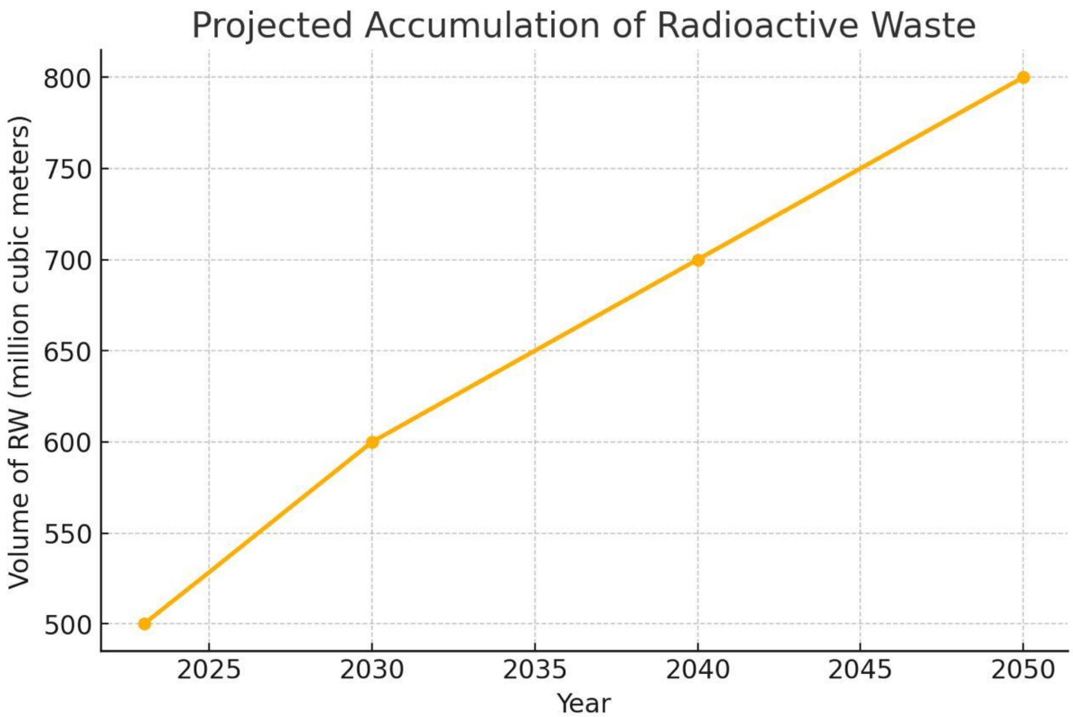



Accumulation of Radioactive Waste in Russia
Volume of RW (million cubic meters)
0
100
200
300
400
500
1960
1970
1980
1990
2000
2010
2020
Year


Classification of Radioactive Waste by Activity
Percentage of Total Volume (%)
0
10
20
30
40
50
60
1-2
3-4
5-6
RW Classes


Accumulated Radioactive Waste by Country
Radioactive Waste Volume (thousand tons)
3000
2500
2000
1500
1000
500
0
USA
Russia
France
Canada
Japan
Countries


Radioactive Waste Activity by Type
Activity (Bq/m³, log scale)
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
LRW
SRW
SNF
RW Types



Conclusion:

The key issue remains the lack of effective radioactive waste (RAW) recycling technologies. Instead of developing waste-free processing methods, waste is treated and placed in storage facilities, which require significant resources and occupy large areas. It is crucial to remember that the development of nuclear energy must be accompanied by careful planning and the creation of reliable solutions for the safe disposal of radioactive waste.

The management of radioactive waste (RAW) and spent nuclear fuel (SNF) is one of the most pressing challenges associated with the development of nuclear energy. With the increasing volume of energy production and, consequently, the accumulation of waste, it is essential to develop and implement effective systems for the safe handling of these materials. Improper disposal of RAW can lead to severe environmental and health consequences, highlighting the need for a comprehensive approach to solving this issue.

Modern technologies and scientific research play a key role in developing sustainable solutions for radioactive waste management. The application of various storage and processing methods, along with the adoption of international standards and best practices, can significantly enhance safety in handling RAW. This is especially relevant in light of growing public concerns and the need for transparency in processes related to nuclear energy.

Despite significant progress in the disposal and storage of radioactive waste, the challenge remains highly complex and requires further investment in research and the development of new technologies. It is crucial for governments and international organizations to collaborate in creating reliable systems that can ensure the safety of both the environment and human health.

In conclusion, effective radioactive waste management is a necessary condition for the sustainable development of nuclear energy. Solving this issue requires a comprehensive approach that incorporates scientific, technical, and social aspects. Only through collaboration and innovation can we hope to create a safe and environmentally sustainable future in nuclear energy.

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Challenges and Future Prospects

INTELLECTUAL ARCHIVE, Vol. 14, No. 1, 2025
DOI: 10.32370/IA_2025_01_5

The Current State of Inclusive Competence Development among Future Professionals in the Socionomic Sector

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Abstract

The rapid democratic transformations and integration of Ukraine into the European and global community have significantly influenced the approach to inclusive education. The development of inclusive competence among future professionals in the socionomic sector—such as psychologists, social workers, and special education professionals—is critical for ensuring effective inclusion in education and social life. This article analyzes the current challenges in fostering inclusive competence within Ukraine’s higher education system, particularly in the context of digitalization, internationalization, and ongoing military conflict. The study highlights the barriers caused by limited resources, war-related disruptions, and the need for interdisciplinary approaches in training. It concludes that adapting educational programs to modern global standards and improving professional training are essential steps toward enhancing inclusive competence among future specialists.

Key words: Inclusive competence, socionomic sector, higher education, inclusive education, professional training, digitalization, educational reform, interdisciplinary approach, internationalization, accessibility, adaptive learning, psychological resilience, educational challenges, social inclusion, special education, professional development.

Problem Statement. Democratic transformations in Ukraine over the past decades, along with the country's integration into the global economic community and the European Union, have led to a shift in societal attitudes toward individuals with special needs. This has underscored the necessity of their broader inclusion into society. However, the effective implementation of integration and inclusion in education and social life largely depends on the qualifications of professionals facilitating this process. This highlights the need for changes in the training of future specialists in the socionomic sector, including psychologists, social workers, physical rehabilitation specialists, and special education professionals. In this context, the development of inclusive competence is of particular importance as a key component of professional training in higher education amidst contemporary educational challenges and limitations.

Analysis of the Latest Research. Numerous studies by Ukrainian scholars have explored the formation of inclusive competence, including the works of S. Alyokhina, I. Belike, Yu. Boychuk, T. Bondar, O. Borodina, O. Budnyk, I. Vdovenko, O. Gnoievska, O. Gordiychuk, A. Davidenko, L. Danilenko, O. Derkachova, S. Illyash, N. Klymenyuk, L. Kalchenko, A. Kolupayeva, H. Kosareva, O. Krasovska, N. Matveeva, O. Nikolaescu, Yu. Pelekh, M. Pantiuk, O. Popadych, I. Sadova, I. Sasina, P. Talanachuk, M. Filonenko, O. Khomyak, M. Chaikovskyi, and Z. Shevtsiv. However, the issue of developing inclusive competence among professionals in the socionomic sector remains insufficiently explored within the context of modern influences.

To substantiate the concept of inclusive competence formation among future professionals in the socionomic sector, it is essential to analyze the current state of the issue within the framework of challenges and constraints in the educational process.

Research Objective. This article aims to analyze the current situation in Ukraine, identifying difficulties in higher education development in the context of inclusion and examining various factors that impact the organization of the educational process for fostering inclusive competence among socionomic professionals.

Recent studies indicate that educational challenges related to systemic reform, technological advancements, and adaptation to European standards have been further complicated by global crises such as the COVID-19 pandemic and the ongoing war with Russia [2; 3; 4; 5; 6]. These factors have introduced numerous limitations, including disruptions in learning processes and a shift to remote education, significantly affecting the development of inclusive competencies among future professionals in the socionomic sector.

Main Content Presentation. An analysis of the modernization of higher education in Ukraine has identified several priority challenges, including *digitalization, internationalization of educational programs, and modernization of educational content*. These require substantial educational resources and financial investments.

The reform of higher education necessitates the active implementation of digital technologies. However, inadequate access to modern technical equipment, unequal internet connectivity, and limited access to digital devices among students pose significant barriers. The availability of data from the National Agency for Higher Education Quality Assurance [1] and an understanding of the actual state of higher education—including statistical data on higher education institutions (HEIs), students at various levels, and academic programs - allow us to comprehend the extent to which Ukraine’s higher education sector is currently experiencing significant challenges due to the ongoing war and economic difficulties.

In the modern world, the integration of Ukrainian education into the European and global educational space has become one of the key objectives, presenting Ukraine with new challenges. This means that the education system must align with international standards and requirements, which in turn includes adapting curricula, teaching methodologies, assessment approaches, and developing new educational standards that reflect the best global practices. This process necessitates the harmonization of Ukraine’s education system with European and international norms, particularly in the areas of education quality, academic mobility, and digitalization.

This requires not only the modernization of existing educational programs but also the continuous professional development of educators so they can effectively implement new teaching approaches. Preparing teaching staff capable of working in accordance with international standards is a crucial step in ensuring high-quality education.

Teaching in the context of globalization requires knowledge of modern methodologies, the ability to integrate new technologies into the learning process, and the development of intercultural competence. Professional development for educators should encompass not only foreign language proficiency and the use of modern information and communication technologies but also flexibility in applying innovative pedagogical approaches. This is particularly important for the training of future professionals in the socionomic sector, including psychologists, social workers, physical rehabilitation specialists, and special education professionals. The creation of interdisciplinary teams and the development of new educational programs (hereinafter referred to as EPs) enable the integration of diverse knowledge and skills essential for effective professional activity in a rapidly changing global environment.

The socionomic sector encompasses various fields such as social work, economics, law, and management, all of which require a comprehensive approach to professional training. This approach fosters the ability of specialists to adapt to emerging challenges in the process of globalization, enhances their capacity to work in multicultural teams, and facilitates the application of interdisciplinary methods to solve complex social issues.

The modernization of educational content involves adapting learning materials to meet contemporary labor market demands and changes in social and economic spheres. This requires an interdisciplinary approach, as the socio-economic sphere encompasses various sectors. Educational programs must be more flexible and aimed at forming comprehensive knowledge that enables professionals to work effectively in interdisciplinary environments. This can include specialized courses and variable disciplines that take into account shifts in social policy, economic processes, government initiatives, and the new challenges facing the socio-economic field. Programs should emphasize experiential learning, allowing students to work on real projects where they can apply their knowledge in practice. Given globalization, educational programs should not only target the national labor market but also prepare students for employment opportunities in international organizations, social projects, and economies in other countries.

Consequently, the integration of Ukrainian education into the European and global educational space is a complex yet essential process that requires not only significant resources but also a profound transformation of the country's educational system. This path will ensure the training of future specialists capable of working effectively in a globalized world and contribute to the overall development of society.

Our research [2; 3; 5] has established that significant challenges in preparing specialists for the socio-economic sector arise from limitations associated with the state of war and russian aggression, including the *migration of students and educators, security issues, the* *psychological impact of war on students and teachers, difficulties in organizing the educational process, limited access to resources and practical training, and financial constraints.*

Military actions have forced many students and educators to leave the country or relocate to other regions, complicating the educational process. Constant shelling, destruction of educational infrastructure, and threats to life compel institutions to transition to distance learning, which reduces the effectiveness of specialist training. The ongoing military actions and the constant threat to life and safety, as well as the loss of loved ones, significantly impact the emotional state of students and educators. This leads to decreased concentration and resilience. Students and teachers are forced to adapt to conditions of constant uncertainty, increasing emotional stress and affecting their motivation to learn and teach.

The shift to distance learning presents additional challenges, such as poor internet connectivity, inadequate access to technical devices, and reduced interactivity between students and educators. As a result of military actions and the occupation of parts of territories, many educational institutions have suffered destruction or shifted to an online format. This limits opportunities for practical training, particularly in working with individuals with disabilities, which is fundamental to inclusive competence.

The economic crisis caused by the war has significantly reduced funding for the educational sector. This complicates the implementation of modern teaching methods, the development of new programs, and the provision of necessary resources for students.

Insufficient financial, material, and methodological resources in the context of martial law hinder the establishment of a comfortable and effective inclusive educational environment in HEI. Additionally, the imperfect regulatory framework poses challenges to the implementation of inclusive programs and principles. Many specialists and education seekers face stereotypical notions about inclusion practices and are unfamiliar with modern methodologies for working with diverse groups of learners. The insufficient development of social partnerships reduces opportunities for enriching students' experiences regarding inclusivity.

Most of these issues are external, forced, and linked to the military events in Ukraine. Overcoming these problems requires a comprehensive approach from the government, educational institutions, and international organizations. In the socio-economic sphere, which includes social services, business, finance, education, healthcare, and other sectors, inclusion is not only a moral obligation but also an economic necessity, as it contributes to the development of society as a whole.

The development of adapted training programs based on a multidisciplinary approach, psychological support for participants in the educational process, and intellectual investments in educational programs, technologies, and methodological resources are key steps for HEI to ensure the formation of inclusive competence among specialists in today's challenging conditions.

Within the framework of our research, achieving the set goal requires further steps, namely: clearly defining the components of inclusive competence and the criteria for its formation in different groups of higher education seekers; developing and justifying pedagogical conditions for effectively forming inclusive competence and assessing the effectiveness of the implemented pedagogical conditions in preparing specialists for the socio-economic sphere.

Conclusion. The development of inclusive competence among future professionals in the socionomic sector is a critical component of modern education in Ukraine. As the country continues to integrate into the European and global community, the demand for well-trained specialists who can effectively implement inclusive practices is increasing. However, numerous challenges hinder this process, including limited resources, the digital divide, war-related disruptions, and the need for interdisciplinary approaches in training.

Addressing these issues requires a comprehensive strategy that includes modernizing educational programs, enhancing digital accessibility, fostering international collaboration, and providing continuous professional development for educators. The adoption of innovative pedagogical methods, practical training opportunities, and interdisciplinary cooperation will significantly contribute to the formation of highly skilled professionals ready to work in inclusive environments.

Ultimately, strengthening inclusive competence in higher education will not only improve the quality of services provided by socionomic professionals but also contribute to the broader goal of social integration and equal opportunities for all individuals in Ukraine. Ensuring sustainable development in this field requires the joint efforts of educational institutions, policymakers, and international partners to create a more inclusive and accessible society.

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12. **The word "References"** (if any). Font size - 12, bold-italics. Position - central alignment.

13. **The text of References** (if any). Font size - 12, regular (not bold).

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