Quasi-chaotic behavior of a linear dynamic system under periodic pulse excitation

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Astract

The paper deals with the problem of the behavior of a harmonic oscillator with a periodic pulse excitation. An analytical solution is proposed using the original step function. It is shown that the action can be accompanied by periodic and quasi-chaotic motion, as well as pulse of a different nature.

Keywords: dynamic system, harmonic oscillator, pulse excitation, chaos.

Consider a dynamic system Consider a dynamic system

$$\ddot{x} + \omega_1^2 x = f(t), \quad f(t) = \frac{1}{2}\left(1 + \frac{\sin \omega_2 t}{\sqrt{\sin^2 \omega_2 t}}\right) , \qquad (1)$$

where x - is the generalized coordinate, ω_1 -is the frequency of natural oscillations, ω_2 - is the frequency of a periodic impulsive perturbing generalized force.

The function sets the rectangular pulse excitation (Fig. 1).

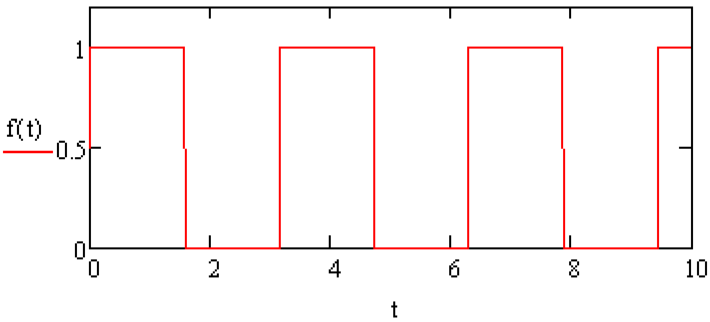


Fig.1. Periodic pulse excitation at $\omega_2 = 2$.

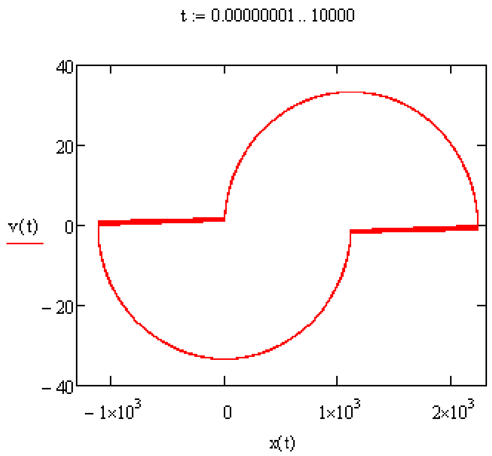
The general solution to equation is (1):

$$x = \left(x_0 - \frac{1}{\omega_1^2} f(t_0)\right)\cos \omega_1 t + \frac{v_0}{\omega_1}\sin \omega_1 t + \frac{1}{\omega_1^2} f(t) .$$

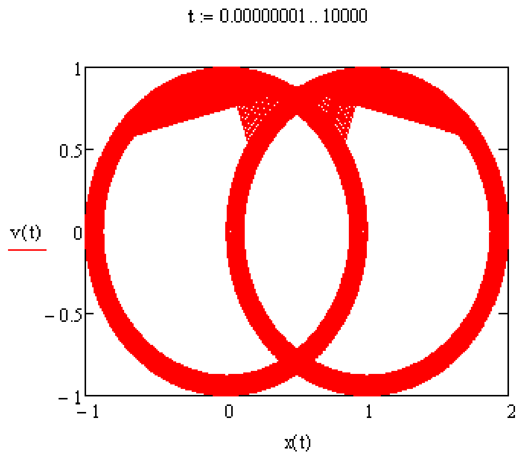
Generalized velocity:

$$v = \dot{x} = -\omega_1\left(x_0 - \frac{1}{\omega_1^2} f(t_0)\right)\sin \omega_1 t + v_0 \cos \omega_1 t .$$

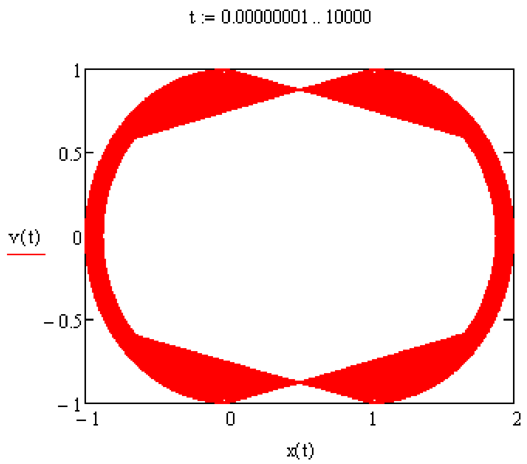
Figure 2 shows the characteristic phase portraits of the asymptotic behavior of the system at different frequencies of the harmonic oscillator and the pulse excitation and under the initial conditions: $x(t_0) = 1, \quad v(t_0) = 1, \quad t_0 = 0.00000001$.



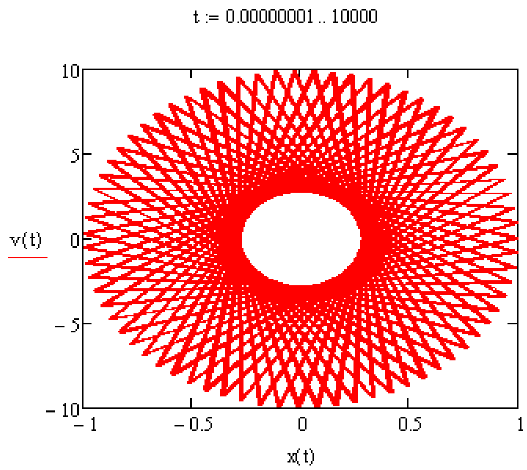
$\omega_1 = 0.03, \ \omega_2 = 0.03$



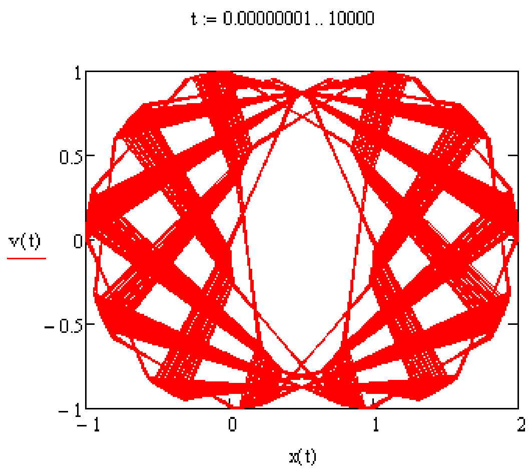
$\omega_1 = 1, \ \ \omega_2 = 0.1$



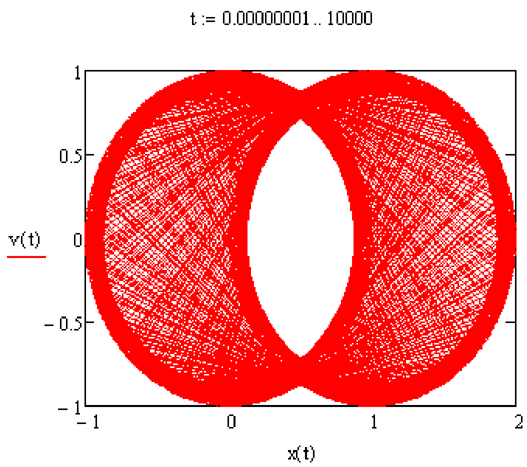
$\omega_1 = 1, \ \ \omega_2 = 1$



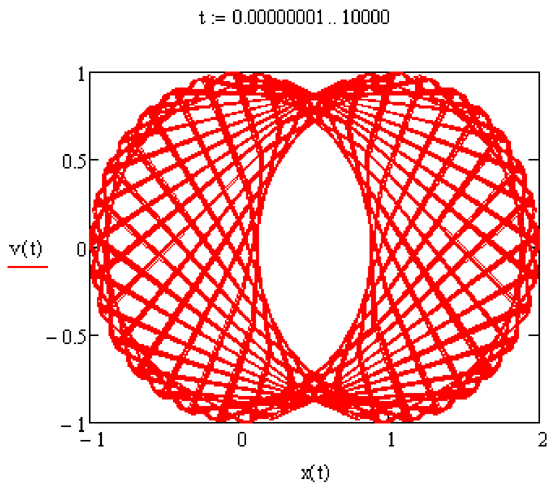
$\omega_1 = 10, \ \ \omega_2 = 1$



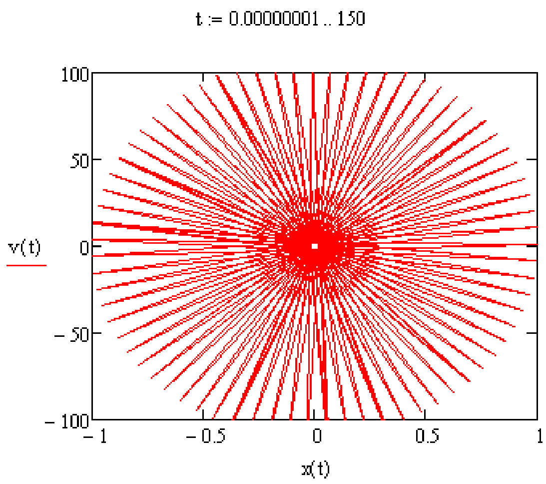
$\omega_1 = 1, \ \ \omega_2 = 10$



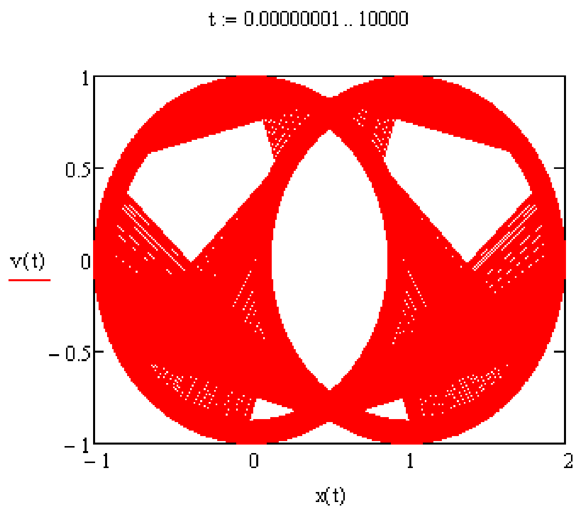
$\omega_1 = 1, \ \ \omega_2 = 1000$



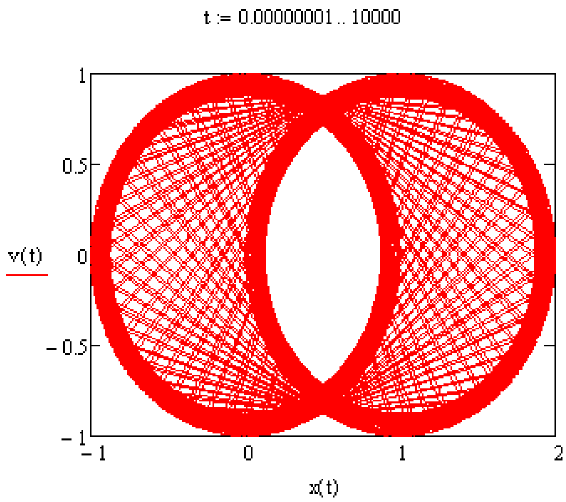
$\omega_1 = 1, \ \omega_2 = 30$



$\omega_1 = 110, \ \omega_2 = 110$



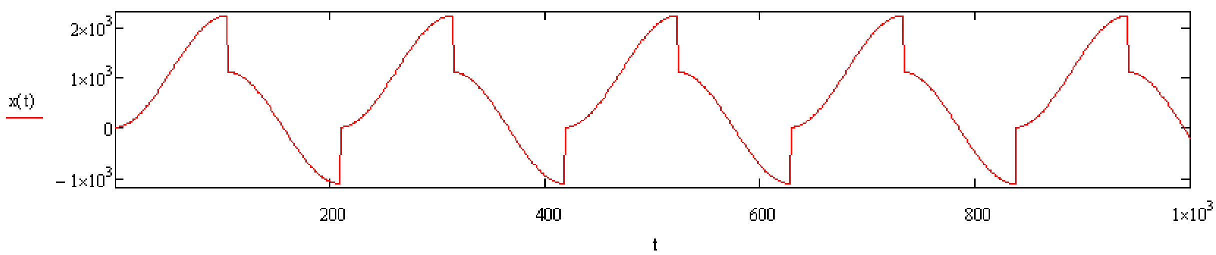
$\omega_1 = 1, \ \omega_2 = 0.3$



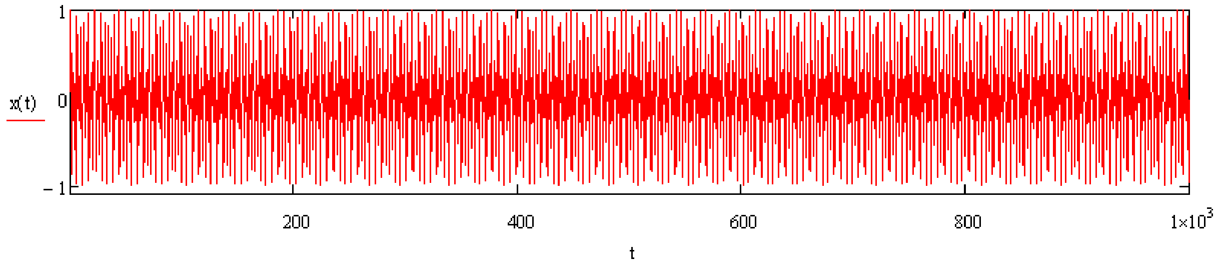
$\omega_1 = 1, \ \omega_2 = 1200$

Fig. 2. Phase portraits of a harmonic oscillator with a periodic pulse excitation.

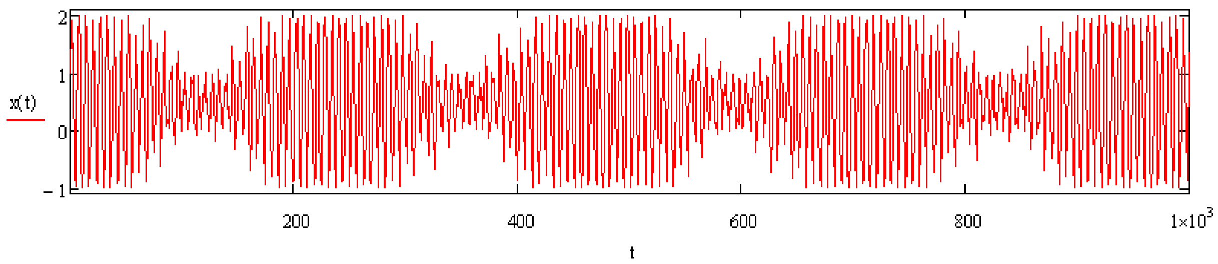
In Figure 3, the graphs of the change of the generalized coordinate at different frequencies of the harmonic oscillator and the pulse excitation.



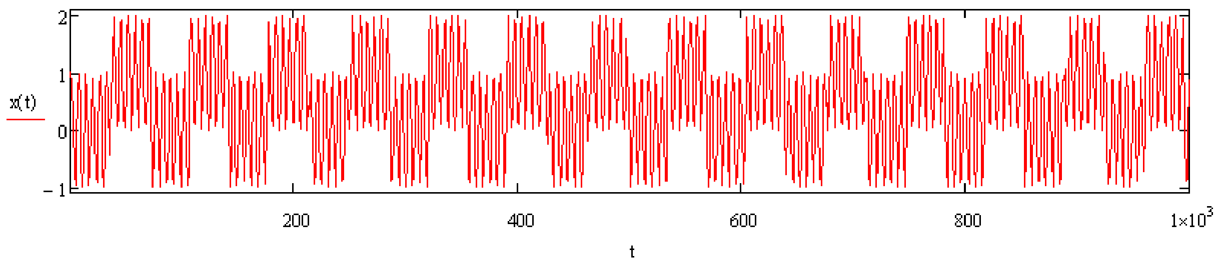
$\omega_1 = 0.03, \ \omega_2 = 0.03$



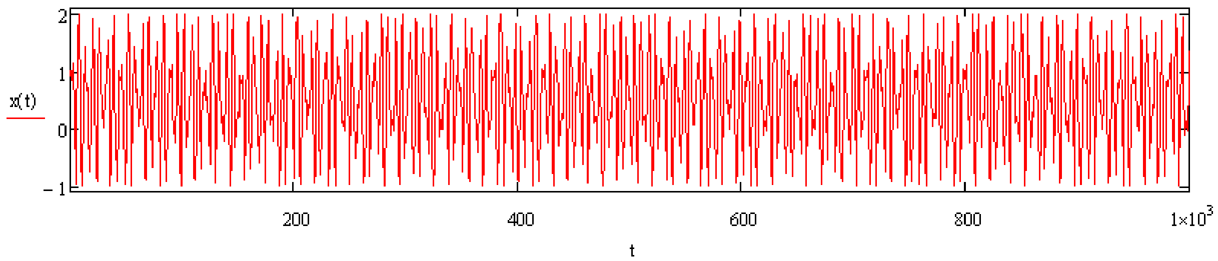
$\omega_1 = 10, \quad \omega_2 = 1$



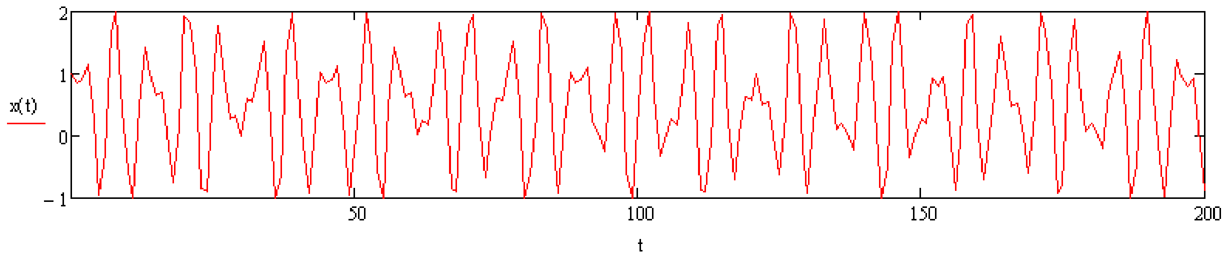
$\omega_1 = 1, \quad \omega_2 = 1000$



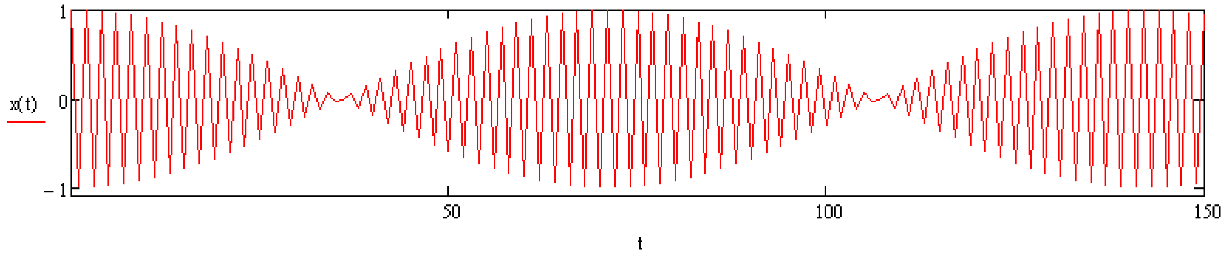
$\omega_1 = 1, \quad \omega_2 = 1200$



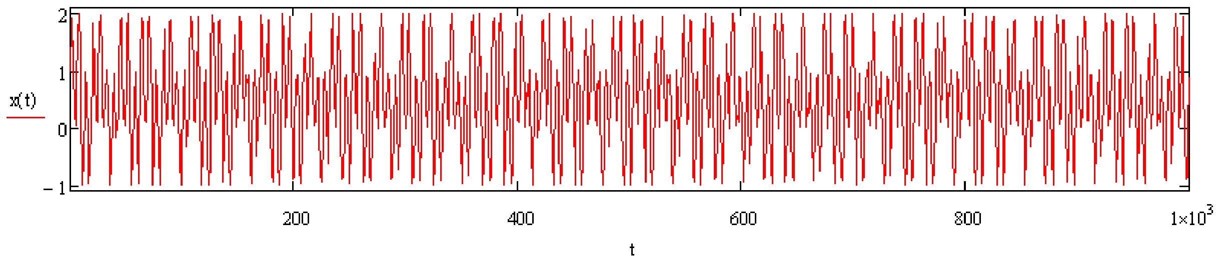
$\omega_1 = 1, \quad \omega_2 = 30$



$\omega_1 = 1, \quad \omega_2 = 30$



$\omega_1 = 110, \quad \omega_2 = 110$



$\omega_1 = 1, \quad \omega_2 = 0.3$

Fig.3. Graphs of changes of the generalized coordinates.

As follows from the presented illustrations, a periodic pulse excitation on a linear dynamic system can be accompanied by a periodic and quasi-chaotic motion, as well as pulses of various kinds.