

# Superluminal Particles and Quantum Theory with non fixed Causal Structure

Marco Zaopo\*

*Dipartimento di Fisica, Università di Pavia, via Bassi 6, 27100 Pavia, Italy\**

(ΩDated: March 20, 2012)

In this paper we assume that it exists at least one particle other than the photon that has an invariant speed. We also assume the speed of this hypothetical particle to be greater than  $c$ . We consider a simple operational framework consisting of two devices that can analyze a generic quantum system. In this simple framework we show that in quantum theory the above assumptions do not lead to causal paradoxes since the causal structure of two outcomes appearing on two distinct devices is always assumed in an absolute way. We then consider a probabilistic theory in which the causal structure of the outcomes seen at two distinct devices is not established in an absolute way but is defined by means of an operational protocol by the observers. If the invariant speed is unique then this situation is equivalent to have a probabilistic theory with absolute causal structure. If there exist more than one invariant speed, then observers performing the protocol with signals having different invariant speeds can assign a different causal structure to the outcomes. We show that quantum theory with pure states is an informationally consistent model of this last situation.

**Assumption 1:** There exists at least one particle other than the photon, having an invariant speed  $v_n > c$ .

**Proposition 1** *Space-time coordinates in two different reference frames are related by a Lorentz transformation with invariant speed  $v$  if and only if they are used particles travelling at speed  $v$  to establish the simultaneity of two events.*

In the following we will denote a photon as PH and a superluminal particle as SLP. We will denote  $v$  the speed of SLPs and  $c$  the speed of PHs. We will suppose that the SLP is a quantum system. We now describe a simple operational framework consisting of two devices,  $\mathcal{A}, \mathcal{B}$ , that can analyze the same observables of a quantum system. In this operational framework  $\mathcal{A}$  and  $\mathcal{B}$  can randomly output the possible mutually exclusive values of two observables,  $A_i, B_j$  that can be chosen by an operator among the set  $\{A_i\}_{i \in \mathcal{A}}$  for device  $\mathcal{A}$  and the set  $\{B_j\}_{j \in \mathcal{B}}$  for device  $\mathcal{B}$ . Since both devices analyze the same set of observable we have that the sets  $\{A_i\}_{i \in \mathcal{A}}$  and  $\{B_j\}_{j \in \mathcal{B}}$  are the same set. The mutually exclusive outcomes outputted by device  $\mathcal{A}$  when observable  $A_i$  is chosen are denoted by  $\{a_i^t\}_{t \in A_i}$  and those possibly outputted by  $\mathcal{B}$  when  $B_j$  is chosen are  $\{b_j^u\}_{u \in B_j}$ . Quantum theory is then seen as a set of mathematical rules that permit to calculate  $p(a_i^t, b_j^u | A_i, B_j)$  for all  $(a_i^t, b_j^u) \in A_i \times B_j$  and for all  $(A_i, B_j) \in \mathcal{A} \times \mathcal{B}$ .

In the context outlined above we now describe a protocol that can be used to establish whether two events, one appearing on device  $\mathcal{A}$  and one on  $\mathcal{B}$  are space-like or time-like.

Alice and Bob must establish whether two events  $a^t, b^u$  appearing on devices  $\mathcal{A}, \mathcal{B}$  are space-like or time-like.

Alice and Bob each possess:

(i) A device that can measure all the observables belonging to a quantum system  $s$ .

(ii) A clock.

(iii) A gun shooting some signal

(iv) A detector for the type of signal shot by the gun.

We will assume that only signals having an invariant speed can be used to perform the above protocol. Alice chooses an observable  $A$  for her device with possible outcomes  $\{a^t\}_{t=1,n}$  while Bob chooses observable  $B$  for his device with possible outcomes  $\{b^u\}_{u=1,n}$ . The devices of Alice and Bob are in the same laboratory and have distance  $x_{AB}$  in the laboratory reference frame. Any two outcomes  $(a^t, b^u)$  appear respectively on Alice's device  $\mathcal{A}$  and Bob's device  $\mathcal{B}$  with probability  $p(a^t, b^u | A, B)$  that can be calculated using the rules of quantum theory.

The protocol runs as follows:

- *Alice's clock time  $t_A$ :* An outcome  $a^t$  appears on Alice's device because a system  $s$  went into or out from her device. Alice immediately sends a signal to the detector of Bob.
- *Bob's clock time  $t_B$ :* An outcome  $b^u$  appears on Bob's device because a system of type  $s$  went into or out from his device. Bob immediately sends a signal to the detector of Alice
- *Alice's clock time  $t'_A$ :* Alice detects the signal sent by Bob.
- *Bob's clock time  $t'_B$ :* Bob detects the signal sent by Alice.
- Alice measures  $\Delta t_A = t'_A - t_A$ , Bob measures  $\Delta t_B = t'_B - t_B$ .

Clearly the order of  $t_A, t_B, t'_A, t'_B$  in the above list does not necessarily follow the order of Alice and Bob actions.

**Proposition 2**  *$\Delta t_A$  and  $\Delta t_B$  cannot be both smaller than 0.*

---

\* marco.zaopo@unipv.it

If this were the case then Alice and Bob would detect a particle that the other sent in a circumstance in which both of them must first send the signal and then detect the signal sent by the other. This generates a contradiction since Alice and Bob would detect a particle that none of them could have sent. ■

In the following proposition we will assume that Alice and Bob use the same type of signal that has an invariant speed  $v$

**Proposition 3** *If  $\Delta t_A \geq 2x_{AB}/v$  then  $\Delta t_B \leq 0$ . Moreover if  $\Delta t_B \geq 2x_{AB}/v$  then  $\Delta t_A \leq 0$ .*

If  $\Delta t_A \geq 2x_{AB}/v$  then Alice detects the SLP sent by Bob after having seen the outcome on her device and having sent her an SLP to Bob. Suppose now that also Bob detected Alice's SLP after having seen the outcome on the device and having sent his SLP to Alice, hence  $\Delta t_B > 0$ . In this case Alice's signal would arrive at Bob's detector after Bob's signal has left to Alice's detector. Alice would then measure  $\Delta t_A = x_{AB}/v$ (time Bob's SL takes to go from Bob to Alice) +  $t$  where  $t \leq x_{AB}/v$  since we are assuming that when Bob's signal is shot by Bob's gun, Alice SLP is not yet arrived. This generates a contradiction with the hypothesis that  $\Delta t_A \geq 2x_{AB}/v$  and proves that if  $\Delta t_A \geq 2x_{AB}/v$  then  $\Delta t_B \leq 0$ . Employing the same argument with the roles of Alice and Bob exchanged we prove that if  $\Delta t_B \geq 2x_{AB}/v$  then  $\Delta t_A \leq 0$ . ■

**Proposition 4**  *$0 < \Delta t_A < 2x_{AB}/v$  if and only if  $0 < \Delta t_B < 2x_{AB}/v$*

*Proof:* We first show that if  $0 < \Delta t_A < 2x_{AB}/v$  then  $0 < \Delta t_B < 2x_{AB}/v$ . It is clear that we cannot have  $0 < \Delta t_A < 2x_{AB}/v$  together with  $\Delta t_B \geq 2x_{AB}/v$  since otherwise we would be in contradiction with proposition ???. Suppose now that  $0 < \Delta t_A < 2x_{AB}/v$  and  $\Delta t_B \leq 0$ . In this case Bob first detects Alice's SLP and then sees the outcome  $b^j$  on his device and shoots his SLP to Alice. Alice would then measure  $\Delta t_A = x_{AB}/v$ (time Bob's SL takes to go from Bob to Alice) +  $t'$  where  $t' > x_{AB}/v$  since we are assuming that Bob shoots an SLP to Alice after having detected Alice's SLP. This generates a contradiction and proves that if  $0 < \Delta t_A < 2x_{AB}/v$  then  $0 < \Delta t_B < 2x_{AB}/v$ . In the same way, exchanging the roles of Alice and Bob it can be proved that if  $0 < \Delta t_B < 2x_{AB}/v$  then  $0 < \Delta t_A < 2x_{AB}/v$ . This proves the thesis. ■

Depending on whether  $\Delta t_A$  and  $\Delta t_B$  satisfy proposition 3 or 4 Alice and Bob establish that the events on their devices  $\mathcal{A}$  and  $\mathcal{B}$  are time-like or space-like.

The protocol described above to establish if two events are time-like or space-like gives an unambiguous answer only in the case the invariant speed is unique. If we assume the existence of a hypothetical particle with invariant speed  $v > c$  then the above protocol gives ambiguous answers. To see this suppose that both Alice and Bob have used photons and that Alice has measured  $2x_{AB}/v < \Delta t_A < 2x_{AB}/c$ . From proposition 4 it follows that Bob measured  $0 < \Delta t_B < 2x_{AB}/c$  thus Alice and Bob conclude that  $a^t$  and  $b^j$  are space-like. However if they performed the protocol using SLP as signals and Alice measured  $2x_{AB}/v < \Delta t_A < 2x_{AB}/c$ , they would find to be in principle possible for event  $a^t$  to be the (probabilistic) cause of event  $b^u$ . This is the case since there is enough time for the SLP to go from Alice's device  $\mathcal{A}$  to Bob's device in a time  $x_{AB}/v$  and for another SLP (shot by Bob when he sees  $b^u$ ) to go to Alice's SLPs detector in a time  $x_{AB}/v$ . Hence performing the protocol with photons we find space-like outcomes that could be time-like outcomes if the protocol were performed with SLP. At first sight this could seem source of paradoxical situations. However in quantum theory it holds the following:

**Assumption: Absoluteness of causal structure.**

Whenever they appear two outcomes  $(a^t, b^u)$  on two devices  $\mathcal{A}$ ,  $\mathcal{B}$ , for which it is defined a joint probability  $p(a^t, b^u|A, B)$  then one of the following must occur for every observer:

- (i)  $a^t$  causes  $b^u$
- (ii)  $b^u$  causes  $a^t$
- (iii)  $a^t$  does not cause  $b^u$  and  $b^u$  does not cause  $a^t$

With the above assumption in mind, suppose the system  $s$  is a SLP going out from Alice's device  $\mathcal{A}$  in state  $a^t$ , travelling a distance  $x_{AB}$  and causing probabilistically a measurement outcome  $b^u$ . It then must hold for every observer that  $a^t$  is the cause of  $b^u$ . Suppose also that Alice and Bob perform the protocol with photons. Then if it is found  $2x_{AB}/v < \Delta t_A < 2x_{AB}/c$  they establish  $a^t$  and  $b^u$  to be space-like. They thus find that it exists a certain point  $z$  of  $x_{AB}$  (the distance between  $\mathcal{A}$  and  $\mathcal{B}$ ) such that if they shoot two photons from  $z$  one in direction of Bob's device and one in direction of Alice's device, one photon reaches Alice's device at the same time outcome  $a^t$  happens and the other photon reaches Bob's device at the same time outcome  $b^u$  happens.